# Computer algebra independent integration tests

1_Algebraic_functions/1.1_Binomial_products/1.1.3General/1.1.3.3(a+bx^n)^p(c+dx^n)^c

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December 6, 2018 Compiled on December 6, 2018 at 4:11am

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# 1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

## 1.1 Listing of CAS systems tested

The following systems were tested at this time.

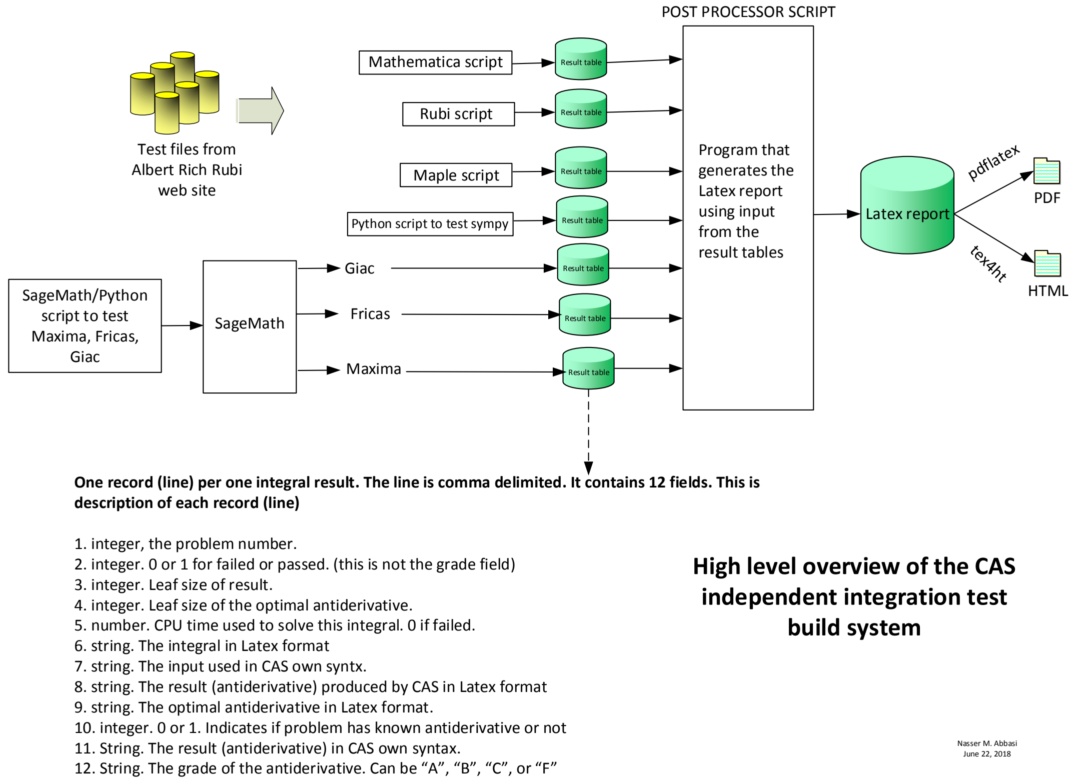
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

## 1.2 Design of the test system

The following diagram gives a high level view of the current test build system.




## 1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expressi

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | solved | Failed |
|---|---|---|
| Rubi | % 100. ( 286 ) | % 0. ( 0 ) |
| Rubi in Sympy | % 79.72 ( 228 ) | % 20.28 ( 58 ) |
| Mathematica | % 97.55 ( 279 ) | % 2.45 ( 7 ) |
| Maple | % 64.34 ( 184 ) | % 35.66 ( 102 ) |
| Maxima | % 18.18 ( 52 ) | % 81.82 ( 234 ) |
| Fricas | % 61.89 ( 177 ) | % 38.11 ( 109 ) |
| Sympy | % 37.06 ( 106 ) | % 62.94 ( 180 ) |
| Giac | % 45.8 ( 131 ) | % 54.2 ( 155 ) |

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

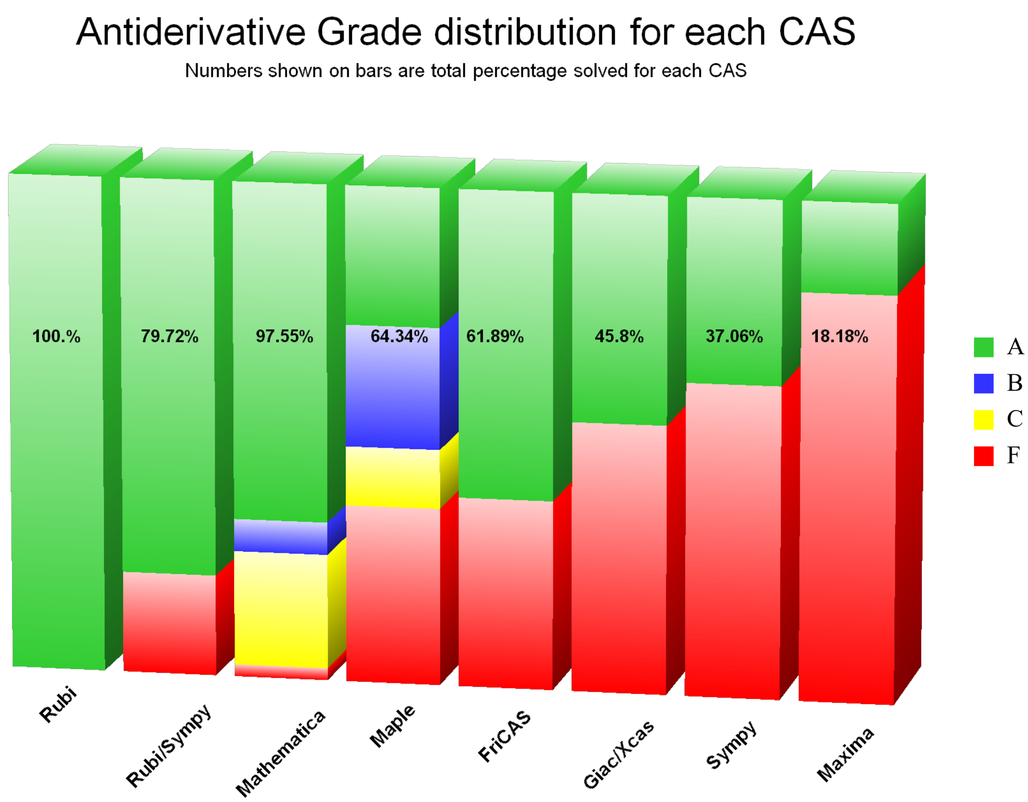
| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

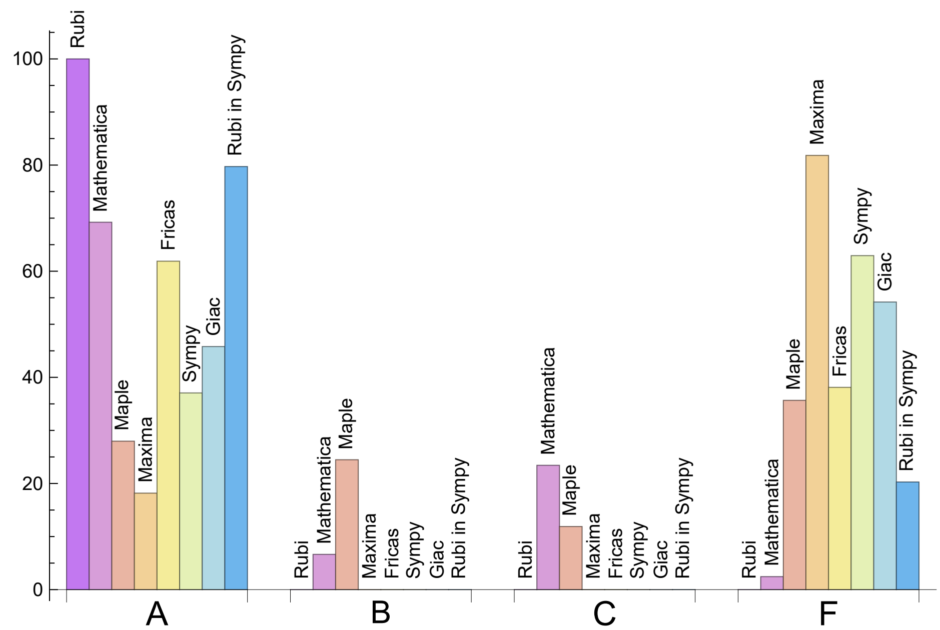
Based on the above, the following table summarizes the grading for this test suite.

| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 100. | 0. | 0. | 0. |
| Rubi in Sympy | 79.72 | 0. | 0. | 20.28 |
| Mathematica | 69.23 | 6.64 | 23.43 | 2.45 |
| Maple | 27.97 | 24.48 | 11.89 | 35.66 |
| Maxima | 18.18 | 0. | 0. | 81.82 |
| Fricas | 61.89 | 0. | 0. | 38.11 |
| Sympy | 37.06 | 0. | 0. | 62.94 |
| Giac | 45.8 | 0. | 0. | 54.2 |

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.




## 1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

| System | Mean time (sec) | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|---|
| Rubi | 0.42 | 172.17 | 1. | 124. | 1. |
| Rubi in Sympy | 42.81 | 134.72 | 0.86 | 104. | 0.87 |
| Mathematica | 0.99 | 194.65 | 1.5 | 150. | 1. |
| Maple | 0.02 | 455.77 | 2.46 | 233.5 | 1.42 |
| Maxima | 1.44 | 127.5 | 1.47 | 111. | 1.38 |
| Fricas | 1.25 | 441.94 | 2.42 | 184. | 1.98 |
| Sympy | 30.54 | 324.75 | 2.78 | 167.5 | 1.43 |
| Giac | 0.24 | 299.06 | 1.99 | 217. | 1.75 |

## 1.8 list of integrals that has no closed form antiderivative

{}

## 1.9 list of integrals not solved by each system

**Not solved by Rubi** {}

**Not solved by Rubi in Sympy** {1, 2, 3, 4, 8, 9, 10, 11, 12, 14, 15, 16, 20, 21, 22, 23, 39, 47, 48, 49, 50, 54, 55, 56, 57, 58, 59, 61, 62, 63, 67, 68, 69, 70, 73, 80, 83, 84, 85, 91, 124, 159, 166, 177, 178, 181, 185, 186, 187, 188, 193, 194, 195, 199, 200, 206, 212, 214}

**Not solved by Mathematica** {228, 237, 281, 282, 283, 284, 286}

**Not solved by Maple** {31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 170, 175, 182, 183, 184, 189, 190, 191, 192, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 281, 282, 283, 284, 285, 286}

**Not solved by Maxima** {5, 6, 7, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 51, 52, 53, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 241, 242, 243, 247, 248, 264, 265, 266, 267, 268, 274, 275, 276, 277, 278, 281, 282, 283, 284, 285, 286}

**Not solved by Fricas** {31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 170, 172, 173, 174, 175, 182, 183, 184, 189, 190, 191, 192, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 226, 227, 228, 229, 234, 235, 236, 237, 283, 284, 285}

**Not solved by Sympy** {19, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 65, 66, 67, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 129, 130, 131, 136, 137, 138, 143, 144, 145, 150, 151, 152, 153, 154, 157, 158, 159, 160, 161, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 182, 183, 184, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 259, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286}

**Not solved by Giac** {27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 65, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 167, 168, 169, 170, 172, 173, 174, 175, 180, 182, 183, 184, 189, 190, 191, 192, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 280, 281, 282, 283, 284, 285, 286}

## 1.10 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Rubi in Sympy** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

## 1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {80, 81, 82, 83, 84}

**Mathematica** {30, 31, 32, 33, 34, 37, 38, 42, 43, 44, 45, 46, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 122, 123, 124, 170, 175, 213, 218, 219, 220, 229, 230, 235, 236, 280, 285}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Rubi in Sympy** Verification phase not implemented yet.

# 2 detailed summary tables of results

## 2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

| Problem 1 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 94 | 94 | 97 | 130 | 1 | 104 | 131 | 0 |
| normalized size | 1 | 1. | 1. | 1.03 | 1.38 | 0.01 | 1.11 | 1.39 | 0. |
| time (sec) | N/A | 0.149 | 0.035 | 0. | 1.41 | 0.18 | 0.145 | 0.218 | 0. |

| Problem 2 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 70 | 70 | 73 | 95 | 1 | 80 | 100 | 0 |
| normalized size | 1 | 1. | 1. | 1.04 | 1.36 | 0.01 | 1.14 | 1.43 | 0. |
| time (sec) | N/A | 0.103 | 0.021 | 0.001 | 1.376 | 0.181 | 0.123 | 0.214 | 0. |

| Problem 3 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 50 | 49 | 65 | 1 | 51 | 68 | 0 |
| normalized size | 1 | 1. | 1. | 0.98 | 1.3 | 0.02 | 1.02 | 1.36 | 0. |
| time (sec) | N/A | 0.074 | 0.014 | 0.001 | 1.361 | 0.181 | 0.109 | 0.213 | 0. |

| Problem 4 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 28 | 25 | 32 | 1 | 26 | 35 | 0 |
| normalized size | 1 | 1. | 1. | 0.89 | 1.14 | 0.04 | 0.93 | 1.25 | 0. |
| time (sec) | N/A | 0.039 | 0.009 | 0.002 | 1.353 | 0.182 | 0.075 | 0.211 | 0. |

| Problem 5 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 144 | 128 | 195 | 0 | 176 | 71 | 217 | 134 |
| normalized size | 1 | 1. | 0.89 | 1.35 | 0. | 1.22 | 0.49 | 1.51 | 0.93 |
| time (sec) | N/A | 0.211 | 0.14 | 0.007 | 0. | 0.215 | 1.937 | 0.217 | 32.683 |

| Problem 6 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 169 | 145 | 221 | 0 | 275 | 97 | 246 | 155 |
| normalized size | 1 | 1. | 0.86 | 1.31 | 0. | 1.63 | 0.57 | 1.46 | 0.92 |
| time (sec) | N/A | 0.195 | 0.17 | 0.011 | 0. | 0.215 | 2.62 | 0.218 | 34.232 |

| Problem 7 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 197 | 197 | 175 | 249 | 0 | 414 | 133 | 273 | 180 |
| normalized size | 1 | 1. | 0.89 | 1.26 | 0. | 2.1 | 0.68 | 1.39 | 0.91 |
| time (sec) | N/A | 0.229 | 0.225 | 0.014 | 0. | 0.214 | 3.771 | 0.221 | 39.124 |

| Problem 8 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 122 | 122 | 125 | 167 | 1 | 139 | 178 | 0 |
| normalized size | 1 | 1. | 1. | 1.02 | 1.37 | 0.01 | 1.14 | 1.46 | 0. |
| time (sec) | N/A | 0.175 | 0.034 | 0. | 1.37 | 0.182 | 0.166 | 0.211 | 0. |

| Problem 9 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 82 | 87 | 111 | 1 | 90 | 123 | 0 |
| normalized size | 1 | 1. | 1. | 1.06 | 1.35 | 0.01 | 1.1 | 1.5 | 0. |
| time (sec) | N/A | 0.117 | 0.023 | 0.001 | 1.407 | 0.181 | 0.138 | 0.211 | 0. |

| Problem 10 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 50 | 49 | 65 | 1 | 51 | 68 | 0 |
| normalized size | 1 | 1. | 1. | 0.98 | 1.3 | 0.02 | 1.02 | 1.36 | 0. |
| time (sec) | N/A | 0.075 | 0.014 | 0. | 1.383 | 0.18 | 0.102 | 0.21 | 0. |

| Problem 11 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 173 | 173 | 167 | 334 | 0 | 258 | 156 | 336 | 0 |
| normalized size | 1 | 1. | 0.97 | 1.93 | 0. | 1.49 | 0.9 | 1.94 | 0. |
| time (sec) | N/A | 0.276 | 0.171 | 0.004 | 0. | 0.212 | 2.861 | 0.219 | 0. |

| Problem 12 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 203 | 203 | 210 | 367 | 0 | 435 | 189 | 358 | 0 |
| normalized size | 1 | 1. | 1.03 | 1.81 | 0. | 2.14 | 0.93 | 1.76 | 0. |
| time (sec) | N/A | 0.495 | 0.379 | 0.014 | 0. | 0.218 | 4.832 | 0.22 | 0. |

| Problem 13 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 258 | 258 | 234 | 388 | 0 | 637 | 233 | 400 | 243 |
| normalized size | 1 | 1. | 0.91 | 1.5 | 0. | 2.47 | 0.9 | 1.55 | 0.94 |
| time (sec) | N/A | 0.477 | 0.466 | 0.015 | 0. | 0.219 | 7.431 | 0.222 | 53.106 |

| Problem 14 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 252 | 252 | 253 | 661 | 0 | 494 | 369 | 622 | 0 |
| normalized size | 1 | 1. | 1. | 2.62 | 0. | 1.96 | 1.46 | 2.47 | 0. |
| time (sec) | N/A | 0.415 | 0.199 | 0.006 | 0. | 0.216 | 5.404 | 0.22 | 0. |

| Problem 15 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 208 | 203 | 486 | 0 | 379 | 255 | 473 | 0 |
| normalized size | 1 | 1. | 0.98 | 2.34 | 0. | 1.82 | 1.23 | 2.27 | 0. |
| time (sec) | N/A | 0.317 | 0.156 | 0.004 | 0. | 0.216 | 4.164 | 0.22 | 0. |

| Problem 16 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 173 | 173 | 167 | 334 | 0 | 259 | 156 | 336 | 0 |
| normalized size | 1 | 1. | 0.97 | 1.93 | 0. | 1.5 | 0.9 | 1.94 | 0. |
| time (sec) | N/A | 0.266 | 0.185 | 0.005 | 0. | 0.212 | 2.995 | 0.218 | 0. |

| Problem 17 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 145 | 129 | 195 | 0 | 184 | 71 | 217 | 134 |
| normalized size | 1 | 1. | 0.89 | 1.34 | 0. | 1.27 | 0.49 | 1.5 | 0.92 |
| time (sec) | N/A | 0.182 | 0.109 | 0.005 | 0. | 0.216 | 2.025 | 0.216 | 37.706 |

| Problem 18 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 288 | 288 | 224 | 222 | 0 | 377 | 447 | 375 | 260 |
| normalized size | 1 | 1. | 0.78 | 0.77 | 0. | 1.31 | 1.55 | 1.3 | 0.9 |
| time (sec) | N/A | 0.321 | 0.22 | 0.009 | 0. | 0.279 | 104.613 | 0.228 | 68.209 |

| Problem 19 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 346 | 346 | 336 | 406 | 0 | 625 | 0 | 598 | 321 |
| normalized size | 1 | 1. | 0.97 | 1.17 | 0. | 1.81 | 0. | 1.73 | 0.93 |
| time (sec) | N/A | 0.613 | 0.351 | 0.018 | 0. | 7.366 | 0. | 0.227 | 108.441 |

| Problem 20 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 320 | 320 | 313 | 905 | 0 | 1000 | 536 | 822 | 0 |
| normalized size | 1 | 1. | 0.98 | 2.83 | 0. | 3.12 | 1.68 | 2.57 | 0. |
| time (sec) | N/A | 0.633 | 0.439 | 0.017 | 0. | 0.223 | 25.775 | 0.222 | 0. |

| Problem 21 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 267 | 267 | 260 | 708 | 0 | 794 | 403 | 643 | 0 |
| normalized size | 1 | 1. | 0.97 | 2.65 | 0. | 2.97 | 1.51 | 2.41 | 0. |
| time (sec) | N/A | 0.498 | 0.364 | 0.016 | 0. | 0.223 | 14.071 | 0.221 | 0. |

| Problem 22 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 234 | 234 | 227 | 529 | 0 | 606 | 289 | 495 | 0 |
| normalized size | 1 | 1. | 0.97 | 2.26 | 0. | 2.59 | 1.24 | 2.12 | 0. |
| time (sec) | N/A | 0.482 | 0.261 | 0.013 | 0. | 0.22 | 8.056 | 0.221 | 0. |

| Problem 23 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 203 | 203 | 205 | 367 | 0 | 427 | 189 | 350 | 0 |
| normalized size | 1 | 1. | 1.01 | 1.81 | 0. | 2.1 | 0.93 | 1.72 | 0. |
| time (sec) | N/A | 0.498 | 0.412 | 0.013 | 0. | 0.219 | 5.08 | 0.218 | 0. |

| Problem 24 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 169 | 145 | 221 | 0 | 277 | 97 | 246 | 155 |
| normalized size | 1 | 1. | 0.86 | 1.31 | 0. | 1.64 | 0.57 | 1.46 | 0.92 |
| time (sec) | N/A | 0.193 | 0.173 | 0.012 | 0. | 0.216 | 2.686 | 0.218 | 34.112 |

| Problem 25 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 346 | 346 | 337 | 406 | 0 | 633 | 0 | 598 | 321 |
| normalized size | 1 | 1. | 0.97 | 1.17 | 0. | 1.83 | 0. | 1.73 | 0.93 |
| time (sec) | N/A | 0.589 | 0.36 | 0.017 | 0. | 7.404 | 0. | 0.229 | 109.102 |

| Problem 26 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 419 | 419 | 381 | 606 | 0 | 1250 | 0 | 896 | 386 |
| normalized size | 1 | 1. | 0.91 | 1.45 | 0. | 2.98 | 0. | 2.14 | 0.92 |
| time (sec) | N/A | 1.078 | 1.638 | 0.023 | 0. | 77.342 | 0. | 0.23 | 177.573 |

| Problem 27 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 120 | 134 | 246 | 224 | 0 | 0 | 102 |
| normalized size | 1 | 1. | 1.1 | 1.23 | 2.26 | 2.06 | 0. | 0. | 0.94 |
| time (sec) | N/A | 0.116 | 0.133 | 0.01 | 1.52 | 0.222 | 0. | 0. | 17.977 |

| Problem 28 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 73 | 76 | 147 | 139 | 0 | 0 | 71 |
| normalized size | 1 | 1. | 0.94 | 0.97 | 1.88 | 1.78 | 0. | 0. | 0.91 |
| time (sec) | N/A | 0.07 | 0.084 | 0.01 | 1.384 | 0.219 | 0. | 0. | 11.286 |

| Problem 29 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 37 | 34 | 69 | 73 | 0 | 0 | 41 |
| normalized size | 1 | 1. | 0.79 | 0.72 | 1.47 | 1.55 | 0. | 0. | 0.87 |
| time (sec) | N/A | 0.036 | 0.045 | 0.005 | 1.361 | 0.218 | 0. | 0. | 5.491 |

| Problem 30 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 674 | 15 | 19 | 31 | 29 | 0 | 12 |
| normalized size | 1 | 1. | 42.12 | 0.94 | 1.19 | 1.94 | 1.81 | 0. | 0.75 |
| time (sec) | N/A | 0.009 | 2.202 | 0.003 | 1.402 | 0.211 | 1.952 | 0. | 1.257 |

| Problem 31 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-1) | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 208 | 168 | 0 | 0 | 0 | 0 | 0 | 185 |
| normalized size | 1 | 1. | 0.81 | 0. | 0. | 0. | 0. | 0. | 0.89 |
| time (sec) | N/A | 0.458 | 0.344 | 0.061 | 0. | 0. | 0. | 0. | 34.618 |

| Problem 32 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 241 | 241 | 198 | 0 | 0 | 0 | 0 | 0 | 216 |
| normalized size | 1 | 1. | 0.82 | 0. | 0. | 0. | 0. | 0. | 0.9 |
| time (sec) | N/A | 0.385 | 0.582 | 0.058 | 0. | 0. | 0. | 0. | 44.998 |

| Problem 33 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 276 | 276 | 219 | 0 | 0 | 0 | 0 | 0 | 252 |
| normalized size | 1 | 1. | 0.79 | 0. | 0. | 0. | 0. | 0. | 0.91 |
| time (sec) | N/A | 0.457 | 0.728 | 0.059 | 0. | 0. | 0. | 0. | 58.583 |

| Problem 34 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 172 | 0 | 0 | 0 | 0 | 0 | 61 |
| normalized size | 1 | 1. | 2.18 | 0. | 0. | 0. | 0. | 0. | 0.77 |
| time (sec) | N/A | 0.107 | 0.355 | 0.114 | 0. | 0. | 0. | 0. | 21.148 |

| Problem 35 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 167 | 176 | 106 | 0 | 0 | 0 | 0 | 0 | 153 |
| normalized size | 1 | 1.05 | 0.63 | 0. | 0. | 0. | 0. | 0. | 0.92 |
| time (sec) | N/A | 0.278 | 0.08 | 0.082 | 0. | 0. | 0. | 0. | 33.094 |

| Problem 36 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 85 | 75 | 0 | 0 | 0 | 0 | 0 | 73 |
| normalized size | 1 | 1.01 | 0.89 | 0. | 0. | 0. | 0. | 0. | 0.87 |
| time (sec) | N/A | 0.098 | 0.031 | 0.048 | 0. | 0. | 0. | 0. | 11.049 |

| Problem 37 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 162 | 0 | 0 | 0 | 0 | 0 | 42 |
| normalized size | 1 | 1. | 2.84 | 0. | 0. | 0. | 0. | 0. | 0.74 |
| time (sec) | N/A | 0.075 | 0.284 | 0.069 | 0. | 0. | 0. | 0. | 21.65 |

| Problem 38 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 162 | 0 | 0 | 0 | 0 | 0 | 44 |
| normalized size | 1 | 1. | 2.84 | 0. | 0. | 0. | 0. | 0. | 0.77 |
| time (sec) | N/A | 0.073 | 0.296 | 0.097 | 0. | 0. | 0. | 0. | 20.499 |

| Problem 39 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 298 | 298 | 137 | 0 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.46 | 0. | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.69 | 0.104 | 0.07 | 0. | 0. | 0. | 0. | 0. |

| Problem 40 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 176 | 176 | 106 | 0 | 0 | 0 | 0 | 0 | 153 |
| normalized size | 1 | 1. | 0.6 | 0. | 0. | 0. | 0. | 0. | 0.87 |
| time (sec) | N/A | 0.282 | 0.057 | 0.07 | 0. | 0. | 0. | 0. | 32.217 |

| Problem 41 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 85 | 75 | 0 | 0 | 0 | 0 | 0 | 73 |
| normalized size | 1 | 0.91 | 0.81 | 0. | 0. | 0. | 0. | 0. | 0.78 |
| time (sec) | N/A | 0.1 | 0.03 | 0.055 | 0. | 0. | 0. | 0. | 10.716 |

| Problem 42 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | A | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 44 | 196 | 0 | 0 | 0 | 34 | 0 | 34 |
| normalized size | 1 | 1. | 4.45 | 0. | 0. | 0. | 0.77 | 0. | 0.77 |
| time (sec) | N/A | 0.027 | 0.287 | 0.035 | 0. | 0. | 54.267 | 0. | 3.922 |

| Problem 43 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 162 | 0 | 0 | 0 | 0 | 0 | 42 |
| normalized size | 1 | 1. | 2.84 | 0. | 0. | 0. | 0. | 0. | 0.74 |
| time (sec) | N/A | 0.071 | 0.327 | 0.076 | 0. | 0. | 0. | 0. | 21.223 |

| Problem 44 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 162 | 0 | 0 | 0 | 0 | 0 | 44 |
| normalized size | 1 | 1. | 2.84 | 0. | 0. | 0. | 0. | 0. | 0.77 |
| time (sec) | N/A | 0.072 | 0.298 | 0.095 | 0. | 0. | 0. | 0. | 19.708 |

| Problem 45 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 162 | 0 | 0 | 0 | 0 | 0 | 44 |
| normalized size | 1 | 1. | 2.84 | 0. | 0. | 0. | 0. | 0. | 0.77 |
| time (sec) | N/A | 0.071 | 0.392 | 0.078 | 0. | 0. | 0. | 0. | 19.867 |

| Problem 46 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 594 | 71 | 0 | 123 | 0 | 0 | 44 |
| normalized size | 1 | 1. | 11.21 | 1.34 | 0. | 2.32 | 0. | 0. | 0.83 |
| time (sec) | N/A | 0.061 | 2.436 | 0.006 | 0. | 0.252 | 0. | 0. | 10.552 |

| Problem 47 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 94 | 94 | 97 | 130 | 1 | 107 | 132 | 0 |
| normalized size | 1 | 1. | 1. | 1.03 | 1.38 | 0.01 | 1.14 | 1.4 | 0. |
| time (sec) | N/A | 0.145 | 0.035 | 0.001 | 1.362 | 0.192 | 0.149 | 0.213 | 0. |

| Problem 48 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 70 | 70 | 73 | 95 | 1 | 76 | 100 | 0 |
| normalized size | 1 | 1. | 1. | 1.04 | 1.36 | 0.01 | 1.09 | 1.43 | 0. |
| time (sec) | N/A | 0.104 | 0.031 | 0.001 | 1.356 | 0.192 | 0.122 | 0.212 | 0. |

| Problem 49 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 50 | 49 | 65 | 1 | 53 | 68 | 0 |
| normalized size | 1 | 1. | 1. | 0.98 | 1.3 | 0.02 | 1.06 | 1.36 | 0. |
| time (sec) | N/A | 0.074 | 0.02 | 0.001 | 1.413 | 0.19 | 0.105 | 0.211 | 0. |

| Problem 50 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 28 | 25 | 32 | 1 | 26 | 35 | 0 |
| normalized size | 1 | 1. | 1. | 0.89 | 1.14 | 0.04 | 0.93 | 1.25 | 0. |
| time (sec) | N/A | 0.038 | 0.009 | 0.001 | 1.364 | 0.189 | 0.071 | 0.211 | 0. |

| Problem 51 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 223 | 223 | 196 | 266 | 0 | 743 | 87 | 331 | 204 |
| normalized size | 1 | 1. | 0.88 | 1.19 | 0. | 3.33 | 0.39 | 1.48 | 0.91 |
| time (sec) | N/A | 0.328 | 0.228 | 0.01 | 0. | 0.24 | 2.235 | 0.22 | 57.162 |

| Problem 52 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 245 | 245 | 212 | 295 | 0 | 838 | 112 | 359 | 226 |
| normalized size | 1 | 1. | 0.87 | 1.2 | 0. | 3.42 | 0.46 | 1.47 | 0.92 |
| time (sec) | N/A | 0.312 | 0.29 | 0.013 | 0. | 0.241 | 3.207 | 0.219 | 59.134 |

| Problem 53 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 273 | 273 | 243 | 314 | 0 | 941 | 151 | 386 | 258 |
| normalized size | 1 | 1. | 0.89 | 1.15 | 0. | 3.45 | 0.55 | 1.41 | 0.95 |
| time (sec) | N/A | 0.357 | 0.372 | 0.016 | 0. | 0.239 | 6.697 | 0.222 | 65.639 |

| Problem 54 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 154 | 154 | 154 | 163 | 213 | 1 | 185 | 234 | 0 |
| normalized size | 1 | 1. | 1. | 1.06 | 1.38 | 0.01 | 1.2 | 1.52 | 0. |
| time (sec) | N/A | 0.236 | 0.062 | 0.001 | 1.372 | 0.189 | 0.183 | 0.21 | 0. |

| Problem 55 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 122 | 122 | 125 | 167 | 1 | 139 | 178 | 0 |
| normalized size | 1 | 1. | 1. | 1.02 | 1.37 | 0.01 | 1.14 | 1.46 | 0. |
| time (sec) | N/A | 0.173 | 0.043 | 0.002 | 1.358 | 0.191 | 0.156 | 0.212 | 0. |

| Problem 56 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 82 | 87 | 111 | 1 | 97 | 123 | 0 |
| normalized size | 1 | 1. | 1. | 1.06 | 1.35 | 0.01 | 1.18 | 1.5 | 0. |
| time (sec) | N/A | 0.119 | 0.032 | 0.001 | 1.378 | 0.19 | 0.138 | 0.209 | 0. |

| Problem 57 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 50 | 49 | 65 | 1 | 53 | 68 | 0 |
| normalized size | 1 | 1. | 1. | 0.98 | 1.3 | 0.02 | 1.06 | 1.36 | 0. |
| time (sec) | N/A | 0.073 | 0.014 | 0.002 | 1.395 | 0.192 | 0.108 | 0.215 | 0. |

| Problem 58 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 253 | 253 | 231 | 436 | 0 | 1457 | 187 | 477 | 0 |
| normalized size | 1 | 1. | 0.91 | 1.72 | 0. | 5.76 | 0.74 | 1.89 | 0. |
| time (sec) | N/A | 0.415 | 0.193 | 0.002 | 0. | 0.241 | 3.589 | 0.221 | 0. |

| Problem 59 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 291 | 291 | 298 | 475 | 0 | 1592 | 219 | 508 | 0 |
| normalized size | 1 | 1. | 1.02 | 1.63 | 0. | 5.47 | 0.75 | 1.75 | 0. |
| time (sec) | N/A | 0.723 | 0.283 | 0.002 | 0. | 0.256 | 6.902 | 0.222 | 0. |

| Problem 60 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 349 | 349 | 319 | 499 | 0 | 1692 | 264 | 549 | 337 |
| normalized size | 1 | 1. | 0.91 | 1.43 | 0. | 4.85 | 0.76 | 1.57 | 0.97 |
| time (sec) | N/A | 0.55 | 0.321 | 0.002 | 0. | 0.258 | 20.563 | 0.223 | 82.463 |

| Problem 61 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 332 | 332 | 322 | 837 | 0 | 2939 | 430 | 833 | 0 |
| normalized size | 1 | 1. | 0.97 | 2.52 | 0. | 8.85 | 1.3 | 2.51 | 0. |
| time (sec) | N/A | 0.581 | 0.329 | 0.008 | 0. | 0.259 | 9.621 | 0.22 | 0. |

| Problem 62 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 288 | 288 | 271 | 627 | 0 | 2198 | 301 | 649 | 0 |
| normalized size | 1 | 1. | 0.94 | 2.18 | 0. | 7.63 | 1.05 | 2.25 | 0. |
| time (sec) | N/A | 0.45 | 0.235 | 0.002 | 0. | 0.251 | 6.181 | 0.218 | 0. |

| Problem 63 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 253 | 253 | 231 | 436 | 0 | 1458 | 187 | 477 | 0 |
| normalized size | 1 | 1. | 0.91 | 1.72 | 0. | 5.76 | 0.74 | 1.89 | 0. |
| time (sec) | N/A | 0.401 | 0.173 | 0.003 | 0. | 0.243 | 3.885 | 0.218 | 0. |

| Problem 64 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 223 | 223 | 196 | 266 | 0 | 743 | 87 | 331 | 204 |
| normalized size | 1 | 1. | 0.88 | 1.19 | 0. | 3.33 | 0.39 | 1.48 | 0.91 |
| time (sec) | N/A | 0.299 | 0.215 | 0.005 | 0. | 0.238 | 2.379 | 0.218 | 61.206 |

| Problem 65 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 449 | 449 | 340 | 320 | 0 | 1472 | 0 | 0 | 400 |
| normalized size | 1 | 1. | 0.76 | 0.71 | 0. | 3.28 | 0. | 0. | 0.89 |
| time (sec) | N/A | 0.567 | 0.282 | 0.002 | 0. | 0.395 | 0. | 0. | 119.517 |

| Problem 66 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 513 | 513 | 498 | 550 | 0 | 3665 | 0 | 900 | 474 |
| normalized size | 1 | 1. | 0.97 | 1.07 | 0. | 7.14 | 0. | 1.75 | 0.92 |
| time (sec) | N/A | 0.884 | 0.562 | 0.003 | 0. | 21.891 | 0. | 0.228 | 174.262 |

| Problem 67 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 407 | 407 | 391 | 1118 | 0 | 3853 | 0 | 1077 | 0 |
| normalized size | 1 | 1. | 0.96 | 2.75 | 0. | 9.47 | 0. | 2.65 | 0. |
| time (sec) | N/A | 0.786 | 0.673 | 0.018 | 0. | 0.268 | 0. | 0.221 | 0. |

| Problem 68 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 357 | 357 | 341 | 885 | 0 | 3083 | 466 | 867 | 0 |
| normalized size | 1 | 1. | 0.96 | 2.48 | 0. | 8.64 | 1.31 | 2.43 | 0. |
| time (sec) | N/A | 0.719 | 0.501 | 0.017 | 0. | 0.257 | 89.201 | 0.221 | 0. |

| Problem 69 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 317 | 317 | 301 | 669 | 0 | 2306 | 335 | 670 | 0 |
| normalized size | 1 | 1. | 0.95 | 2.11 | 0. | 7.27 | 1.06 | 2.11 | 0. |
| time (sec) | N/A | 0.611 | 0.4 | 0.001 | 0. | 0.248 | 20.583 | 0.222 | 0. |

| Problem 70 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 291 | 291 | 297 | 475 | 0 | 1592 | 219 | 508 | 0 |
| normalized size | 1 | 1. | 1.02 | 1.63 | 0. | 5.47 | 0.75 | 1.75 | 0. |
| time (sec) | N/A | 0.724 | 0.3 | 0.014 | 0. | 0.245 | 7.239 | 0.222 | 0. |

| Problem 71 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 245 | 245 | 212 | 295 | 0 | 838 | 112 | 359 | 226 |
| normalized size | 1 | 1. | 0.87 | 1.2 | 0. | 3.42 | 0.46 | 1.47 | 0.92 |
| time (sec) | N/A | 0.317 | 0.304 | 0.011 | 0. | 0.23 | 3.409 | 0.22 | 62.06 |

| Problem 72 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 513 | 513 | 499 | 550 | 0 | 3665 | 0 | 900 | 474 |
| normalized size | 1 | 1. | 0.97 | 1.07 | 0. | 7.14 | 0. | 1.75 | 0.92 |
| time (sec) | N/A | 0.903 | 0.545 | 0.018 | 0. | 22.335 | 0. | 0.228 | 168.513 |

| Problem 73 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | F(-1) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 596 | 596 | 561 | 784 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.94 | 1.32 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 1.468 | 3.493 | 0.003 | 0. | 0. | 0. | 0. | 0. |

| Problem 74 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 321 | 321 | 385 | 408 | 0 | 0 | 0 | 0 | 292 |
| normalized size | 1 | 1. | 1.2 | 1.27 | 0. | 0. | 0. | 0. | 0.91 |
| time (sec) | N/A | 0.914 | 1.729 | 0.069 | 0. | 0. | 0. | 0. | 151.844 |

| Problem 75 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 277 | 277 | 419 | 311 | 0 | 0 | 0 | 0 | 248 |
| normalized size | 1 | 1. | 1.51 | 1.12 | 0. | 0. | 0. | 0. | 0.9 |
| time (sec) | N/A | 0.679 | 0.743 | 0.027 | 0. | 0. | 0. | 0. | 110.42 |

| Problem 76 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 240 | 240 | 155 | 259 | 0 | 0 | 0 | 0 | 211 |
| normalized size | 1 | 1. | 0.65 | 1.08 | 0. | 0. | 0. | 0. | 0.88 |
| time (sec) | N/A | 0.449 | 0.277 | 0.025 | 0. | 0. | 0. | 0. | 73.184 |

| Problem 77 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 162 | 156 | 183 | 0 | 0 | 0 | 0 | 143 |
| normalized size | 1 | 1. | 0.96 | 1.13 | 0. | 0. | 0. | 0. | 0.88 |
| time (sec) | N/A | 0.326 | 0.235 | 0.023 | 0. | 0. | 0. | 0. | 52.459 |

| Problem 78 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 281 | 281 | 329 | 301 | 0 | 0 | 0 | 0 | 241 |
| normalized size | 1 | 1. | 1.17 | 1.07 | 0. | 0. | 0. | 0. | 0.86 |
| time (sec) | N/A | 0.609 | 0.603 | 0.043 | 0. | 0. | 0. | 0. | 106.733 |

| Problem 79 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 334 | 334 | 396 | 361 | 0 | 0 | 0 | 0 | 298 |
| normalized size | 1 | 1. | 1.19 | 1.08 | 0. | 0. | 0. | 0. | 0.89 |
| time (sec) | N/A | 0.962 | 1.568 | 0.047 | 0. | 0. | 0. | 0. | 165.584 |

| Problem 80 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F | F | F(-1) |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 973 | 973 | 435 | 322 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.45 | 0.33 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 3.707 | 1.147 | 0.036 | 0. | 0. | 0. | 0. | 0. |

| Problem 81 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 932 | 932 | 161 | 273 | 0 | 0 | 0 | 0 | 814 |
| normalized size | 1 | 1. | 0.17 | 0.29 | 0. | 0. | 0. | 0. | 0.87 |
| time (sec) | N/A | 1.766 | 0.238 | 0.023 | 0. | 0. | 0. | 0. | 149.324 |

| Problem 82 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 775 | 775 | 161 | 191 | 0 | 0 | 0 | 0 | 690 |
| normalized size | 1 | 1. | 0.21 | 0.25 | 0. | 0. | 0. | 0. | 0.89 |
| time (sec) | N/A | 1.234 | 0.081 | 0.021 | 0. | 0. | 0. | 0. | 100.558 |

| Problem 83 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F | F | F(-1) |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 969 | 969 | 342 | 313 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.35 | 0.32 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 2.406 | 0.495 | 0.043 | 0. | 0. | 0. | 0. | 0. |

| Problem 84 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F | F | F(-1) |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1029 | 1029 | 406 | 371 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.39 | 0.36 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 4.384 | 1.641 | 0.047 | 0. | 0. | 0. | 0. | 0. |

| Problem 85 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F(-1) | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 426 | 426 | 580 | 540 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.36 | 1.27 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 1.391 | 1.826 | 0.041 | 0. | 0. | 0. | 0. | 0. |

| Problem 86 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 365 | 365 | 491 | 412 | 0 | 0 | 0 | 0 | 333 |
| normalized size | 1 | 1. | 1.35 | 1.13 | 0. | 0. | 0. | 0. | 0.91 |
| time (sec) | N/A | 1.049 | 1.362 | 0.04 | 0. | 0. | 0. | 0. | 168.627 |

| Problem 87 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 309 | 309 | 423 | 329 | 0 | 0 | 0 | 0 | 279 |
| normalized size | 1 | 1. | 1.37 | 1.06 | 0. | 0. | 0. | 0. | 0.9 |
| time (sec) | N/A | 0.746 | 0.594 | 0.036 | 0. | 0. | 0. | 0. | 107.566 |

| Problem 88 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 276 | 276 | 310 | 294 | 0 | 0 | 0 | 0 | 243 |
| normalized size | 1 | 1. | 1.12 | 1.07 | 0. | 0. | 0. | 0. | 0.88 |
| time (sec) | N/A | 0.621 | 0.269 | 0.033 | 0. | 0. | 0. | 0. | 97.147 |

| Problem 89 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 310 | 310 | 349 | 322 | 0 | 0 | 0 | 0 | 270 |
| normalized size | 1 | 1. | 1.13 | 1.04 | 0. | 0. | 0. | 0. | 0.87 |
| time (sec) | N/A | 0.688 | 0.991 | 0.035 | 0. | 0. | 0. | 0. | 109.539 |

| Problem 90 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 362 | 362 | 465 | 375 | 0 | 0 | 0 | 0 | 323 |
| normalized size | 1 | 1. | 1.28 | 1.04 | 0. | 0. | 0. | 0. | 0.89 |
| time (sec) | N/A | 1.074 | 1.186 | 0.058 | 0. | 0. | 0. | 0. | 175.193 |

| Problem 91 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F(-1) | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 439 | 439 | 617 | 484 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.41 | 1.1 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 1.439 | 2.184 | 0.057 | 0. | 0. | 0. | 0. | 0. |

| Problem 92 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 155 | 103 | 0 | 450 | 0 | 0 | 94 |
| normalized size | 1 | 1. | 1.5 | 1. | 0. | 4.37 | 0. | 0. | 0.91 |
| time (sec) | N/A | 0.141 | 0.242 | 0.024 | 0. | 0.592 | 0. | 0. | 22.737 |

| Problem 93 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 155 | 158 | 0 | 500 | 0 | 0 | 100 |
| normalized size | 1 | 1. | 1.34 | 1.36 | 0. | 4.31 | 0. | 0. | 0.86 |
| time (sec) | N/A | 0.079 | 0.272 | 0.025 | 0. | 0.587 | 0. | 0. | 12.654 |

| Problem 94 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 211 | 211 | 396 | 0 | 0 | 2776 | 0 | 0 | 192 |
| normalized size | 1 | 1. | 1.88 | 0. | 0. | 13.16 | 0. | 0. | 0.91 |
| time (sec) | N/A | 0.523 | 1.806 | 0.084 | 0. | 2.723 | 0. | 0. | 62.95 |

| Problem 95 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 173 | 173 | 161 | 0 | 0 | 1006 | 0 | 0 | 148 |
| normalized size | 1 | 1. | 0.93 | 0. | 0. | 5.82 | 0. | 0. | 0.86 |
| time (sec) | N/A | 0.24 | 0.255 | 0.055 | 0. | 0.294 | 0. | 0. | 37.997 |

| Problem 96 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-1) | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 105 | 122 | 0 | 0 | 0 | 0 | 0 | 90 |
| normalized size | 1 | 1. | 1.16 | 0. | 0. | 0. | 0. | 0. | 0.86 |
| time (sec) | N/A | 0.136 | 0.124 | 0.05 | 0. | 0. | 0. | 0. | 23.604 |

| Problem 97 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-1) | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 153 | 0 | 0 | 0 | 0 | 0 | 117 |
| normalized size | 1 | 1. | 1.14 | 0. | 0. | 0. | 0. | 0. | 0.87 |
| time (sec) | N/A | 0.236 | 0.368 | 0.055 | 0. | 0. | 0. | 0. | 34.045 |

| Problem 98 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 180 | 184 | 0 | 0 | 0 | 0 | 0 | 158 |
| normalized size | 1 | 1. | 1.02 | 0. | 0. | 0. | 0. | 0. | 0.88 |
| time (sec) | N/A | 0.49 | 0.563 | 0.057 | 0. | 0. | 0. | 0. | 80.616 |

| Problem 99 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 233 | 233 | 231 | 0 | 0 | 0 | 0 | 0 | 212 |
| normalized size | 1 | 1. | 0.99 | 0. | 0. | 0. | 0. | 0. | 0.91 |
| time (sec) | N/A | 0.73 | 0.684 | 0.061 | 0. | 0. | 0. | 0. | 133.593 |

| Problem 100 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 316 | 316 | 396 | 0 | 0 | 0 | 0 | 0 | 282 |
| normalized size | 1 | 1. | 1.25 | 0. | 0. | 0. | 0. | 0. | 0.89 |
| time (sec) | N/A | 0.922 | 1.725 | 0.098 | 0. | 0. | 0. | 0. | 112.517 |

| Problem 101 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 274 | 274 | 435 | 0 | 0 | 0 | 0 | 0 | 235 |
| normalized size | 1 | 1. | 1.59 | 0. | 0. | 0. | 0. | 0. | 0.86 |
| time (sec) | N/A | 0.493 | 0.714 | 0.094 | 0. | 0. | 0. | 0. | 62.65 |

| Problem 102 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 166 | 160 | 0 | 0 | 0 | 0 | 0 | 143 |
| normalized size | 1 | 1. | 0.96 | 0. | 0. | 0. | 0. | 0. | 0.86 |
| time (sec) | N/A | 0.293 | 0.247 | 0.055 | 0. | 0. | 0. | 0. | 41.329 |

| Problem 103 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 259 | 259 | 161 | 0 | 0 | 0 | 0 | 0 | 221 |
| normalized size | 1 | 1. | 0.62 | 0. | 0. | 0. | 0. | 0. | 0.85 |
| time (sec) | N/A | 0.471 | 0.087 | 0.052 | 0. | 0. | 0. | 0. | 62.698 |

| Problem 104 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 304 | 304 | 343 | 0 | 0 | 0 | 0 | 0 | 265 |
| normalized size | 1 | 1. | 1.13 | 0. | 0. | 0. | 0. | 0. | 0.87 |
| time (sec) | N/A | 0.698 | 0.536 | 0.054 | 0. | 0. | 0. | 0. | 99.62 |

| Problem 105 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 357 | 357 | 407 | 0 | 0 | 0 | 0 | 0 | 318 |
| normalized size | 1 | 1. | 1.14 | 0. | 0. | 0. | 0. | 0. | 0.89 |
| time (sec) | N/A | 1.08 | 1.561 | 0.058 | 0. | 0. | 0. | 0. | 161.555 |

| Problem 106 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 280 | 280 | 735 | 0 | 0 | 3868 | 0 | 0 | 255 |
| normalized size | 1 | 1. | 2.62 | 0. | 0. | 13.81 | 0. | 0. | 0.91 |
| time (sec) | N/A | 0.893 | 4.048 | 0.096 | 0. | 19.954 | 0. | 0. | 101.332 |

| Problem 107 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 230 | 230 | 462 | 0 | 0 | 1972 | 0 | 0 | 206 |
| normalized size | 1 | 1. | 2.01 | 0. | 0. | 8.57 | 0. | 0. | 0.9 |
| time (sec) | N/A | 0.439 | 0.819 | 0.059 | 0. | 1.425 | 0. | 0. | 63.515 |

| Problem 108 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 135 | 152 | 0 | 0 | 0 | 0 | 0 | 119 |
| normalized size | 1 | 1. | 1.13 | 0. | 0. | 0. | 0. | 0. | 0.88 |
| time (sec) | N/A | 0.204 | 0.351 | 0.056 | 0. | 0. | 0. | 0. | 31.236 |

| Problem 109 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 162 | 171 | 0 | 0 | 0 | 0 | 0 | 141 |
| normalized size | 1 | 1. | 1.06 | 0. | 0. | 0. | 0. | 0. | 0.87 |
| time (sec) | N/A | 0.301 | 0.285 | 0.066 | 0. | 0. | 0. | 0. | 36.648 |

| Problem 110 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 205 | 205 | 204 | 0 | 0 | 0 | 0 | 0 | 180 |
| normalized size | 1 | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0.88 |
| time (sec) | N/A | 0.495 | 0.449 | 0.059 | 0. | 0. | 0. | 0. | 77. |

| Problem 111 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 266 | 266 | 242 | 0 | 0 | 0 | 0 | 0 | 240 |
| normalized size | 1 | 1. | 0.91 | 0. | 0. | 0. | 0. | 0. | 0.9 |
| time (sec) | N/A | 0.885 | 0.822 | 0.064 | 0. | 0. | 0. | 0. | 144.937 |

| Problem 112 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 353 | 353 | 506 | 0 | 0 | 0 | 0 | 0 | 316 |
| normalized size | 1 | 1. | 1.43 | 0. | 0. | 0. | 0. | 0. | 0.9 |
| time (sec) | N/A | 0.987 | 1.294 | 0.094 | 0. | 0. | 0. | 0. | 112.913 |

| Problem 113 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 298 | 298 | 440 | 0 | 0 | 0 | 0 | 0 | 260 |
| normalized size | 1 | 1. | 1.48 | 0. | 0. | 0. | 0. | 0. | 0.87 |
| time (sec) | N/A | 0.704 | 0.646 | 0.059 | 0. | 0. | 0. | 0. | 83.37 |

| Problem 114 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 308 | 308 | 322 | 0 | 0 | 0 | 0 | 0 | 267 |
| normalized size | 1 | 1. | 1.05 | 0. | 0. | 0. | 0. | 0. | 0.87 |
| time (sec) | N/A | 0.637 | 0.288 | 0.058 | 0. | 0. | 0. | 0. | 85.741 |

| Problem 115 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 330 | 330 | 341 | 0 | 0 | 0 | 0 | 0 | 292 |
| normalized size | 1 | 1. | 1.03 | 0. | 0. | 0. | 0. | 0. | 0.88 |
| time (sec) | N/A | 0.753 | 0.891 | 0.059 | 0. | 0. | 0. | 0. | 96.992 |

| Problem 116 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 390 | 390 | 485 | 0 | 0 | 0 | 0 | 0 | 350 |
| normalized size | 1 | 1. | 1.24 | 0. | 0. | 0. | 0. | 0. | 0.9 |
| time (sec) | N/A | 1.164 | 1.038 | 0.06 | 0. | 0. | 0. | 0. | 157.932 |

| Problem 117 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 70 | 0 | 0 | 273 | 0 | 0 | 49 |
| normalized size | 1 | 1. | 1.32 | 0. | 0. | 5.15 | 0. | 0. | 0.92 |
| time (sec) | N/A | 0.057 | 0.085 | 0.038 | 0. | 4.313 | 0. | 0. | 4.794 |

| Problem 118 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-1) | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 76 | 0 | 0 | 0 | 0 | 0 | 49 |
| normalized size | 1 | 1. | 1.33 | 0. | 0. | 0. | 0. | 0. | 0.86 |
| time (sec) | N/A | 0.069 | 0.11 | 0.076 | 0. | 0. | 0. | 0. | 9.04 |

| Problem 119 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 172 | 0 | 0 | 0 | 0 | 0 | 61 |
| normalized size | 1 | 1. | 2.18 | 0. | 0. | 0. | 0. | 0. | 0.77 |
| time (sec) | N/A | 0.123 | 0.359 | 0.115 | 0. | 0. | 0. | 0. | 21.945 |

| Problem 120 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 176 | 176 | 106 | 0 | 0 | 0 | 0 | 0 | 153 |
| normalized size | 1 | 1. | 0.6 | 0. | 0. | 0. | 0. | 0. | 0.87 |
| time (sec) | N/A | 0.316 | 0.079 | 0.057 | 0. | 0. | 0. | 0. | 32.823 |

| Problem 121 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 85 | 75 | 0 | 0 | 0 | 0 | 0 | 73 |
| normalized size | 1 | 0.91 | 0.81 | 0. | 0. | 0. | 0. | 0. | 0.78 |
| time (sec) | N/A | 0.111 | 0.035 | 0.049 | 0. | 0. | 0. | 0. | 11.371 |

| Problem 122 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 162 | 0 | 0 | 0 | 0 | 0 | 42 |
| normalized size | 1 | 1. | 2.84 | 0. | 0. | 0. | 0. | 0. | 0.74 |
| time (sec) | N/A | 0.084 | 0.281 | 0.072 | 0. | 0. | 0. | 0. | 21.903 |

| Problem 123 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 162 | 0 | 0 | 0 | 0 | 0 | 44 |
| normalized size | 1 | 1. | 2.84 | 0. | 0. | 0. | 0. | 0. | 0.77 |
| time (sec) | N/A | 0.083 | 0.308 | 0.064 | 0. | 0. | 0. | 0. | 20.222 |

| Problem 124 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 545 | 545 | 379 | 0 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.7 | 0. | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 2.311 | 1.626 | 0.055 | 0. | 0. | 0. | 0. | 0. |

| Problem 125 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 143 | 143 | 131 | 248 | 0 | 1 | 491 | 0 | 131 |
| normalized size | 1 | 1. | 0.92 | 1.73 | 0. | 0.01 | 3.43 | 0. | 0.92 |
| time (sec) | N/A | 0.417 | 0.205 | 0.02 | 0. | 0.26 | 15.493 | 0. | 50.01 |

| Problem 126 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 99 | 99 | 85 | 191 | 0 | 1 | 156 | 0 | 82 |
| normalized size | 1 | 1. | 0.86 | 1.93 | 0. | 0.01 | 1.58 | 0. | 0.83 |
| time (sec) | N/A | 0.218 | 0.134 | 0.017 | 0. | 0.257 | 12.042 | 0. | 20.394 |

| Problem 127 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 63 | 163 | 0 | 1 | 121 | 0 | 63 |
| normalized size | 1 | 1. | 0.85 | 2.2 | 0. | 0.01 | 1.64 | 0. | 0.85 |
| time (sec) | N/A | 0.146 | 0.122 | 0.016 | 0. | 0.259 | 21.053 | 0. | 12.818 |

| Problem 128 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 50 | 74 | 0 | 1 | 42 | 86 | 31 |
| normalized size | 1 | 1. | 1.28 | 1.9 | 0. | 0.03 | 1.08 | 2.21 | 0.79 |
| time (sec) | N/A | 0.057 | 0.035 | 0.006 | 0. | 0.257 | 6.494 | 0.233 | 5.903 |

| Problem 129 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 104 | 153 | 286 | 0 | 1 | 0 | 0 | 90 |
| normalized size | 1 | 1. | 1.47 | 2.75 | 0. | 0.01 | 0. | 0. | 0.87 |
| time (sec) | N/A | 0.355 | 0.359 | 0.03 | 0. | 0.288 | 0. | 0. | 41.254 |

| Problem 130 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | F | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 147 | 197 | 939 | 0 | 1 | 0 | 0 | 122 |
| normalized size | 1 | 1. | 1.34 | 6.39 | 0. | 0.01 | 0. | 0. | 0.83 |
| time (sec) | N/A | 0.607 | 0.599 | 0.028 | 0. | 0.3 | 0. | 0. | 67.723 |

| Problem 131 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | F(-1) | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 213 | 213 | 275 | 1963 | 0 | 1 | 0 | 4 | 182 |
| normalized size | 1 | 1. | 1.29 | 9.22 | 0. | 0. | 0. | 0.02 | 0.85 |
| time (sec) | N/A | 0.968 | 0.789 | 0.028 | 0. | 0.402 | 0. | 0.561 | 111.143 |

| Problem 132 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 164 | 164 | 169 | 332 | 0 | 1 | 1862 | 0 | 151 |
| normalized size | 1 | 1. | 1.03 | 2.02 | 0. | 0.01 | 11.35 | 0. | 0.92 |
| time (sec) | N/A | 0.448 | 0.259 | 0.02 | 0. | 0.255 | 41.585 | 0. | 48.464 |

| Problem 133 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 126 | 115 | 244 | 0 | 1 | 576 | 0 | 107 |
| normalized size | 1 | 1. | 0.91 | 1.94 | 0. | 0.01 | 4.57 | 0. | 0.85 |
| time (sec) | N/A | 0.249 | 0.239 | 0.019 | 0. | 0.259 | 26.834 | 0. | 22.461 |

| Problem 134 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 100 | 84 | 194 | 0 | 1 | 204 | 0 | 87 |
| normalized size | 1 | 1. | 0.84 | 1.94 | 0. | 0.01 | 2.04 | 0. | 0.87 |
| time (sec) | N/A | 0.182 | 0.163 | 0.016 | 0. | 0.259 | 32.568 | 0. | 14.757 |

| Problem 135 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 56 | 94 | 0 | 1 | 92 | 0 | 44 |
| normalized size | 1 | 1. | 1.04 | 1.74 | 0. | 0.02 | 1.7 | 0. | 0.81 |
| time (sec) | N/A | 0.077 | 0.046 | 0.006 | 0. | 0.264 | 9.18 | 0. | 7.711 |

| Problem 136 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 106 | 156 | 528 | 0 | 1 | 0 | 0 | 92 |
| normalized size | 1 | 1. | 1.47 | 4.98 | 0. | 0.01 | 0. | 0. | 0.87 |
| time (sec) | N/A | 0.384 | 0.494 | 0.02 | 0. | 0.296 | 0. | 0. | 37.956 |

| Problem 137 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | F | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 231 | 1202 | 0 | 1 | 0 | 0 | 129 |
| normalized size | 1 | 1. | 1.48 | 7.71 | 0. | 0.01 | 0. | 0. | 0.83 |
| time (sec) | N/A | 0.67 | 0.479 | 0.02 | 0. | 0.293 | 0. | 0. | 68.384 |

| Problem 138 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | F | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 209 | 209 | 256 | 2373 | 0 | 1 | 0 | 4 | 182 |
| normalized size | 1 | 1. | 1.22 | 11.35 | 0. | 0. | 0. | 0.02 | 0.87 |
| time (sec) | N/A | 0.984 | 0.512 | 0.021 | 0. | 0.328 | 0. | 0.551 | 106.166 |

| Problem 139 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 198 | 198 | 212 | 434 | 0 | 1 | 5557 | 0 | 184 |
| normalized size | 1 | 1. | 1.07 | 2.19 | 0. | 0.01 | 28.07 | 0. | 0.93 |
| time (sec) | N/A | 0.5 | 0.355 | 0.022 | 0. | 0.261 | 82.497 | 0. | 54.841 |

| Problem 140 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 152 | 152 | 156 | 318 | 0 | 1 | 1884 | 0 | 131 |
| normalized size | 1 | 1. | 1.03 | 2.09 | 0. | 0.01 | 12.39 | 0. | 0.86 |
| time (sec) | N/A | 0.299 | 0.25 | 0.02 | 0. | 0.252 | 48.817 | 0. | 26.754 |

| Problem 141 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 125 | 105 | 240 | 0 | 1 | 561 | 0 | 110 |
| normalized size | 1 | 1. | 0.84 | 1.92 | 0. | 0.01 | 4.49 | 0. | 0.88 |
| time (sec) | N/A | 0.209 | 0.252 | 0.016 | 0. | 0.263 | 47.699 | 0. | 17.605 |

| Problem 142 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 71 | 112 | 0 | 1 | 99 | 0 | 60 |
| normalized size | 1 | 1. | 1. | 1.58 | 0. | 0.01 | 1.39 | 0. | 0.85 |
| time (sec) | N/A | 0.101 | 0.102 | 0.005 | 0. | 0.256 | 13.361 | 0. | 9.588 |

| Problem 143 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 172 | 859 | 0 | 1 | 0 | 0 | 114 |
| normalized size | 1 | 1. | 1.28 | 6.41 | 0. | 0.01 | 0. | 0. | 0.85 |
| time (sec) | N/A | 0.639 | 0.228 | 0.023 | 0. | 0.436 | 0. | 0. | 62.059 |

| Problem 144 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | F | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 166 | 219 | 2014 | 0 | 1 | 0 | 0 | 138 |
| normalized size | 1 | 1. | 1.32 | 12.13 | 0. | 0.01 | 0. | 0. | 0.83 |
| time (sec) | N/A | 0.669 | 0.604 | 0.024 | 0. | 0.373 | 0. | 0. | 63.203 |

| Problem 145 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | F | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 237 | 237 | 304 | 2807 | 0 | 1 | 0 | 4 | 204 |
| normalized size | 1 | 1. | 1.28 | 11.84 | 0. | 0. | 0. | 0.02 | 0.86 |
| time (sec) | N/A | 1.022 | 0.651 | 0.022 | 0. | 0.342 | 0. | 0.549 | 106.304 |

| Problem 146 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 126 | 99 | 540 | 0 | 1 | 377 | 207 | 114 |
| normalized size | 1 | 1. | 0.79 | 4.29 | 0. | 0.01 | 2.99 | 1.64 | 0.9 |
| time (sec) | N/A | 0.284 | 0.289 | 0.021 | 0. | 0.253 | 15.563 | 0.257 | 25.537 |

| Problem 147 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 75 | 354 | 0 | 1 | 105 | 131 | 60 |
| normalized size | 1 | 1. | 1.03 | 4.85 | 0. | 0.01 | 1.44 | 1.79 | 0.82 |
| time (sec) | N/A | 0.181 | 0.161 | 0.02 | 0. | 0.248 | 12.382 | 0.256 | 16.515 |

| Problem 148 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 62 | 179 | 0 | 1 | 73 | 99 | 42 |
| normalized size | 1 | 1. | 1.22 | 3.51 | 0. | 0.02 | 1.43 | 1.94 | 0.82 |
| time (sec) | N/A | 0.11 | 0.075 | 0.017 | 0. | 0.25 | 25.068 | 0.251 | 9.442 |

| Problem 149 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 53 | 70 | 0 | 1 | 44 | 96 | 32 |
| normalized size | 1 | 1. | 1.23 | 1.63 | 0. | 0.02 | 1.02 | 2.23 | 0.74 |
| time (sec) | N/A | 0.059 | 0.047 | 0.005 | 0. | 0.247 | 7.839 | 0.237 | 5.357 |

| Problem 150 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 108 | 157 | 389 | 0 | 1 | 0 | 174 | 92 |
| normalized size | 1 | 1. | 1.45 | 3.6 | 0. | 0.01 | 0. | 1.61 | 0.85 |
| time (sec) | N/A | 0.317 | 0.467 | 0.019 | 0. | 0.277 | 0. | 0.253 | 40.759 |

| Problem 151 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | F(-2) | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 172 | 224 | 1137 | 0 | 1 | 0 | 393 | 141 |
| normalized size | 1 | 1. | 1.3 | 6.61 | 0. | 0.01 | 0. | 2.28 | 0.82 |
| time (sec) | N/A | 0.65 | 0.841 | 0.029 | 0. | 0.36 | 0. | 0.257 | 68.013 |

| Problem 152 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | F(-2) | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 250 | 250 | 301 | 2269 | 0 | 1 | 0 | 459 | 218 |
| normalized size | 1 | 1. | 1.2 | 9.08 | 0. | 0. | 0. | 1.84 | 0.87 |
| time (sec) | N/A | 1.083 | 0.95 | 0.025 | 0. | 0.767 | 0. | 0.261 | 117.385 |

| Problem 153 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 132 | 123 | 976 | 0 | 1 | 0 | 297 | 122 |
| normalized size | 1 | 1. | 0.93 | 7.39 | 0. | 0.01 | 0. | 2.25 | 0.92 |
| time (sec) | N/A | 0.327 | 0.179 | 0.025 | 0. | 0.257 | 0. | 0.252 | 27.207 |

| Problem 154 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 90 | 100 | 796 | 0 | 1 | 0 | 217 | 82 |
| normalized size | 1 | 0.96 | 1.06 | 8.47 | 0. | 0.01 | 0. | 2.31 | 0.87 |
| time (sec) | N/A | 0.248 | 0.188 | 0.022 | 0. | 0.243 | 0. | 0.252 | 18.821 |

| Problem 155 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 81 | 394 | 0 | 1 | 224 | 165 | 66 |
| normalized size | 1 | 1. | 1.07 | 5.18 | 0. | 0.01 | 2.95 | 2.17 | 0.87 |
| time (sec) | N/A | 0.155 | 0.117 | 0.018 | 0. | 0.247 | 21.432 | 0.254 | 12.511 |

| Problem 156 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 61 | 67 | 203 | 0 | 1 | 71 | 116 | 51 |
| normalized size | 1 | 1. | 1.1 | 3.33 | 0. | 0.02 | 1.16 | 1.9 | 0.84 |
| time (sec) | N/A | 0.083 | 0.1 | 0.005 | 0. | 0.251 | 11.453 | 0.25 | 8.025 |

| Problem 157 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 147 | 197 | 1480 | 0 | 1 | 0 | 261 | 122 |
| normalized size | 1 | 1. | 1.34 | 10.07 | 0. | 0.01 | 0. | 1.78 | 0.83 |
| time (sec) | N/A | 0.619 | 1.228 | 0.026 | 0. | 0.386 | 0. | 0.254 | 67.665 |

| Problem 158 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | F(-2) | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 224 | 224 | 290 | 3121 | 0 | 1 | 0 | 560 | 194 |
| normalized size | 1 | 1. | 1.29 | 13.93 | 0. | 0. | 0. | 2.5 | 0.87 |
| time (sec) | N/A | 0.962 | 1.108 | 0.026 | 0. | 0.812 | 0. | 0.256 | 110.149 |

| Problem 159 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | F(-1) | A | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 320 | 320 | 385 | 5164 | 0 | 1 | 0 | 678 | 0 |
| normalized size | 1 | 1. | 1.2 | 16.14 | 0. | 0. | 0. | 2.12 | 0. |
| time (sec) | N/A | 1.524 | 1.824 | 0.027 | 0. | 2.248 | 0. | 0.261 | 0. |

| Problem 160 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 143 | 143 | 145 | 1157 | 0 | 1 | 0 | 267 | 138 |
| normalized size | 1 | 1. | 1.01 | 8.09 | 0. | 0.01 | 0. | 1.87 | 0.97 |
| time (sec) | N/A | 0.429 | 0.242 | 0.023 | 0. | 0.255 | 0. | 0.26 | 27.837 |

| Problem 161 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 118 | 123 | 595 | 0 | 1 | 0 | 217 | 107 |
| normalized size | 1 | 0.97 | 1.01 | 4.88 | 0. | 0.01 | 0. | 1.78 | 0.88 |
| time (sec) | N/A | 0.296 | 0.189 | 0.02 | 0. | 0.254 | 0. | 0.258 | 22.725 |

| Problem 162 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 102 | 548 | 0 | 1 | 1479 | 185 | 90 |
| normalized size | 1 | 1. | 0.99 | 5.32 | 0. | 0.01 | 14.36 | 1.8 | 0.87 |
| time (sec) | N/A | 0.199 | 0.141 | 0.019 | 0. | 0.242 | 39.109 | 0.256 | 16.362 |

| Problem 163 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 82 | 276 | 0 | 1 | 774 | 132 | 70 |
| normalized size | 1 | 1. | 1. | 3.37 | 0. | 0.01 | 9.44 | 1.61 | 0.85 |
| time (sec) | N/A | 0.113 | 0.134 | 0.006 | 0. | 0.242 | 17.9 | 0.254 | 11.539 |

| Problem 164 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 201 | 201 | 250 | 2490 | 0 | 1 | 0 | 332 | 173 |
| normalized size | 1 | 1. | 1.24 | 12.39 | 0. | 0. | 0. | 1.65 | 0.86 |
| time (sec) | N/A | 0.946 | 0.963 | 0.024 | 0. | 0.903 | 0. | 0.258 | 106.153 |

| Problem 165 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | F(-2) | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 287 | 287 | 364 | 4648 | 0 | 1 | 0 | 779 | 252 |
| normalized size | 1 | 1. | 1.27 | 16.2 | 0. | 0. | 0. | 2.71 | 0.88 |
| time (sec) | N/A | 1.409 | 1.844 | 0.028 | 0. | 2.542 | 0. | 0.281 | 160.074 |

| Problem 166 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | F(-1) | A | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 409 | 409 | 465 | 7306 | 0 | 1 | 0 | 703 | 0 |
| normalized size | 1 | 1. | 1.14 | 17.86 | 0. | 0. | 0. | 1.72 | 0. |
| time (sec) | N/A | 2.072 | 3.82 | 0.031 | 0. | 6.637 | 0. | 0.266 | 0. |

| Problem 167 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 123 | 168 | 253 | 0 | 1 | 0 | 0 | 104 |
| normalized size | 1 | 1. | 1.37 | 2.06 | 0. | 0.01 | 0. | 0. | 0.85 |
| time (sec) | N/A | 0.354 | 0.355 | 0.033 | 0. | 0.729 | 0. | 0. | 32.876 |

| Problem 168 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 98 | 155 | 0 | 1 | 0 | 0 | 65 |
| normalized size | 1 | 1. | 1.21 | 1.91 | 0. | 0.01 | 0. | 0. | 0.8 |
| time (sec) | N/A | 0.193 | 0.195 | 0.034 | 0. | 0.327 | 0. | 0. | 14.874 |

| Problem 169 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 122 | 112 | 280 | 0 | 1 | 0 | 0 | 114 |
| normalized size | 1 | 1. | 0.92 | 2.3 | 0. | 0.01 | 0. | 0. | 0.93 |
| time (sec) | N/A | 0.301 | 0.214 | 0.049 | 0. | 0.329 | 0. | 0. | 24.182 |

| Problem 170 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 206 | 0 | 0 | 0 | 0 | 0 | 66 |
| normalized size | 1 | 1. | 2.15 | 0. | 0. | 0. | 0. | 0. | 0.69 |
| time (sec) | N/A | 0.181 | 0.493 | 0.142 | 0. | 0. | 0. | 0. | 23.234 |

| Problem 171 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 40 | 45 | 0 | 1 | 82 | 45 | 34 |
| normalized size | 1 | 1. | 1.03 | 1.15 | 0. | 0.03 | 2.1 | 1.15 | 0.87 |
| time (sec) | N/A | 0.072 | 0.044 | 0.007 | 0. | 0.235 | 1.665 | 0.214 | 11.336 |

| Problem 172 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 233 | 233 | 205 | 277 | 0 | 0 | 0 | 0 | 202 |
| normalized size | 1 | 1. | 0.88 | 1.19 | 0. | 0. | 0. | 0. | 0.87 |
| time (sec) | N/A | 0.634 | 0.52 | 0.05 | 0. | 0. | 0. | 0. | 62.946 |

| Problem 173 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 232 | 232 | 86 | 94 | 0 | 0 | 0 | 0 | 199 |
| normalized size | 1 | 1. | 0.37 | 0.41 | 0. | 0. | 0. | 0. | 0.86 |
| time (sec) | N/A | 0.588 | 0.102 | 0.029 | 0. | 0. | 0. | 0. | 56.681 |

| Problem 174 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 262 | 262 | 191 | 185 | 0 | 0 | 0 | 0 | 230 |
| normalized size | 1 | 1. | 0.73 | 0.71 | 0. | 0. | 0. | 0. | 0.88 |
| time (sec) | N/A | 0.836 | 0.377 | 0.065 | 0. | 0. | 0. | 0. | 80.5 |

| Problem 175 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 252 | 0 | 0 | 0 | 0 | 0 | 63 |
| normalized size | 1 | 1. | 3.19 | 0. | 0. | 0. | 0. | 0. | 0.8 |
| time (sec) | N/A | 0.234 | 0.701 | 0.141 | 0. | 0. | 0. | 0. | 31.42 |

| Problem 176 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 145 | 129 | 195 | 0 | 184 | 71 | 217 | 134 |
| normalized size | 1 | 1. | 0.89 | 1.34 | 0. | 1.27 | 0.49 | 1.5 | 0.92 |
| time (sec) | N/A | 0.258 | 0.166 | 0.007 | 0. | 0.239 | 2.223 | 0.22 | 37.577 |

| Problem 177 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 51 | 59 | 63 | 65 | 82 | 66 | 0 |
| normalized size | 1 | 1. | 1.04 | 1.2 | 1.29 | 1.33 | 1.67 | 1.35 | 0. |
| time (sec) | N/A | 0.122 | 0.044 | 0.007 | 1.706 | 0.24 | 0.818 | 0.216 | 0. |

| Problem 178 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 26 | 21 | 27 | 27 | 24 | 27 | 0 |
| normalized size | 1 | 1. | 1. | 0.81 | 1.04 | 1.04 | 0.92 | 1.04 | 0. |
| time (sec) | N/A | 0.046 | 0.01 | 0.004 | 1.4 | 0.23 | 0.472 | 0.215 | 0. |

| Problem 179 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 31 | 24 | 31 | 31 | 27 | 32 | 15 |
| normalized size | 1 | 1. | 1.82 | 1.41 | 1.82 | 1.82 | 1.59 | 1.88 | 0.88 |
| time (sec) | N/A | 0.05 | 0.01 | 0.005 | 1.391 | 0.227 | 0.649 | 0.217 | 10.383 |

| Problem 180 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 104 | 102 | 66 | 90 | 123 | 102 | 0 | 99 |
| normalized size | 1 | 1. | 0.98 | 0.63 | 0.87 | 1.18 | 0.98 | 0. | 0.95 |
| time (sec) | N/A | 0.203 | 0.05 | 0.009 | 1.648 | 0.242 | 20.267 | 0. | 19.88 |

| Problem 181 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 30 | 23 | 30 | 30 | 26 | 31 | 0 |
| normalized size | 1 | 1. | 1. | 0.77 | 1. | 1. | 0.87 | 1.03 | 0. |
| time (sec) | N/A | 0.055 | 0.011 | 0.004 | 1.376 | 0.237 | 0.405 | 0.216 | 0. |

| Problem 182 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 151 | 0 | 0 | 0 | 0 | 0 | 66 |
| normalized size | 1 | 1. | 1.91 | 0. | 0. | 0. | 0. | 0. | 0.84 |
| time (sec) | N/A | 0.083 | 0.262 | 0.118 | 0. | 0. | 0. | 0. | 19.689 |

| Problem 183 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 116 | 0 | 0 | 0 | 0 | 0 | 63 |
| normalized size | 1 | 1. | 1.53 | 0. | 0. | 0. | 0. | 0. | 0.83 |
| time (sec) | N/A | 0.076 | 0.147 | 0.068 | 0. | 0. | 0. | 0. | 19.437 |

| Problem 184 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 72 | 0 | 0 | 0 | 0 | 0 | 56 |
| normalized size | 1 | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0.78 |
| time (sec) | N/A | 0.073 | 0.126 | 0.242 | 0. | 0. | 0. | 0. | 21.4 |

| Problem 185 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 132 | 123 | 138 | 0 | 711 | 2744 | 1073 | 0 |
| normalized size | 1 | 1. | 0.93 | 1.05 | 0. | 5.39 | 20.79 | 8.13 | 0. |
| time (sec) | N/A | 0.251 | 0.19 | 0.018 | 0. | 0.263 | 9.819 | 0.229 | 0. |

| Problem 186 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 99 | 99 | 92 | 104 | 0 | 431 | 1540 | 656 | 0 |
| normalized size | 1 | 1. | 0.93 | 1.05 | 0. | 4.35 | 15.56 | 6.63 | 0. |
| time (sec) | N/A | 0.167 | 0.145 | 0.016 | 0. | 0.248 | 5.26 | 0.225 | 0. |

| Problem 187 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 70 | 65 | 74 | 0 | 236 | 726 | 342 | 0 |
| normalized size | 1 | 1. | 0.93 | 1.06 | 0. | 3.37 | 10.37 | 4.89 | 0. |
| time (sec) | N/A | 0.114 | 0.109 | 0.014 | 0. | 0.257 | 2.792 | 0.221 | 0. |

| Problem 188 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 37 | 43 | 0 | 93 | 236 | 126 | 0 |
| normalized size | 1 | 1. | 0.92 | 1.08 | 0. | 2.32 | 5.9 | 3.15 | 0. |
| time (sec) | N/A | 0.059 | 0.083 | 0.012 | 0. | 0.25 | 1.312 | 0.214 | 0. |

| Problem 189 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 40 | 0 | 0 | 0 | 0 | 0 | 31 |
| normalized size | 1 | 1. | 0.93 | 0. | 0. | 0. | 0. | 0. | 0.72 |
| time (sec) | N/A | 0.055 | 0.037 | 0.083 | 0. | 0. | 0. | 0. | 7.203 |

| Problem 190 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 68 | 0 | 0 | 0 | 0 | 0 | 53 |
| normalized size | 1 | 1. | 0.93 | 0. | 0. | 0. | 0. | 0. | 0.73 |
| time (sec) | N/A | 0.091 | 0.079 | 0.064 | 0. | 0. | 0. | 0. | 9.663 |

| Problem 191 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 96 | 0 | 0 | 0 | 0 | 0 | 60 |
| normalized size | 1 | 1. | 1.23 | 0. | 0. | 0. | 0. | 0. | 0.77 |
| time (sec) | N/A | 0.092 | 0.108 | 0.076 | 0. | 0. | 0. | 0. | 10.156 |

| Problem 192 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 136 | 0 | 0 | 0 | 0 | 0 | 60 |
| normalized size | 1 | 1. | 1.74 | 0. | 0. | 0. | 0. | 0. | 0.77 |
| time (sec) | N/A | 0.09 | 0.165 | 0.093 | 0. | 0. | 0. | 0. | 10.142 |

| Problem 193 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-2) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 158 | 149 | 164 | 0 | 900 | 0 | 1 | 0 |
| normalized size | 1 | 1. | 0.94 | 1.04 | 0. | 5.7 | 0. | 0.01 | 0. |
| time (sec) | N/A | 0.291 | 0.251 | 0.019 | 0. | 0.253 | 0. | 0.227 | 0. |

| Problem 194 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-2) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 112 | 105 | 117 | 0 | 500 | 0 | 782 | 0 |
| normalized size | 1 | 1. | 0.94 | 1.04 | 0. | 4.46 | 0. | 6.98 | 0. |
| time (sec) | N/A | 0.187 | 0.259 | 0.017 | 0. | 0.256 | 0. | 0.227 | 0. |

| Problem 195 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 70 | 65 | 74 | 0 | 236 | 726 | 342 | 0 |
| normalized size | 1 | 1. | 0.93 | 1.06 | 0. | 3.37 | 10.37 | 4.89 | 0. |
| time (sec) | N/A | 0.114 | 0.111 | 0.014 | 0. | 0.255 | 2.771 | 0.22 | 0. |

| Problem 196 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 82 | 0 | 0 | 0 | 0 | 0 | 70 |
| normalized size | 1 | 1. | 0.98 | 0. | 0. | 0. | 0. | 0. | 0.83 |
| time (sec) | N/A | 0.219 | 0.069 | 0.062 | 0. | 0. | 0. | 0. | 22.834 |

| Problem 197 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 95 | 0 | 0 | 0 | 0 | 0 | 95 |
| normalized size | 1 | 1. | 0.83 | 0. | 0. | 0. | 0. | 0. | 0.83 |
| time (sec) | N/A | 0.245 | 0.191 | 0.067 | 0. | 0. | 0. | 0. | 23.097 |

| Problem 198 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 160 | 133 | 0 | 0 | 0 | 0 | 0 | 141 |
| normalized size | 1 | 1. | 0.83 | 0. | 0. | 0. | 0. | 0. | 0.88 |
| time (sec) | N/A | 0.359 | 0.173 | 0.079 | 0. | 0. | 0. | 0. | 27.925 |

| Problem 199 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 310 | 310 | 146 | 0 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.47 | 0. | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 1.128 | 0.185 | 0.072 | 0. | 0. | 0. | 0. | 0. |

| Problem 200 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 173 | 173 | 104 | 0 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.6 | 0. | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.614 | 0.507 | 0.068 | 0. | 0. | 0. | 0. | 0. |

| Problem 201 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 82 | 0 | 0 | 0 | 0 | 0 | 70 |
| normalized size | 1 | 1. | 0.98 | 0. | 0. | 0. | 0. | 0. | 0.83 |
| time (sec) | N/A | 0.236 | 0.071 | 0.062 | 0. | 0. | 0. | 0. | 22.161 |

| Problem 202 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 40 | 0 | 0 | 0 | 0 | 0 | 31 |
| normalized size | 1 | 1. | 0.95 | 0. | 0. | 0. | 0. | 0. | 0.74 |
| time (sec) | N/A | 0.053 | 0.033 | 0.058 | 0. | 0. | 0. | 0. | 6.355 |

| Problem 203 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 64 | 0 | 0 | 0 | 0 | 0 | 53 |
| normalized size | 1 | 1. | 0.89 | 0. | 0. | 0. | 0. | 0. | 0.74 |
| time (sec) | N/A | 0.079 | 0.055 | 0.091 | 0. | 0. | 0. | 0. | 10.966 |

| Problem 204 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 123 | 121 | 0 | 0 | 0 | 0 | 0 | 100 |
| normalized size | 1 | 1. | 0.98 | 0. | 0. | 0. | 0. | 0. | 0.81 |
| time (sec) | N/A | 0.36 | 0.217 | 0.165 | 0. | 0. | 0. | 0. | 48.486 |

| Problem 205 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 210 | 210 | 210 | 0 | 0 | 0 | 0 | 0 | 218 |
| normalized size | 1 | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 1.04 |
| time (sec) | N/A | 0.819 | 0.334 | 0.175 | 0. | 0. | 0. | 0. | 112.827 |

| Problem 206 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 341 | 341 | 217 | 0 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.64 | 0. | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 1.287 | 0.337 | 0.078 | 0. | 0. | 0. | 0. | 0. |

| Problem 207 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 200 | 167 | 0 | 0 | 0 | 0 | 0 | 172 |
| normalized size | 1 | 1. | 0.84 | 0. | 0. | 0. | 0. | 0. | 0.86 |
| time (sec) | N/A | 0.61 | 0.458 | 0.073 | 0. | 0. | 0. | 0. | 65.611 |

| Problem 208 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 96 | 0 | 0 | 0 | 0 | 0 | 97 |
| normalized size | 1 | 1. | 0.83 | 0. | 0. | 0. | 0. | 0. | 0.84 |
| time (sec) | N/A | 0.237 | 0.2 | 0.066 | 0. | 0. | 0. | 0. | 22.598 |

| Problem 209 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 68 | 0 | 0 | 0 | 0 | 0 | 53 |
| normalized size | 1 | 1. | 0.94 | 0. | 0. | 0. | 0. | 0. | 0.74 |
| time (sec) | N/A | 0.084 | 0.077 | 0.065 | 0. | 0. | 0. | 0. | 9. |

| Problem 210 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 122 | 108 | 0 | 0 | 0 | 0 | 0 | 100 |
| normalized size | 1 | 1. | 0.89 | 0. | 0. | 0. | 0. | 0. | 0.82 |
| time (sec) | N/A | 0.344 | 0.255 | 0.154 | 0. | 0. | 0. | 0. | 48.624 |

| Problem 211 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 193 | 147 | 0 | 0 | 0 | 0 | 0 | 162 |
| normalized size | 1 | 1. | 0.76 | 0. | 0. | 0. | 0. | 0. | 0.84 |
| time (sec) | N/A | 0.695 | 0.344 | 0.153 | 0. | 0. | 0. | 0. | 105.36 |

| Problem 212 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 233 | 0 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.78 | 0. | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 1.305 | 0.536 | 0.187 | 0. | 0. | 0. | 0. | 0. |

| Problem 213 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 190 | 0 | 0 | 0 | 0 | 0 | 63 |
| normalized size | 1 | 1. | 2.35 | 0. | 0. | 0. | 0. | 0. | 0.78 |
| time (sec) | N/A | 0.129 | 0.662 | 0.207 | 0. | 0. | 0. | 0. | 21.667 |

| Problem 214 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-2) | F(-2) | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 402 | 401 | 168 | 0 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.42 | 0. | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 1.285 | 0.444 | 0.159 | 0. | 0. | 0. | 0. | 0. |

| Problem 215 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-2) | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 202 | 197 | 140 | 0 | 0 | 0 | 0 | 0 | 173 |
| normalized size | 1 | 0.98 | 0.69 | 0. | 0. | 0. | 0. | 0. | 0.86 |
| time (sec) | N/A | 0.624 | 0.248 | 0.137 | 0. | 0. | 0. | 0. | 36.28 |

| Problem 216 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-2) | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 89 | 85 | 0 | 0 | 0 | 0 | 0 | 80 |
| normalized size | 1 | 0.91 | 0.87 | 0. | 0. | 0. | 0. | 0. | 0.82 |
| time (sec) | N/A | 0.12 | 0.103 | 0.135 | 0. | 0. | 0. | 0. | 11.037 |

| Problem 217 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 46 | 0 | 0 | 0 | 0 | 0 | 36 |
| normalized size | 1 | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0.78 |
| time (sec) | N/A | 0.032 | 0.027 | 0.002 | 0. | 0. | 0. | 0. | 3.704 |

| Problem 218 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 180 | 0 | 0 | 0 | 0 | 0 | 44 |
| normalized size | 1 | 1. | 3.05 | 0. | 0. | 0. | 0. | 0. | 0.75 |
| time (sec) | N/A | 0.084 | 0.418 | 0.122 | 0. | 0. | 0. | 0. | 21.433 |

| Problem 219 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 180 | 0 | 0 | 0 | 0 | 0 | 46 |
| normalized size | 1 | 1. | 3.05 | 0. | 0. | 0. | 0. | 0. | 0.78 |
| time (sec) | N/A | 0.083 | 0.474 | 0.103 | 0. | 0. | 0. | 0. | 19.946 |

| Problem 220 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 180 | 0 | 0 | 0 | 0 | 0 | 46 |
| normalized size | 1 | 1. | 3.05 | 0. | 0. | 0. | 0. | 0. | 0.78 |
| time (sec) | N/A | 0.084 | 0.692 | 0.112 | 0. | 0. | 0. | 0. | 20.116 |

| Problem 221 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 93 | 93 | 0 | 0 | 0 | 0 | 0 | 83 |
| normalized size | 1 | 1. | 1. | 0. | 0. | 0. | 0. | 0. | 0.89 |
| time (sec) | N/A | 0.073 | 0.558 | 0.249 | 0. | 0. | 0. | 0. | 8.191 |

| Problem 222 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F(-1) | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 178 | 178 | 198 | 0 | 0 | 645 | 0 | 0 | 156 |
| normalized size | 1 | 1. | 1.11 | 0. | 0. | 3.62 | 0. | 0. | 0.88 |
| time (sec) | N/A | 0.241 | 0.608 | 0.203 | 0. | 0.255 | 0. | 0. | 35.596 |

| Problem 223 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F(-1) | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 139 | 0 | 0 | 312 | 0 | 0 | 100 |
| normalized size | 1 | 1. | 1.2 | 0. | 0. | 2.69 | 0. | 0. | 0.86 |
| time (sec) | N/A | 0.113 | 0.314 | 0.177 | 0. | 0.257 | 0. | 0. | 18.101 |

| Problem 224 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F(-1) | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 82 | 0 | 0 | 115 | 0 | 0 | 48 |
| normalized size | 1 | 1. | 1.41 | 0. | 0. | 1.98 | 0. | 0. | 0.83 |
| time (sec) | N/A | 0.048 | 0.169 | 0.165 | 0. | 0.267 | 0. | 0. | 6.749 |

| Problem 225 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 18 | 53 | 0 | 42 | 0 | 0 | 12 |
| normalized size | 1 | 1. | 1. | 2.94 | 0. | 2.33 | 0. | 0. | 0.67 |
| time (sec) | N/A | 0.012 | 0.04 | 0.036 | 0. | 0.249 | 0. | 0. | 1.391 |

| Problem 226 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 79 | 0 | 0 | 0 | 0 | 0 | 39 |
| normalized size | 1 | 1. | 1.49 | 0. | 0. | 0. | 0. | 0. | 0.74 |
| time (sec) | N/A | 0.048 | 0.301 | 0.12 | 0. | 0. | 0. | 0. | 6.259 |

| Problem 227 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 53 | 0 | 0 | 0 | 0 | 0 | 42 |
| normalized size | 1 | 1. | 0.98 | 0. | 0. | 0. | 0. | 0. | 0.78 |
| time (sec) | N/A | 0.05 | 0.135 | 0.112 | 0. | 0. | 0. | 0. | 6.155 |

| Problem 228 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 0 | 0 | 0 | 0 | 0 | 0 | 44 |
| normalized size | 1 | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0.79 |
| time (sec) | N/A | 0.052 | 179.999 | 0.115 | 0. | 0. | 0. | 0. | 6.47 |

| Problem 229 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 179 | 1414 | 0 | 0 | 0 | 0 | 0 | 155 |
| normalized size | 1 | 0.93 | 7.33 | 0. | 0. | 0. | 0. | 0. | 0.8 |
| time (sec) | N/A | 0.206 | 53.257 | 0.264 | 0. | 0. | 0. | 0. | 25.243 |

| Problem 230 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 461 | 0 | 0 | 146 | 0 | 0 | 41 |
| normalized size | 1 | 1. | 8.09 | 0. | 0. | 2.56 | 0. | 0. | 0.72 |
| time (sec) | N/A | 0.085 | 1.939 | 0.306 | 0. | 0.265 | 0. | 0. | 13.522 |

| Problem 231 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F(-1) | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 327 | 327 | 153 | 0 | 0 | 540 | 0 | 0 | 286 |
| normalized size | 1 | 1. | 0.47 | 0. | 0. | 1.65 | 0. | 0. | 0.87 |
| time (sec) | N/A | 0.548 | 0.305 | 0.21 | 0. | 0.26 | 0. | 0. | 81.854 |

| Problem 232 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F(-1) | F(-2) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 127 | 96 | 0 | 0 | 234 | 0 | 0 | 105 |
| normalized size | 1 | 1. | 0.76 | 0. | 0. | 1.84 | 0. | 0. | 0.83 |
| time (sec) | N/A | 0.173 | 0.139 | 0.18 | 0. | 0.261 | 0. | 0. | 17.83 |

| Problem 233 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 55 | 0 | 0 | 92 | 0 | 0 | 39 |
| normalized size | 1 | 1. | 1.1 | 0. | 0. | 1.84 | 0. | 0. | 0.78 |
| time (sec) | N/A | 0.04 | 0.037 | 0.131 | 0. | 0.256 | 0. | 0. | 3.682 |

| Problem 234 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 95 | 95 | 153 | 0 | 0 | 0 | 0 | 0 | 70 |
| normalized size | 1 | 1. | 1.61 | 0. | 0. | 0. | 0. | 0. | 0.74 |
| time (sec) | N/A | 0.106 | 2.236 | 0.126 | 0. | 0. | 0. | 0. | 13.62 |

| Problem 235 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 127 | 1070 | 0 | 0 | 0 | 0 | 0 | 95 |
| normalized size | 1 | 1. | 8.43 | 0. | 0. | 0. | 0. | 0. | 0.75 |
| time (sec) | N/A | 0.151 | 62.191 | 0.116 | 0. | 0. | 0. | 0. | 19.246 |

| Problem 236 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 1241 | 0 | 0 | 0 | 0 | 0 | 105 |
| normalized size | 1 | 1. | 9.47 | 0. | 0. | 0. | 0. | 0. | 0.8 |
| time (sec) | N/A | 0.162 | 43.768 | 0.113 | 0. | 0. | 0. | 0. | 19.697 |

| Problem 237 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 133 | 133 | 0 | 0 | 0 | 0 | 0 | 0 | 107 |
| normalized size | 1 | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0.8 |
| time (sec) | N/A | 0.171 | 179.999 | 0.137 | 0. | 0. | 0. | 0. | 20.657 |

| Problem 238 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 152 | 164 | 109 | 92 | 240 | 741 | 0 | 309 | 150 |
| normalized size | 1 | 1.08 | 0.72 | 0.61 | 1.58 | 4.88 | 0. | 2.03 | 0.99 |
| time (sec) | N/A | 0.407 | 0.094 | 0.01 | 1.392 | 0.743 | 0. | 0.257 | 25.853 |

| Problem 239 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 118 | 88 | 68 | 167 | 582 | 0 | 227 | 105 |
| normalized size | 1 | 1.08 | 0.81 | 0.62 | 1.53 | 5.34 | 0. | 2.08 | 0.96 |
| time (sec) | N/A | 0.291 | 0.075 | 0.009 | 1.381 | 0.413 | 0. | 0.231 | 17.54 |

| Problem 240 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 72 | 62 | 44 | 95 | 421 | 0 | 126 | 61 |
| normalized size | 1 | 1.07 | 0.93 | 0.66 | 1.42 | 6.28 | 0. | 1.88 | 0.91 |
| time (sec) | N/A | 0.151 | 0.058 | 0.006 | 1.378 | 0.28 | 0. | 0.217 | 9.894 |

| Problem 241 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 75 | 174 | 0 | 383 | 0 | 109 | 65 |
| normalized size | 1 | 1. | 0.94 | 2.17 | 0. | 4.79 | 0. | 1.36 | 0.81 |
| time (sec) | N/A | 0.286 | 0.143 | 0.02 | 0. | 0.25 | 0. | 0.225 | 17.384 |

| Problem 242 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | F | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 114 | 105 | 182 | 0 | 419 | 0 | 212 | 95 |
| normalized size | 1 | 1.19 | 1.09 | 1.9 | 0. | 4.36 | 0. | 2.21 | 0.99 |
| time (sec) | N/A | 0.32 | 0.197 | 0.02 | 0. | 0.251 | 0. | 0.24 | 18.789 |

| Problem 243 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | F(-2) | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 121 | 164 | 132 | 226 | 0 | 539 | 0 | 437 | 141 |
| normalized size | 1 | 1.36 | 1.09 | 1.87 | 0. | 4.45 | 0. | 3.61 | 1.17 |
| time (sec) | N/A | 0.399 | 0.179 | 0.023 | 0. | 0.263 | 0. | 0.244 | 22.663 |

| Problem 244 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 208 | 146 | 298 | 356 | 976 | 0 | 394 | 187 |
| normalized size | 1 | 1. | 0.7 | 1.43 | 1.71 | 4.69 | 0. | 1.89 | 0.9 |
| time (sec) | N/A | 0.476 | 0.178 | 0.032 | 1.411 | 0.652 | 0. | 0.306 | 34.333 |

| Problem 245 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 159 | 159 | 120 | 240 | 284 | 741 | 0 | 311 | 139 |
| normalized size | 1 | 1. | 0.75 | 1.51 | 1.79 | 4.66 | 0. | 1.96 | 0.87 |
| time (sec) | N/A | 0.377 | 0.122 | 0.018 | 1.389 | 0.346 | 0. | 0.29 | 23.938 |

| Problem 246 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 114 | 96 | 182 | 205 | 517 | 0 | 204 | 129 |
| normalized size | 1 | 1. | 0.84 | 1.6 | 1.8 | 4.54 | 0. | 1.79 | 1.13 |
| time (sec) | N/A | 0.161 | 0.083 | 0.016 | 1.435 | 0.26 | 0. | 0.279 | 19.364 |

| Problem 247 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | A | F | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 104 | 81 | 153 | 0 | 379 | 0 | 149 | 90 |
| normalized size | 1 | 1. | 0.78 | 1.47 | 0. | 3.64 | 0. | 1.43 | 0.87 |
| time (sec) | N/A | 0.256 | 0.107 | 0.021 | 0. | 0.242 | 0. | 0.23 | 16.744 |

| Problem 248 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | A | F(-2) | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 78 | 153 | 0 | 313 | 0 | 231 | 70 |
| normalized size | 1 | 1. | 0.93 | 1.82 | 0. | 3.73 | 0. | 2.75 | 0.83 |
| time (sec) | N/A | 0.26 | 0.162 | 0.022 | 0. | 0.238 | 0. | 0.234 | 15.525 |

| Problem 249 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 125 | 102 | 191 | 231 | 581 | 216 | 205 | 116 |
| normalized size | 1 | 1. | 0.82 | 1.53 | 1.85 | 4.65 | 1.73 | 1.64 | 0.93 |
| time (sec) | N/A | 0.335 | 0.137 | 0.05 | 1.486 | 0.27 | 134.074 | 0.247 | 17.229 |

| Problem 250 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 61 | 57 | 128 | 333 | 216 | 130 | 95 |
| normalized size | 1 | 1. | 0.59 | 0.55 | 1.24 | 3.23 | 2.1 | 1.26 | 0.92 |
| time (sec) | N/A | 0.266 | 0.063 | 0.009 | 1.394 | 0.254 | 89.855 | 0.218 | 13.39 |

| Problem 251 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 87 | 83 | 147 | 177 | 406 | 212 | 151 | 80 |
| normalized size | 1 | 1. | 0.95 | 1.69 | 2.03 | 4.67 | 2.44 | 1.74 | 0.92 |
| time (sec) | N/A | 0.262 | 0.094 | 0.028 | 1.381 | 0.244 | 86.612 | 0.249 | 12.753 |

| Problem 252 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 43 | 38 | 73 | 216 | 202 | 74 | 58 |
| normalized size | 1 | 1. | 0.66 | 0.58 | 1.12 | 3.32 | 3.11 | 1.14 | 0.89 |
| time (sec) | N/A | 0.142 | 0.049 | 0.006 | 1.417 | 0.243 | 57.389 | 0.22 | 8.283 |

| Problem 253 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 63 | 103 | 120 | 220 | 182 | 96 | 44 |
| normalized size | 1 | 1. | 1.34 | 2.19 | 2.55 | 4.68 | 3.87 | 2.04 | 0.94 |
| time (sec) | N/A | 0.09 | 0.054 | 0.023 | 1.425 | 0.242 | 37.1 | 0.246 | 8.934 |

| Problem 254 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 47 | 62 | 42 | 149 | 162 | 61 | 41 |
| normalized size | 1 | 1. | 1.02 | 1.35 | 0.91 | 3.24 | 3.52 | 1.33 | 0.89 |
| time (sec) | N/A | 0.211 | 0.065 | 0.026 | 1.707 | 0.242 | 36.489 | 0.218 | 9.713 |

| Problem 255 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 53 | 77 | 68 | 109 | 148 | 78 | 27 |
| normalized size | 1 | 1. | 1.61 | 2.33 | 2.06 | 3.3 | 4.48 | 2.36 | 0.82 |
| time (sec) | N/A | 0.183 | 0.055 | 0.029 | 1.694 | 0.24 | 42.474 | 0.223 | 8.576 |

| Problem 256 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 61 | 84 | 66 | 223 | 141 | 154 | 49 |
| normalized size | 1 | 1. | 1.02 | 1.4 | 1.1 | 3.72 | 2.35 | 2.57 | 0.82 |
| time (sec) | N/A | 0.216 | 0.089 | 0.028 | 1.544 | 0.242 | 56.108 | 0.229 | 9.672 |

| Problem 257 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 42 | 37 | 73 | 140 | 146 | 157 | 51 |
| normalized size | 1 | 1. | 0.68 | 0.6 | 1.18 | 2.26 | 2.35 | 2.53 | 0.82 |
| time (sec) | N/A | 0.217 | 0.052 | 0.008 | 1.613 | 0.23 | 84.097 | 0.225 | 9.318 |

| Problem 258 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 99 | 99 | 80 | 125 | 120 | 423 | 148 | 362 | 88 |
| normalized size | 1 | 1. | 0.81 | 1.26 | 1.21 | 4.27 | 1.49 | 3.66 | 0.89 |
| time (sec) | N/A | 0.286 | 0.133 | 0.029 | 1.587 | 0.245 | 124.583 | 0.229 | 13.774 |

| Problem 259 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F(-1) | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 164 | 164 | 123 | 240 | 289 | 734 | 0 | 246 | 146 |
| normalized size | 1 | 1. | 0.75 | 1.46 | 1.76 | 4.48 | 0. | 1.5 | 0.89 |
| time (sec) | N/A | 0.422 | 0.171 | 0.04 | 1.398 | 0.361 | 0. | 0.258 | 26.19 |

| Problem 260 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 118 | 74 | 68 | 167 | 420 | 240 | 151 | 105 |
| normalized size | 1 | 1. | 0.63 | 0.58 | 1.42 | 3.56 | 2.03 | 1.28 | 0.89 |
| time (sec) | N/A | 0.306 | 0.08 | 0.009 | 1.375 | 0.275 | 136.219 | 0.222 | 17.574 |

| Problem 261 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 118 | 96 | 182 | 216 | 508 | 236 | 176 | 104 |
| normalized size | 1 | 1. | 0.81 | 1.54 | 1.83 | 4.31 | 2. | 1.49 | 0.88 |
| time (sec) | N/A | 0.318 | 0.106 | 0.03 | 1.386 | 0.253 | 117.765 | 0.255 | 17.997 |

| Problem 262 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 48 | 43 | 93 | 262 | 223 | 82 | 61 |
| normalized size | 1 | 1. | 0.67 | 0.6 | 1.29 | 3.64 | 3.1 | 1.14 | 0.85 |
| time (sec) | N/A | 0.159 | 0.056 | 0.006 | 1.384 | 0.241 | 76.023 | 0.218 | 9.97 |

| Problem 263 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 71 | 124 | 140 | 261 | 199 | 107 | 75 |
| normalized size | 1 | 1. | 1.04 | 1.82 | 2.06 | 3.84 | 2.93 | 1.57 | 1.1 |
| time (sec) | N/A | 0.116 | 0.067 | 0.025 | 1.396 | 0.238 | 43.666 | 0.256 | 13.259 |

| Problem 264 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 55 | 108 | 0 | 176 | 178 | 74 | 44 |
| normalized size | 1 | 1. | 0.98 | 1.93 | 0. | 3.14 | 3.18 | 1.32 | 0.79 |
| time (sec) | N/A | 0.242 | 0.1 | 0.026 | 0. | 0.241 | 44.724 | 0.22 | 12.883 |

| Problem 265 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 60 | 97 | 0 | 117 | 165 | 89 | 46 |
| normalized size | 1 | 1. | 1.05 | 1.7 | 0. | 2.05 | 2.89 | 1.56 | 0.81 |
| time (sec) | N/A | 0.227 | 0.073 | 0.029 | 0. | 0.237 | 51.506 | 0.225 | 11.526 |

| Problem 266 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 103 | 158 | 0 | 279 | 162 | 190 | 63 |
| normalized size | 1 | 1. | 1.36 | 2.08 | 0. | 3.67 | 2.13 | 2.5 | 0.83 |
| time (sec) | N/A | 0.261 | 0.139 | 0.029 | 0. | 0.245 | 68.558 | 0.23 | 13.362 |

| Problem 267 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 56 | 49 | 0 | 162 | 170 | 185 | 63 |
| normalized size | 1 | 1. | 0.75 | 0.65 | 0. | 2.16 | 2.27 | 2.47 | 0.84 |
| time (sec) | N/A | 0.254 | 0.074 | 0.008 | 0. | 0.238 | 108.996 | 0.234 | 11.989 |

| Problem 268 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 123 | 135 | 227 | 0 | 544 | 172 | 439 | 107 |
| normalized size | 1 | 1. | 1.1 | 1.85 | 0. | 4.42 | 1.4 | 3.57 | 0.87 |
| time (sec) | N/A | 0.343 | 0.196 | 0.03 | 0. | 0.272 | 157.614 | 0.24 | 19.736 |

| Problem 269 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 161 | 150 | 316 | 289 | 702 | 233 | 290 | 146 |
| normalized size | 1 | 1. | 0.93 | 1.96 | 1.8 | 4.36 | 1.45 | 1.8 | 0.91 |
| time (sec) | N/A | 0.418 | 0.196 | 0.047 | 1.435 | 0.245 | 102.515 | 0.243 | 26.809 |

| Problem 270 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 72 | 68 | 166 | 354 | 226 | 194 | 102 |
| normalized size | 1 | 1. | 0.63 | 0.59 | 1.44 | 3.08 | 1.97 | 1.69 | 0.89 |
| time (sec) | N/A | 0.316 | 0.09 | 0.008 | 1.404 | 0.255 | 82.293 | 0.236 | 18.613 |

| Problem 271 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 152 | 152 | 121 | 254 | 211 | 448 | 212 | 208 | 134 |
| normalized size | 1 | 1. | 0.8 | 1.67 | 1.39 | 2.95 | 1.39 | 1.37 | 0.88 |
| time (sec) | N/A | 0.381 | 0.161 | 0.033 | 1.414 | 0.246 | 80.972 | 0.241 | 24.675 |

| Problem 272 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 45 | 43 | 93 | 196 | 201 | 127 | 65 |
| normalized size | 1 | 1. | 0.59 | 0.57 | 1.22 | 2.58 | 2.64 | 1.67 | 0.86 |
| time (sec) | N/A | 0.182 | 0.076 | 0.005 | 1.381 | 0.247 | 71.83 | 0.229 | 11.335 |

| Problem 273 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 100 | 160 | 115 | 153 | 182 | 153 | 94 |
| normalized size | 1 | 1. | 1.59 | 2.54 | 1.83 | 2.43 | 2.89 | 2.43 | 1.49 |
| time (sec) | N/A | 0.115 | 0.093 | 0.027 | 1.418 | 0.237 | 70.116 | 0.232 | 21.082 |

| Problem 274 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 96 | 188 | 0 | 223 | 172 | 155 | 53 |
| normalized size | 1 | 1. | 1.48 | 2.89 | 0. | 3.43 | 2.65 | 2.38 | 0.82 |
| time (sec) | N/A | 0.266 | 0.159 | 0.034 | 0. | 0.242 | 93.709 | 0.253 | 14.22 |

| Problem 275 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 67 | 51 | 48 | 0 | 117 | 165 | 296 | 56 |
| normalized size | 1 | 1. | 0.76 | 0.72 | 0. | 1.75 | 2.46 | 4.42 | 0.84 |
| time (sec) | N/A | 0.239 | 0.085 | 0.007 | 0. | 0.237 | 152.285 | 0.278 | 11.399 |

| Problem 276 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | F(-1) | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 117 | 126 | 315 | 0 | 474 | 0 | 285 | 100 |
| normalized size | 1 | 1. | 1.08 | 2.69 | 0. | 4.05 | 0. | 2.44 | 0.85 |
| time (sec) | N/A | 0.348 | 0.29 | 0.036 | 0. | 0.255 | 0. | 0.296 | 21.452 |

| Problem 277 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F(-1) | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 119 | 77 | 73 | 0 | 193 | 0 | 327 | 105 |
| normalized size | 1 | 1. | 0.65 | 0.61 | 0. | 1.62 | 0. | 2.75 | 0.88 |
| time (sec) | N/A | 0.324 | 0.108 | 0.008 | 0. | 0.236 | 0. | 0.314 | 17.908 |

| Problem 278 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | F(-1) | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 166 | 157 | 387 | 0 | 707 | 0 | 543 | 150 |
| normalized size | 1 | 1. | 0.95 | 2.33 | 0. | 4.26 | 0. | 3.27 | 0.9 |
| time (sec) | N/A | 0.459 | 0.382 | 0.038 | 0. | 0.305 | 0. | 0.345 | 30.958 |

| Problem 279 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 42 | 53 | 34 | 127 | 148 | 54 | 34 |
| normalized size | 1 | 1. | 1.05 | 1.32 | 0.85 | 3.18 | 3.7 | 1.35 | 0.85 |
| time (sec) | N/A | 0.199 | 0.054 | 0.024 | 1.529 | 0.241 | 31.06 | 0.22 | 9.29 |

| Problem 280 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 1424 | 66 | 107 | 88 | 0 | 0 | 42 |
| normalized size | 1 | 1. | 26.87 | 1.25 | 2.02 | 1.66 | 0. | 0. | 0.79 |
| time (sec) | N/A | 0.298 | 4.263 | 0.014 | 1.859 | 0.262 | 0. | 0. | 10.087 |

| Problem 281 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F(-2) | A |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 36 | 0 | 0 | 0 | 93 | 0 | 0 | 31 |
| normalized size | 1 | 1. | 0. | 0. | 0. | 2.58 | 0. | 0. | 0.86 |
| time (sec) | N/A | 0.071 | 0.948 | 0.062 | 0. | 0.236 | 0. | 0. | 8.379 |

| Problem 282 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | A |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 0 | 0 | 0 | 68 | 0 | 0 | 65 |
| normalized size | 1 | 1. | 0. | 0. | 0. | 0.91 | 0. | 0. | 0.87 |
| time (sec) | N/A | 0.146 | 1.963 | 0.059 | 0. | 0.242 | 0. | 0. | 18.21 |

| Problem 283 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 113 | 0 | 0 | 0 | 0 | 0 | 0 | 88 |
| normalized size | 1 | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0.78 |
| time (sec) | N/A | 0.235 | 0.37 | 0.813 | 0. | 0. | 0. | 0. | 37.859 |

| Problem 284 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 87 | 0 | 0 | 0 | 0 | 0 | 0 | 70 |
| normalized size | 1 | 1. | 0. | 0. | 0. | 0. | 0. | 0. | 0.8 |
| time (sec) | N/A | 0.15 | 0.278 | 0.806 | 0. | 0. | 0. | 0. | 32.525 |

| Problem 285 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 258 | 0 | 0 | 0 | 0 | 0 | 58 |
| normalized size | 1 | 1. | 3.39 | 0. | 0. | 0. | 0. | 0. | 0.76 |
| time (sec) | N/A | 0.168 | 0.493 | 0.139 | 0. | 0. | 0. | 0. | 35.352 |

| Problem 286 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Rubi in Sympy |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F | A |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 104 | 0 | 0 | 0 | 243 | 0 | 0 | 102 |
| normalized size | 1 | 1.08 | 0. | 0. | 0. | 2.53 | 0. | 0. | 1.06 |
| time (sec) | N/A | 0.361 | 0.929 | 1.044 | 0. | 0.289 | 0. | 0. | 51.438 |

## 2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [180] had the largest ratio of [ 0.5294 ]

Table 1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 2 | 1 | 1. | 17 | 0.059 |
| 2 | A | 2 | 1 | 1. | 17 | 0.059 |
| 3 | A | 2 | 1 | 1. | 17 | 0.059 |
| 4 | A | 2 | 1 | 1. | 15 | 0.067 |
| 5 | A | 7 | 7 | 1. | 17 | 0.412 |
| 6 | A | 7 | 7 | 1. | 17 | 0.412 |
| 7 | A | 8 | 8 | 1. | 17 | 0.471 |
| 8 | A | 2 | 1 | 1. | 19 | 0.053 |
| 9 | A | 2 | 1 | 1. | 19 | 0.053 |
| 10 | A | 2 | 1 | 1. | 17 | 0.059 |
| 11 | A | 8 | 7 | 1. | 19 | 0.368 |
| 12 | A | 9 | 8 | 1. | 19 | 0.421 |
| 13 | A | 8 | 8 | 1. | 19 | 0.421 |
| 14 | A | 8 | 7 | 1. | 19 | 0.368 |
| 15 | A | 8 | 7 | 1. | 19 | 0.368 |
| 16 | A | 8 | 7 | 1. | 19 | 0.368 |
| 17 | A | 7 | 7 | 1. | 17 | 0.412 |
| 18 | A | 13 | 7 | 1. | 19 | 0.368 |
| 19 | A | 14 | 8 | 1. | 19 | 0.421 |
| 20 | A | 9 | 8 | 1. | 19 | 0.421 |
| 21 | A | 9 | 8 | 1. | 19 | 0.421 |
| 22 | A | 9 | 8 | 1. | 19 | 0.421 |
| 23 | A | 9 | 8 | 1. | 19 | 0.421 |
| 24 | A | 7 | 7 | 1. | 17 | 0.412 |
| 25 | A | 14 | 8 | 1. | 19 | 0.421 |
| 26 | A | 15 | 9 | 1. | 19 | 0.474 |
| 27 | A | 4 | 2 | 1. | 21 | 0.095 |
| 28 | A | 3 | 2 | 1. | 21 | 0.095 |

Continued on next page

Table 1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 29 | A | 2 | 2 | 1. | 19 | 0.105 |
| 30 | A | 1 | 1 | 1. | 11 | 0.091 |
| 31 | A | 7 | 7 | 1. | 21 | 0.333 |
| 32 | A | 8 | 8 | 1. | 21 | 0.381 |
| 33 | A | 9 | 8 | 1. | 21 | 0.381 |
| 34 | A | 3 | 2 | 1. | 19 | 0.105 |
| 35 | A | 4 | 4 | 1.05 | 19 | 0.21 |
| 36 | A | 3 | 3 | 1.01 | 17 | 0.176 |
| 37 | A | 2 | 2 | 1. | 19 | 0.105 |
| 38 | A | 2 | 2 | 1. | 19 | 0.105 |
| 39 | A | 5 | 5 | 1. | 19 | 0.263 |
| 40 | A | 4 | 4 | 1. | 19 | 0.21 |
| 41 | A | 3 | 3 | 0.91 | 17 | 0.176 |
| 42 | A | 2 | 2 | 1. | 9 | 0.222 |
| 43 | A | 2 | 2 | 1. | 19 | 0.105 |
| 44 | A | 2 | 2 | 1. | 19 | 0.105 |
| 45 | A | 2 | 2 | 1. | 19 | 0.105 |
| 46 | A | 1 | 1 | 1. | 50 | 0.02 |
| 47 | A | 2 | 1 | 1. | 17 | 0.059 |
| 48 | A | 2 | 1 | 1. | 17 | 0.059 |
| 49 | A | 2 | 1 | 1. | 17 | 0.059 |
| 50 | A | 2 | 1 | 1. | 15 | 0.067 |
| 51 | A | 10 | 7 | 1. | 17 | 0.412 |
| 52 | A | 10 | 7 | 1. | 17 | 0.412 |
| 53 | A | 11 | 8 | 1. | 17 | 0.471 |
| 54 | A | 2 | 1 | 1. | 19 | 0.053 |
| 55 | A | 2 | 1 | 1. | 19 | 0.053 |
| 56 | A | 2 | 1 | 1. | 19 | 0.053 |
| 57 | A | 2 | 1 | 1. | 17 | 0.059 |
| 58 | A | 11 | 7 | 1. | 19 | 0.368 |
| 59 | A | 12 | 8 | 1. | 19 | 0.421 |
| 60 | A | 11 | 8 | 1. | 19 | 0.421 |
| 61 | A | 11 | 7 | 1. | 19 | 0.368 |
| 62 | A | 11 | 7 | 1. | 19 | 0.368 |
| 63 | A | 11 | 7 | 1. | 19 | 0.368 |
| 64 | A | 10 | 7 | 1. | 17 | 0.412 |

Continued on next page

Table 1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 65 | A | 19 | 7 | 1. | 19 | 0.368 |
| 66 | A | 20 | 8 | 1. | 19 | 0.421 |
| 67 | A | 12 | 8 | 1. | 19 | 0.421 |
| 68 | A | 12 | 8 | 1. | 19 | 0.421 |
| 69 | A | 12 | 8 | 1. | 19 | 0.421 |
| 70 | A | 12 | 8 | 1. | 19 | 0.421 |
| 71 | A | 10 | 7 | 1. | 17 | 0.412 |
| 72 | A | 20 | 8 | 1. | 19 | 0.421 |
| 73 | A | 21 | 9 | 1. | 19 | 0.474 |
| 74 | A | 10 | 8 | 1. | 23 | 0.348 |
| 75 | A | 9 | 7 | 1. | 23 | 0.304 |
| 76 | A | 8 | 6 | 1. | 23 | 0.261 |
| 77 | A | 5 | 3 | 1. | 23 | 0.13 |
| 78 | A | 9 | 7 | 1. | 23 | 0.304 |
| 79 | A | 10 | 8 | 1. | 23 | 0.348 |
| 80 | A | 8 | 5 | 1. | 21 | 0.238 |
| 81 | A | 7 | 4 | 1. | 21 | 0.19 |
| 82 | A | 5 | 3 | 1. | 21 | 0.143 |
| 83 | A | 8 | 5 | 1. | 21 | 0.238 |
| 84 | A | 9 | 6 | 1. | 21 | 0.286 |
| 85 | A | 11 | 8 | 1. | 23 | 0.348 |
| 86 | A | 10 | 8 | 1. | 23 | 0.348 |
| 87 | A | 9 | 7 | 1. | 23 | 0.304 |
| 88 | A | 9 | 7 | 1. | 23 | 0.304 |
| 89 | A | 9 | 7 | 1. | 23 | 0.304 |
| 90 | A | 10 | 8 | 1. | 23 | 0.348 |
| 91 | A | 11 | 8 | 1. | 23 | 0.348 |
| 92 | A | 4 | 4 | 1. | 25 | 0.16 |
| 93 | A | 1 | 1 | 1. | 25 | 0.04 |
| 94 | A | 10 | 9 | 1. | 21 | 0.429 |
| 95 | A | 9 | 8 | 1. | 21 | 0.381 |
| 96 | A | 4 | 4 | 1. | 21 | 0.19 |
| 97 | A | 5 | 5 | 1. | 21 | 0.238 |
| 98 | A | 7 | 7 | 1. | 21 | 0.333 |
| 99 | A | 8 | 7 | 1. | 21 | 0.333 |
| 100 | A | 11 | 10 | 1. | 21 | 0.476 |

Continued on next page

Table 1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 101 | A | 10 | 9 | 1. | 21 | 0.429 |
| 102 | A | 4 | 3 | 1. | 21 | 0.143 |
| 103 | A | 9 | 8 | 1. | 21 | 0.381 |
| 104 | A | 10 | 9 | 1. | 21 | 0.429 |
| 105 | A | 11 | 10 | 1. | 21 | 0.476 |
| 106 | A | 11 | 10 | 1. | 21 | 0.476 |
| 107 | A | 10 | 9 | 1. | 21 | 0.429 |
| 108 | A | 5 | 5 | 1. | 21 | 0.238 |
| 109 | A | 5 | 5 | 1. | 21 | 0.238 |
| 110 | A | 7 | 7 | 1. | 21 | 0.333 |
| 111 | A | 8 | 7 | 1. | 21 | 0.333 |
| 112 | A | 11 | 10 | 1. | 21 | 0.476 |
| 113 | A | 10 | 9 | 1. | 21 | 0.429 |
| 114 | A | 10 | 9 | 1. | 21 | 0.429 |
| 115 | A | 10 | 9 | 1. | 21 | 0.429 |
| 116 | A | 11 | 10 | 1. | 21 | 0.476 |
| 117 | A | 4 | 4 | 1. | 17 | 0.235 |
| 118 | A | 4 | 4 | 1. | 26 | 0.154 |
| 119 | A | 3 | 2 | 1. | 19 | 0.105 |
| 120 | A | 4 | 4 | 1. | 19 | 0.21 |
| 121 | A | 3 | 3 | 0.91 | 17 | 0.176 |
| 122 | A | 2 | 2 | 1. | 19 | 0.105 |
| 123 | A | 2 | 2 | 1. | 19 | 0.105 |
| 124 | A | 7 | 7 | 1. | 21 | 0.333 |
| 125 | A | 6 | 6 | 1. | 21 | 0.286 |
| 126 | A | 6 | 6 | 1. | 21 | 0.286 |
| 127 | A | 5 | 5 | 1. | 19 | 0.263 |
| 128 | A | 4 | 4 | 1. | 11 | 0.364 |
| 129 | A | 7 | 6 | 1. | 21 | 0.286 |
| 130 | A | 8 | 7 | 1. | 21 | 0.333 |
| 131 | A | 9 | 7 | 1. | 21 | 0.333 |
| 132 | A | 7 | 7 | 1. | 21 | 0.333 |
| 133 | A | 7 | 6 | 1. | 21 | 0.286 |
| 134 | A | 6 | 5 | 1. | 19 | 0.263 |
| 135 | A | 5 | 5 | 1. | 11 | 0.454 |
| 136 | A | 7 | 6 | 1. | 21 | 0.286 |

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Table 1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 137 | A | 8 | 7 | 1. | 21 | 0.333 |
| 138 | A | 9 | 7 | 1. | 21 | 0.333 |
| 139 | A | 8 | 7 | 1. | 21 | 0.333 |
| 140 | A | 8 | 6 | 1. | 21 | 0.286 |
| 141 | A | 7 | 5 | 1. | 19 | 0.263 |
| 142 | A | 6 | 5 | 1. | 11 | 0.454 |
| 143 | A | 8 | 7 | 1. | 21 | 0.333 |
| 144 | A | 8 | 7 | 1. | 21 | 0.333 |
| 145 | A | 9 | 8 | 1. | 21 | 0.381 |
| 146 | A | 5 | 5 | 1. | 21 | 0.238 |
| 147 | A | 5 | 5 | 1. | 21 | 0.238 |
| 148 | A | 4 | 4 | 1. | 19 | 0.21 |
| 149 | A | 4 | 4 | 1. | 11 | 0.364 |
| 150 | A | 7 | 6 | 1. | 21 | 0.286 |
| 151 | A | 8 | 7 | 1. | 21 | 0.333 |
| 152 | A | 9 | 7 | 1. | 21 | 0.333 |
| 153 | A | 5 | 5 | 1. | 21 | 0.238 |
| 154 | A | 5 | 5 | 0.96 | 21 | 0.238 |
| 155 | A | 5 | 5 | 1. | 19 | 0.263 |
| 156 | A | 5 | 4 | 1. | 11 | 0.364 |
| 157 | A | 8 | 7 | 1. | 21 | 0.333 |
| 158 | A | 9 | 8 | 1. | 21 | 0.381 |
| 159 | A | 10 | 8 | 1. | 21 | 0.381 |
| 160 | A | 5 | 5 | 1. | 21 | 0.238 |
| 161 | A | 6 | 6 | 0.97 | 21 | 0.286 |
| 162 | A | 6 | 5 | 1. | 19 | 0.263 |
| 163 | A | 6 | 4 | 1. | 11 | 0.364 |
| 164 | A | 9 | 7 | 1. | 21 | 0.333 |
| 165 | A | 10 | 8 | 1. | 21 | 0.381 |
| 166 | A | 11 | 8 | 1. | 21 | 0.381 |
| 167 | A | 7 | 6 | 1. | 23 | 0.261 |
| 168 | A | 4 | 4 | 1. | 23 | 0.174 |
| 169 | A | 5 | 5 | 1. | 23 | 0.217 |
| 170 | A | 3 | 3 | 1. | 19 | 0.158 |
| 171 | A | 3 | 3 | 1. | 17 | 0.176 |
| 172 | A | 6 | 6 | 1. | 23 | 0.261 |

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Table 1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 173 | A | 7 | 7 | 1. | 23 | 0.304 |
| 174 | A | 7 | 7 | 1. | 23 | 0.304 |
| 175 | A | 4 | 3 | 1. | 19 | 0.158 |
| 176 | A | 8 | 8 | 1. | 17 | 0.471 |
| 177 | A | 3 | 2 | 1. | 21 | 0.095 |
| 178 | A | 3 | 2 | 1. | 17 | 0.118 |
| 179 | A | 4 | 4 | 1. | 17 | 0.235 |
| 180 | A | 9 | 9 | 1. | 17 | 0.529 |
| 181 | A | 4 | 3 | 1. | 17 | 0.176 |
| 182 | A | 3 | 3 | 1. | 24 | 0.125 |
| 183 | A | 3 | 3 | 1. | 24 | 0.125 |
| 184 | A | 3 | 3 | 1. | 20 | 0.15 |
| 185 | A | 2 | 1 | 1. | 17 | 0.059 |
| 186 | A | 2 | 1 | 1. | 17 | 0.059 |
| 187 | A | 2 | 1 | 1. | 17 | 0.059 |
| 188 | A | 2 | 1 | 1. | 15 | 0.067 |
| 189 | A | 2 | 2 | 1. | 17 | 0.118 |
| 190 | A | 2 | 2 | 1. | 17 | 0.118 |
| 191 | A | 2 | 2 | 1. | 17 | 0.118 |
| 192 | A | 2 | 2 | 1. | 17 | 0.118 |
| 193 | A | 2 | 1 | 1. | 19 | 0.053 |
| 194 | A | 2 | 1 | 1. | 19 | 0.053 |
| 195 | A | 2 | 1 | 1. | 17 | 0.059 |
| 196 | A | 3 | 3 | 1. | 19 | 0.158 |
| 197 | A | 3 | 3 | 1. | 19 | 0.158 |
| 198 | A | 3 | 3 | 1. | 19 | 0.158 |
| 199 | A | 5 | 4 | 1. | 19 | 0.21 |
| 200 | A | 4 | 4 | 1. | 19 | 0.21 |
| 201 | A | 3 | 3 | 1. | 19 | 0.158 |
| 202 | A | 2 | 2 | 1. | 17 | 0.118 |
| 203 | A | 3 | 2 | 1. | 19 | 0.105 |
| 204 | A | 4 | 3 | 1. | 19 | 0.158 |
| 205 | A | 5 | 4 | 1. | 19 | 0.21 |
| 206 | A | 5 | 4 | 1. | 19 | 0.21 |
| 207 | A | 4 | 4 | 1. | 19 | 0.21 |
| 208 | A | 3 | 3 | 1. | 19 | 0.158 |

Continued on next page

Table 1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 209 | A | 2 | 2 | 1. | 17 | 0.118 |
| 210 | A | 4 | 3 | 1. | 19 | 0.158 |
| 211 | A | 5 | 4 | 1. | 19 | 0.21 |
| 212 | A | 6 | 4 | 1. | 19 | 0.21 |
| 213 | A | 3 | 2 | 1. | 19 | 0.105 |
| 214 | A | 5 | 5 | 1. | 19 | 0.263 |
| 215 | A | 4 | 4 | 0.98 | 19 | 0.21 |
| 216 | A | 3 | 3 | 0.91 | 17 | 0.176 |
| 217 | A | 2 | 2 | 1. | 9 | 0.222 |
| 218 | A | 2 | 2 | 1. | 19 | 0.105 |
| 219 | A | 2 | 2 | 1. | 19 | 0.105 |
| 220 | A | 2 | 2 | 1. | 19 | 0.105 |
| 221 | A | 1 | 1 | 1. | 28 | 0.036 |
| 222 | A | 4 | 2 | 1. | 25 | 0.08 |
| 223 | A | 3 | 2 | 1. | 25 | 0.08 |
| 224 | A | 2 | 2 | 1. | 23 | 0.087 |
| 225 | A | 1 | 1 | 1. | 15 | 0.067 |
| 226 | A | 1 | 1 | 1. | 23 | 0.043 |
| 227 | A | 1 | 1 | 1. | 25 | 0.04 |
| 228 | A | 1 | 1 | 1. | 25 | 0.04 |
| 229 | A | 2 | 2 | 0.93 | 28 | 0.071 |
| 230 | A | 1 | 1 | 1. | 69 | 0.014 |
| 231 | A | 5 | 3 | 1. | 25 | 0.12 |
| 232 | A | 3 | 3 | 1. | 23 | 0.13 |
| 233 | A | 2 | 2 | 1. | 15 | 0.133 |
| 234 | A | 2 | 2 | 1. | 25 | 0.08 |
| 235 | A | 2 | 2 | 1. | 23 | 0.087 |
| 236 | A | 2 | 2 | 1. | 25 | 0.08 |
| 237 | A | 2 | 2 | 1. | 25 | 0.08 |
| 238 | A | 6 | 4 | 1.08 | 31 | 0.129 |
| 239 | A | 4 | 4 | 1.08 | 31 | 0.129 |
| 240 | A | 2 | 2 | 1.07 | 29 | 0.069 |
| 241 | A | 5 | 5 | 1. | 31 | 0.161 |
| 242 | A | 5 | 5 | 1.19 | 31 | 0.161 |
| 243 | A | 5 | 4 | 1.36 | 31 | 0.129 |
| 244 | A | 8 | 7 | 1. | 31 | 0.226 |

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Table 1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 245 | A | 6 | 6 | 1. | 31 | 0.194 |
| 246 | A | 4 | 4 | 1. | 28 | 0.143 |
| 247 | A | 4 | 4 | 1. | 31 | 0.129 |
| 248 | A | 5 | 5 | 1. | 31 | 0.161 |
| 249 | A | 5 | 5 | 1. | 29 | 0.172 |
| 250 | A | 4 | 4 | 1. | 29 | 0.138 |
| 251 | A | 3 | 3 | 1. | 29 | 0.103 |
| 252 | A | 2 | 2 | 1. | 27 | 0.074 |
| 253 | A | 2 | 2 | 1. | 26 | 0.077 |
| 254 | A | 3 | 3 | 1. | 29 | 0.103 |
| 255 | A | 2 | 2 | 1. | 29 | 0.069 |
| 256 | A | 3 | 3 | 1. | 29 | 0.103 |
| 257 | A | 2 | 2 | 1. | 29 | 0.069 |
| 258 | A | 5 | 5 | 1. | 29 | 0.172 |
| 259 | A | 7 | 6 | 1. | 31 | 0.194 |
| 260 | A | 4 | 4 | 1. | 31 | 0.129 |
| 261 | A | 5 | 5 | 1. | 31 | 0.161 |
| 262 | A | 2 | 2 | 1. | 29 | 0.069 |
| 263 | A | 3 | 3 | 1. | 28 | 0.107 |
| 264 | A | 3 | 3 | 1. | 31 | 0.097 |
| 265 | A | 3 | 3 | 1. | 31 | 0.097 |
| 266 | A | 3 | 3 | 1. | 31 | 0.097 |
| 267 | A | 2 | 2 | 1. | 31 | 0.065 |
| 268 | A | 5 | 5 | 1. | 31 | 0.161 |
| 269 | A | 7 | 7 | 1. | 31 | 0.226 |
| 270 | A | 4 | 4 | 1. | 31 | 0.129 |
| 271 | A | 6 | 6 | 1. | 31 | 0.194 |
| 272 | A | 2 | 2 | 1. | 29 | 0.069 |
| 273 | A | 3 | 3 | 1. | 28 | 0.107 |
| 274 | A | 3 | 3 | 1. | 31 | 0.097 |
| 275 | A | 2 | 2 | 1. | 31 | 0.065 |
| 276 | A | 5 | 5 | 1. | 31 | 0.161 |
| 277 | A | 4 | 4 | 1. | 31 | 0.129 |
| 278 | A | 7 | 7 | 1. | 31 | 0.226 |
| 279 | A | 3 | 3 | 1. | 31 | 0.097 |
| 280 | A | 1 | 1 | 1. | 57 | 0.018 |

Continued on next page

Table 1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 281 | A | 3 | 3 | 1. | 32 | 0.094 |
| 282 | A | 3 | 3 | 1. | 41 | 0.073 |
| 283 | A | 4 | 3 | 1. | 31 | 0.097 |
| 284 | A | 4 | 4 | 1. | 35 | 0.114 |
| 285 | A | 3 | 3 | 1. | 31 | 0.097 |
| 286 | A | 2 | 2 | 1.08 | 76 | 0.026 |

# 3 Listing of integrals

## 3.1 $\int (a+bx^3)(c+dx^3)^4\,dx$

**Optimal**. Leaf size=94

$$\frac{1}{4}c^3x^4(4ad+bc)+\frac{2}{7}c^2dx^7(3ad+2bc)+\frac{1}{13}d^3x^{13}(ad+4bc)+\frac{1}{5}cd^2x^{10}(2ad+3bc)+ac^4x+\frac{1}{16}bd^4x^{16}$$

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^
7)/7 + (c*d^2*(3*b*c + 2*a*d)*x^10)/5 + (d^3*(4*b*c + a*d)*x^13)/
13 + (b*d^4*x^16)/16
```

---

**Rubi [A]** time = 0.14945, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{1}{4}c^3x^4(4ad+bc)+\frac{2}{7}c^2dx^7(3ad+2bc)+\frac{1}{13}d^3x^{13}(ad+4bc)+\frac{1}{5}cd^2x^{10}(2ad+3bc)+ac^4x+\frac{1}{16}bd^4x^{16}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)*(c + d*x^3)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^
7)/7 + (c*d^2*(3*b*c + 2*a*d)*x^10)/5 + (d^3*(4*b*c + a*d)*x^13)/
13 + (b*d^4*x^16)/16
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bd^4x^{16}}{16}+c^4\int a\,dx+\frac{c^3x^4\,(4ad+bc)}{4}+\frac{2c^2dx^7\,(3ad+2bc)}{7}+\frac{cd^2x^{10}\,(2ad+3bc)}{5}+\frac{d^3x^{13}\,(ad+4bc)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)*(d*x**3+c)**4,x)
```

```
[Out] b*d**4*x**16/16 + c**4*Integral(a, x) + c**3*x**4*(4*a*d + b*c)/4
 + 2*c**2*d*x**7*(3*a*d + 2*b*c)/7 + c*d**2*x**10*(2*a*d + 3*b*c)
/5 + d**3*x**13*(a*d + 4*b*c)/13
```

---

**Mathematica [A]** time = 0.0345473, size = 94, normalized size = 1.

$$\frac{1}{4}c^3x^4(4ad+bc)+\frac{2}{7}c^2dx^7(3ad+2bc)+\frac{1}{13}d^3x^{13}(ad+4bc)+\frac{1}{5}cd^2x^{10}(2ad+3bc)+ac^4x+\frac{1}{16}bd^4x^{16}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)*(c + d*x^3)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^
7)/7 + (c*d^2*(3*b*c + 2*a*d)*x^10)/5 + (d^3*(4*b*c + a*d)*x^13)/
13 + (b*d^4*x^16)/16
```

---

**Maple [A]** time = 0., size = 97, normalized size = 1.

$$\frac{bd^4x^{16}}{16}+\frac{\left(ad^4+4\,bcd^3\right)x^{13}}{13}+\frac{\left(4\,acd^3+6\,c^2d^2b\right)x^{10}}{10}+\frac{\left(6\,ac^2d^2+4\,c^3db\right)x^7}{7}+\frac{\left(4\,ac^3d+bc^4\right)x^4}{4}+ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)*(d*x^3+c)^4,x)
```

```
[Out] 1/16*b*d^4*x^16+1/13*(a*d^4+4*b*c*d^3)*x^13+1/10*(4*a*c*d^3+6*b*c
^2*d^2)*x^10+1/7*(6*a*c^2*d^2+4*b*c^3*d)*x^7+1/4*(4*a*c^3*d+b*c^4
)*x^4+a*c^4*x
```

---

**Maxima [A]** time = 1.40995, size = 130, normalized size = 1.38

$$\frac{1}{16}\,bd^4x^{16}+\frac{1}{13}\left(4\,bcd^3+ad^4\right)x^{13}+\frac{1}{5}\left(3\,bc^2d^2+2\,acd^3\right)x^{10}+\frac{2}{7}\left(2\,bc^3d+3\,ac^2d^2\right)x^7+ac^4x+\frac{1}{4}\left(bc^4+4\,ac^3d\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(d*x^3 + c)^4,x, algorithm="maxima")
```

```
[Out] 1/16*b*d^4*x^16 + 1/13*(4*b*c*d^3 + a*d^4)*x^13 + 1/5*(3*b*c^2*d^
2 + 2*a*c*d^3)*x^10 + 2/7*(2*b*c^3*d + 3*a*c^2*d^2)*x^7 + a*c^4*x
 + 1/4*(b*c^4 + 4*a*c^3*d)*x^4
```

---

**Fricas [A]** time = 0.179713, size = 1, normalized size = 0.01

$$\frac{1}{16}x^{16}d^4b+\frac{4}{13}x^{13}d^3cb+\frac{1}{13}x^{13}d^4a+\frac{3}{5}x^{10}d^2c^2b+\frac{2}{5}x^{10}d^3ca+\frac{4}{7}x^7dc^3b+\frac{6}{7}x^7d^2c^2a+\frac{1}{4}x^4c^4b+x^4dc^3a+xc^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(d*x^3 + c)^4,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*d^4*b + 4/13*x^13*d^3*c*b + 1/13*x^13*d^4*a + 3/5*x^10*
d^2*c^2*b + 2/5*x^10*d^3*c*a + 4/7*x^7*d*c^3*b + 6/7*x^7*d^2*c^2*
a + 1/4*x^4*c^4*b + x^4*d*c^3*a + x*c^4*a
```

---

**Sympy [A]** time = 0.144622, size = 104, normalized size = 1.11

$$ac^4x + \frac{bd^4x^{16}}{16} + x^{13}\left(\frac{ad^4}{13} + \frac{4bcd^3}{13}\right) + x^{10}\left(\frac{2acd^3}{5} + \frac{3bc^2d^2}{5}\right) + x^7\left(\frac{6ac^2d^2}{7} + \frac{4bc^3d}{7}\right) + x^4\left(ac^3d + \frac{bc^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)*(d*x**3+c)**4,x)
```

```
[Out] a*c**4*x + b*d**4*x**16/16 + x**13*(a*d**4/13 + 4*b*c*d**3/13) +
x**10*(2*a*c*d**3/5 + 3*b*c**2*d**2/5) + x**7*(6*a*c**2*d**2/7 +
4*b*c**3*d/7) + x**4*(a*c**3*d + b*c**4/4)
```

---

**GIAC/XCAS [A]** time = 0.217729, size = 131, normalized size = 1.39

$$\frac{1}{16}\,bd^4x^{16} + \frac{4}{13}\,bcd^3x^{13} + \frac{1}{13}\,ad^4x^{13} + \frac{3}{5}\,bc^2d^2x^{10} + \frac{2}{5}\,acd^3x^{10} + \frac{4}{7}\,bc^3dx^7 + \frac{6}{7}\,ac^2d^2x^7 + \frac{1}{4}\,bc^4x^4 + ac^3dx^4 + ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(d*x^3 + c)^4,x, algorithm="giac")
```

```
[Out] 1/16*b*d^4*x^16 + 4/13*b*c*d^3*x^13 + 1/13*a*d^4*x^13 + 3/5*b*c^2
*d^2*x^10 + 2/5*a*c*d^3*x^10 + 4/7*b*c^3*d*x^7 + 6/7*a*c^2*d^2*x^
7 + 1/4*b*c^4*x^4 + a*c^3*d*x^4 + a*c^4*x
```

## 3.2 $\int (a+bx^3)(c+dx^3)^3\,dx$

**Optimal**. Leaf size=70

$$\frac{1}{4}c^2x^4(3ad+bc)+\frac{1}{10}d^2x^{10}(ad+3bc)+\frac{3}{7}cdx^7(ad+bc)+ac^3x+\frac{1}{13}bd^3x^{13}$$

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 +
 (d^2*(3*b*c + a*d)*x^10)/10 + (b*d^3*x^13)/13
```

---

**Rubi [A]** time = 0.102641, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{1}{4}c^2x^4(3ad+bc)+\frac{1}{10}d^2x^{10}(ad+3bc)+\frac{3}{7}cdx^7(ad+bc)+ac^3x+\frac{1}{13}bd^3x^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)*(c + d*x^3)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 +
 (d^2*(3*b*c + a*d)*x^10)/10 + (b*d^3*x^13)/13
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bd^3x^{13}}{13}+c^3\int a\,dx+\frac{c^2x^4\,(3ad+bc)}{4}+\frac{3cdx^7\,(ad+bc)}{7}+\frac{d^2x^{10}\,(ad+3bc)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)*(d*x**3+c)**3,x)
```

```
[Out] b*d**3*x**13/13 + c**3*Integral(a, x) + c**2*x**4*(3*a*d + b*c)/4
 + 3*c*d*x**7*(a*d + b*c)/7 + d**2*x**10*(a*d + 3*b*c)/10
```

---

**Mathematica [A]** time = 0.0205167, size = 70, normalized size = 1.

$$\frac{1}{4}c^2x^4(3ad+bc)+\frac{1}{10}d^2x^{10}(ad+3bc)+\frac{3}{7}cdx^7(ad+bc)+ac^3x+\frac{1}{13}bd^3x^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)*(c + d*x^3)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 +
 (d^2*(3*b*c + a*d)*x^10)/10 + (b*d^3*x^13)/13
```

---

**Maple [A]** time = 0.001, size = 73, normalized size = 1.

$$\frac{bd^3x^{13}}{13} + \frac{\left(ad^3 + 3\,bcd^2\right)x^{10}}{10} + \frac{\left(3\,acd^2 + 3\,bc^2d\right)x^7}{7} + \frac{\left(3\,ac^2d + bc^3\right)x^4}{4} + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)*(d*x^3+c)^3,x)
```

```
[Out] 1/13*b*d^3*x^13+1/10*(a*d^3+3*b*c*d^2)*x^10+1/7*(3*a*c*d^2+3*b*c^
2*d)*x^7+1/4*(3*a*c^2*d+b*c^3)*x^4+a*c^3*x
```

---

**Maxima [A]** time = 1.37586, size = 95, normalized size = 1.36

$$\frac{1}{13}\,bd^3x^{13} + \frac{1}{10}\left(3\,bcd^2 + ad^3\right)x^{10} + \frac{3}{7}\left(bc^2d + acd^2\right)x^7 + ac^3x + \frac{1}{4}\left(bc^3 + 3\,ac^2d\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(d*x^3 + c)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b*d^3*x^13 + 1/10*(3*b*c*d^2 + a*d^3)*x^10 + 3/7*(b*c^2*d +
a*c*d^2)*x^7 + a*c^3*x + 1/4*(b*c^3 + 3*a*c^2*d)*x^4
```

---

**Fricas [A]** time = 0.180698, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}d^3b + \frac{3}{10}x^{10}d^2cb + \frac{1}{10}x^{10}d^3a + \frac{3}{7}x^7dc^2b + \frac{3}{7}x^7d^2ca + \frac{1}{4}x^4c^3b + \frac{3}{4}x^4dc^2a + xc^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(d*x^3 + c)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^3*b + 3/10*x^10*d^2*c*b + 1/10*x^10*d^3*a + 3/7*x^7*d
*c^2*b + 3/7*x^7*d^2*c*a + 1/4*x^4*c^3*b + 3/4*x^4*d*c^2*a + x*c^
3*a
```

**Sympy [A]** time = 0.123204, size = 80, normalized size = 1.14

$$ac^3x + \frac{bd^3x^{13}}{13} + x^{10}\left(\frac{ad^3}{10} + \frac{3bcd^2}{10}\right) + x^7\left(\frac{3acd^2}{7} + \frac{3bc^2d}{7}\right) + x^4\left(\frac{3ac^2d}{4} + \frac{bc^3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)*(d*x**3+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**13/13 + x**10*(a*d**3/10 + 3*b*c*d**2/10) +
x**7*(3*a*c*d**2/7 + 3*b*c**2*d/7) + x**4*(3*a*c**2*d/4 + b*c**3/
4)
```

**GIAC/XCAS [A]** time = 0.213986, size = 100, normalized size = 1.43

$$\frac{1}{13}\,bd^3x^{13} + \frac{3}{10}\,bcd^2x^{10} + \frac{1}{10}\,ad^3x^{10} + \frac{3}{7}\,bc^2dx^7 + \frac{3}{7}\,acd^2x^7 + \frac{1}{4}\,bc^3x^4 + \frac{3}{4}\,ac^2dx^4 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(d*x^3 + c)^3,x, algorithm="giac")
```

```
[Out] 1/13*b*d^3*x^13 + 3/10*b*c*d^2*x^10 + 1/10*a*d^3*x^10 + 3/7*b*c^2
*d*x^7 + 3/7*a*c*d^2*x^7 + 1/4*b*c^3*x^4 + 3/4*a*c^2*d*x^4 + a*c^
3*x
```

## 3.3 $\int (a+bx^3)(c+dx^3)^2\,dx$

**Optimal**. Leaf size=50

$$\frac{1}{7}dx^7(ad+2bc)+\frac{1}{4}cx^4(2ad+bc)+ac^2x+\frac{1}{10}bd^2x^{10}$$

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*
d^2*x^10)/10
```

---

**Rubi [A]** time = 0.0742223, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{1}{7}dx^7(ad+2bc)+\frac{1}{4}cx^4(2ad+bc)+ac^2x+\frac{1}{10}bd^2x^{10}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)*(c + d*x^3)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*
d^2*x^10)/10
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bd^2x^{10}}{10}+c^2\int a\,dx+\frac{cx^4\,(2ad+bc)}{4}+\frac{dx^7\,(ad+2bc)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)*(d*x**3+c)**2,x)
```

```
[Out] b*d**2*x**10/10 + c**2*Integral(a, x) + c*x**4*(2*a*d + b*c)/4 +
d*x**7*(a*d + 2*b*c)/7
```

---

**Mathematica [A]** time = 0.0141192, size = 50, normalized size = 1.

$$\frac{1}{7}dx^7(ad+2bc)+\frac{1}{4}cx^4(2ad+bc)+ac^2x+\frac{1}{10}bd^2x^{10}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)*(c + d*x^3)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*
d^2*x^10)/10
```

---

**Maple [A]** time = 0.001, size = 49, normalized size = 1.

$$\frac{bd^2x^{10}}{10} + \frac{\left(ad^2 + 2\,bcd\right)x^7}{7} + \frac{\left(2\,acd + bc^2\right)x^4}{4} + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)*(d*x^3+c)^2,x)
```

```
[Out] 1/10*b*d^2*x^10+1/7*(a*d^2+2*b*c*d)*x^7+1/4*(2*a*c*d+b*c^2)*x^4+a
*c^2*x
```

---

**Maxima [A]** time = 1.36066, size = 65, normalized size = 1.3

$$\frac{1}{10}\,bd^2x^{10} + \frac{1}{7}\left(2\,bcd + ad^2\right)x^7 + \frac{1}{4}\left(bc^2 + 2\,acd\right)x^4 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(d*x^3 + c)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b*d^2*x^10 + 1/7*(2*b*c*d + a*d^2)*x^7 + 1/4*(b*c^2 + 2*a*c*
d)*x^4 + a*c^2*x
```

---

**Fricas [A]** time = 0.180838, size = 1, normalized size = 0.02

$$\frac{1}{10}x^{10}d^2b + \frac{2}{7}x^7dcb + \frac{1}{7}x^7d^2a + \frac{1}{4}x^4c^2b + \frac{1}{2}x^4dca + xc^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(d*x^3 + c)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d^2*b + 2/7*x^7*d*c*b + 1/7*x^7*d^2*a + 1/4*x^4*c^2*b +
 1/2*x^4*d*c*a + x*c^2*a
```

---

**Sympy [A]** time = 0.108516, size = 51, normalized size = 1.02

$$ac^2x + \frac{bd^2x^{10}}{10} + x^7\left(\frac{ad^2}{7} + \frac{2bcd}{7}\right) + x^4\left(\frac{acd}{2} + \frac{bc^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)*(d*x**3+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**10/10 + x**7*(a*d**2/7 + 2*b*c*d/7) + x**4*(
a*c*d/2 + b*c**2/4)
```

---

**GIAC/XCAS [A]** time = 0.213265, size = 68, normalized size = 1.36

$$\frac{1}{10}\,bd^2x^{10} + \frac{2}{7}\,bcdx^7 + \frac{1}{7}\,ad^2x^7 + \frac{1}{4}\,bc^2x^4 + \frac{1}{2}\,acdx^4 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(d*x^3 + c)^2,x, algorithm="giac")
```

```
[Out] 1/10*b*d^2*x^10 + 2/7*b*c*d*x^7 + 1/7*a*d^2*x^7 + 1/4*b*c^2*x^4 +
 1/2*a*c*d*x^4 + a*c^2*x
```

## 3.4 $\int \left(a+bx^3\right) \left(c+dx^3\right) \, dx$

**Optimal**. Leaf size=28

$$\frac{1}{4}x^4(ad+bc)+acx+\frac{1}{7}bdx^7$$

```
[Out] a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7
```

---

**Rubi [A]** time = 0.0393509, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{1}{4}x^4(ad+bc)+acx+\frac{1}{7}bdx^7$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)*(c + d*x^3),x]

[Out] a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bdx^7}{7}+c\int a\,dx+x^4\left(\frac{ad}{4}+\frac{bc}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)*(d*x**3+c),x)

[Out] b*d*x**7/7 + c*Integral(a, x) + x**4*(a*d/4 + b*c/4)
```

---

**Mathematica [A]** time = 0.00892945, size = 28, normalized size = 1.

$$\frac{1}{4}x^4(ad+bc)+acx+\frac{1}{7}bdx^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)*(c + d*x^3),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7
```

---

**Maple [A]** time = 0.002, size = 25, normalized size = 0.9

$$acx + \frac{(ad + bc)x^4}{4} + \frac{bdx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)*(d*x^3+c),x)
```

```
[Out] a*c*x+1/4*(a*d+b*c)*x^4+1/7*b*d*x^7
```

---

**Maxima [A]** time = 1.35272, size = 32, normalized size = 1.14

$$\frac{1}{7}\,bdx^7 + \frac{1}{4}\,(bc + ad)x^4 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(d*x^3 + c),x, algorithm="maxima")
```

```
[Out] 1/7*b*d*x^7 + 1/4*(b*c + a*d)*x^4 + a*c*x
```

---

**Fricas [A]** time = 0.18203, size = 1, normalized size = 0.04

$$\frac{1}{7}x^7db + \frac{1}{4}x^4cb + \frac{1}{4}x^4da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(d*x^3 + c),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*d*b + 1/4*x^4*c*b + 1/4*x^4*d*a + x*c*a
```

---

**Sympy [A]** time = 0.074719, size = 26, normalized size = 0.93

$$acx + \frac{bdx^7}{7} + x^4\left(\frac{ad}{4} + \frac{bc}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)*(d*x**3+c),x)
```

```
[Out] a*c*x + b*d*x**7/7 + x**4*(a*d/4 + b*c/4)
```

---

**GIAC/XCAS [A]** time = 0.211328, size = 35, normalized size = 1.25

$$\frac{1}{7}\,bdx^7 + \frac{1}{4}\,bcx^4 + \frac{1}{4}\,adx^4 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(d*x^3 + c),x, algorithm="giac")
```

```
[Out] 1/7*b*d*x^7 + 1/4*b*c*x^4 + 1/4*a*d*x^4 + a*c*x
```

## 3.5 $\int \frac{a+bx^3}{c+dx^3}\,dx$

**Optimal**. Leaf size=144

$$\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}-\frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}}+\frac{bx}{d}$$

```
[Out] (b*x)/d + ((b*c - a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^
(1/3))])/(Sqrt[3]*c^(2/3)*d^(4/3)) - ((b*c - a*d)*Log[c^(1/3) + d
^(1/3)*x])/(3*c^(2/3)*d^(4/3)) + ((b*c - a*d)*Log[c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3))
```

---

**Rubi [A]** time = 0.211318, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412

$$\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}-\frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)/(c + d*x^3),x]
```

```
[Out] (b*x)/d + ((b*c - a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^
(1/3))])/(Sqrt[3]*c^(2/3)*d^(4/3)) - ((b*c - a*d)*Log[c^(1/3) + d
^(1/3)*x])/(3*c^(2/3)*d^(4/3)) + ((b*c - a*d)*Log[c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3))
```

---

**Rubi in Sympy [A]** time = 32.6833, size = 134, normalized size = 0.93

$$\frac{bx}{d}+\frac{(ad-bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{\frac{2}{3}}d^{\frac{4}{3}}}-\frac{(ad-bc)\log\left(c^{\frac{2}{3}}-\sqrt[3]{c}\sqrt[3]{d}x+d^{\frac{2}{3}}x^2\right)}{6c^{\frac{2}{3}}d^{\frac{4}{3}}}-\frac{\sqrt{3}\,(ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3}-\frac{2\sqrt[3]{d}x}{3}\right)}{\sqrt[3]{c}}\right)}{3c^{\frac{2}{3}}d^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)/(d*x**3+c),x)
```

```
[Out] b*x/d + (a*d - b*c)*log(c**(1/3) + d**(1/3)*x)/(3*c**(2/3)*d**(4/
3)) - (a*d - b*c)*log(c**(2/3) - c**(1/3)*d**(1/3)*x + d**(2/3)*x
```

```
**2)/(6*c**(2/3)*d**(4/3)) - sqrt(3)*(a*d - b*c)*atan(sqrt(3)*(c*
*(1/3)/3 - 2*d**(1/3)*x/3)/c**(1/3))/(3*c**(2/3)*d**(4/3))
```

---

**Mathematica [A]** time = 0.140246, size = 128, normalized size = 0.89

$$\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)-2(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)+2\sqrt{3}(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)+6bc^{2/3}\sqrt[3]{d}x}{6c^{2/3}d^{4/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)/(c + d*x^3),x]
```

```
[Out] (6*b*c^(2/3)*d^(1/3)*x + 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*d^(
1/3)*x)/c^(1/3))/Sqrt[3]] - 2*(b*c - a*d)*Log[c^(1/3) + d^(1/3)*x
] + (b*c - a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(2/3)*d^(4/3))
```

---

**Maple [A]** time = 0.007, size = 195, normalized size = 1.4

$$\frac{bx}{d}+\frac{a}{3\,d}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{bc}{3\,d^2}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}$$
$$-\frac{a}{6\,d}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{bc}{6\,d^2}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}$$
$$+\frac{\sqrt{3}a}{3\,d}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{\sqrt{3}bc}{3\,d^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)/(d*x^3+c),x)
```

```
[Out] b*x/d+1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a-1/3/d^2/(c/d)^(2/3)*l
n(x+(c/d)^(1/3))*b*c-1/6/d/(c/d)^(2/3)*ln(x^2-x*(c/d)^(1/3)+(c/d)
^(2/3))*a+1/6/d^2/(c/d)^(2/3)*ln(x^2-x*(c/d)^(1/3)+(c/d)^(2/3))*b
*c+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-
1))*a-1/3/d^2/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/
3)*x-1))*b*c
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)/(d*x^3 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.215331, size = 176, normalized size = 1.22

$$\frac{\sqrt{3}\left(6\sqrt{3}\left(c^2d\right)^{\frac{1}{3}}bx+\sqrt{3}(bc-ad)\log\left(\left(c^2d\right)^{\frac{2}{3}}x^2-\left(c^2d\right)^{\frac{1}{3}}cx+c^2\right)-2\sqrt{3}(bc-ad)\log\left(\left(c^2d\right)^{\frac{1}{3}}x+c\right)-6\,(bc-ad)\arctan\right.}{18\left(c^2d\right)^{\frac{1}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)/(d*x^3 + c),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(6*sqrt(3)*(c^2*d)^(1/3)*b*x + sqrt(3)*(b*c - a*d)*l
og((c^2*d)^(2/3)*x^2 - (c^2*d)^(1/3)*c*x + c^2) - 2*sqrt(3)*(b*c
- a*d)*log((c^2*d)^(1/3)*x + c) - 6*(b*c - a*d)*arctan(1/3*(2*sqr
t(3)*(c^2*d)^(1/3)*x - sqrt(3)*c)/c))/((c^2*d)^(1/3)*d)
```

---

**Sympy [A]** time = 1.93706, size = 71, normalized size = 0.49

$$\frac{bx}{d}+\operatorname{RootSum}\left(27t^3c^2d^4-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3,\left(t\mapsto t\log\left(\frac{3tcd}{ad-bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)/(d*x**3+c),x)
```

```
[Out] b*x/d + RootSum(27*_t**3*c**2*d**4 - a**3*d**3 + 3*a**2*b*c*d**2
- 3*a*b**2*c**2*d + b**3*c**3, Lambda(_t, _t*log(3*_t*c*d/(a*d -
b*c) + x)))
```

---

**GIAC/XCAS [A]** time = 0.21749, size = 217, normalized size = 1.51

$$\frac{bx}{d} + \frac{(bc-ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd} - \frac{\sqrt{3}\left(\left(-cd^2\right)^{\frac{1}{3}}bc-\left(-cd^2\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd^2} - \frac{\left(\left(-cd^2\right)^{\frac{1}{3}}bc-\left(-cd^2\right)^{\frac{1}{3}}ad\right)\ln\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)/(d*x^3 + c),x, algorithm="giac")
```

```
[Out] b*x/d + 1/3*(b*c - a*d)*(-c/d)^(1/3)*ln(abs(x - (-c/d)^(1/3)))/(c
*d) - 1/3*sqrt(3)*((-c*d^2)^(1/3)*b*c - (-c*d^2)^(1/3)*a*d)*arcta
n(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(c*d^2) - 1/6*((
-c*d^2)^(1/3)*b*c - (-c*d^2)^(1/3)*a*d)*ln(x^2 + x*(-c/d)^(1/3) +
 (-c/d)^(2/3))/(c*d^2)
```

## 3.6 $\int \frac{a+bx^3}{(c+dx^3)^2}\,dx$

**Optimal**. Leaf size=169

$$-\frac{(2ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}}+\frac{(2ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{4/3}}-\frac{(2ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{4/3}}-\frac{x(bc-ad)}{3cd\,(c+dx^3)}$$

```
[Out] -((b*c - a*d)*x)/(3*c*d*(c + d*x^3)) - ((b*c + 2*a*d)*ArcTan[(c^(
1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(5/3)*d^(4/3
)) + ((b*c + 2*a*d)*Log[c^(1/3) + d^(1/3)*x])/(9*c^(5/3)*d^(4/3))
 - ((b*c + 2*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])
/(18*c^(5/3)*d^(4/3))
```

---

**Rubi [A]** time = 0.195134, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412

$$-\frac{(2ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}}+\frac{(2ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{4/3}}-\frac{(2ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{4/3}}-\frac{x(bc-ad)}{3cd\,(c+dx^3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)/(c + d*x^3)^2,x]
```

```
[Out] -((b*c - a*d)*x)/(3*c*d*(c + d*x^3)) - ((b*c + 2*a*d)*ArcTan[(c^(
1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(5/3)*d^(4/3
)) + ((b*c + 2*a*d)*Log[c^(1/3) + d^(1/3)*x])/(9*c^(5/3)*d^(4/3))
 - ((b*c + 2*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])
/(18*c^(5/3)*d^(4/3))
```

---

**Rubi in Sympy [A]** time = 34.2323, size = 155, normalized size = 0.92

$$\frac{x(ad-bc)}{3cd(c+dx^3)}+\frac{(2ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{\frac{5}{3}}d^{\frac{4}{3}}}-\frac{(2ad+bc)\log\left(c^{\frac{2}{3}}-\sqrt[3]{c}\sqrt[3]{d}x+d^{\frac{2}{3}}x^2\right)}{18c^{\frac{5}{3}}d^{\frac{4}{3}}}-\frac{\sqrt{3}(2ad+bc)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3}-\frac{2\sqrt[3]{d}x}{3}\right)}{\sqrt[3]{c}}\right)}{9c^{\frac{5}{3}}d^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)/(d*x**3+c)**2,x)
```

```
[Out] x*(a*d - b*c)/(3*c*d*(c + d*x**3)) + (2*a*d + b*c)*log(c**(1/3) +
 d**(1/3)*x)/(9*c**(5/3)*d**(4/3)) - (2*a*d + b*c)*log(c**(2/3) -
 c**(1/3)*d**(1/3)*x + d**(2/3)*x**2)/(18*c**(5/3)*d**(4/3)) - sq
rt(3)*(2*a*d + b*c)*atan(sqrt(3)*(c**(1/3)/3 - 2*d**(1/3)*x/3)/c*
*(1/3))/(9*c**(5/3)*d**(4/3))
```

---

**Mathematica [A]** time = 0.1699, size = 145, normalized size = 0.86

$$\frac{-(2ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)-\frac{6c^{2/3}\sqrt[3]{d}x(bc-ad)}{c+dx^3}+2(2ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)-2\sqrt{3}(2ad+bc)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{18c^{5/3}d^{4/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)/(c + d*x^3)^2,x]
```

```
[Out] ((-6*c^(2/3)*d^(1/3)*(b*c - a*d)*x)/(c + d*x^3) - 2*Sqrt[3]*(b*c
+ 2*a*d)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*(b*c + 2
*a*d)*Log[c^(1/3) + d^(1/3)*x] - (b*c + 2*a*d)*Log[c^(2/3) - c^(1
/3)*d^(1/3)*x + d^(2/3)*x^2])/(18*c^(5/3)*d^(4/3))
```

---

**Maple [A]** time = 0.011, size = 221, normalized size = 1.3

$$\frac{(ad-bc)\,x}{3\,cd\,(dx^3+c)}+\frac{2\,a}{9\,cd}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{b}{9\,d^2}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}$$
$$-\frac{a}{9\,cd}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{b}{18\,d^2}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}$$
$$+\frac{2\,\sqrt{3}a}{9\,cd}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}b}{9\,d^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)/(d*x^3+c)^2,x)
```

```
[Out] 1/3*(a*d-b*c)/c/d*x/(d*x^3+c)+2/9/c/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3
))*a+1/9/d^2/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*b-1/9/c/d/(c/d)^(2/3)*
ln(x^2-x*(c/d)^(1/3)+(c/d)^(2/3))*a-1/18/d^2/(c/d)^(2/3)*ln(x^2-x
*(c/d)^(1/3)+(c/d)^(2/3))*b+2/9/c/d/(c/d)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a+1/9/d^2/(c/d)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)/(d*x^3 + c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.214528, size = 275, normalized size = 1.63

$$-\frac{\sqrt{3}\Big(6\,\sqrt{3}\,\left(c^2d\right)^{\frac{1}{3}}(bc-ad)x+\sqrt{3}\left(\left(bcd+2\,ad^2\right)x^3+bc^2+2\,acd\right)\log\left(\left(c^2d\right)^{\frac{2}{3}}x^2-\left(c^2d\right)^{\frac{1}{3}}cx+c^2\right)-2\,\sqrt{3}\left(\left(bcd+2\,ad^2\right)x\right.}{54\,(cd^2x^3+c^2d)\left(c^2d\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)/(d*x^3 + c)^2,x, algorithm="fricas")
```

```
[Out] -1/54*sqrt(3)*(6*sqrt(3)*(c^2*d)^(1/3)*(b*c - a*d)*x + sqrt(3)*((
b*c*d + 2*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*log((c^2*d)^(2/3)*x^2 - (
c^2*d)^(1/3)*c*x + c^2) - 2*sqrt(3)*((b*c*d + 2*a*d^2)*x^3 + b*c^
2 + 2*a*c*d)*log((c^2*d)^(1/3)*x + c) - 6*((b*c*d + 2*a*d^2)*x^3
+ b*c^2 + 2*a*c*d)*arctan(1/3*(2*sqrt(3)*(c^2*d)^(1/3)*x - sqrt(3
)*c)/c))/((c*d^2*x^3 + c^2*d)*(c^2*d)^(1/3))
```

---

**Sympy [A]** time = 2.61964, size = 97, normalized size = 0.57

$$\frac{x(ad-bc)}{3c^2d+3cd^2x^3}+\operatorname{RootSum}\left(729t^3c^5d^4-8a^3d^3-12a^2bcd^2-6ab^2c^2d-b^3c^3,\left(t\mapsto t\log\left(\frac{9tc^2d}{2ad+bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)/(d*x**3+c)**2,x)
```

```
[Out] x*(a*d - b*c)/(3*c**2*d + 3*c*d**2*x**3) + RootSum(729*_t**3*c**5
*d**4 - 8*a**3*d**3 - 12*a**2*b*c*d**2 - 6*a*b**2*c**2*d - b**3*c
**3, Lambda(_t, _t*log(9*_t*c**2*d/(2*a*d + b*c) + x)))
```

---

**GIAC/XCAS [A]** time = 0.217759, size = 246, normalized size = 1.46

$$-\frac{(bc+2\,ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\,c^2d}+\frac{\sqrt{3}\left(\left(-cd^2\right)^{\frac{1}{3}}bc+2\,\left(-cd^2\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,c^2d^2}-\frac{bcx-adx}{3\,(dx^3+c)cd}+\frac{\left(\left(-cd^2\right)^{\frac{1}{3}}bc+2\,\left(-cd^2\right)^{\frac{1}{3}}ad\right)\ln\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)/(d*x^3 + c)^2,x, algorithm="giac")
```

```
[Out] -1/9*(b*c + 2*a*d)*(-c/d)^(1/3)*ln(abs(x - (-c/d)^(1/3)))/(c^2*d)
 + 1/9*sqrt(3)*((-c*d^2)^(1/3)*b*c + 2*(-c*d^2)^(1/3)*a*d)*arctan
(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(c^2*d^2) - 1/3*(
b*c*x - a*d*x)/((d*x^3 + c)*c*d) + 1/18*((-c*d^2)^(1/3)*b*c + 2*(
-c*d^2)^(1/3)*a*d)*ln(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(c^2*d
^2)
```

## 3.7 $\int \frac{a+bx^3}{(c+dx^3)^3} \, dx$

**Optimal**. Leaf size=197

$$-\frac{(5ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{4/3}}+\frac{(5ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{4/3}}-\frac{(5ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{4/3}}+\frac{x(5ad+bc)}{18c^2d\,(c+dx^3)}-\frac{x(bc-ad)}{6cd\,(c+dx^3)^2}$$

```
[Out] -((b*c - a*d)*x)/(6*c*d*(c + d*x^3)^2) + ((b*c + 5*a*d)*x)/(18*c^
2*d*(c + d*x^3)) - ((b*c + 5*a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/
(Sqrt[3]*c^(1/3))])/(9*Sqrt[3]*c^(8/3)*d^(4/3)) + ((b*c + 5*a*d)*
Log[c^(1/3) + d^(1/3)*x])/(27*c^(8/3)*d^(4/3)) - ((b*c + 5*a*d)*L
og[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(4/3
))
```

---

**Rubi [A]** time = 0.22915, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$

$$-\frac{(5ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{4/3}}+\frac{(5ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{4/3}}-\frac{(5ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{4/3}}+\frac{x(5ad+bc)}{18c^2d\,(c+dx^3)}-\frac{x(bc-ad)}{6cd\,(c+dx^3)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)/(c + d*x^3)^3,x]
```

```
[Out] -((b*c - a*d)*x)/(6*c*d*(c + d*x^3)^2) + ((b*c + 5*a*d)*x)/(18*c^
2*d*(c + d*x^3)) - ((b*c + 5*a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/
(Sqrt[3]*c^(1/3))])/(9*Sqrt[3]*c^(8/3)*d^(4/3)) + ((b*c + 5*a*d)*
Log[c^(1/3) + d^(1/3)*x])/(27*c^(8/3)*d^(4/3)) - ((b*c + 5*a*d)*L
og[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(4/3
))
```

---

**Rubi in Sympy [A]** time = 39.1236, size = 180, normalized size = 0.91

$$\frac{x(ad-bc)}{6cd(c+dx^3)^2}+\frac{x(5ad+bc)}{18c^2d(c+dx^3)}+\frac{(5ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{\frac{8}{3}}d^{\frac{4}{3}}}-\frac{(5ad+bc)\log\left(c^{\frac{2}{3}}-\sqrt[3]{c}\sqrt[3]{d}x+d^{\frac{2}{3}}x^2\right)}{54c^{\frac{8}{3}}d^{\frac{4}{3}}}-\frac{\sqrt{3}(5ad+bc)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3}-\frac{2\sqrt[3]{d}x}{3}\right)}{\sqrt[3]{c}}\right)}{27c^{\frac{8}{3}}d^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)/(d*x**3+c)**3,x)
```

```
[Out] x*(a*d - b*c)/(6*c*d*(c + d*x**3)**2) + x*(5*a*d + b*c)/(18*c**2*
d*(c + d*x**3)) + (5*a*d + b*c)*log(c**(1/3) + d**(1/3)*x)/(27*c*
*(8/3)*d**(4/3)) - (5*a*d + b*c)*log(c**(2/3) - c**(1/3)*d**(1/3)
*x + d**(2/3)*x**2)/(54*c**(8/3)*d**(4/3)) - sqrt(3)*(5*a*d + b*c
)*atan(sqrt(3)*(c**(1/3)/3 - 2*d**(1/3)*x/3)/c**(1/3))/(27*c**(8/
3)*d**(4/3))
```

---

**Mathematica [A]** time = 0.225062, size = 175, normalized size = 0.89

$$\frac{-(5ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)-\frac{9c^{5/3}\sqrt[3]{d}x(bc-ad)}{(c+dx^3)^2}+\frac{3c^{2/3}\sqrt[3]{d}x(5ad+bc)}{c+dx^3}+2(5ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)-2\sqrt{3}(5ad+}{54c^{8/3}d^{4/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)/(c + d*x^3)^3,x]
```

```
[Out] ((-9*c^(5/3)*d^(1/3)*(b*c - a*d)*x)/(c + d*x^3)^2 + (3*c^(2/3)*d^
(1/3)*(b*c + 5*a*d)*x)/(c + d*x^3) - 2*Sqrt[3]*(b*c + 5*a*d)*ArcT
an[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*(b*c + 5*a*d)*Log[c^(
1/3) + d^(1/3)*x] - (b*c + 5*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x
 + d^(2/3)*x^2])/(54*c^(8/3)*d^(4/3))
```

---

**Maple [A]** time = 0.014, size = 249, normalized size = 1.3

$$\frac{1}{\left(dx^{3}+c\right)^{2}}\left(\frac{\left(5\,ad+bc\right)x^{4}}{18\,c^{2}}+\frac{\left(4\,ad-bc\right)x}{9\,cd}\right)+\frac{5\,a}{27\,c^{2}d}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{b}{27\,cd^{2}}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{5\,a}{54\,c^{2}d}\ln\left(x^{2}-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{b}{54\,cd^{2}}\ln\left(x^{2}-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{5\,\sqrt{3}a}{27\,c^{2}d}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}b}{27\,cd^{2}}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)/(d*x^3+c)^3,x)
```

```
[Out] (1/18*(5*a*d+b*c)/c^2*x^4+1/9*(4*a*d-b*c)/c/d*x)/(d*x^3+c)^2+5/27
/c^2/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a+1/27/c/d^2/(c/d)^(2/3)*ln(
x+(c/d)^(1/3))*b-5/54/c^2/d/(c/d)^(2/3)*ln(x^2-x*(c/d)^(1/3)+(c/d
)^(2/3))*a-1/54/c/d^2/(c/d)^(2/3)*ln(x^2-x*(c/d)^(1/3)+(c/d)^(2/3
))*b+5/27/c^2/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(
1/3)*x-1))*a+1/27/c/d^2/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(c/d)^(1/3)*x-1))*b
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)/(d*x^3 + c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.214195, size = 414, normalized size = 2.1

$$-\frac{\sqrt{3}\left(\sqrt{3}\left(\left(bcd^{2}+5\,ad^{3}\right)x^{6}+bc^{3}+5\,ac^{2}d+2\left(bc^{2}d+5\,acd^{2}\right)x^{3}\right)\log\left(\left(c^{2}d\right)^{\frac{2}{3}}x^{2}-\left(c^{2}d\right)^{\frac{1}{3}}cx+c^{2}\right)-2\,\sqrt{3}\left(\left(bcd^{2}+5\,ad^{3}\right)x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)/(d*x^3 + c)^3,x, algorithm="fricas")
```

```
[Out] -1/162*sqrt(3)*(sqrt(3)*((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^
2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3)*log((c^2*d)^(2/3)*x^2 - (c^2*d
)^(1/3)*c*x + c^2) - 2*sqrt(3)*((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 +
 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3)*log((c^2*d)^(1/3)*x + c
) - 6*((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d +
 5*a*c*d^2)*x^3)*arctan(1/3*(2*sqrt(3)*(c^2*d)^(1/3)*x - sqrt(3)*
c)/c) - 3*sqrt(3)*((b*c*d + 5*a*d^2)*x^4 - 2*(b*c^2 - 4*a*c*d)*x)
*(c^2*d)^(1/3))/((c^2*d^3*x^6 + 2*c^3*d^2*x^3 + c^4*d)*(c^2*d)^(1
/3))
```

---

**Sympy [A]** time = 3.77063, size = 133, normalized size = 0.68

$$\frac{x^4\left(5ad^2+bcd\right)+x\left(8acd-2bc^2\right)}{18c^4d+36c^3d^2x^3+18c^2d^3x^6}+\operatorname{RootSum}\left(19683t^3c^8d^4-125a^3d^3-75a^2bcd^2-15ab^2c^2d-b^3c^3,\left(t\mapsto t\log\left(\frac{27tc^3d}{5ad+bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)/(d*x**3+c)**3,x)
```

```
[Out] (x**4*(5*a*d**2 + b*c*d) + x*(8*a*c*d - 2*b*c**2))/(18*c**4*d + 3
6*c**3*d**2*x**3 + 18*c**2*d**3*x**6) + RootSum(19683*_t**3*c**8*
d**4 - 125*a**3*d**3 - 75*a**2*b*c*d**2 - 15*a*b**2*c**2*d - b**3
*c**3, Lambda(_t, _t*log(27*_t*c**3*d/(5*a*d + b*c) + x)))
```

---

**GIAC/XCAS [A]** time = 0.220801, size = 273, normalized size = 1.39

$$-\frac{(bc+5\,ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{27\,c^3d}+\frac{\sqrt{3}\left(\left(-cd^2\right)^{\frac{1}{3}}bc+5\,\left(-cd^2\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,c^3d^2}+\frac{\left(\left(-cd^2\right)^{\frac{1}{3}}bc+5\,\left(-cd^2\right)^{\frac{1}{3}}ad\right)\ln\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,c^3d^2}+\frac{bcdx^4+5\,ad^2x^4-2\,bc^2x+8\,acdx}{18\,(dx^3+c)^2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)/(d*x^3 + c)^3,x, algorithm="giac")
```

```
[Out] -1/27*(b*c + 5*a*d)*(-c/d)^(1/3)*ln(abs(x - (-c/d)^(1/3)))/(c^3*d
) + 1/27*sqrt(3)*((-c*d^2)^(1/3)*b*c + 5*(-c*d^2)^(1/3)*a*d)*arct
an(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(c^3*d^2) + 1/5
4*((-c*d^2)^(1/3)*b*c + 5*(-c*d^2)^(1/3)*a*d)*ln(x^2 + x*(-c/d)^(
1/3) + (-c/d)^(2/3))/(c^3*d^2) + 1/18*(b*c*d*x^4 + 5*a*d^2*x^4 -
2*b*c^2*x + 8*a*c*d*x)/((d*x^3 + c)^2*c^2*d)
```

## 3.8 $\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^3 \, dx$

**Optimal**. Leaf size=122

$$\frac{1}{10}dx^{10}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{4}ac^2x^4(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{16}b^2d^3x^{16}$$

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^4)/4 + (c*(b^2*c^2 + 6*a*b*c
*d + 3*a^2*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^1
0)/10 + (b*d^2*(3*b*c + 2*a*d)*x^13)/13 + (b^2*d^3*x^16)/16
```

---

**Rubi [A]** time = 0.175244, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{1}{10}dx^{10}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{4}ac^2x^4(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{16}b^2d^3x^{16}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^2*(c + d*x^3)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^4)/4 + (c*(b^2*c^2 + 6*a*b*c
*d + 3*a^2*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^1
0)/10 + (b*d^2*(3*b*c + 2*a*d)*x^13)/13 + (b^2*d^3*x^16)/16
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{ac^2x^4\left(3ad+2bc\right)}{4}+\frac{b^2d^3x^{16}}{16}+\frac{bd^2x^{13}\left(2ad+3bc\right)}{13}+c^3\int a^2\,dx+\frac{cx^7\left(3a^2d^2+6abcd+b^2c^2\right)}{7}+\frac{dx^{10}\left(a^2d^2+6abcd+3b^2c^2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**2*(d*x**3+c)**3,x)
```

```
[Out] a*c**2*x**4*(3*a*d + 2*b*c)/4 + b**2*d**3*x**16/16 + b*d**2*x**13
*(2*a*d + 3*b*c)/13 + c**3*Integral(a**2, x) + c*x**7*(3*a**2*d**
2 + 6*a*b*c*d + b**2*c**2)/7 + d*x**10*(a**2*d**2 + 6*a*b*c*d + 3
*b**2*c**2)/10
```

**Mathematica [A]** time = 0.0342443, size = 122, normalized size = 1.

$$\frac{1}{10}dx^{10}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{4}ac^2x^4(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{16}b^2d^3x^{16}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)^2*(c + d*x^3)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^4)/4 + (c*(b^2*c^2 + 6*a*b*c
*d + 3*a^2*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^1
0)/10 + (b*d^2*(3*b*c + 2*a*d)*x^13)/13 + (b^2*d^3*x^16)/16
```

**Maple [A]** time = 0., size = 125, normalized size = 1.

$$\frac{b^2d^3x^{16}}{16}+\frac{\left(2\,abd^3+3\,b^2cd^2\right)x^{13}}{13}+\frac{\left(a^2d^3+6\,abcd^2+3\,b^2c^2d\right)x^{10}}{10}+\frac{\left(3\,a^2cd^2+6\,abc^2d+b^2c^3\right)x^7}{7}+\frac{\left(3\,a^2c^2d+2\,abc^3\right)x^4}{4}+a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^2*(d*x^3+c)^3,x)
```

```
[Out] 1/16*b^2*d^3*x^16+1/13*(2*a*b*d^3+3*b^2*c*d^2)*x^13+1/10*(a^2*d^3
+6*a*b*c*d^2+3*b^2*c^2*d)*x^10+1/7*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c
^3)*x^7+1/4*(3*a^2*c^2*d+2*a*b*c^3)*x^4+a^2*c^3*x
```

**Maxima [A]** time = 1.36992, size = 167, normalized size = 1.37

$$\frac{1}{16}\,b^2d^3x^{16}+\frac{1}{13}\left(3\,b^2cd^2+2\,abd^3\right)x^{13}+\frac{1}{10}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{10}+\frac{1}{7}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^7+a^2c^3x+\frac{1}{4}\left(2\,abc^3+3\,a^2c^2d\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(d*x^3 + c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^2*d^3*x^16 + 1/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^13 + 1/10*(3
*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1/7*(b^2*c^3 + 6*a*b*c
```

```
^2*d + 3*a^2*c*d^2)*x^7 + a^2*c^3*x + 1/4*(2*a*b*c^3 + 3*a^2*c^2*
d)*x^4
```

---

**Fricas [A]** time = 0.182125, size = 1, normalized size = 0.01

$$\frac{1}{16}x^{16}d^3b^2 + \frac{3}{13}x^{13}d^2cb^2 + \frac{2}{13}x^{13}d^3ba + \frac{3}{10}x^{10}dc^2b^2 + \frac{3}{5}x^{10}d^2cba + \frac{1}{10}x^{10}d^3a^2 + \frac{1}{7}x^7c^3b^2 + \frac{6}{7}x^7dc^2ba + \frac{3}{7}x^7d^2ca^2 + \frac{1}{2}x^4c^3ba + \frac{3}{4}x^4dc^2a^2 + xc^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(d*x^3 + c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*d^3*b^2 + 3/13*x^13*d^2*c*b^2 + 2/13*x^13*d^3*b*a + 3/1
0*x^10*d*c^2*b^2 + 3/5*x^10*d^2*c*b*a + 1/10*x^10*d^3*a^2 + 1/7*x
^7*c^3*b^2 + 6/7*x^7*d*c^2*b*a + 3/7*x^7*d^2*c*a^2 + 1/2*x^4*c^3*
b*a + 3/4*x^4*d*c^2*a^2 + x*c^3*a^2
```

---

**Sympy [A]** time = 0.165601, size = 139, normalized size = 1.14

$$a^2c^3x + \frac{b^2d^3x^{16}}{16} + x^{13}\left(\frac{2abd^3}{13} + \frac{3b^2cd^2}{13}\right) + x^{10}\left(\frac{a^2d^3}{10} + \frac{3abcd^2}{5} + \frac{3b^2c^2d}{10}\right) + x^7\left(\frac{3a^2cd^2}{7} + \frac{6abc^2d}{7} + \frac{b^2c^3}{7}\right) + x^4\left(\frac{3a^2c^2d}{4} + \frac{abc^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**2*(d*x**3+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**16/16 + x**13*(2*a*b*d**3/13 + 3*b**2*
c*d**2/13) + x**10*(a**2*d**3/10 + 3*a*b*c*d**2/5 + 3*b**2*c**2*d
/10) + x**7*(3*a**2*c*d**2/7 + 6*a*b*c**2*d/7 + b**2*c**3/7) + x*
*4*(3*a**2*c**2*d/4 + a*b*c**3/2)
```

---

**GIAC/XCAS [A]** time = 0.211415, size = 178, normalized size = 1.46

$$\frac{1}{16}\,b^2d^3x^{16} + \frac{3}{13}\,b^2cd^2x^{13} + \frac{2}{13}\,abd^3x^{13} + \frac{3}{10}\,b^2c^2dx^{10} + \frac{3}{5}\,abcd^2x^{10} + \frac{1}{10}\,a^2d^3x^{10} + \frac{1}{7}\,b^2c^3x^7 + \frac{6}{7}\,abc^2dx^7 + \frac{3}{7}\,a^2cd^2x^7 + \frac{1}{2}\,abc^3x^4 + \frac{3}{4}\,a^2c^2dx^4 + a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(d*x^3 + c)^3,x, algorithm="giac")
```

```
[Out] 1/16*b^2*d^3*x^16 + 3/13*b^2*c*d^2*x^13 + 2/13*a*b*d^3*x^13 + 3/1
0*b^2*c^2*d*x^10 + 3/5*a*b*c*d^2*x^10 + 1/10*a^2*d^3*x^10 + 1/7*b
^2*c^3*x^7 + 6/7*a*b*c^2*d*x^7 + 3/7*a^2*c*d^2*x^7 + 1/2*a*b*c^3*
x^4 + 3/4*a^2*c^2*d*x^4 + a^2*c^3*x
```

## 3.9 $\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^2 \, dx$

**Optimal**. Leaf size=82

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{2}acx^4(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

```
[Out] a^2*c^2*x + (a*c*(b*c + a*d)*x^4)/2 + ((b^2*c^2 + 4*a*b*c*d + a^2
*d^2)*x^7)/7 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^13)/13
```

---

**Rubi [A]** time = 0.116628, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{2}acx^4(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^2*(c + d*x^3)^2,x]
```

```
[Out] a^2*c^2*x + (a*c*(b*c + a*d)*x^4)/2 + ((b^2*c^2 + 4*a*b*c*d + a^2
*d^2)*x^7)/7 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^13)/13
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{acx^4\left(ad+bc\right)}{2}+\frac{b^2d^2x^{13}}{13}+\frac{bdx^{10}\left(ad+bc\right)}{5}+c^2\int a^2\,dx+x^7\left(\frac{a^2d^2}{7}+\frac{4abcd}{7}+\frac{b^2c^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**2*(d*x**3+c)**2,x)
```

```
[Out] a*c*x**4*(a*d + b*c)/2 + b**2*d**2*x**13/13 + b*d*x**10*(a*d + b*
c)/5 + c**2*Integral(a**2, x) + x**7*(a**2*d**2/7 + 4*a*b*c*d/7 +
 b**2*c**2/7)
```

---

**Mathematica [A]** time = 0.0233658, size = 82, normalized size = 1.

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{2}acx^4(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)^2*(c + d*x^3)^2,x]
```

```
[Out] a^2*c^2*x + (a*c*(b*c + a*d)*x^4)/2 + ((b^2*c^2 + 4*a*b*c*d + a^2
*d^2)*x^7)/7 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^13)/13
```

---

**Maple [A]** time = 0.001, size = 87, normalized size = 1.1

$$\frac{b^2d^2x^{13}}{13} + \frac{\left(2\,abd^2 + 2\,b^2cd\right)x^{10}}{10} + \frac{\left(a^2d^2 + 4\,cabd + b^2c^2\right)x^7}{7} + \frac{\left(2\,a^2cd + 2\,abc^2\right)x^4}{4} + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^2*(d*x^3+c)^2,x)
```

```
[Out] 1/13*b^2*d^2*x^13+1/10*(2*a*b*d^2+2*b^2*c*d)*x^10+1/7*(a^2*d^2+4*
a*b*c*d+b^2*c^2)*x^7+1/4*(2*a^2*c*d+2*a*b*c^2)*x^4+a^2*c^2*x
```

---

**Maxima [A]** time = 1.40654, size = 111, normalized size = 1.35

$$\frac{1}{13}\,b^2d^2x^{13} + \frac{1}{5}\left(b^2cd + abd^2\right)x^{10} + \frac{1}{7}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^7 + a^2c^2x + \frac{1}{2}\left(abc^2 + a^2cd\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(d*x^3 + c)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d^2*x^13 + 1/5*(b^2*c*d + a*b*d^2)*x^10 + 1/7*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^7 + a^2*c^2*x + 1/2*(a*b*c^2 + a^2*c*d)*x
^4
```

---

**Fricas [A]** time = 0.180726, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}d^2b^2 + \frac{1}{5}x^{10}dcb^2 + \frac{1}{5}x^{10}d^2ba + \frac{1}{7}x^7c^2b^2 + \frac{4}{7}x^7dcba + \frac{1}{7}x^7d^2a^2 + \frac{1}{2}x^4c^2ba + \frac{1}{2}x^4dca^2 + xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(d*x^3 + c)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^2*b^2 + 1/5*x^10*d*c*b^2 + 1/5*x^10*d^2*b*a + 1/7*x^7
*c^2*b^2 + 4/7*x^7*d*c*b*a + 1/7*x^7*d^2*a^2 + 1/2*x^4*c^2*b*a +
1/2*x^4*d*c*a^2 + x*c^2*a^2
```

**Sympy [A]** time = 0.138084, size = 90, normalized size = 1.1

$$a^{2}c^{2}x+\frac{b^{2}d^{2}x^{13}}{13}+x^{10}\left(\frac{abd^{2}}{5}+\frac{b^{2}cd}{5}\right)+x^{7}\left(\frac{a^{2}d^{2}}{7}+\frac{4abcd}{7}+\frac{b^{2}c^{2}}{7}\right)+x^{4}\left(\frac{a^{2}cd}{2}+\frac{abc^{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**2*(d*x**3+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**13/13 + x**10*(a*b*d**2/5 + b**2*c*d/5
) + x**7*(a**2*d**2/7 + 4*a*b*c*d/7 + b**2*c**2/7) + x**4*(a**2*c
*d/2 + a*b*c**2/2)
```

**GIAC/XCAS [A]** time = 0.211426, size = 123, normalized size = 1.5

$$\frac{1}{13}\,b^{2}d^{2}x^{13}+\frac{1}{5}\,b^{2}cdx^{10}+\frac{1}{5}\,abd^{2}x^{10}+\frac{1}{7}\,b^{2}c^{2}x^{7}+\frac{4}{7}\,abcdx^{7}+\frac{1}{7}\,a^{2}d^{2}x^{7}+\frac{1}{2}\,abc^{2}x^{4}+\frac{1}{2}\,a^{2}cdx^{4}+a^{2}c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(d*x^3 + c)^2,x, algorithm="giac")
```

```
[Out] 1/13*b^2*d^2*x^13 + 1/5*b^2*c*d*x^10 + 1/5*a*b*d^2*x^10 + 1/7*b^2
*c^2*x^7 + 4/7*a*b*c*d*x^7 + 1/7*a^2*d^2*x^7 + 1/2*a*b*c^2*x^4 +
1/2*a^2*c*d*x^4 + a^2*c^2*x
```

## 3.10 $\int \left(a+bx^3\right)^2 \left(c+dx^3\right)\, dx$

**Optimal**. Leaf size=50

$$a^2cx+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{4}ax^4(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^
2*d*x^10)/10
```

---

**Rubi [A]** time = 0.0754555, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$a^2cx+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{4}ax^4(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^2*(c + d*x^3),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^
2*d*x^10)/10
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$a^2\int c\,dx+\frac{ax^4\,(ad+2bc)}{4}+\frac{b^2dx^{10}}{10}+\frac{bx^7\,(2ad+bc)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**2*(d*x**3+c),x)
```

```
[Out] a**2*Integral(c, x) + a*x**4*(a*d + 2*b*c)/4 + b**2*d*x**10/10 +
b*x**7*(2*a*d + b*c)/7
```

---

**Mathematica [A]** time = 0.0136847, size = 50, normalized size = 1.

$$a^2cx+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{4}ax^4(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)^2*(c + d*x^3),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^
2*d*x^10)/10
```

---

**Maple [A]** time = 0., size = 49, normalized size = 1.

$$\frac{b^2dx^{10}}{10} + \frac{\left(2\,abd + b^2c\right)\,x^7}{7} + \frac{\left(a^2d + 2\,abc\right)\,x^4}{4} + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^2*(d*x^3+c),x)
```

```
[Out] 1/10*b^2*d*x^10+1/7*(2*a*b*d+b^2*c)*x^7+1/4*(a^2*d+2*a*b*c)*x^4+a
^2*c*x
```

---

**Maxima [A]** time = 1.38326, size = 65, normalized size = 1.3

$$\frac{1}{10}\,b^2dx^{10} + \frac{1}{7}\left(b^2c + 2\,abd\right)x^7 + \frac{1}{4}\left(2\,abc + a^2d\right)x^4 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(d*x^3 + c),x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d*x^10 + 1/7*(b^2*c + 2*a*b*d)*x^7 + 1/4*(2*a*b*c + a^2*
d)*x^4 + a^2*c*x
```

---

**Fricas [A]** time = 0.179662, size = 1, normalized size = 0.02

$$\frac{1}{10}x^{10}db^2 + \frac{1}{7}x^7cb^2 + \frac{2}{7}x^7dba + \frac{1}{2}x^4cba + \frac{1}{4}x^4da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(d*x^3 + c),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d*b^2 + 1/7*x^7*c*b^2 + 2/7*x^7*d*b*a + 1/2*x^4*c*b*a +
 1/4*x^4*d*a^2 + x*c*a^2
```

---

**Sympy [A]** time = 0.102264, size = 51, normalized size = 1.02

$$a^2cx + \frac{b^2dx^{10}}{10} + x^7\left(\frac{2abd}{7} + \frac{b^2c}{7}\right) + x^4\left(\frac{a^2d}{4} + \frac{abc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**2*(d*x**3+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**10/10 + x**7*(2*a*b*d/7 + b**2*c/7) + x**4*(
a**2*d/4 + a*b*c/2)
```

---

**GIAC/XCAS [A]** time = 0.209987, size = 68, normalized size = 1.36

$$\frac{1}{10}\,b^2dx^{10} + \frac{1}{7}\,b^2cx^7 + \frac{2}{7}\,abdx^7 + \frac{1}{2}\,abcx^4 + \frac{1}{4}\,a^2dx^4 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(d*x^3 + c),x, algorithm="giac")
```

```
[Out] 1/10*b^2*d*x^10 + 1/7*b^2*c*x^7 + 2/7*a*b*d*x^7 + 1/2*a*b*c*x^4 +
 1/4*a^2*d*x^4 + a^2*c*x
```

## 3.11 $\int \frac{(a+bx^3)^2}{c+dx^3} \, dx$

**Optimal**. Leaf size=173

$$-\frac{(bc-ad)^2\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{7/3}}-\frac{bx(bc-2ad)}{d^2}+\frac{b^2x^4}{4d}$$

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^4)/(4*d) - ((b*c - a*d)^2*Arc
Tan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*
d^(7/3)) + ((b*c - a*d)^2*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^
(7/3)) - ((b*c - a*d)^2*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2])/(6*c^(2/3)*d^(7/3))
```

---

**Rubi [A]** time = 0.27559, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368

$$-\frac{(bc-ad)^2\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{7/3}}-\frac{bx(bc-2ad)}{d^2}+\frac{b^2x^4}{4d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^2/(c + d*x^3),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^4)/(4*d) - ((b*c - a*d)^2*Arc
Tan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*
d^(7/3)) + ((b*c - a*d)^2*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^
(7/3)) - ((b*c - a*d)^2*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2])/(6*c^(2/3)*d^(7/3))
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{b^2x^4}{4d}+\frac{(2ad-bc)\int b\,dx}{d^2}+\frac{(ad-bc)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{\frac{2}{3}}d^{\frac{7}{3}}}-\frac{(ad-bc)^2\log\left(c^{\frac{2}{3}}-\sqrt[3]{c}\sqrt[3]{d}x+d^{\frac{2}{3}}x^2\right)}{6c^{\frac{2}{3}}d^{\frac{7}{3}}}-\frac{\sqrt{3}\,(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3}-\frac{2\sqrt[3]{d}x}{3}\right)}{\sqrt[3]{c}}\right)}{3c^{\frac{2}{3}}d^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**2/(d*x**3+c),x)
```

```
[Out] b**2*x**4/(4*d) + (2*a*d - b*c)*Integral(b, x)/d**2 + (a*d - b*c)
**2*log(c**(1/3) + d**(1/3)*x)/(3*c**(2/3)*d**(7/3)) - (a*d - b*c
)**2*log(c**(2/3) - c**(1/3)*d**(1/3)*x + d**(2/3)*x**2)/(6*c**(2
/3)*d**(7/3)) - sqrt(3)*(a*d - b*c)**2*atan(sqrt(3)*(c**(1/3)/3 -
 2*d**(1/3)*x/3)/c**(1/3))/(3*c**(2/3)*d**(7/3))
```

---

**Mathematica [A]** time = 0.170788, size = 167, normalized size = 0.97

$$\frac{-2(bc-ad)^2\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)-12bc^{2/3}\sqrt[3]{d}x(bc-2ad)+4(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)+4\sqrt{3}(bc-ad)^2\tan^{-1}\left(\right.}{12c^{2/3}d^{7/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)^2/(c + d*x^3),x]
```

```
[Out] (-12*b*c^(2/3)*d^(1/3)*(b*c - 2*a*d)*x + 3*b^2*c^(2/3)*d^(4/3)*x^
4 + 4*Sqrt[3]*(b*c - a*d)^2*ArcTan[(-c^(1/3) + 2*d^(1/3)*x)/(Sqrt
[3]*c^(1/3))] + 4*(b*c - a*d)^2*Log[c^(1/3) + d^(1/3)*x] - 2*(b*c
 - a*d)^2*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(12*c^(
2/3)*d^(7/3))
```

---

**Maple [B]** time = 0.004, size = 334, normalized size = 1.9

$$\frac{b^2x^4}{4d}+2\frac{abx}{d}-\frac{b^2xc}{d^2}+\frac{a^2}{3d}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{2abc}{3d^2}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{b^2c^2}{3d^3}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{a^2}{6d}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{abc}{3d^2}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{b^2c^2}{6d^3}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}a^2}{3d}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{2\sqrt{3}cab}{3d^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}b^2c^2}{3d^3}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^2/(d*x^3+c),x)
```

```
[Out] 1/4*b^2*x^4/d+2*b/d*a*x-b^2/d^2*x*c+1/3/d/(c/d)^(2/3)*ln(x+(c/d)^
(1/3))*a^2-2/3/d^2/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*c*a*b+1/3/d^3/(c
/d)^(2/3)*ln(x+(c/d)^(1/3))*b^2*c^2-1/6/d/(c/d)^(2/3)*ln(x^2-x*(c
/d)^(1/3)+(c/d)^(2/3))*a^2+1/3/d^2/(c/d)^(2/3)*ln(x^2-x*(c/d)^(1/
3)+(c/d)^(2/3))*c*a*b-1/6/d^3/(c/d)^(2/3)*ln(x^2-x*(c/d)^(1/3)+(c
/d)^(2/3))*b^2*c^2+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(c/d)^(1/3)*x-1))*a^2-2/3/d^2/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/(c/d)^(1/3)*x-1))*c*a*b+1/3/d^3/(c/d)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b^2*c^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2/(d*x^3 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.212159, size = 258, normalized size = 1.49

$$-\frac{\sqrt{3}\left(2\sqrt{3}\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\left(c^2d\right)^{\frac{2}{3}}x^2-\left(c^2d\right)^{\frac{1}{3}}cx+c^2\right)-4\sqrt{3}\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\left(c^2d\right)^{\frac{1}{3}}x+c\right)-12\,(\cdots\right.}{36\,\left(c^2d\right)^{\frac{1}{3}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2/(d*x^3 + c),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(3)*(2*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log((c^2
*d)^(2/3)*x^2 - (c^2*d)^(1/3)*c*x + c^2) - 4*sqrt(3)*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*log((c^2*d)^(1/3)*x + c) - 12*(b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*arctan(1/3*(2*sqrt(3)*(c^2*d)^(1/3)*x - sqrt(3)*
c)/c) - 3*sqrt(3)*(b^2*d*x^4 - 4*(b^2*c - 2*a*b*d)*x)*(c^2*d)^(1/
3))/((c^2*d)^(1/3)*d^2)
```

---

**Sympy [A]** time = 2.86124, size = 156, normalized size = 0.9

$$\frac{b^2x^4}{4d}$$
$$+\mathrm{RootSum}\left(27t^3c^2d^7-a^6d^6+6a^5bcd^5-15a^4b^2c^2d^4+20a^3b^3c^3d^3-15a^2b^4c^4d^2+6ab^5c^5d-b^6c^6,\left(t\mapsto t\log\left(\frac{3tc\cdots}{a^2d^2-2ab\cdots}\right.\right.\right.$$
$$+\frac{x\left(2abd-b^2c\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**2/(d*x**3+c),x)
```

```
[Out] b**2*x**4/(4*d) + RootSum(27*_t**3*c**2*d**7 - a**6*d**6 + 6*a**5
*b*c*d**5 - 15*a**4*b**2*c**2*d**4 + 20*a**3*b**3*c**3*d**3 - 15*
a**2*b**4*c**4*d**2 + 6*a*b**5*c**5*d - b**6*c**6, Lambda(_t, _t*
log(3*_t*c*d**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x))) + x*(2
*a*b*d - b**2*c)/d**2
```

---

**GIAC/XCAS [A]** time = 0.218936, size = 336, normalized size = 1.94

$$\frac{\sqrt{3}\left(\left(-cd^{2}\right)^{\frac{1}{3}}b^{2}c^{2}-2\left(-cd^{2}\right)^{\frac{1}{3}}abcd+\left(-cd^{2}\right)^{\frac{1}{3}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3cd^{3}}+\frac{\left(\left(-cd^{2}\right)^{\frac{1}{3}}b^{2}c^{2}-2\left(-cd^{2}\right)^{\frac{1}{3}}abcd+\left(-cd^{2}\right)^{\frac{1}{3}}a^{2}d^{2}\right)\ln\left(x^{2}+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6cd^{3}}-\frac{\left(b^{2}c^{2}d^{2}-2abcd^{3}+a^{2}d^{4}\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3cd^{4}}+\frac{b^{2}d^{3}x^{4}-4b^{2}cd^{2}x+8abd^{3}x}{4d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2/(d*x^3 + c),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-c*d^2)^(1/3)*b^2*c^2 - 2*(-c*d^2)^(1/3)*a*b*c*d +
(-c*d^2)^(1/3)*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(
-c/d)^(1/3))/(c*d^3) + 1/6*((-c*d^2)^(1/3)*b^2*c^2 - 2*(-c*d^2)^(
1/3)*a*b*c*d + (-c*d^2)^(1/3)*a^2*d^2)*ln(x^2 + x*(-c/d)^(1/3) +
(-c/d)^(2/3))/(c*d^3) - 1/3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*(-c/d)^(1/3)*ln(abs(x - (-c/d)^(1/3)))/(c*d^4) + 1/4*(b^2*d^3*x^
4 - 4*b^2*c*d^2*x + 8*a*b*d^3*x)/d^4
```

## 3.12 $\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx$

**Optimal**. Leaf size=203

$$\frac{(bc-ad)(ad+2bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{9c^{5/3}d^{7/3}}-\frac{2(bc-ad)(ad+2bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{7/3}}+\frac{2(bc-ad)(ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{7/3}}+\frac{x(bc-ad)^2}{3cd^2\left(c+dx^3\right)}+\frac{b^2x}{d^2}$$

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(3*c*d^2*(c + d*x^3)) + (2*(b*c -
 a*d)*(2*b*c + a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/
3))])/(3*Sqrt[3]*c^(5/3)*d^(7/3)) - (2*(b*c - a*d)*(2*b*c + a*d)*
Log[c^(1/3) + d^(1/3)*x])/(9*c^(5/3)*d^(7/3)) + ((b*c - a*d)*(2*b
*c + a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(9*c^(5
/3)*d^(7/3))
```

---

**Rubi [A]** time = 0.495065, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$\frac{(bc-ad)(ad+2bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{9c^{5/3}d^{7/3}}-\frac{2(bc-ad)(ad+2bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{7/3}}+\frac{2(bc-ad)(ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{7/3}}+\frac{x(bc-ad)^2}{3cd^2\left(c+dx^3\right)}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^2/(c + d*x^3)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(3*c*d^2*(c + d*x^3)) + (2*(b*c -
 a*d)*(2*b*c + a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/
3))])/(3*Sqrt[3]*c^(5/3)*d^(7/3)) - (2*(b*c - a*d)*(2*b*c + a*d)*
Log[c^(1/3) + d^(1/3)*x])/(9*c^(5/3)*d^(7/3)) + ((b*c - a*d)*(2*b
*c + a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(9*c^(5
/3)*d^(7/3))
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int b^2\,dx}{d^2}+\frac{x\,(ad-bc)^2}{3cd^2\,(c+dx^3)}+\frac{2\,(ad-bc)\,(ad+2bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{\frac{5}{3}}d^{\frac{7}{3}}}-\frac{(ad-bc)\,(ad+2bc)\log\left(c^{\frac{2}{3}}-\sqrt[3]{c}\sqrt[3]{d}x+d^{\frac{2}{3}}x^2\right)}{9c^{\frac{5}{3}}d^{\frac{7}{3}}}-\frac{2\sqrt{3}\,(ad-bc)\,(ad+2bc)\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3}-\frac{2\sqrt[3]{d}x}{3}\right)}{\sqrt[3]{c}}\right)}{9c^{\frac{5}{3}}d^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**2/(d*x**3+c)**2,x)
```

```
[Out] Integral(b**2, x)/d**2 + x*(a*d - b*c)**2/(3*c*d**2*(c + d*x**3))
 + 2*(a*d - b*c)*(a*d + 2*b*c)*log(c**(1/3) + d**(1/3)*x)/(9*c**(
5/3)*d**(7/3)) - (a*d - b*c)*(a*d + 2*b*c)*log(c**(2/3) - c**(1/3
)*d**(1/3)*x + d**(2/3)*x**2)/(9*c**(5/3)*d**(7/3)) - 2*sqrt(3)*(
a*d - b*c)*(a*d + 2*b*c)*atan(sqrt(3)*(c**(1/3)/3 - 2*d**(1/3)*x/
3)/c**(1/3))/(9*c**(5/3)*d**(7/3))
```

---

**Mathematica [A]** time = 0.379356, size = 210, normalized size = 1.03

$$\frac{-\frac{2\left(-a^2d^2-abcd+2b^2c^2\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{5/3}}+\frac{2\sqrt{3}\left(-a^2d^2-abcd+2b^2c^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}}+\frac{\left(-a^2d^2-abcd+2b^2c^2\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{5/3}}+\frac{3\sqrt[3]{d}x}{c(c}}{9d^{7/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)^2/(c + d*x^3)^2,x]
```

```
[Out] (9*b^2*d^(1/3)*x + (3*d^(1/3)*(b*c - a*d)^2*x)/(c*(c + d*x^3)) +
(2*Sqrt[3]*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*ArcTan[(1 - (2*d^(1/3)
*x)/c^(1/3))/Sqrt[3]])/c^(5/3) - (2*(2*b^2*c^2 - a*b*c*d - a^2*d^
2)*Log[c^(1/3) + d^(1/3)*x]])/c^(5/3) + ((2*b^2*c^2 - a*b*c*d - a^
2*d^2)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(5/3))/(
9*d^(7/3))
```

---

**Maple [B]** time = 0.014, size = 367, normalized size = 1.8

$$\frac{b^2x}{d^2}+\frac{xa^2}{3c(dx^3+c)}-\frac{2axb}{3d(dx^3+c)}+\frac{cxb^2}{3d^2(dx^3+c)}+\frac{2a^2}{9cd}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{2ab}{9d^2}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{4b^2c}{9d^3}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{a^2}{9cd}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{ab}{9d^2}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{2b^2c}{9d^3}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{2\sqrt{3}a^2}{9cd}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{2\sqrt{3}ab}{9d^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{4c\sqrt{3}b^2}{9d^3}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^2/(d*x^3+c)^2,x)
```

```
[Out] b^2*x/d^2+1/3/c*x/(d*x^3+c)*a^2-2/3/d*x/(d*x^3+c)*a*b+1/3/d^2*c*x
/(d*x^3+c)*b^2+2/9/d/c/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a^2+2/9/d^2/
(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a*b-4/9/d^3*c/(c/d)^(2/3)*ln(x+(c/d
)^(1/3))*b^2-1/9/d/c/(c/d)^(2/3)*ln(x^2-x*(c/d)^(1/3)+(c/d)^(2/3)
)*a^2-1/9/d^2/(c/d)^(2/3)*ln(x^2-x*(c/d)^(1/3)+(c/d)^(2/3))*a*b+2
/9/d^3*c/(c/d)^(2/3)*ln(x^2-x*(c/d)^(1/3)+(c/d)^(2/3))*b^2+2/9/d/
c/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a^2
+2/9/d^2/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-
1))*a*b-4/9/d^3*c/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)
^(1/3)*x-1))*b^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2/(d*x^3 + c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.218018, size = 435, normalized size = 2.14

$$\frac{\sqrt{3}\left(\sqrt{3}\left(2\,b^2c^3 - abc^2d - a^2cd^2 + \left(2\,b^2c^2d - abcd^2 - a^2d^3\right)x^3\right)\log\left(\left(c^2d\right)^{\frac{2}{3}}x^2 - \left(c^2d\right)^{\frac{1}{3}}cx + c^2\right) - 2\,\sqrt{3}\left(2\,b^2c^3 - abc^2d - a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2/(d*x^3 + c)^2,x, algorithm="fricas")
```

```
[Out] 1/27*sqrt(3)*(sqrt(3)*(2*b^2*c^3 - a*b*c^2*d - a^2*c*d^2 + (2*b^2
*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*log((c^2*d)^(2/3)*x^2 - (c^2*d
)^(1/3)*c*x + c^2) - 2*sqrt(3)*(2*b^2*c^3 - a*b*c^2*d - a^2*c*d^2
 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*log((c^2*d)^(1/3)*x +
 c) - 6*(2*b^2*c^3 - a*b*c^2*d - a^2*c*d^2 + (2*b^2*c^2*d - a*b*c
*d^2 - a^2*d^3)*x^3)*arctan(1/3*(2*sqrt(3)*(c^2*d)^(1/3)*x - sqrt
(3)*c)/c) + 3*sqrt(3)*(3*b^2*c*d*x^4 + (4*b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*x)*(c^2*d)^(1/3))/((c*d^3*x^3 + c^2*d^2)*(c^2*d)^(1/3))
```

---

**Sympy [A]** time = 4.8321, size = 189, normalized size = 0.93

$$\frac{b^2x}{d^2} + \frac{x\left(a^2d^2 - 2abcd + b^2c^2\right)}{3c^2d^2 + 3cd^3x^3}$$
$$+\text{RootSum}\left(729t^3c^5d^7 - 8a^6d^6 - 24a^5bcd^5 + 24a^4b^2c^2d^4 + 88a^3b^3c^3d^3 - 48a^2b^4c^4d^2 - 96ab^5c^5d + 64b^6c^6, \left(t \mapsto t\log\left(\frac{}{2a^2c\cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**2/(d*x**3+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*c**2*d**2
+ 3*c*d**3*x**3) + RootSum(729*_t**3*c**5*d**7 - 8*a**6*d**6 - 24
*a**5*b*c*d**5 + 24*a**4*b**2*c**2*d**4 + 88*a**3*b**3*c**3*d**3
- 48*a**2*b**4*c**4*d**2 - 96*a*b**5*c**5*d + 64*b**6*c**6, Lambd
a(_t, _t*log(9*_t*c**2*d**2/(2*a**2*d**2 + 2*a*b*c*d - 4*b**2*c**
2) + x)))
```

---

**GIAC/XCAS [A]** time = 0.219868, size = 358, normalized size = 1.76

$$\frac{b^2x}{d^2} + \frac{2\left(2\,b^2c^2 - abcd - a^2d^2\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\,c^2d^2} - \frac{2\,\sqrt{3}\left(2\,\left(-cd^2\right)^{\frac{1}{3}}b^2c^2 - \left(-cd^2\right)^{\frac{1}{3}}abcd - \left(-cd^2\right)^{\frac{1}{3}}a^2d^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,c^2d^3} + \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{3\,(dx^3 + c)cd^2} - \frac{\left(2\,\left(-cd^2\right)^{\frac{1}{3}}b^2c^2 - \left(-cd^2\right)^{\frac{1}{3}}abcd - \left(-cd^2\right)^{\frac{1}{3}}a^2d^2\right)\ln\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\,c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2/(d*x^3 + c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 + 2/9*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*(-c/d)^(1/3)*ln(a
bs(x - (-c/d)^(1/3)))/(c^2*d^2) - 2/9*sqrt(3)*(2*(-c*d^2)^(1/3)*b
^2*c^2 - (-c*d^2)^(1/3)*a*b*c*d - (-c*d^2)^(1/3)*a^2*d^2)*arctan(
1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(c^2*d^3) + 1/3*(b
^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^3 + c)*c*d^2) - 1/9*(2*
(-c*d^2)^(1/3)*b^2*c^2 - (-c*d^2)^(1/3)*a*b*c*d - (-c*d^2)^(1/3)*
a^2*d^2)*ln(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(c^2*d^3)
```

## 3.13 $\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx$

**Optimal**. Leaf size=258

$$-\frac{(5a^2d^2+2abcd+2b^2c^2)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{7/3}}+\frac{(5a^2d^2+2abcd+2b^2c^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{7/3}}-\frac{(5a^2d^2+2abcd+2b^2c^2)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{7/3}}-\frac{x(bc-ad)(5ad+4bc)}{18c^2d^2(c+dx^3)}-\frac{x(a+bx^3)(bc-ad)}{6cd(c+dx^3)^2}$$

```
[Out] -((b*c - a*d)*x*(a + b*x^3))/(6*c*d*(c + d*x^3)^2) - ((b*c - a*d)
*(4*b*c + 5*a*d)*x)/(18*c^2*d^2*(c + d*x^3)) - ((2*b^2*c^2 + 2*a*
b*c*d + 5*a^2*d^2)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3
))])/(9*Sqrt[3]*c^(8/3)*d^(7/3)) + ((2*b^2*c^2 + 2*a*b*c*d + 5*a^
2*d^2)*Log[c^(1/3) + d^(1/3)*x])/(27*c^(8/3)*d^(7/3)) - ((2*b^2*c
^2 + 2*a*b*c*d + 5*a^2*d^2)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(
2/3)*x^2])/(54*c^(8/3)*d^(7/3))
```

---

**Rubi [A]** time = 0.477267, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$-\frac{(5a^2d^2+2abcd+2b^2c^2)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{7/3}}+\frac{(5a^2d^2+2abcd+2b^2c^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{7/3}}-\frac{(5a^2d^2+2abcd+2b^2c^2)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{7/3}}-\frac{x(bc-ad)(5ad+4bc)}{18c^2d^2(c+dx^3)}-\frac{x(a+bx^3)(bc-ad)}{6cd(c+dx^3)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^2/(c + d*x^3)^3,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^3))/(6*c*d*(c + d*x^3)^2) - ((b*c - a*d)
*(4*b*c + 5*a*d)*x)/(18*c^2*d^2*(c + d*x^3)) - ((2*b^2*c^2 + 2*a*
b*c*d + 5*a^2*d^2)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3
))])/(9*Sqrt[3]*c^(8/3)*d^(7/3)) + ((2*b^2*c^2 + 2*a*b*c*d + 5*a^
2*d^2)*Log[c^(1/3) + d^(1/3)*x])/(27*c^(8/3)*d^(7/3)) - ((2*b^2*c
^2 + 2*a*b*c*d + 5*a^2*d^2)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(
2/3)*x^2])/(54*c^(8/3)*d^(7/3))
```

---

**Rubi in Sympy [A]** time = 53.106, size = 243, normalized size = 0.94

$$\frac{x\left(a+bx^3\right)(ad-bc)}{6cd\left(c+dx^3\right)^2}+\frac{x(ad-bc)(5ad+4bc)}{18c^2d^2\left(c+dx^3\right)}+\frac{(ad(5ad+bc)+bc(ad+2bc))\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{\frac{8}{3}}d^{\frac{7}{3}}}$$
$$-\frac{(ad(5ad+bc)+bc(ad+2bc))\log\left(c^{\frac{2}{3}}-\sqrt[3]{c}\sqrt[3]{d}x+d^{\frac{2}{3}}x^2\right)}{54c^{\frac{8}{3}}d^{\frac{7}{3}}}$$
$$-\frac{\sqrt{3}(ad(5ad+bc)+bc(ad+2bc))\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3}-\frac{2\sqrt[3]{d}x}{3}\right)}{\sqrt[3]{c}}\right)}{27c^{\frac{8}{3}}d^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**2/(d*x**3+c)**3,x)
```

```
[Out] x*(a + b*x**3)*(a*d - b*c)/(6*c*d*(c + d*x**3)**2) + x*(a*d - b*c
)*(5*a*d + 4*b*c)/(18*c**2*d**2*(c + d*x**3)) + (a*d*(5*a*d + b*c
) + b*c*(a*d + 2*b*c))*log(c**(1/3) + d**(1/3)*x)/(27*c**(8/3)*d*
*(7/3)) - (a*d*(5*a*d + b*c) + b*c*(a*d + 2*b*c))*log(c**(2/3) -
c**(1/3)*d**(1/3)*x + d**(2/3)*x**2)/(54*c**(8/3)*d**(7/3)) - sqr
t(3)*(a*d*(5*a*d + b*c) + b*c*(a*d + 2*b*c))*atan(sqrt(3)*(c**(1/
3)/3 - 2*d**(1/3)*x/3)/c**(1/3))/(27*c**(8/3)*d**(7/3))
```

---

**Mathematica [A]** time = 0.465576, size = 234, normalized size = 0.91

$$\frac{2\left(5a^2d^2+2abcd+2b^2c^2\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)-2\sqrt{3}\left(5a^2d^2+2abcd+2b^2c^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)-\frac{3c^{2/3}\sqrt[3]{d}x\left(-a^2d^2\left(8c+5dx^3\right)+2abcd\right.}{\left(c+dx^3\right)^2}}{54c^{8/3}d^{7/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)^2/(c + d*x^3)^3,x]
```

```
[Out] ((-3*c^(2/3)*d^(1/3)*x*(2*a*b*c*d*(2*c - d*x^3) - a^2*d^2*(8*c +
5*d*x^3) + b^2*c^2*(4*c + 7*d*x^3)))/(c + d*x^3)^2 - 2*Sqrt[3]*(2
*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/
3))/Sqrt[3]] + 2*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Log[c^(1/3)
+ d^(1/3)*x] - (2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Log[c^(2/3) -
c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(7/3))
```

---

**Maple [A]** time = 0.015, size = 388, normalized size = 1.5

$$\frac{1}{(dx^3+c)^2}\left(\frac{(5a^2d^2+2cabd-7b^2c^2)\,x^4}{18c^2d}+\frac{(4a^2d^2-2cabd-2b^2c^2)\,x}{9d^2c}\right)+\frac{5a^2}{27c^2d}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{2ab}{27d^2c}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{2b^2}{27d^3}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{5a^2}{54c^2d}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{ab}{27d^2c}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{b^2}{27d^3}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{5\sqrt{3}a^2}{27c^2d}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{2\sqrt{3}ab}{27d^2c}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{2\sqrt{3}b^2}{27d^3}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^2/(d*x^3+c)^3,x)
```

```
[Out] (1/18*(5*a^2*d^2+2*a*b*c*d-7*b^2*c^2)/c^2/d*x^4+2/9*(2*a^2*d^2-a*
b*c*d-b^2*c^2)/d^2/c*x)/(d*x^3+c)^2+5/27/c^2/d/(c/d)^(2/3)*ln(x+(
c/d)^(1/3))*a^2+2/27/c/d^2/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a*b+2/27
/d^3/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*b^2-5/54/c^2/d/(c/d)^(2/3)*ln(
x^2-x*(c/d)^(1/3)+(c/d)^(2/3))*a^2-1/27/c/d^2/(c/d)^(2/3)*ln(x^2-
x*(c/d)^(1/3)+(c/d)^(2/3))*a*b-1/27/d^3/(c/d)^(2/3)*ln(x^2-x*(c/d
)^(1/3)+(c/d)^(2/3))*b^2+5/27/c^2/d/(c/d)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a^2+2/27/c/d^2/(c/d)^(2/3)*3^(1/2)
*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a*b+2/27/d^3/(c/d)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2/(d*x^3 + c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.218697, size = 637, normalized size = 2.47

$$-\frac{\sqrt{3}\left(\sqrt{3}\left(\left(2\,b^2c^2d^2 + 2\,abcd^3 + 5\,a^2d^4\right)x^6 + 2\,b^2c^4 + 2\,abc^3d + 5\,a^2c^2d^2 + 2\,\left(2\,b^2c^3d + 2\,abc^2d^2 + 5\,a^2cd^3\right)x^3\right)\log\left(\left(c^2d\right)^{\frac{2}{3}}x^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2/(d*x^3 + c)^3,x, algorithm="fricas")
```

```
[Out] -1/162*sqrt(3)*(sqrt(3)*((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4
)*x^6 + 2*b^2*c^4 + 2*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d
+ 2*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*log((c^2*d)^(2/3)*x^2 - (c^2*
d)^(1/3)*c*x + c^2) - 2*sqrt(3)*((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5
*a^2*d^4)*x^6 + 2*b^2*c^4 + 2*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(2*b^
2*c^3*d + 2*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*log((c^2*d)^(1/3)*x +
 c) - 6*((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^2*c^
4 + 2*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2
+ 5*a^2*c*d^3)*x^3)*arctan(1/3*(2*sqrt(3)*(c^2*d)^(1/3)*x - sqrt(
3)*c)/c) + 3*sqrt(3)*((7*b^2*c^2*d - 2*a*b*c*d^2 - 5*a^2*d^3)*x^4
 + 4*(b^2*c^3 + a*b*c^2*d - 2*a^2*c*d^2)*x)*(c^2*d)^(1/3))/((c^2*
d^4*x^6 + 2*c^3*d^3*x^3 + c^4*d^2)*(c^2*d)^(1/3))
```

---

**Sympy [A]** time = 7.43143, size = 233, normalized size = 0.9

$$\frac{x^4\left(5a^2d^3 + 2abcd^2 - 7b^2c^2d\right) + x\left(8a^2cd^2 - 4abc^2d - 4b^2c^3\right)}{18c^4d^2 + 36c^3d^3x^3 + 18c^2d^4x^6}$$
$$+\mathrm{RootSum}\left(19683t^3c^8d^7 - 125a^6d^6 - 150a^5bcd^5 - 210a^4b^2c^2d^4 - 128a^3b^3c^3d^3 - 84a^2b^4c^4d^2 - 24ab^5c^5d - 8b^6c^6, \left(t \mapsto t\,\mathrm{lo}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**2/(d*x**3+c)**3,x)
```

```
[Out] (x**4*(5*a**2*d**3 + 2*a*b*c*d**2 - 7*b**2*c**2*d) + x*(8*a**2*c*
d**2 - 4*a*b*c**2*d - 4*b**2*c**3))/(18*c**4*d**2 + 36*c**3*d**3*
x**3 + 18*c**2*d**4*x**6) + RootSum(19683*_t**3*c**8*d**7 - 125*a
**6*d**6 - 150*a**5*b*c*d**5 - 210*a**4*b**2*c**2*d**4 - 128*a**3
*b**3*c**3*d**3 - 84*a**2*b**4*c**4*d**2 - 24*a*b**5*c**5*d - 8*b
**6*c**6, Lambda(_t, _t*log(27*_t*c**3*d**2/(5*a**2*d**2 + 2*a*b*
c*d + 2*b**2*c**2) + x)))
```

---

**GIAC/XCAS [A]** time = 0.222457, size = 400, normalized size = 1.55

$$-\frac{\left(2\,b^{2}c^{2}+2\,abcd+5\,a^{2}d^{2}\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{27\,c^{3}d^{2}}+\frac{\sqrt{3}\left(2\,\left(-cd^{2}\right)^{\frac{1}{3}}b^{2}c^{2}+2\,\left(-cd^{2}\right)^{\frac{1}{3}}abcd+5\,\left(-cd^{2}\right)^{\frac{1}{3}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,c^{3}d^{3}}+\frac{\left(2\,\left(-cd^{2}\right)^{\frac{1}{3}}b^{2}c^{2}+2\,\left(-cd^{2}\right)^{\frac{1}{3}}abcd+5\,\left(-cd^{2}\right)^{\frac{1}{3}}a^{2}d^{2}\right)\ln\left(x^{2}+x\,\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,c^{3}d^{3}}-\frac{7\,b^{2}c^{2}dx^{4}-2\,abcd^{2}x^{4}-5\,a^{2}d^{3}x^{4}+4\,b^{2}c^{3}x+4\,abc^{2}dx-8\,a^{2}cd^{2}x}{18\,\left(dx^{3}+c\right)^{2}c^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2/(d*x^3 + c)^3,x, algorithm="giac")
```

```
[Out] -1/27*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*(-c/d)^(1/3)*ln(abs(x -
 (-c/d)^(1/3)))/(c^3*d^2) + 1/27*sqrt(3)*(2*(-c*d^2)^(1/3)*b^2*c^
2 + 2*(-c*d^2)^(1/3)*a*b*c*d + 5*(-c*d^2)^(1/3)*a^2*d^2)*arctan(1
/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(c^3*d^3) + 1/54*(2
*(-c*d^2)^(1/3)*b^2*c^2 + 2*(-c*d^2)^(1/3)*a*b*c*d + 5*(-c*d^2)^(
1/3)*a^2*d^2)*ln(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(c^3*d^3) -
 1/18*(7*b^2*c^2*d*x^4 - 2*a*b*c*d^2*x^4 - 5*a^2*d^3*x^4 + 4*b^2*
c^3*x + 4*a*b*c^2*d*x - 8*a^2*c*d^2*x)/((d*x^3 + c)^2*c^2*d^2)
```

## 3.14 $\int \frac{(c+dx^3)^4}{a+bx^3} \, dx$

**Optimal**. Leaf size=252

$$-\frac{(bc-ad)^4 \log \left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{13/3}}+\frac{(bc-ad)^4 \log \left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{13/3}} -\frac{(bc-ad)^4 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{13/3}}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4} +\frac{d^2x^4\left(a^2d^2-4abcd+6b^2c^2\right)}{4b^3}+\frac{d^3x^7(4bc-ad)}{7b^2}+\frac{d^4x^{10}}{10b}$$

```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*
(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^4)/(4*b^3) + (d^3*(4*b*c - a*
d)*x^7)/(7*b^2) + (d^4*x^10)/(10*b) - ((b*c - a*d)^4*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(13/3))
 + ((b*c - a*d)^4*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(13/3))
- ((b*c - a*d)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
(6*a^(2/3)*b^(13/3))
```

---

**Rubi [A]** time = 0.415438, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368

$$-\frac{(bc-ad)^4 \log \left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{13/3}}+\frac{(bc-ad)^4 \log \left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{13/3}} -\frac{(bc-ad)^4 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{13/3}}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4} +\frac{d^2x^4\left(a^2d^2-4abcd+6b^2c^2\right)}{4b^3}+\frac{d^3x^7(4bc-ad)}{7b^2}+\frac{d^4x^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3)^4/(a + b*x^3),x]
```


```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*
(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^4)/(4*b^3) + (d^3*(4*b*c - a*
d)*x^7)/(7*b^2) + (d^4*x^10)/(10*b) - ((b*c - a*d)^4*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(13/3))
 + ((b*c - a*d)^4*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(13/3))
- ((b*c - a*d)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
(6*a^(2/3)*b^(13/3))
```


---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{d^4x^{10}}{10b} - \frac{d^3x^7\,(ad-4bc)}{7b^2} + \frac{d^2x^4\,(a^2d^2-4abcd+6b^2c^2)}{4b^3} - \frac{(ad-2bc)\,(a^2d^2-2abcd+2b^2c^2)\int d\,dx}{b^4} + \frac{(ad-bc)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{2}{3}}b^{\frac{13}{3}}} - \frac{(ad-bc)^4\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}b^{\frac{13}{3}}} - \frac{\sqrt{3}\,(ad-bc)^4\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}b^{\frac{13}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c)**4/(b*x**3+a),x)
```

```
[Out] d**4*x**10/(10*b) - d**3*x**7*(a*d - 4*b*c)/(7*b**2) + d**2*x**4*
(a**2*d**2 - 4*a*b*c*d + 6*b**2*c**2)/(4*b**3) - (a*d - 2*b*c)*(a
**2*d**2 - 2*a*b*c*d + 2*b**2*c**2)*Integral(d, x)/b**4 + (a*d -
b*c)**4*log(a**(1/3) + b**(1/3)*x)/(3*a**(2/3)*b**(13/3)) - (a*d
- b*c)**4*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*
a**(2/3)*b**(13/3)) - sqrt(3)*(a*d - b*c)**4*atan(sqrt(3)*(a**(1/
3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(2/3)*b**(13/3))
```

---

**Mathematica [A]** time = 0.199474, size = 253, normalized size = 1.

$$\frac{-\frac{70(bc-ad)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}}+\frac{140(bc-ad)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}}+\frac{140\sqrt{3}(bc-ad)^4\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}}+105b^{4/3}d^2x^4\left(a^2d^2-4abcd+\right.}{420b^{13/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3)^4/(a + b*x^3),x]
```

```
[Out] (420*b^(1/3)*d*(4*b^3*c^3 - 6*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d
^3)*x + 105*b^(4/3)*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^4 + 6
0*b^(7/3)*d^3*(4*b*c - a*d)*x^7 + 42*b^(10/3)*d^4*x^10 + (140*Sqr
t[3]*(b*c - a*d)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/a^(2/3) + (140*(b*c - a*d)^4*Log[a^(1/3) + b^(1/3)*x])/a^(2
/3) - (70*(b*c - a*d)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/a^(2/3))/(420*b^(13/3))
```

---

**Maple [B]** time = 0.006, size = 661, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c)^4/(b*x^3+a),x)
```

```
[Out] -4/3/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a*c^3*d+2/3/b^4/(a/b)^(2/3
)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*a^3*c*d^3+2/3/b^2/(a/b)^(2/3)
*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*a*c^3*d+1/3/b^5/(a/b)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^4*d^4-1/b^3/(a/b)
^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*a^2*c^2*d^2-4/3/b^4/(a/b
)^(2/3)*ln(x+(a/b)^(1/3))*a^3*c*d^3+2/b^3/(a/b)^(2/3)*ln(x+(a/b)^
(1/3))*a^2*c^2*d^2+1/3/b^5/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a^4*d^4-
d^3/b^2*x^4*a*c+4*d^3/b^3*a^2*c*x-6*d^2/b^2*a*c^2*x-4/3/b^4/(a/b)
^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^3*c*d^3+
2/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
*a^2*c^2*d^2-1/6/b^5/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3)
)*a^4*d^4+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))*c^4+4/7*d^3/b*x^7*c-1/7*d^4/b^2*x^7*a+3/2*d^2/b*x^4*c^
2-d^4/b^4*a^3*x+4*d/b*c^3*x+1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c
^4-1/6/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c^4+1/4*d^
4/b^3*x^4*a^2-4/3/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1))*a*c^3*d+1/10*d^4*x^10/b
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^4/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.216267, size = 494, normalized size = 1.96

$$-\frac{\sqrt{3}\Big(70\sqrt{3}\big(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\big)\log\Big(\big(a^2b\big)^{\frac{2}{3}}x^2 - \big(a^2b\big)^{\frac{1}{3}}ax + a^2\Big) - 140\sqrt{3}\big(b^4c^4 - 4\,ab^3c^3d +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^4/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] -1/1260*sqrt(3)*(70*sqrt(3)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log((a^2*b)^(2/3)*x^2 - (a^2*b
)^(1/3)*a*x + a^2) - 140*sqrt(3)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2
```

```
*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log((a^2*b)^(1/3)*x + a)
- 420*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^
3 + a^4*d^4)*arctan(1/3*(2*sqrt(3)*(a^2*b)^(1/3)*x - sqrt(3)*a)/a
) - 3*sqrt(3)*(14*b^3*d^4*x^10 + 20*(4*b^3*c*d^3 - a*b^2*d^4)*x^7
 + 35*(6*b^3*c^2*d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^4 + 140*(4*b^
3*c^3*d - 6*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3 - a^3*d^4)*x)*(a^2*b)^(
1/3))/((a^2*b)^(1/3)*b^4)
```

---

**Sympy [A]** time = 5.40425, size = 369, normalized size = 1.46

$$\mathrm{RootSum}\left(27t^3a^2b^{13} - a^{12}d^{12} + 12a^{11}bcd^{11} - 66a^{10}b^2c^2d^{10} + 220a^9b^3c^3d^9 - 495a^8b^4c^4d^8 + 792a^7b^5c^5d^7 - 924a^6b^6c^6d^6 + 79\right.$$

$$+ \frac{d^4x^{10}}{10b} - \frac{x^7\left(ad^4 - 4bcd^3\right)}{7b^2} + \frac{x^4\left(a^2d^4 - 4abcd^3 + 6b^2c^2d^2\right)}{4b^3} - \frac{x\left(a^3d^4 - 4a^2bcd^3 + 6ab^2c^2d^2 - 4b^3c^3d\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c)**4/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**13 - a**12*d**12 + 12*a**11*b*c*d**11 -
66*a**10*b**2*c**2*d**10 + 220*a**9*b**3*c**3*d**9 - 495*a**8*b**
4*c**4*d**8 + 792*a**7*b**5*c**5*d**7 - 924*a**6*b**6*c**6*d**6 +
 792*a**5*b**7*c**7*d**5 - 495*a**4*b**8*c**8*d**4 + 220*a**3*b**
9*c**9*d**3 - 66*a**2*b**10*c**10*d**2 + 12*a*b**11*c**11*d - b**
12*c**12, Lambda(_t, _t*log(3*_t*a*b**4/(a**4*d**4 - 4*a**3*b*c*d
**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4) + x)))
 + d**4*x**10/(10*b) - x**7*(a*d**4 - 4*b*c*d**3)/(7*b**2) + x**4
*(a**2*d**4 - 4*a*b*c*d**3 + 6*b**2*c**2*d**2)/(4*b**3) - x*(a**3
*d**4 - 4*a**2*b*c*d**3 + 6*a*b**2*c**2*d**2 - 4*b**3*c**3*d)/b**
4
```

---

**GIAC/XCAS [A]** time = 0.220205, size = 622, normalized size = 2.47

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^4c^4 - 4\left(-ab^2\right)^{\frac{1}{3}}ab^3c^3d + 6\left(-ab^2\right)^{\frac{1}{3}}a^2b^2c^2d^2 - 4\left(-ab^2\right)^{\frac{1}{3}}a^3bcd^3 + \left(-ab^2\right)^{\frac{1}{3}}a^4d^4\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^5}$$

$$+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^4c^4 - 4\left(-ab^2\right)^{\frac{1}{3}}ab^3c^3d + 6\left(-ab^2\right)^{\frac{1}{3}}a^2b^2c^2d^2 - 4\left(-ab^2\right)^{\frac{1}{3}}a^3bcd^3 + \left(-ab^2\right)^{\frac{1}{3}}a^4d^4\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)\right.}{6ab^5}$$

$$-\frac{\left(b^{10}c^4 - 4ab^9c^3d + 6a^2b^8c^2d^2 - 4a^3b^7cd^3 + a^4b^6d^4\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^{10}}$$

$$+\frac{14b^9d^4x^{10} + 80b^9cd^3x^7 - 20ab^8d^4x^7 + 210b^9c^2d^2x^4 - 140ab^8cd^3x^4 + 35a^2b^7d^4x^4 + 560b^9c^3dx - 840ab^8c^2d^2x + 560a^2}{140b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^4/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^4*c^4 - 4*(-a*b^2)^(1/3)*a*b^3*c^3*
d + 6*(-a*b^2)^(1/3)*a^2*b^2*c^2*d^2 - 4*(-a*b^2)^(1/3)*a^3*b*c*d
^3 + (-a*b^2)^(1/3)*a^4*d^4)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/(a*b^5) + 1/6*((-a*b^2)^(1/3)*b^4*c^4 - 4*(-a*b
^2)^(1/3)*a*b^3*c^3*d + 6*(-a*b^2)^(1/3)*a^2*b^2*c^2*d^2 - 4*(-a*
b^2)^(1/3)*a^3*b*c*d^3 + (-a*b^2)^(1/3)*a^4*d^4)*ln(x^2 + x*(-a/b
)^(1/3) + (-a/b)^(2/3))/(a*b^5) - 1/3*(b^10*c^4 - 4*a*b^9*c^3*d +
 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*(-a/b)^(1/3)*
ln(abs(x - (-a/b)^(1/3)))/(a*b^10) + 1/140*(14*b^9*d^4*x^10 + 80*
b^9*c*d^3*x^7 - 20*a*b^8*d^4*x^7 + 210*b^9*c^2*d^2*x^4 - 140*a*b^
8*c*d^3*x^4 + 35*a^2*b^7*d^4*x^4 + 560*b^9*c^3*d*x - 840*a*b^8*c^
2*d^2*x + 560*a^2*b^7*c*d^3*x - 140*a^3*b^6*d^4*x)/b^10
```

## 3.15 $\int \frac{(c+dx^3)^3}{a+bx^3}\,dx$

**Optimal**. Leaf size=208

$$-\frac{(bc-ad)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}}+\frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{10/3}}+\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{d^2x^4(3bc-ad)}{4b^2}+\frac{d^3x^7}{7b}$$

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*
x^4)/(4*b^2) + (d^3*x^7)/(7*b) - ((b*c - a*d)^3*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(10/3)) + ((
b*c - a*d)^3*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(10/3)) - ((b
*c - a*d)^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^
(2/3)*b^(10/3))
```

---

**Rubi [A]** time = 0.31675, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368

$$-\frac{(bc-ad)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}}+\frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{10/3}}+\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{d^2x^4(3bc-ad)}{4b^2}+\frac{d^3x^7}{7b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3)^3/(a + b*x^3),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*
x^4)/(4*b^2) + (d^3*x^7)/(7*b) - ((b*c - a*d)^3*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(10/3)) + ((
b*c - a*d)^3*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(10/3)) - ((b
*c - a*d)^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^
(2/3)*b^(10/3))
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{d^3x^7}{7b} - \frac{d^2x^4(ad-3bc)}{4b^2} + \frac{(a^2d^2-3abcd+3b^2c^2)\int d\,dx}{b^3} - \frac{(ad-bc)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{2}{3}}b^{\frac{10}{3}}} + \frac{(ad-bc)^3\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}b^{\frac{10}{3}}} + \frac{\sqrt{3}(ad-bc)^3\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}b^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c)**3/(b*x**3+a),x)
```

```
[Out] d**3*x**7/(7*b) - d**2*x**4*(a*d - 3*b*c)/(4*b**2) + (a**2*d**2 -
 3*a*b*c*d + 3*b**2*c**2)*Integral(d, x)/b**3 - (a*d - b*c)**3*lo
g(a**(1/3) + b**(1/3)*x)/(3*a**(2/3)*b**(10/3)) + (a*d - b*c)**3*
log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*a**(2/3)*b
**(10/3)) + sqrt(3)*(a*d - b*c)**3*atan(sqrt(3)*(a**(1/3)/3 - 2*b
**(1/3)*x/3)/a**(1/3))/(3*a**(2/3)*b**(10/3))
```

---

**Mathematica [A]** time = 0.155733, size = 203, normalized size = 0.98

$$\frac{\frac{14(ad-bc)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}} + \frac{28(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + \frac{28\sqrt{3}(bc-ad)^3\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}} + 84\sqrt[3]{b}dx\left(a^2d^2-3abcd+3b^2c^2\right)}{84b^{10/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3)^3/(a + b*x^3),x]
```

```
[Out] (84*b^(1/3)*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x + 21*b^(4/3)*d^
2*(3*b*c - a*d)*x^4 + 12*b^(7/3)*d^3*x^7 + (28*Sqrt[3]*(b*c - a*d
)^3*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(2/3) +
 (28*(b*c - a*d)^3*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (14*(-(b*c
) + a*d)^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3
))/(84*b^(10/3))
```

---

**Maple [B]** time = 0.004, size = 486, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c)^3/(b*x^3+a),x)
```

```
[Out] 1/7*d^3*x^7/b-1/4*d^3/b^2*x^4*a+3/4*d^2/b*x^4*c+d^3/b^3*a^2*x-3*d
^2/b^2*a*c*x+3*d/b*c^2*x-1/3/b^4/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a^
3*d^3+1/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a^2*c*d^2-1/b^2/(a/b)^(
2/3)*ln(x+(a/b)^(1/3))*a*c^2*d+1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3)
)*c^3+1/6/b^4/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*a^3*d
^3-1/2/b^3/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*a^2*c*d^
2+1/2/b^2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*a*c^2*d-1
/6/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c^3-1/3/b^4/(a
/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^3*d^3
+1/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)*a^2*c*d^2-1/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))*a*c^2*d+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1))*c^3
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.215725, size = 379, normalized size = 1.82

$$\frac{\sqrt{3}\left(14\sqrt{3}\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\log\left(\left(-a^2b\right)^{\frac{2}{3}}x^2 + \left(-a^2b\right)^{\frac{1}{3}}ax + a^2\right) - 28\sqrt{3}\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/252*sqrt(3)*(14*sqrt(3)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^
2 - a^3*d^3)*log((-a^2*b)^(2/3)*x^2 + (-a^2*b)^(1/3)*a*x + a^2) -
 28*sqrt(3)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*l
og((-a^2*b)^(1/3)*x - a) + 84*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*
c*d^2 - a^3*d^3)*arctan(1/3*(2*sqrt(3)*(-a^2*b)^(1/3)*x + sqrt(3)
*a)/a) + 3*sqrt(3)*(4*b^2*d^3*x^7 + 7*(3*b^2*c*d^2 - a*b*d^3)*x^4
 + 28*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*x)*(-a^2*b)^(1/3))/((
-a^2*b)^(1/3)*b^3)
```

**Sympy [A]** time = 4.16369, size = 255, normalized size = 1.23

$$\mathrm{RootSum}\left(27t^3a^2b^{10} + a^9d^9 - 9a^8bcd^8 + 36a^7b^2c^2d^7 - 84a^6b^3c^3d^6 + 126a^5b^4c^4d^5 - 126a^4b^5c^5d^4 + 84a^3b^6c^6d^3 - 36a^2b^7c^7d\right.$$
$$+ \frac{d^3x^7}{7b} - \frac{x^4\left(ad^3 - 3bcd^2\right)}{4b^2} + \frac{x\left(a^2d^3 - 3abcd^2 + 3b^2c^2d\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c)**3/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**10 + a**9*d**9 - 9*a**8*b*c*d**8 + 36*a*
*7*b**2*c**2*d**7 - 84*a**6*b**3*c**3*d**6 + 126*a**5*b**4*c**4*d
**5 - 126*a**4*b**5*c**5*d**4 + 84*a**3*b**6*c**6*d**3 - 36*a**2*
b**7*c**7*d**2 + 9*a*b**8*c**8*d - b**9*c**9, Lambda(_t, _t*log(-
3*_t*a*b**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3
*c**3) + x))) + d**3*x**7/(7*b) - x**4*(a*d**3 - 3*b*c*d**2)/(4*b
**2) + x*(a**2*d**3 - 3*a*b*c*d**2 + 3*b**2*c**2*d)/b**3
```

**GIAC/XCAS [A]** time = 0.21963, size = 473, normalized size = 2.27

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c^3 - 3\left(-ab^2\right)^{\frac{1}{3}}ab^2c^2d + 3\left(-ab^2\right)^{\frac{1}{3}}a^2bcd^2 - \left(-ab^2\right)^{\frac{1}{3}}a^3d^3\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^4}$$
$$+ \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c^3 - 3\left(-ab^2\right)^{\frac{1}{3}}ab^2c^2d + 3\left(-ab^2\right)^{\frac{1}{3}}a^2bcd^2 - \left(-ab^2\right)^{\frac{1}{3}}a^3d^3\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^4}$$
$$- \frac{\left(b^7c^3 - 3ab^6c^2d + 3a^2b^5cd^2 - a^3b^4d^3\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^7}$$
$$+ \frac{4b^6d^3x^7 + 21b^6cd^2x^4 - 7ab^5d^3x^4 + 84b^6c^2dx - 84ab^5cd^2x + 28a^2b^4d^3x}{28b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c^3 - 3*(-a*b^2)^(1/3)*a*b^2*c^2*
d + 3*(-a*b^2)^(1/3)*a^2*b*c*d^2 - (-a*b^2)^(1/3)*a^3*d^3)*arctan
(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^4) + 1/6*((-
a*b^2)^(1/3)*b^3*c^3 - 3*(-a*b^2)^(1/3)*a*b^2*c^2*d + 3*(-a*b^2)^
(1/3)*a^2*b*c*d^2 - (-a*b^2)^(1/3)*a^3*d^3)*ln(x^2 + x*(-a/b)^(1/
3) + (-a/b)^(2/3))/(a*b^4) - 1/3*(b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2
*b^5*c*d^2 - a^3*b^4*d^3)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/
(a*b^7) + 1/28*(4*b^6*d^3*x^7 + 21*b^6*c*d^2*x^4 - 7*a*b^5*d^3*x^
```

```
4 + 84*b^6*c^2*d*x - 84*a*b^5*c*d^2*x + 28*a^2*b^4*d^3*x)/b^7
```

## 3.16 $\int \frac{(c+dx^3)^2}{a+bx^3} dx$

**Optimal**. Leaf size=173

$$-\frac{(bc-ad)^2 \log \left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}}+\frac{(bc-ad)^2 \log \left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}} -\frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{7/3}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^4}{4b}$$

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^4)/(4*b) - ((b*c - a*d)^2*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(
7/3)) + ((b*c - a*d)^2*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(7/
3)) - ((b*c - a*d)^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/(6*a^(2/3)*b^(7/3))
```

---

**Rubi [A]** time = 0.265867, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$-\frac{(bc-ad)^2 \log \left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}}+\frac{(bc-ad)^2 \log \left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}} -\frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{7/3}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^4}{4b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3)^2/(a + b*x^3),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^4)/(4*b) - ((b*c - a*d)^2*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(
7/3)) + ((b*c - a*d)^2*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(7/
3)) - ((b*c - a*d)^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/(6*a^(2/3)*b^(7/3))
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{d^2x^4}{4b} - \frac{(ad-2bc)\int d\,dx}{b^2} + \frac{(ad-bc)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{2}{3}}b^{\frac{7}{3}}} - \frac{(ad-bc)^2\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}b^{\frac{7}{3}}} - \frac{\sqrt{3}\,(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}b^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c)**2/(b*x**3+a),x)
```

```
[Out] d**2*x**4/(4*b) - (a*d - 2*b*c)*Integral(d, x)/b**2 + (a*d - b*c)
**2*log(a**(1/3) + b**(1/3)*x)/(3*a**(2/3)*b**(7/3)) - (a*d - b*c
)**2*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*a**(2
/3)*b**(7/3)) - sqrt(3)*(a*d - b*c)**2*atan(sqrt(3)*(a**(1/3)/3 -
 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(2/3)*b**(7/3))
```

---

**Mathematica [A]** time = 0.185366, size = 167, normalized size = 0.97

$$\frac{-2(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+3a^{2/3}b^{4/3}d^2x^4-12a^{2/3}\sqrt[3]{b}dx(ad-2bc)+4(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+4\sqrt{3}(b}{12a^{2/3}b^{7/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3)^2/(a + b*x^3),x]
```

```
[Out] (-12*a^(2/3)*b^(1/3)*d*(-2*b*c + a*d)*x + 3*a^(2/3)*b^(4/3)*d^2*x
^4 + 4*Sqrt[3]*(b*c - a*d)^2*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))] + 4*(b*c - a*d)^2*Log[a^(1/3) + b^(1/3)*x] - 2*(b*
c - a*d)^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(12*a^
(2/3)*b^(7/3))
```

---

**Maple [B]** time = 0.005, size = 334, normalized size = 1.9

$$\frac{d^2x^4}{4b} - \frac{ad^2x}{b^2} + 2\frac{dxc}{b} + \frac{a^2d^2}{3b^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{2acd}{3b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{c^2}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{a^2d^2}{6b^3}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{acd}{3b^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{c^2}{6b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{\sqrt{3}a^2d^2}{3b^3}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{2\sqrt{3}cad}{3b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{\sqrt{3}c^2}{3b}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c)^2/(b*x^3+a),x)
```

```
[Out] 1/4*d^2*x^4/b-d^2/b^2*a*x+2*d/b*x*c+1/3/b^3/(a/b)^(2/3)*ln(x+(a/b
)^(1/3))*a^2*d^2-2/3/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c*a*d+1/3/
b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^2-1/6/b^3/(a/b)^(2/3)*ln(x^2-x*
(a/b)^(1/3)+(a/b)^(2/3))*a^2*d^2+1/3/b^2/(a/b)^(2/3)*ln(x^2-x*(a/
b)^(1/3)+(a/b)^(2/3))*c*a*d-1/6/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3
)+(a/b)^(2/3))*c^2+1/3/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1))*a^2*d^2-2/3/b^2/(a/b)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c*a*d+1/3/b/(a/b)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.212444, size = 259, normalized size = 1.5

$$-\frac{\sqrt{3}\left(2\sqrt{3}\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\left(a^2b\right)^{\frac{2}{3}}x^2-\left(a^2b\right)^{\frac{1}{3}}ax+a^2\right)-4\sqrt{3}\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\left(a^2b\right)^{\frac{1}{3}}x+a\right)-12\right.}{36\,\left(a^2b\right)^{\frac{1}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(3)*(2*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log((a^2
*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a*x + a^2) - 4*sqrt(3)*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*log((a^2*b)^(1/3)*x + a) - 12*(b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*arctan(1/3*(2*sqrt(3)*(a^2*b)^(1/3)*x - sqrt(3)*
a)/a) - 3*sqrt(3)*(b*d^2*x^4 + 4*(2*b*c*d - a*d^2)*x)*(a^2*b)^(1/
3))/((a^2*b)^(1/3)*b^2)
```

---

**Sympy [A]** time = 2.99486, size = 156, normalized size = 0.9

$$\operatorname{RootSum}\left(27t^3a^2b^7-a^6d^6+6a^5bcd^5-15a^4b^2c^2d^4+20a^3b^3c^3d^3-15a^2b^4c^4d^2+6ab^5c^5d-b^6c^6,\left(t\mapsto t\log\left(\frac{3tab^2}{a^2d^2-2abcd\ldots}\right.\right.\right.$$
$$+\frac{d^2x^4}{4b}-\frac{x\left(ad^2-2bcd\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c)**2/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**7 - a**6*d**6 + 6*a**5*b*c*d**5 - 15*a**
4*b**2*c**2*d**4 + 20*a**3*b**3*c**3*d**3 - 15*a**2*b**4*c**4*d**
2 + 6*a*b**5*c**5*d - b**6*c**6, Lambda(_t, _t*log(3*_t*a*b**2/(a
**2*d**2 - 2*a*b*c*d + b**2*c**2) + x))) + d**2*x**4/(4*b) - x*(a
*d**2 - 2*b*c*d)/b**2
```

---

**GIAC/XCAS [A]** time = 0.218411, size = 336, normalized size = 1.94

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c^2-2\,\left(-ab^2\right)^{\frac{1}{3}}abcd+\left(-ab^2\right)^{\frac{1}{3}}a^2d^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c^2-2\,\left(-ab^2\right)^{\frac{1}{3}}abcd+\left(-ab^2\right)^{\frac{1}{3}}a^2d^2\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^3}-\frac{\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^4}+\frac{b^3d^2x^4+8\,b^3cdx-4\,ab^2d^2x}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^2*c^2 - 2*(-a*b^2)^(1/3)*a*b*c*d +
(-a*b^2)^(1/3)*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(
-a/b)^(1/3))/(a*b^3) + 1/6*((-a*b^2)^(1/3)*b^2*c^2 - 2*(-a*b^2)^(
1/3)*a*b*c*d + (-a*b^2)^(1/3)*a^2*d^2)*ln(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/(a*b^3) - 1/3*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)
*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a*b^4) + 1/4*(b^3*d^2*x^
4 + 8*b^3*c*d*x - 4*a*b^2*d^2*x)/b^4
```

## 3.17 $\int \frac{c+dx^3}{a+bx^3}\,dx$

**Optimal**. Leaf size=145

$$-\frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}}+\frac{dx}{b}$$

```
[Out] (d*x)/b - ((b*c - a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(2/3)*b^(4/3)) + ((b*c - a*d)*Log[a^(1/3) + b
^(1/3)*x])/(3*a^(2/3)*b^(4/3)) - ((b*c - a*d)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/3))
```

---

**Rubi [A]** time = 0.182217, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$

$$-\frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3)/(a + b*x^3),x]
```

```
[Out] (d*x)/b - ((b*c - a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(2/3)*b^(4/3)) + ((b*c - a*d)*Log[a^(1/3) + b
^(1/3)*x])/(3*a^(2/3)*b^(4/3)) - ((b*c - a*d)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/3))
```

---

**Rubi in Sympy [A]** time = 37.7057, size = 134, normalized size = 0.92

$$\frac{dx}{b}-\frac{(ad-bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{2}{3}}b^{\frac{4}{3}}}+\frac{(ad-bc)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}b^{\frac{4}{3}}}+\frac{\sqrt{3}\,(ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}b^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c)/(b*x**3+a),x)
```

```
[Out] d*x/b - (a*d - b*c)*log(a**(1/3) + b**(1/3)*x)/(3*a**(2/3)*b**(4/
3)) + (a*d - b*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x
```

```
**2)/(6*a**(2/3)*b**(4/3)) + sqrt(3)*(a*d - b*c)*atan(sqrt(3)*(a*
*(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(2/3)*b**(4/3))
```

**Mathematica [A]** time = 0.109031, size = 129, normalized size = 0.89

$$\frac{-(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+6a^{2/3}\sqrt[3]{b}dx+2(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3)/(a + b*x^3),x]
```

```
[Out] (6*a^(2/3)*b^(1/3)*d*x - 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*b^(
1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(b*c - a*d)*Log[a^(1/3) + b^(1/3)*x
] - (b*c - a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
6*a^(2/3)*b^(4/3))
```

**Maple [A]** time = 0.005, size = 195, normalized size = 1.3

$$\frac{dx}{b}-\frac{ad}{3\,b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{c}{3\,b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{ad}{6\,b^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{6\,b}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$-\frac{\sqrt{3}ad}{3\,b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}c}{3\,b}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c)/(b*x^3+a),x)
```

```
[Out] d*x/b-1/3/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a*d+1/3/b/(a/b)^(2/3)
*ln(x+(a/b)^(1/3))*c+1/6/b^2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/
b)^(2/3))*a*d-1/6/b/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))
*c-1/3/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1))*a*d+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x-1))*c
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.215956, size = 184, normalized size = 1.27

$$\frac{\sqrt{3}\left(6\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}dx + \sqrt{3}(bc-ad)\log\left(\left(-a^2b\right)^{\frac{2}{3}}x^2 + \left(-a^2b\right)^{\frac{1}{3}}ax + a^2\right) - 2\sqrt{3}(bc-ad)\log\left(\left(-a^2b\right)^{\frac{1}{3}}x - a\right) + 6\,(bc-ad\right.}{18\,\left(-a^2b\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(6*sqrt(3)*(-a^2*b)^(1/3)*d*x + sqrt(3)*(b*c - a*d)*
log((-a^2*b)^(2/3)*x^2 + (-a^2*b)^(1/3)*a*x + a^2) - 2*sqrt(3)*(b
*c - a*d)*log((-a^2*b)^(1/3)*x - a) + 6*(b*c - a*d)*arctan(1/3*(2
*sqrt(3)*(-a^2*b)^(1/3)*x + sqrt(3)*a)/a))/((-a^2*b)^(1/3)*b)
```

---

**Sympy [A]** time = 2.02482, size = 71, normalized size = 0.49

$$\operatorname{RootSum}\left(27t^3a^2b^4 + a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3, \left(t \mapsto t\log\left(-\frac{3tab}{ad-bc} + x\right)\right)\right) + \frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**4 + a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b*
*2*c**2*d - b**3*c**3, Lambda(_t, _t*log(-3*_t*a*b/(a*d - b*c) +
x))) + d*x/b
```

---

**GIAC/XCAS [A]** time = 0.215884, size = 217, normalized size = 1.5

$$\frac{dx}{b} - \frac{(bc-ad)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bc-\left(-ab^2\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}bc-\left(-ab^2\right)^{\frac{1}{3}}ad\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] d*x/b - 1/3*(b*c - a*d)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a
*b) + 1/3*sqrt(3)*((-a*b^2)^(1/3)*b*c - (-a*b^2)^(1/3)*a*d)*arcta
n(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/6*((
-a*b^2)^(1/3)*b*c - (-a*b^2)^(1/3)*a*d)*ln(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/(a*b^2)
```

## 3.18 $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

**Optimal**. Leaf size=288

$$-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)}+\frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)}-\frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)}+\frac{d^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)}$$

```
[Out] -((b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*a^(2/3)*(b*c - a*d))) + (d^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3
)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*(b*c - a*d)) + (b^(2/3)
*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*(b*c - a*d)) - (d^(2/3)*Log
[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c - a*d)) - (b^(2/3)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*(b*c - a*d))
+ (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(
2/3)*(b*c - a*d))
```

---

**Rubi [A]** time = 0.320915, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)}+\frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)}-\frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)}+\frac{d^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -((b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*a^(2/3)*(b*c - a*d))) + (d^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3
)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*(b*c - a*d)) + (b^(2/3)
*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*(b*c - a*d)) - (d^(2/3)*Log
[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c - a*d)) - (b^(2/3)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*(b*c - a*d))
+ (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(
2/3)*(b*c - a*d))
```

---

**Rubi in Sympy [A]** time = 68.2094, size = 260, normalized size = 0.9

$$\frac{d^{\frac{2}{3}}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{\frac{2}{3}}\left(ad-bc\right)} - \frac{d^{\frac{2}{3}}\log\left(c^{\frac{2}{3}}-\sqrt[3]{c}\sqrt[3]{d}x+d^{\frac{2}{3}}x^2\right)}{6c^{\frac{2}{3}}\left(ad-bc\right)} - \frac{\sqrt{3}d^{\frac{2}{3}}\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3}-\frac{2\sqrt[3]{d}x}{3}\right)}{\sqrt[3]{c}}\right)}{3c^{\frac{2}{3}}\left(ad-bc\right)}$$
$$- \frac{b^{\frac{2}{3}}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{2}{3}}\left(ad-bc\right)} + \frac{b^{\frac{2}{3}}\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}\left(ad-bc\right)} + \frac{\sqrt{3}b^{\frac{2}{3}}\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] d**(2/3)*log(c**(1/3) + d**(1/3)*x)/(3*c**(2/3)*(a*d - b*c)) - d*
*(2/3)*log(c**(2/3) - c**(1/3)*d**(1/3)*x + d**(2/3)*x**2)/(6*c**
(2/3)*(a*d - b*c)) - sqrt(3)*d**(2/3)*atan(sqrt(3)*(c**(1/3)/3 -
2*d**(1/3)*x/3)/c**(1/3))/(3*c**(2/3)*(a*d - b*c)) - b**(2/3)*log
(a**(1/3) + b**(1/3)*x)/(3*a**(2/3)*(a*d - b*c)) + b**(2/3)*log(a
**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(6*a**(2/3)*(a*d -
 b*c)) + sqrt(3)*b**(2/3)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x
/3)/a**(1/3))/(3*a**(2/3)*(a*d - b*c))
```

---

**Mathematica [A]** time = 0.219728, size = 224, normalized size = 0.78

$$\frac{\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}} - \frac{2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + \frac{2\sqrt{3}b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{2/3}} + \frac{2d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{2/3}} - \cdots}{6ad-6bc}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/
a^(2/3) - (2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/S
qrt[3]])/c^(2/3) - (2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) +
 (2*d^(2/3)*Log[c^(1/3) + d^(1/3)*x])/c^(2/3) + (b^(2/3)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3) - (d^(2/3)*Log[c^
(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(2/3))/(-6*b*c + 6*a*
d)
```

---

**Maple [A]** time = 0.009, size = 222, normalized size = 0.8

$$-\frac{1}{3\,ad-3\,bc}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{1}{6\,ad-6\,bc}\ln\left(x^{2}-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{\sqrt{3}}{3\,ad-3\,bc}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{1}{3\,ad-3\,bc}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{1}{6\,ad-6\,bc}\ln\left(x^{2}-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}}{3\,ad-3\,bc}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3/(a*d-b*c)/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/6/(a*d-b*c)/(a/b)^
(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))-1/3/(a*d-b*c)/(a/b)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/(a*d-b*c)/(c
/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/(a*d-b*c)/(c/d)^(2/3)*ln(x^2-x*(c
/d)^(1/3)+(c/d)^(2/3))+1/3/(a*d-b*c)/(c/d)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^3 + a)*(d*x^3 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.279112, size = 377, normalized size = 1.31

$$\sqrt{3}\left(\sqrt{3}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x^{2}+abx\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}+a^{2}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}\right)+\sqrt{3}\left(\frac{d^{2}}{c^{2}}\right)^{\frac{1}{3}}\log\left(d^{2}x^{2}-cdx\left(\frac{d^{2}}{c^{2}}\right)^{\frac{1}{3}}+c^{2}\left(\frac{d^{2}}{c^{2}}\right)^{\frac{2}{3}}\right)-2\sqrt{3}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\right.$$

18

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^3 + a)*(d*x^3 + c)),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(sqrt(3)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/
a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + sqrt(3)*(d^2/c^2)^(1/3)*log(
d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) - 2*sqrt(3
)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) - 2*sqrt(3)*(d^2
/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) + 6*(-b^2/a^2)^(1/3)*arc
tan(1/3*(2*sqrt(3)*b*x + sqrt(3)*a*(-b^2/a^2)^(1/3))/(a*(-b^2/a^2
)^(1/3))) + 6*(d^2/c^2)^(1/3)*arctan(-1/3*(2*sqrt(3)*d*x - sqrt(3
)*c*(d^2/c^2)^(1/3))/(c*(d^2/c^2)^(1/3))))/(b*c - a*d)
```

---

**Sympy [A]** time = 104.613, size = 447, normalized size = 1.55

$$\operatorname{RootSum}\left(t^{3}\left(27a^{5}d^{3}-81a^{4}bcd^{2}+81a^{3}b^{2}c^{2}d-27a^{2}b^{3}c^{3}\right)+b^{2},\left(t\mapsto t\log\left(x+\frac{81t^{4}a^{7}c^{2}d^{5}-243t^{4}a^{6}bc^{3}d^{4}+162t^{4}a^{5}b^{2}c^{4}\cdots}{\cdots}\right.\right.\right.$$
$$+\operatorname{RootSum}\left(t^{3}\left(27a^{3}c^{2}d^{3}-81a^{2}bc^{3}d^{2}+81ab^{2}c^{4}d-27b^{3}c^{5}\right)-d^{2},\left(t\mapsto t\log\left(x+\frac{81t^{4}a^{7}c^{2}d^{5}-243t^{4}a^{6}bc^{3}d^{4}+162t^{4}a^{5}b^{2}c\cdots}{\cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**5*d**3 - 81*a**4*b*c*d**2 + 81*a**3*b**2*c**
2*d - 27*a**2*b**3*c**3) + b**2, Lambda(_t, _t*log(x + (81*_t**4*
a**7*c**2*d**5 - 243*_t**4*a**6*b*c**3*d**4 + 162*_t**4*a**5*b**2
*c**4*d**3 + 162*_t**4*a**4*b**3*c**5*d**2 - 243*_t**4*a**3*b**4*
c**6*d + 81*_t**4*a**2*b**5*c**7 - 3*_t*a**4*d**4 + 3*_t*a**3*b*c
*d**3 + 3*_t*a*b**3*c**3*d - 3*_t*b**4*c**4)/(a**2*b*d**3 + b**3*
c**2*d)))) + RootSum(_t**3*(27*a**3*c**2*d**3 - 81*a**2*b*c**3*d*
*2 + 81*a*b**2*c**4*d - 27*b**3*c**5) - d**2, Lambda(_t, _t*log(x
 + (81*_t**4*a**7*c**2*d**5 - 243*_t**4*a**6*b*c**3*d**4 + 162*_t
**4*a**5*b**2*c**4*d**3 + 162*_t**4*a**4*b**3*c**5*d**2 - 243*_t*
*4*a**3*b**4*c**6*d + 81*_t**4*a**2*b**5*c**7 - 3*_t*a**4*d**4 +
3*_t*a**3*b*c*d**3 + 3*_t*a*b**3*c**3*d - 3*_t*b**4*c**4)/(a**2*b
*d**3 + b**3*c**2*d))))
```

---

**GIAC/XCAS [A]** time = 0.227861, size = 375, normalized size = 1.3

$$-\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(abc-a^2d)}+\frac{d\left(-\frac{c}{d}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}abc-\sqrt{3}a^2d}-\frac{\left(-cd^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2-\sqrt{3}acd}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(abc-a^2d)}-\frac{\left(-cd^2\right)^{\frac{1}{3}}\ln\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^2-acd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^3 + a)*(d*x^3 + c)),x, algorithm="giac")
```

```
[Out] -1/3*b*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1
/3*d*(-c/d)^(1/3)*ln(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a
*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))
/(sqrt(3)*a*b*c - sqrt(3)*a^2*d) - (-c*d^2)^(1/3)*arctan(1/3*sqrt
(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(sqrt(3)*b*c^2 - sqrt(3)*a
*c*d) + 1/6*(-a*b^2)^(1/3)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)
)/(a*b*c - a^2*d) - 1/6*(-c*d^2)^(1/3)*ln(x^2 + x*(-c/d)^(1/3) +
(-c/d)^(2/3))/(b*c^2 - a*c*d)
```

## 3.19 $\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx$

**Optimal**. Leaf size=346

$$-\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)^2}+\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)^2}-\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)^2}+\frac{d^{2/3}(5bc-2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}(bc-ad)^2}-\frac{d^{2/3}(5bc-2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^2}+\frac{d^{2/3}(5bc-2ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^2}-\frac{dx}{3c\left(c+dx^3\right)(bc-ad)}$$

```
[Out] -(d*x)/(3*c*(b*c - a*d)*(c + d*x^3)) - (b^(5/3)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(b*c - a*d)^2)
 + (d^(2/3)*(5*b*c - 2*a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[
3]*c^(1/3))])/(3*Sqrt[3]*c^(5/3)*(b*c - a*d)^2) + (b^(5/3)*Log[a^
(1/3) + b^(1/3)*x])/(3*a^(2/3)*(b*c - a*d)^2) - (d^(2/3)*(5*b*c -
 2*a*d)*Log[c^(1/3) + d^(1/3)*x])/(9*c^(5/3)*(b*c - a*d)^2) - (b^
(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*
(b*c - a*d)^2) + (d^(2/3)*(5*b*c - 2*a*d)*Log[c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2])/(18*c^(5/3)*(b*c - a*d)^2)
```

---

**Rubi [A]** time = 0.613004, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$-\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)^2}+\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)^2}-\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)^2}+\frac{d^{2/3}(5bc-2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}(bc-ad)^2}-\frac{d^{2/3}(5bc-2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^2}+\frac{d^{2/3}(5bc-2ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^2}-\frac{dx}{3c\left(c+dx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^3)*(c + d*x^3)^2),x]
```

```
[Out] -(d*x)/(3*c*(b*c - a*d)*(c + d*x^3)) - (b^(5/3)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(b*c - a*d)^2)
 + (d^(2/3)*(5*b*c - 2*a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[
3]*c^(1/3))])/(3*Sqrt[3]*c^(5/3)*(b*c - a*d)^2) + (b^(5/3)*Log[a^
(1/3) + b^(1/3)*x])/(3*a^(2/3)*(b*c - a*d)^2) - (d^(2/3)*(5*b*c -
```

```
 2*a*d)*Log[c^(1/3) + d^(1/3)*x])/(9*c^(5/3)*(b*c - a*d)^2) - (b^
(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*
(b*c - a*d)^2) + (d^(2/3)*(5*b*c - 2*a*d)*Log[c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2])/(18*c^(5/3)*(b*c - a*d)^2)
```

---

**Rubi in Sympy [A]** time = 108.441, size = 321, normalized size = 0.93

$$\frac{dx}{3c\left(c+dx^{3}\right)\left(ad-bc\right)}+\frac{d^{\frac{2}{3}}\left(2ad-5bc\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{\frac{5}{3}}\left(ad-bc\right)^{2}}-\frac{d^{\frac{2}{3}}\left(2ad-5bc\right)\log\left(c^{\frac{2}{3}}-\sqrt[3]{c}\sqrt[3]{d}x+d^{\frac{2}{3}}x^{2}\right)}{18c^{\frac{5}{3}}\left(ad-bc\right)^{2}}-\frac{\sqrt{3}d^{\frac{2}{3}}\left(2ad-5bc\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3}-\frac{2\sqrt[3]{d}x}{3}\right)}{\sqrt[3]{c}}\right)}{9c^{\frac{5}{3}}\left(ad-bc\right)^{2}}+\frac{b^{\frac{5}{3}}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{\frac{2}{3}}\left(ad-bc\right)^{2}}-\frac{b^{\frac{5}{3}}\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}\right)}{6a^{\frac{2}{3}}\left(ad-bc\right)^{2}}-\frac{\sqrt{3}b^{\frac{5}{3}}\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}\left(ad-bc\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**3+a)/(d*x**3+c)**2,x)
```

```
[Out] d*x/(3*c*(c + d*x**3)*(a*d - b*c)) + d**(2/3)*(2*a*d - 5*b*c)*log
(c**(1/3) + d**(1/3)*x)/(9*c**(5/3)*(a*d - b*c)**2) - d**(2/3)*(2
*a*d - 5*b*c)*log(c**(2/3) - c**(1/3)*d**(1/3)*x + d**(2/3)*x**2)
/(18*c**(5/3)*(a*d - b*c)**2) - sqrt(3)*d**(2/3)*(2*a*d - 5*b*c)*
atan(sqrt(3)*(c**(1/3)/3 - 2*d**(1/3)*x/3)/c**(1/3))/(9*c**(5/3)*
(a*d - b*c)**2) + b**(5/3)*log(a**(1/3) + b**(1/3)*x)/(3*a**(2/3)
*(a*d - b*c)**2) - b**(5/3)*log(a**(2/3) - a**(1/3)*b**(1/3)*x +
b**(2/3)*x**2)/(6*a**(2/3)*(a*d - b*c)**2) - sqrt(3)*b**(5/3)*ata
n(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(3*a**(2/3)*(a*
d - b*c)**2)
```

---

**Mathematica [A]** time = 0.351284, size = 336, normalized size = 0.97

$$-3b^{5/3}c^{5/3}\left(c+dx^{3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^{2}\right)+a^{2/3}d^{2/3}\left(c+dx^{3}\right)\left(5bc-2ad\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^{2}\right)+6a^{2/3}c^{2/}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^3)*(c + d*x^3)^2),x]
```

```
[Out] (6*a^(2/3)*c^(2/3)*d*(-(b*c) + a*d)*x - 6*Sqrt[3]*b^(5/3)*c^(5/3)
*(c + d*x^3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Sqrt
[3]*a^(2/3)*d^(2/3)*(-5*b*c + 2*a*d)*(c + d*x^3)*ArcTan[(1 - (2*d
^(1/3)*x)/c^(1/3))/Sqrt[3]] + 6*b^(5/3)*c^(5/3)*(c + d*x^3)*Log[a
^(1/3) + b^(1/3)*x] + 2*a^(2/3)*d^(2/3)*(-5*b*c + 2*a*d)*(c + d*x
^3)*Log[c^(1/3) + d^(1/3)*x] - 3*b^(5/3)*c^(5/3)*(c + d*x^3)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(2/3)*d^(2/3)*(5*b
*c - 2*a*d)*(c + d*x^3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2])/(18*a^(2/3)*c^(5/3)*(b*c - a*d)^2*(c + d*x^3))
```

---

**Maple [A]** time = 0.018, size = 406, normalized size = 1.2

$$\frac{b}{3\,(ad-bc)^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{b}{6\,(ad-bc)^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$
$$+\frac{b\sqrt{3}}{3\,(ad-bc)^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{d^2xa}{3\,(ad-bc)^2\,c\,(dx^3+c)}$$
$$-\frac{dxb}{3\,(ad-bc)^2\,(dx^3+c)}+\frac{2\,ad}{9\,(ad-bc)^2\,c}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}$$
$$-\frac{5\,b}{9\,(ad-bc)^2}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{ad}{9\,(ad-bc)^2\,c}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}$$
$$+\frac{5\,b}{18\,(ad-bc)^2}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}$$
$$+\frac{2\,d\sqrt{3}a}{9\,(ad-bc)^2\,c}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}$$
$$-\frac{5\,b\sqrt{3}}{9\,(ad-bc)^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^3+a)/(d*x^3+c)^2,x)
```

```
[Out] 1/3*b/(a*d-b*c)^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*b/(a*d-b*c)^2
/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))+1/3*b/(a*d-b*c)^2/
(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*d
^2/(a*d-b*c)^2/c*x/(d*x^3+c)*a-1/3*d/(a*d-b*c)^2*x/(d*x^3+c)*b+2/
9*d/(a*d-b*c)^2/c/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a-5/9/(a*d-b*c)^2
/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*b-1/9*d/(a*d-b*c)^2/c/(c/d)^(2/3)*
```

```
ln(x^2-x*(c/d)^(1/3)+(c/d)^(2/3))*a+5/18/(a*d-b*c)^2/(c/d)^(2/3)*
ln(x^2-x*(c/d)^(1/3)+(c/d)^(2/3))*b+2/9*d/(a*d-b*c)^2/c/(c/d)^(2/
3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a-5/9/(a*d-b*c
)^2/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^3 + a)*(d*x^3 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 7.36613, size = 625, normalized size = 1.81

$$-\frac{\sqrt{3}\left(3\sqrt{3}\left(bcdx^3+bc^2\right)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2-abx\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)-\sqrt{3}\left(\left(5\,bcd-2\,ad^2\right)x^3+5\,bc^2-2\,acd\right)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(d^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^3 + a)*(d*x^3 + c)^2),x, algorithm="fricas")
```

```
[Out] -1/54*sqrt(3)*(3*sqrt(3)*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*log(
b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) - sqrt(3)*
((5*b*c*d - 2*a*d^2)*x^3 + 5*b*c^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*log
(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) - 6*sqrt(
3)*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3
)) + 2*sqrt(3)*((5*b*c*d - 2*a*d^2)*x^3 + 5*b*c^2 - 2*a*c*d)*(d^2
/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) + 6*sqrt(3)*(b*c*d - a*d
^2)*x + 18*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*arctan(-1/3*(2*sqr
t(3)*b*x - sqrt(3)*a*(b^2/a^2)^(1/3))/(a*(b^2/a^2)^(1/3))) - 6*((
5*b*c*d - 2*a*d^2)*x^3 + 5*b*c^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*arcta
n(-1/3*(2*sqrt(3)*d*x - sqrt(3)*c*(d^2/c^2)^(1/3))/(c*(d^2/c^2)^(
1/3))))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b
*c^2*d^2 + a^2*c*d^3)*x^3)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**3+a)/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [A]** time = 0.227324, size = 598, normalized size = 1.73

$$-\frac{b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(ab^2c^2-2\,a^2bcd+a^3d^2)}+\frac{\left(-ab^2\right)^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}ab^2c^2-2\,\sqrt{3}a^2bcd+\sqrt{3}a^3d^2}+\frac{\left(-ab^2\right)^{\frac{1}{3}}b\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(ab^2c^2-2\,a^2bcd+a^3d^2)}+\frac{\left(5\,bcd-2\,ad^2\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\,(b^2c^4-2\,abc^3d+a^2c^2d^2)}-\frac{\left(5\left(-cd^2\right)^{\frac{1}{3}}bc-2\left(-cd^2\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(\sqrt{3}b^2c^4-2\,\sqrt{3}abc^3d+\sqrt{3}a^2c^2d^2\right)}-\frac{\left(5\left(-cd^2\right)^{\frac{1}{3}}bc-2\left(-cd^2\right)^{\frac{1}{3}}ad\right)\ln\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\,(b^2c^4-2\,abc^3d+a^2c^2d^2)}-\frac{dx}{3\,(dx^3+c)(bc^2-acd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^3 + a)*(d*x^3 + c)^2),x, algorithm="giac")
```

```
[Out] -1/3*b^2*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a*b^2*c^2 - 2*a^
2*b*c*d + a^3*d^2) + (-a*b^2)^(1/3)*b*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b^2*c^2 - 2*sqrt(3)*a^2*b*c
*d + sqrt(3)*a^3*d^2) + 1/6*(-a*b^2)^(1/3)*b*ln(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2) + 1/9*(5*
b*c*d - 2*a*d^2)*(-c/d)^(1/3)*ln(abs(x - (-c/d)^(1/3)))/(b^2*c^4
- 2*a*b*c^3*d + a^2*c^2*d^2) - 1/3*(5*(-c*d^2)^(1/3)*b*c - 2*(-c*
d^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1
/3))/(sqrt(3)*b^2*c^4 - 2*sqrt(3)*a*b*c^3*d + sqrt(3)*a^2*c^2*d^2
) - 1/18*(5*(-c*d^2)^(1/3)*b*c - 2*(-c*d^2)^(1/3)*a*d)*ln(x^2 + x
*(-c/d)^(1/3) + (-c/d)^(2/3))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^
2) - 1/3*d*x/((d*x^3 + c)*(b*c^2 - a*c*d))
```

## 3.20 $\int \frac{\left(c+dx^3\right)^5}{\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=320

$$-\frac{(bc-ad)^4(13ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{16/3}}+\frac{(bc-ad)^4(13ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{16/3}}-\frac{(bc-ad)^4(13ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{16/3}}+\frac{d^3x^4\left(3a^2d^2-10abcd+10b^2c^2\right)}{4b^4}+\frac{d^2x\left(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3\right)}{b^5}+\frac{x(bc-ad)^5}{3ab^5\left(a+bx^3\right)}+\frac{d^4x^7(5bc-2ad)}{7b^3}+\frac{d^5x^{10}}{10b^2}$$

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x
)/b^5 + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4)/(4*b^4) +
 (d^4*(5*b*c - 2*a*d)*x^7)/(7*b^3) + (d^5*x^10)/(10*b^2) + ((b*c
- a*d)^5*x)/(3*a*b^5*(a + b*x^3)) - ((b*c - a*d)^4*(2*b*c + 13*a*
d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*
a^(5/3)*b^(16/3)) + ((b*c - a*d)^4*(2*b*c + 13*a*d)*Log[a^(1/3) +
 b^(1/3)*x])/(9*a^(5/3)*b^(16/3)) - ((b*c - a*d)^4*(2*b*c + 13*a*
d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^
(16/3))
```

---

**Rubi [A]** time = 0.63287, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421

$$-\frac{(bc-ad)^4(13ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{16/3}}+\frac{(bc-ad)^4(13ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{16/3}}-\frac{(bc-ad)^4(13ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{16/3}}+\frac{d^3x^4\left(3a^2d^2-10abcd+10b^2c^2\right)}{4b^4}+\frac{d^2x\left(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3\right)}{b^5}+\frac{x(bc-ad)^5}{3ab^5\left(a+bx^3\right)}+\frac{d^4x^7(5bc-2ad)}{7b^3}+\frac{d^5x^{10}}{10b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3)^5/(a + b*x^3)^2,x]
```



```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x
)/b^5 + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4)/(4*b^4) +
 (d^4*(5*b*c - 2*a*d)*x^7)/(7*b^3) + (d^5*x^10)/(10*b^2) + ((b*c
- a*d)^5*x)/(3*a*b^5*(a + b*x^3)) - ((b*c - a*d)^4*(2*b*c + 13*a*
d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*
a^(5/3)*b^(16/3)) + ((b*c - a*d)^4*(2*b*c + 13*a*d)*Log[a^(1/3) +
 b^(1/3)*x])/(9*a^(5/3)*b^(16/3)) - ((b*c - a*d)^4*(2*b*c + 13*a*
d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^
```

```
(16/3))
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-d^{2}\left(4a^{3}d^{3}-15a^{2}bcd^{2}+20ab^{2}c^{2}d-10b^{3}c^{3}\right)\int\frac{1}{b^{5}}\,dx+\frac{d^{5}x^{10}}{10b^{2}}-\frac{d^{4}x^{7}\left(2ad-5bc\right)}{7b^{3}}+\frac{d^{3}x^{4}\left(3a^{2}d^{2}-10abcd+10b^{2}c^{2}\right)}{4b^{4}}-\frac{x\left(ad-bc\right)^{5}}{3ab^{5}\left(a+bx^{3}\right)}+\frac{\left(ad-bc\right)^{4}\left(13ad+2bc\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{5}{3}}b^{\frac{16}{3}}}-\frac{\left(ad-bc\right)^{4}\left(13ad+2bc\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^{2}\right)}{18a^{\frac{5}{3}}b^{\frac{16}{3}}}-\frac{\sqrt{3}\left(ad-bc\right)^{4}\left(13ad+2bc\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{5}{3}}b^{\frac{16}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c)**5/(b*x**3+a)**2,x)
```

```
[Out] -d**2*(4*a**3*d**3 - 15*a**2*b*c*d**2 + 20*a*b**2*c**2*d - 10*b**
3*c**3)*Integral(b**(-5), x) + d**5*x**10/(10*b**2) - d**4*x**7*(
2*a*d - 5*b*c)/(7*b**3) + d**3*x**4*(3*a**2*d**2 - 10*a*b*c*d + 1
0*b**2*c**2)/(4*b**4) - x*(a*d - b*c)**5/(3*a*b**5*(a + b*x**3))
+ (a*d - b*c)**4*(13*a*d + 2*b*c)*log(a**(1/3) + b**(1/3)*x)/(9*a
**(5/3)*b**(16/3)) - (a*d - b*c)**4*(13*a*d + 2*b*c)*log(a**(2/3)
 - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(18*a**(5/3)*b**(16/3)) -
 sqrt(3)*(a*d - b*c)**4*(13*a*d + 2*b*c)*atan(sqrt(3)*(a**(1/3)/3
 - 2*b**(1/3)*x/3)/a**(1/3))/(9*a**(5/3)*b**(16/3))
```

---

**Mathematica [A]** time = 0.438508, size = 313, normalized size = 0.98

$$-\frac{70(bc-ad)^{4}(13ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^{2}\right)}{a^{5/3}}+\frac{140(bc-ad)^{4}(13ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}}+\frac{140\sqrt{3}(bc-ad)^{4}(13ad+2bc)\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}+3$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3)^5/(a + b*x^3)^2,x]
```

```
[Out] (1260*b^(1/3)*d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 -
 4*a^3*d^3)*x + 315*b^(4/3)*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*
d^2)*x^4 + 180*b^(7/3)*d^4*(5*b*c - 2*a*d)*x^7 + 126*b^(10/3)*d^5
```

```
*x^10 + (420*b^(1/3)*(b*c - a*d)^5*x)/(a*(a + b*x^3)) + (140*Sqrt
[3]*(b*c - a*d)^4*(2*b*c + 13*a*d)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (140*(b*c - a*d)^4*(2*b*c + 13*a*
d)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (70*(b*c - a*d)^4*(2*b*c +
 13*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))
/(1260*b^(16/3))
```

---

**Maple [B]** time = 0.017, size = 905, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c)^5/(b*x^3+a)^2,x)
```

```
[Out] 13/9/b^6*a^4/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d^5+5/9/b^2/(a/b)^(2/3
)*ln(x+(a/b)^(1/3))*c^4*d+2/9/b/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c
^5-13/18/b^6*a^4/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d^
5-5/18/b^2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c^4*d-1/
9/b/a/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c^5-20*d^3/b^
3*a*c^2*x-5/2*d^4/b^3*x^4*a*c+15*d^4/b^4*a^2*c*x-1/3/b^5*x*a^4/(b
*x^3+a)*d^5+70/9/b^4*a^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^2*d^3-40
/9/b^3*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^3*d^2+25/9/b^5*a^3/(a/b)
^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c*d^4-35/9/b^4*a^2/(a/b)
^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c^2*d^3+1/3*x/a/(b*x^3+a
)*c^5+10*d^2/b^2*c^3*x-2/7*d^5/b^3*x^7*a+5/7*d^4/b^2*x^7*c+3/4*d^
5/b^4*x^4*a^2+5/2*d^3/b^2*x^4*c^2-4*d^5/b^5*a^3*x+20/9/b^3*a/(a/b
)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c^3*d^2-5/3/b*x/(b*x^3+
a)*c^4*d-50/9/b^5*a^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1))*c*d^4+70/9/b^4*a^2/(a/b)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^2*d^3-40/9/b^3*a/(a/b)^(2/3)*3^(
1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^3*d^2+1/10*d^5*x^1
0/b^2+2/9/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/
3)*x-1))*c^5+13/9/b^6*a^4/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*x-1))*d^5+5/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))*c^4*d-50/9/b^5*a^3/(a/b)^(2/3)*ln(x+(
a/b)^(1/3))*c*d^4+5/3/b^4*x*a^3/(b*x^3+a)*c*d^4-10/3/b^3*x*a^2/(b
*x^3+a)*c^2*d^3+10/3/b^2*x*a/(b*x^3+a)*c^3*d^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^5/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.223337, size = 1000, normalized size = 3.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^5/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] -1/3780*sqrt(3)*(70*sqrt(3)*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d - 40*a
^3*b^3*c^3*d^2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5
 + (2*b^6*c^5 + 5*a*b^5*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3*c
^2*d^3 - 50*a^4*b^2*c*d^4 + 13*a^5*b*d^5)*x^3)*log((a^2*b)^(2/3)*
x^2 - (a^2*b)^(1/3)*a*x + a^2) - 140*sqrt(3)*(2*a*b^5*c^5 + 5*a^2
*b^4*c^4*d - 40*a^3*b^3*c^3*d^2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c
*d^4 + 13*a^6*d^5 + (2*b^6*c^5 + 5*a*b^5*c^4*d - 40*a^2*b^4*c^3*d
^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*b^2*c*d^4 + 13*a^5*b*d^5)*x^3)*l
og((a^2*b)^(1/3)*x + a) - 420*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d - 40
*a^3*b^3*c^3*d^2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d
^5 + (2*b^6*c^5 + 5*a*b^5*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3
*c^2*d^3 - 50*a^4*b^2*c*d^4 + 13*a^5*b*d^5)*x^3)*arctan(1/3*(2*sq
rt(3)*(a^2*b)^(1/3)*x - sqrt(3)*a)/a) - 3*sqrt(3)*(42*a*b^4*d^5*x
^13 + 6*(50*a*b^4*c*d^4 - 13*a^2*b^3*d^5)*x^10 + 15*(70*a*b^4*c^2
*d^3 - 50*a^2*b^3*c*d^4 + 13*a^3*b^2*d^5)*x^7 + 105*(40*a*b^4*c^3
*d^2 - 70*a^2*b^3*c^2*d^3 + 50*a^3*b^2*c*d^4 - 13*a^4*b*d^5)*x^4
+ 140*(b^5*c^5 - 5*a*b^4*c^4*d + 40*a^2*b^3*c^3*d^2 - 70*a^3*b^2*
c^2*d^3 + 50*a^4*b*c*d^4 - 13*a^5*d^5)*x)*(a^2*b)^(1/3))/((a*b^6*
x^3 + a^2*b^5)*(a^2*b)^(1/3))
```

---

**Sympy [A]** time = 25.775, size = 536, normalized size = 1.68

$$-\frac{x\left(a^5d^5-5a^4bcd^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5ab^4c^4d-b^5c^5\right)}{3a^2b^5+3ab^6x^3}$$
$$+\operatorname{RootSum}\Bigl(729t^3a^5b^{16}-2197a^{15}d^{15}+25350a^{14}bcd^{14}-132990a^{13}b^2c^2d^{13}+418280a^{12}b^3c^3d^{12}-874635a^{11}b^4c^4d^{11}+12718$$
$$+\frac{d^5x^{10}}{10b^2}-\frac{x^7\left(2ad^5-5bcd^4\right)}{7b^3}+\frac{x^4\left(3a^2d^5-10abcd^4+10b^2c^2d^3\right)}{4b^4}$$
$$-\frac{x\left(4a^3d^5-15a^2bcd^4+20ab^2c^2d^3-10b^3c^3d^2\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c)**5/(b*x**3+a)**2,x)
```

```
[Out] -x*(a**5*d**5 - 5*a**4*b*c*d**4 + 10*a**3*b**2*c**2*d**3 - 10*a**
2*b**3*c**3*d**2 + 5*a*b**4*c**4*d - b**5*c**5)/(3*a**2*b**5 + 3*
```

```
a*b**6*x**3) + RootSum(729*_t**3*a**5*b**16 - 2197*a**15*d**15 +
25350*a**14*b*c*d**14 - 132990*a**13*b**2*c**2*d**13 + 418280*a**
12*b**3*c**3*d**12 - 874635*a**11*b**4*c**4*d**11 + 1271886*a**10
*b**5*c**5*d**10 - 1302400*a**9*b**6*c**6*d**9 + 922680*a**8*b**7
*c**7*d**8 - 422235*a**7*b**8*c**8*d**7 + 97570*a**6*b**9*c**9*d*
*6 + 7194*a**5*b**10*c**10*d**5 - 10200*a**4*b**11*c**11*d**4 + 1
435*a**3*b**12*c**12*d**3 + 330*a**2*b**13*c**13*d**2 - 60*a*b**1
4*c**14*d - 8*b**15*c**15, Lambda(_t, _t*log(9*_t*a**2*b**5/(13*a
**5*d**5 - 50*a**4*b*c*d**4 + 70*a**3*b**2*c**2*d**3 - 40*a**2*b*
*3*c**3*d**2 + 5*a*b**4*c**4*d + 2*b**5*c**5) + x))) + d**5*x**10
/(10*b**2) - x**7*(2*a*d**5 - 5*b*c*d**4)/(7*b**3) + x**4*(3*a**2
*d**5 - 10*a*b*c*d**4 + 10*b**2*c**2*d**3)/(4*b**4) - x*(4*a**3*d
**5 - 15*a**2*b*c*d**4 + 20*a*b**2*c**2*d**3 - 10*b**3*c**3*d**2)
/b**5
```

---

**GIAC/XCAS [A]** time = 0.221869, size = 822, normalized size = 2.57

$$-\frac{\left(2\,b^{5}c^{5}+5\,ab^{4}c^{4}d-40\,a^{2}b^{3}c^{3}d^{2}+70\,a^{3}b^{2}c^{2}d^{3}-50\,a^{4}bcd^{4}+13\,a^{5}d^{5}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{2}b^{5}}$$

$$+\frac{\sqrt{3}\Big(2\,\left(-ab^{2}\right)^{\frac{1}{3}}b^{5}c^{5}+5\,\left(-ab^{2}\right)^{\frac{1}{3}}ab^{4}c^{4}d-40\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}b^{3}c^{3}d^{2}+70\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}b^{2}c^{2}d^{3}-50\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{4}bcd^{4}+13\,\left(-ab^{2}\right.}{9\,a^{2}b^{6}}$$

$$+\frac{b^{5}c^{5}x-5\,ab^{4}c^{4}dx+10\,a^{2}b^{3}c^{3}d^{2}x-10\,a^{3}b^{2}c^{2}d^{3}x+5\,a^{4}bcd^{4}x-a^{5}d^{5}x}{3\,(bx^{3}+a)ab^{5}}$$

$$+\frac{\Big(2\,\left(-ab^{2}\right)^{\frac{1}{3}}b^{5}c^{5}+5\,\left(-ab^{2}\right)^{\frac{1}{3}}ab^{4}c^{4}d-40\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{2}b^{3}c^{3}d^{2}+70\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}b^{2}c^{2}d^{3}-50\,\left(-ab^{2}\right)^{\frac{1}{3}}a^{4}bcd^{4}+13\,\left(-ab^{2}\right)^{\frac{1}{3}}}{18\,a^{2}b^{6}}$$

$$+\frac{14\,b^{18}d^{5}x^{10}+100\,b^{18}cd^{4}x^{7}-40\,ab^{17}d^{5}x^{7}+350\,b^{18}c^{2}d^{3}x^{4}-350\,ab^{17}cd^{4}x^{4}+105\,a^{2}b^{16}d^{5}x^{4}+1400\,b^{18}c^{3}d^{2}x-2800\,ab^{17}c^{2}}{140\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^5/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2
*c^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*(-a/b)^(1/3)*ln(abs(x - (
-a/b)^(1/3)))/(a^2*b^5) + 1/9*sqrt(3)*(2*(-a*b^2)^(1/3)*b^5*c^5 +
 5*(-a*b^2)^(1/3)*a*b^4*c^4*d - 40*(-a*b^2)^(1/3)*a^2*b^3*c^3*d^2
 + 70*(-a*b^2)^(1/3)*a^3*b^2*c^2*d^3 - 50*(-a*b^2)^(1/3)*a^4*b*c*
d^4 + 13*(-a*b^2)^(1/3)*a^5*d^5)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/(a^2*b^6) + 1/3*(b^5*c^5*x - 5*a*b^4*c^4*d*
x + 10*a^2*b^3*c^3*d^2*x - 10*a^3*b^2*c^2*d^3*x + 5*a^4*b*c*d^4*x
 - a^5*d^5*x)/((b*x^3 + a)*a*b^5) + 1/18*(2*(-a*b^2)^(1/3)*b^5*c^
5 + 5*(-a*b^2)^(1/3)*a*b^4*c^4*d - 40*(-a*b^2)^(1/3)*a^2*b^3*c^3*
d^2 + 70*(-a*b^2)^(1/3)*a^3*b^2*c^2*d^3 - 50*(-a*b^2)^(1/3)*a^4*b
*c*d^4 + 13*(-a*b^2)^(1/3)*a^5*d^5)*ln(x^2 + x*(-a/b)^(1/3) + (-a
/b)^(2/3))/(a^2*b^6) + 1/140*(14*b^18*d^5*x^10 + 100*b^18*c*d^4*x
^7 - 40*a*b^17*d^5*x^7 + 350*b^18*c^2*d^3*x^4 - 350*a*b^17*c*d^4*
```

```
x^4 + 105*a^2*b^16*d^5*x^4 + 1400*b^18*c^3*d^2*x - 2800*a*b^17*c^
2*d^3*x + 2100*a^2*b^16*c*d^4*x - 560*a^3*b^15*d^5*x)/b^20
```

## 3.21 $\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx$

**Optimal**. Leaf size=267

$$-\frac{(bc-ad)^3(5ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{13/3}}+\frac{2(bc-ad)^3(5ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{13/3}}-\frac{2(bc-ad)^3(5ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{13/3}}+\frac{d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)}{b^4}+\frac{x(bc-ad)^4}{3ab^4\left(a+bx^3\right)}+\frac{d^3x^4(2bc-ad)}{2b^3}+\frac{d^4x^7}{7b^2}$$

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (d^3*(2*b*c - a
*d)*x^4)/(2*b^3) + (d^4*x^7)/(7*b^2) + ((b*c - a*d)^4*x)/(3*a*b^4
*(a + b*x^3)) - (2*(b*c - a*d)^3*(b*c + 5*a*d)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(13/3)) + (
2*(b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3
)*b^(13/3)) - ((b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*b^(13/3))
```

---

**Rubi [A]** time = 0.497773, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$-\frac{(bc-ad)^3(5ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{13/3}}+\frac{2(bc-ad)^3(5ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{13/3}}-\frac{2(bc-ad)^3(5ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{13/3}}+\frac{d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)}{b^4}+\frac{x(bc-ad)^4}{3ab^4\left(a+bx^3\right)}+\frac{d^3x^4(2bc-ad)}{2b^3}+\frac{d^4x^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3)^4/(a + b*x^3)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (d^3*(2*b*c - a
*d)*x^4)/(2*b^3) + (d^4*x^7)/(7*b^2) + ((b*c - a*d)^4*x)/(3*a*b^4
*(a + b*x^3)) - (2*(b*c - a*d)^3*(b*c + 5*a*d)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(13/3)) + (
2*(b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3
)*b^(13/3)) - ((b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*b^(13/3))
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$d^2\left(3a^2d^2 - 8abcd + 6b^2c^2\right)\int \frac{1}{b^4}\,dx + \frac{d^4x^7}{7b^2} - \frac{d^3x^4\,(ad-2bc)}{2b^3} + \frac{x\,(ad-bc)^4}{3ab^4\,(a+bx^3)} - \frac{2\,(ad-bc)^3\,(5ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{5}{3}}b^{\frac{13}{3}}} + \frac{(ad-bc)^3\,(5ad+bc)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{9a^{\frac{5}{3}}b^{\frac{13}{3}}} + \frac{2\sqrt{3}\,(ad-bc)^3\,(5ad+bc)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{5}{3}}b^{\frac{13}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c)**4/(b*x**3+a)**2,x)
```

```
[Out] d**2*(3*a**2*d**2 - 8*a*b*c*d + 6*b**2*c**2)*Integral(b**(-4), x)
 + d**4*x**7/(7*b**2) - d**3*x**4*(a*d - 2*b*c)/(2*b**3) + x*(a*d
 - b*c)**4/(3*a*b**4*(a + b*x**3)) - 2*(a*d - b*c)**3*(5*a*d + b*
c)*log(a**(1/3) + b**(1/3)*x)/(9*a**(5/3)*b**(13/3)) + (a*d - b*c
)**3*(5*a*d + b*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*
x**2)/(9*a**(5/3)*b**(13/3)) + 2*sqrt(3)*(a*d - b*c)**3*(5*a*d +
b*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(9*a**(
5/3)*b**(13/3))
```

---

**Mathematica [A]** time = 0.363644, size = 260, normalized size = 0.97

$$\frac{\frac{14(ad-bc)^3(5ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}} + \frac{28(bc-ad)^3(5ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}} + \frac{28\sqrt{3}(bc-ad)^3(5ad+bc)\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}} + 126\sqrt[3]{b}d^2x}{126b^{13/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3)^4/(a + b*x^3)^2,x]
```

```
[Out] (126*b^(1/3)*d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x + 63*b^(4/
3)*d^3*(2*b*c - a*d)*x^4 + 18*b^(7/3)*d^4*x^7 + (42*b^(1/3)*(b*c
- a*d)^4*x)/(a*(a + b*x^3)) + (28*Sqrt[3]*(b*c - a*d)^3*(b*c + 5*
a*d)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3)
+ (28*(b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(1/3) + b^(1/3)*x])/a^(5/
3) + (14*(-(b*c) + a*d)^3*(b*c + 5*a*d)*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/a^(5/3))/(126*b^(13/3))
```

---

**Maple [B]** time = 0.016, size = 708, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c)^4/(b*x^3+a)^2,x)
```

```
[Out] -8*d^3/b^3*a*c*x+5/9/b^5*a^3/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/
b)^(2/3))*d^4-2/9/b^2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3
))*c^3*d+28/9/b^4*a^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1))*c*d^3-8/3/b^3*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))*c^2*d^2-1/2*d^4/b^3*x^4*a+d^3/b^2*x^4
*c+3*d^4/b^4*a^2*x+6*d^2/b^2*c^2*x+1/3*x/a/(b*x^3+a)*c^4-1/9/b/a/
(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c^4-10/9/b^5*a^3/(a
/b)^(2/3)*ln(x+(a/b)^(1/3))*d^4+4/9/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1
/3))*c^3*d+2/9/b/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^4+1/3/b^4*x*a^
3/(b*x^3+a)*d^4-4/3/b*x/(b*x^3+a)*c^3*d+1/7*d^4*x^7/b^2+28/9/b^4*
a^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c*d^3-8/3/b^3*a/(a/b)^(2/3)*ln(
x+(a/b)^(1/3))*c^2*d^2-14/9/b^4*a^2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1
/3)+(a/b)^(2/3))*c*d^3+4/3/b^3*a/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)
+(a/b)^(2/3))*c^2*d^2-10/9/b^5*a^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d^4+2/9/b/a/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^4+2/b^2*x*a/(b*x^3+a)*c^2*
d^2+4/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))*c^3*d-4/3/b^3*x*a^2/(b*x^3+a)*c*d^3
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^4/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.22305, size = 794, normalized size = 2.97

$$\frac{\sqrt{3}\left(14\sqrt{3}\left(ab^4c^4 + 2\,a^2b^3c^3d - 12\,a^3b^2c^2d^2 + 14\,a^4bcd^3 - 5\,a^5d^4 + \left(b^5c^4 + 2\,ab^4c^3d - 12\,a^2b^3c^2d^2 + 14\,a^3b^2cd^3 - 5\,a^4bd^4\right)x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^4/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/378*sqrt(3)*(14*sqrt(3)*(a*b^4*c^4 + 2*a^2*b^3*c^3*d - 12*a^3*b
^2*c^2*d^2 + 14*a^4*b*c*d^3 - 5*a^5*d^4 + (b^5*c^4 + 2*a*b^4*c^3*
d - 12*a^2*b^3*c^2*d^2 + 14*a^3*b^2*c*d^3 - 5*a^4*b*d^4)*x^3)*log
((-a^2*b)^(2/3)*x^2 + (-a^2*b)^(1/3)*a*x + a^2) - 28*sqrt(3)*(a*b
^4*c^4 + 2*a^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d^3 -
5*a^5*d^4 + (b^5*c^4 + 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*a^
3*b^2*c*d^3 - 5*a^4*b*d^4)*x^3)*log((-a^2*b)^(1/3)*x - a) + 84*(a
*b^4*c^4 + 2*a^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d^3
- 5*a^5*d^4 + (b^5*c^4 + 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*
a^3*b^2*c*d^3 - 5*a^4*b*d^4)*x^3)*arctan(1/3*(2*sqrt(3)*(-a^2*b)^
(1/3)*x + sqrt(3)*a)/a) + 3*sqrt(3)*(6*a*b^3*d^4*x^10 + 3*(14*a*b
^3*c*d^3 - 5*a^2*b^2*d^4)*x^7 + 21*(12*a*b^3*c^2*d^2 - 14*a^2*b^2
*c*d^3 + 5*a^3*b*d^4)*x^4 + 14*(b^4*c^4 - 4*a*b^3*c^3*d + 24*a^2*
b^2*c^2*d^2 - 28*a^3*b*c*d^3 + 10*a^4*d^4)*x)*(-a^2*b)^(1/3))/((a
*b^5*x^3 + a^2*b^4)*(-a^2*b)^(1/3))
```

---

**Sympy [A]** time = 14.071, size = 403, normalized size = 1.51

$$\frac{x\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)}{3a^2b^4+3ab^5x^3}$$
$$+\text{RootSum}\Big(729t^3a^5b^{13}+1000a^{12}d^{12}-8400a^{11}bcd^{11}+30720a^{10}b^2c^2d^{10}-63472a^9b^3c^3d^9+79848a^8b^4c^4d^8-60192a^7b^5c^5d$$
$$+\frac{d^4x^7}{7b^2}-\frac{x^4\left(ad^4-2bcd^3\right)}{2b^3}+\frac{x\left(3a^2d^4-8abcd^3+6b^2c^2d^2\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c)**4/(b*x**3+a)**2,x)
```

```
[Out] x*(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3
*c**3*d + b**4*c**4)/(3*a**2*b**4 + 3*a*b**5*x**3) + RootSum(729*
_t**3*a**5*b**13 + 1000*a**12*d**12 - 8400*a**11*b*c*d**11 + 3072
0*a**10*b**2*c**2*d**10 - 63472*a**9*b**3*c**3*d**9 + 79848*a**8*
b**4*c**4*d**8 - 60192*a**7*b**5*c**5*d**7 + 22848*a**6*b**6*c**6
*d**6 + 288*a**5*b**7*c**7*d**5 - 3528*a**4*b**8*c**8*d**4 + 752*
a**3*b**9*c**9*d**3 + 192*a**2*b**10*c**10*d**2 - 48*a*b**11*c**1
1*d - 8*b**12*c**12, Lambda(_t, _t*log(-9*_t*a**2*b**4/(10*a**4*d
**4 - 28*a**3*b*c*d**3 + 24*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d
 - 2*b**4*c**4) + x))) + d**4*x**7/(7*b**2) - x**4*(a*d**4 - 2*b*
c*d**3)/(2*b**3) + x*(3*a**2*d**4 - 8*a*b*c*d**3 + 6*b**2*c**2*d*
*2)/b**4
```

---

**GIAC/XCAS [A]** time = 0.220824, size = 643, normalized size = 2.41

$$-\frac{2\left(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^4}$$
$$+\frac{2\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^4c^4+2\left(-ab^2\right)^{\frac{1}{3}}ab^3c^3d-12\left(-ab^2\right)^{\frac{1}{3}}a^2b^2c^2d^2+14\left(-ab^2\right)^{\frac{1}{3}}a^3bcd^3-5\left(-ab^2\right)^{\frac{1}{3}}a^4d^4\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\cdots\right.\right.}{3\left(-\frac{a}{b}\cdots\right.}\right.}{9\,a^2b^5}$$
$$+\frac{b^4c^4x-4\,ab^3c^3dx+6\,a^2b^2c^2d^2x-4\,a^3bcd^3x+a^4d^4x}{3\,(bx^3+a)ab^4}$$
$$+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^4c^4+2\left(-ab^2\right)^{\frac{1}{3}}ab^3c^3d-12\left(-ab^2\right)^{\frac{1}{3}}a^2b^2c^2d^2+14\left(-ab^2\right)^{\frac{1}{3}}a^3bcd^3-5\left(-ab^2\right)^{\frac{1}{3}}a^4d^4\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(\cdots\right.\right.}{9\,a^2b^5}$$
$$+\frac{2\,b^{12}d^4x^7+14\,b^{12}cd^3x^4-7\,ab^{11}d^4x^4+84\,b^{12}c^2d^2x-112\,ab^{11}cd^3x+42\,a^2b^{10}d^4x}{14\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^4/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] -2/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d
^3 - 5*a^4*d^4)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^2*b^4)
+ 2/9*sqrt(3)*((-a*b^2)^(1/3)*b^4*c^4 + 2*(-a*b^2)^(1/3)*a*b^3*c^
3*d - 12*(-a*b^2)^(1/3)*a^2*b^2*c^2*d^2 + 14*(-a*b^2)^(1/3)*a^3*b
*c*d^3 - 5*(-a*b^2)^(1/3)*a^4*d^4)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/(a^2*b^5) + 1/3*(b^4*c^4*x - 4*a*b^3*c^3*
d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^3*x + a^4*d^4*x)/((b*x^3
+ a)*a*b^4) + 1/9*((-a*b^2)^(1/3)*b^4*c^4 + 2*(-a*b^2)^(1/3)*a*b^
3*c^3*d - 12*(-a*b^2)^(1/3)*a^2*b^2*c^2*d^2 + 14*(-a*b^2)^(1/3)*a
^3*b*c*d^3 - 5*(-a*b^2)^(1/3)*a^4*d^4)*ln(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/(a^2*b^5) + 1/14*(2*b^12*d^4*x^7 + 14*b^12*c*d^3*x^
4 - 7*a*b^11*d^4*x^4 + 84*b^12*c^2*d^2*x - 112*a*b^11*c*d^3*x + 4
2*a^2*b^10*d^4*x)/b^14
```

## 3.22 $\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx$

**Optimal**. Leaf size=234

$$-\frac{(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}}+\frac{(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}}-\frac{(bc-ad)^2(7ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{10/3}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{3ab^3(a+bx^3)}+\frac{d^3x^4}{4b^2}$$

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^4)/(4*b^2) + ((b*c - a*d)^3*
x)/(3*a*b^3*(a + b*x^3)) - ((b*c - a*d)^2*(2*b*c + 7*a*d)*ArcTan[
(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^
(10/3)) + ((b*c - a*d)^2*(2*b*c + 7*a*d)*Log[a^(1/3) + b^(1/3)*x]
)/(9*a^(5/3)*b^(10/3)) - ((b*c - a*d)^2*(2*b*c + 7*a*d)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(10/3))
```

---

**Rubi [A]** time = 0.482122, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421

$$-\frac{(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}}+\frac{(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}}-\frac{(bc-ad)^2(7ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{10/3}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{3ab^3(a+bx^3)}+\frac{d^3x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3)^3/(a + b*x^3)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^4)/(4*b^2) + ((b*c - a*d)^3*
x)/(3*a*b^3*(a + b*x^3)) - ((b*c - a*d)^2*(2*b*c + 7*a*d)*ArcTan[
(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^
(10/3)) + ((b*c - a*d)^2*(2*b*c + 7*a*d)*Log[a^(1/3) + b^(1/3)*x]
)/(9*a^(5/3)*b^(10/3)) - ((b*c - a*d)^2*(2*b*c + 7*a*d)*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(10/3))
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-d^2\left(2ad-3bc\right)\int\frac{1}{b^3}\,dx+\frac{d^3x^4}{4b^2}-\frac{x\left(ad-bc\right)^3}{3ab^3\left(a+bx^3\right)}+\frac{\left(ad-bc\right)^2\left(7ad+2bc\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{5}{3}}b^{\frac{10}{3}}}-\frac{\left(ad-bc\right)^2\left(7ad+2bc\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{18a^{\frac{5}{3}}b^{\frac{10}{3}}}-\frac{\sqrt{3}\left(ad-bc\right)^2\left(7ad+2bc\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{5}{3}}b^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c)**3/(b*x**3+a)**2,x)
```

```
[Out] -d**2*(2*a*d - 3*b*c)*Integral(b**(-3), x) + d**3*x**4/(4*b**2) -
 x*(a*d - b*c)**3/(3*a*b**3*(a + b*x**3)) + (a*d - b*c)**2*(7*a*d
 + 2*b*c)*log(a**(1/3) + b**(1/3)*x)/(9*a**(5/3)*b**(10/3)) - (a*
d - b*c)**2*(7*a*d + 2*b*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x +
b**(2/3)*x**2)/(18*a**(5/3)*b**(10/3)) - sqrt(3)*(a*d - b*c)**2*(
7*a*d + 2*b*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3
))/(9*a**(5/3)*b**(10/3))
```

---

**Mathematica [A]** time = 0.261119, size = 227, normalized size = 0.97

$$\frac{-\frac{2(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}}+\frac{4(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}}+\frac{4\sqrt{3}(bc-ad)^2(7ad+2bc)\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}+36\sqrt[3]{b}d^2x}{36b^{10/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3)^3/(a + b*x^3)^2,x]
```

```
[Out] (36*b^(1/3)*d^2*(3*b*c - 2*a*d)*x + 9*b^(4/3)*d^3*x^4 + (12*b^(1/
3)*(b*c - a*d)^3*x)/(a*(a + b*x^3)) + (4*Sqrt[3]*(b*c - a*d)^2*(2
*b*c + 7*a*d)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])
/a^(5/3) + (4*(b*c - a*d)^2*(2*b*c + 7*a*d)*Log[a^(1/3) + b^(1/3)
*x])/a^(5/3) - (2*(b*c - a*d)^2*(2*b*c + 7*a*d)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(36*b^(10/3))
```

---

**Maple [B]** time = 0.013, size = 529, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c)^3/(b*x^3+a)^2,x)
```

```
[Out] 1/4*d^3*x^4/b^2-2*d^3/b^3*a*x+3*d^2/b^2*x*c-1/3/b^3*x*a^2/(b*x^3+
a)*d^3+1/b^2*x*a/(b*x^3+a)*c*d^2-1/b*x/(b*x^3+a)*c^2*d+1/3*x/a/(b
*x^3+a)*c^3+7/9/b^4*a^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d^3-4/3/b^3
*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c*d^2+1/3/b^2/(a/b)^(2/3)*ln(x+(
a/b)^(1/3))*c^2*d+2/9/b/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^3-7/18/
b^4*a^2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d^3+2/3/b^3
*a/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c*d^2-1/6/b^2/(a
/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c^2*d-1/9/b/a/(a/b)^(
2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c^3+7/9/b^4*a^2/(a/b)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d^3-4/3/b^3*a/(
a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c*d^2+
1/3/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))*c^2*d+2/9/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^
(1/3)*x-1))*c^3
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.219973, size = 606, normalized size = 2.59

$$-\frac{\sqrt{3}\Big(2\sqrt{3}\big(2\,ab^3c^3+3\,a^2b^2c^2d-12\,a^3bcd^2+7\,a^4d^3+\big(2\,b^4c^3+3\,ab^3c^2d-12\,a^2b^2cd^2+7\,a^3bd^3\big)x^3\big)\log\Big(\big(a^2b\big)^{\frac{2}{3}}x^2-\big(a^2b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] -1/108*sqrt(3)*(2*sqrt(3)*(2*a*b^3*c^3 + 3*a^2*b^2*c^2*d - 12*a^3
*b*c*d^2 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 12*a^2*b^2*c*
d^2 + 7*a^3*b*d^3)*x^3)*log((a^2*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a*x
 + a^2) - 4*sqrt(3)*(2*a*b^3*c^3 + 3*a^2*b^2*c^2*d - 12*a^3*b*c*d
^2 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 +
7*a^3*b*d^3)*x^3)*log((a^2*b)^(1/3)*x + a) - 12*(2*a*b^3*c^3 + 3*
```

```
a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b^3
*c^2*d - 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)*arctan(1/3*(2*sqrt(
3)*(a^2*b)^(1/3)*x - sqrt(3)*a)/a) - 3*sqrt(3)*(3*a*b^2*d^3*x^7 +
 3*(12*a*b^2*c*d^2 - 7*a^2*b*d^3)*x^4 + 4*(b^3*c^3 - 3*a*b^2*c^2*
d + 12*a^2*b*c*d^2 - 7*a^3*d^3)*x)*(a^2*b)^(1/3))/((a*b^4*x^3 + a
^2*b^3)*(a^2*b)^(1/3))
```

---

**Sympy [A]** time = 8.05575, size = 289, normalized size = 1.24

$$-\frac{x\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{3a^2b^3+3ab^4x^3}+\mathrm{RootSum}\left(729t^3a^5b^{10}-343a^9d^9+1764a^8bcd^8-3465a^7b^2c^2d^7+2946a^6b^3c^3d^6-477a^5b^4c^4d^5-792a^4b^5c^5d^4+321a^3b^6c^6\right.$$
$$+\frac{d^3x^4}{4b^2}-\frac{x\left(2ad^3-3bcd^2\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c)**3/(b*x**3+a)**2,x)
```

```
[Out] -x*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(3
*a**2*b**3 + 3*a*b**4*x**3) + RootSum(729*_t**3*a**5*b**10 - 343*
a**9*d**9 + 1764*a**8*b*c*d**8 - 3465*a**7*b**2*c**2*d**7 + 2946*
a**6*b**3*c**3*d**6 - 477*a**5*b**4*c**4*d**5 - 792*a**4*b**5*c**
5*d**4 + 321*a**3*b**6*c**6*d**3 + 90*a**2*b**7*c**7*d**2 - 36*a*
b**8*c**8*d - 8*b**9*c**9, Lambda(_t, _t*log(9*_t*a**2*b**3/(7*a*
*3*d**3 - 12*a**2*b*c*d**2 + 3*a*b**2*c**2*d + 2*b**3*c**3) + x))
) + d**3*x**4/(4*b**2) - x*(2*a*d**3 - 3*b*c*d**2)/b**3
```

---

**GIAC/XCAS [A]** time = 0.220806, size = 495, normalized size = 2.12

$$-\frac{\left(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^3}$$
$$+\frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}b^3c^3+3\left(-ab^2\right)^{\frac{1}{3}}ab^2c^2d-12\left(-ab^2\right)^{\frac{1}{3}}a^2bcd^2+7\left(-ab^2\right)^{\frac{1}{3}}a^3d^3\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^4}$$
$$+\frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{3\,(bx^3+a)ab^3}$$
$$+\frac{\left(2\left(-ab^2\right)^{\frac{1}{3}}b^3c^3+3\left(-ab^2\right)^{\frac{1}{3}}ab^2c^2d-12\left(-ab^2\right)^{\frac{1}{3}}a^2bcd^2+7\left(-ab^2\right)^{\frac{1}{3}}a^3d^3\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b^4}$$
$$+\frac{b^6d^3x^4+12\,b^6cd^2x-8\,ab^5d^3x}{4\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*(-a
/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^2*b^3) + 1/9*sqrt(3)*(2*(-
a*b^2)^(1/3)*b^3*c^3 + 3*(-a*b^2)^(1/3)*a*b^2*c^2*d - 12*(-a*b^2)
^(1/3)*a^2*b*c*d^2 + 7*(-a*b^2)^(1/3)*a^3*d^3)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^4) + 1/3*(b^3*c^3*x -
3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x - a^3*d^3*x)/((b*x^3 + a)*a*b^3
) + 1/18*(2*(-a*b^2)^(1/3)*b^3*c^3 + 3*(-a*b^2)^(1/3)*a*b^2*c^2*d
 - 12*(-a*b^2)^(1/3)*a^2*b*c*d^2 + 7*(-a*b^2)^(1/3)*a^3*d^3)*ln(x
^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4) + 1/4*(b^6*d^3*x^4
+ 12*b^6*c*d^2*x - 8*a*b^5*d^3*x)/b^8
```

## 3.23 $\int \frac{(c+dx^3)^2}{(a+bx^3)^2} dx$

**Optimal**. Leaf size=203

$$-\frac{(bc-ad)(2ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}}+\frac{2(bc-ad)(2ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{7/3}}-\frac{2(bc-ad)(2ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{7/3}}+\frac{x(bc-ad)^2}{3ab^2(a+bx^3)}+\frac{d^2x}{b^2}$$

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(3*a*b^2*(a + b*x^3)) - (2*(b*c -
 a*d)*(b*c + 2*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(3*Sqrt[3]*a^(5/3)*b^(7/3)) + (2*(b*c - a*d)*(b*c + 2*a*d)*
Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(7/3)) - ((b*c - a*d)*(b*c
 + 2*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5
/3)*b^(7/3))
```

---

**Rubi [A]** time = 0.497695, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$-\frac{(bc-ad)(2ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}}+\frac{2(bc-ad)(2ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{7/3}}-\frac{2(bc-ad)(2ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{7/3}}+\frac{x(bc-ad)^2}{3ab^2(a+bx^3)}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3)^2/(a + b*x^3)^2,x]
```


```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(3*a*b^2*(a + b*x^3)) - (2*(b*c -
 a*d)*(b*c + 2*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(3*Sqrt[3]*a^(5/3)*b^(7/3)) + (2*(b*c - a*d)*(b*c + 2*a*d)*
Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(7/3)) - ((b*c - a*d)*(b*c
 + 2*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5
/3)*b^(7/3))
```


---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$d^2 \int \frac{1}{b^2}\, dx + \frac{x\,(ad-bc)^2}{3ab^2\,(a+bx^3)} - \frac{2\,(ad-bc)\,(2ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{5}{3}}b^{\frac{7}{3}}} + \frac{(ad-bc)\,(2ad+bc)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{9a^{\frac{5}{3}}b^{\frac{7}{3}}} + \frac{2\sqrt{3}\,(ad-bc)\,(2ad+bc)\,\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{5}{3}}b^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c)**2/(b*x**3+a)**2,x)
```

```
[Out] d**2*Integral(b**(-2), x) + x*(a*d - b*c)**2/(3*a*b**2*(a + b*x**
3)) - 2*(a*d - b*c)*(2*a*d + b*c)*log(a**(1/3) + b**(1/3)*x)/(9*a
**(5/3)*b**(7/3)) + (a*d - b*c)*(2*a*d + b*c)*log(a**(2/3) - a**(
1/3)*b**(1/3)*x + b**(2/3)*x**2)/(9*a**(5/3)*b**(7/3)) + 2*sqrt(3
)*(a*d - b*c)*(2*a*d + b*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)
*x/3)/a**(1/3))/(9*a**(5/3)*b**(7/3))
```

---

**Mathematica [A]** time = 0.412179, size = 205, normalized size = 1.01

$$\frac{\frac{2\left(-2a^2d^2+abcd+b^2c^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}} - \frac{2\sqrt{3}\left(-2a^2d^2+abcd+b^2c^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} - \frac{\left(-2a^2d^2+abcd+b^2c^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}} + \frac{3\sqrt[3]{b}x(b\cdots}{a(a+\cdots}}{9b^{7/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3)^2/(a + b*x^3)^2,x]
```

```
[Out] (9*b^(1/3)*d^2*x + (3*b^(1/3)*(b*c - a*d)^2*x)/(a*(a + b*x^3)) -
(2*Sqrt[3]*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*ArcTan[(1 - (2*b^(1/3)
*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*(b^2*c^2 + a*b*c*d - 2*a^2*d^
2)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - ((b^2*c^2 + a*b*c*d - 2*a^
2*d^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(
9*b^(7/3))
```

---

**Maple [B]** time = 0.013, size = 367, normalized size = 1.8

$$\frac{d^2x}{b^2}+\frac{axd^2}{3\,b^2\,(bx^3+a)}-\frac{2\,cxd}{3\,b\,(bx^3+a)}+\frac{xc^2}{3\,a\,(bx^3+a)}-\frac{4\,ad^2}{9\,b^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2\,cd}{9\,b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2\,c^2}{9\,ab}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2\,ad^2}{9\,b^3}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{cd}{9\,b^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c^2}{9\,ab}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{4\,a\sqrt{3}d^2}{9\,b^3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2\,\sqrt{3}cd}{9\,b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2\,\sqrt{3}c^2}{9\,ab}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c)^2/(b*x^3+a)^2,x)
```

```
[Out] d^2*x/b^2+1/3/b^2*x*a/(b*x^3+a)*d^2-2/3/b*x/(b*x^3+a)*c*d+1/3*x/a
/(b*x^3+a)*c^2-4/9/b^3*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d^2+2/9/b^
2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c*d+2/9/b/a/(a/b)^(2/3)*ln(x+(a/b
)^(1/3))*c^2+2/9/b^3*a/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/
3))*d^2-1/9/b^2/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c*d
-1/9/b/a/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c^2-4/9/b^
3*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d
^2+2/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1))*c*d+2/9/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))*c^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.218815, size = 427, normalized size = 2.1

$$\frac{\sqrt{3}\left(\sqrt{3}\left(ab^2c^2 + a^2bcd - 2\,a^3d^2 + \left(b^3c^2 + ab^2cd - 2\,a^2bd^2\right)x^3\right)\log\left(\left(-a^2b\right)^{\frac{2}{3}}x^2 + \left(-a^2b\right)^{\frac{1}{3}}ax + a^2\right) - 2\,\sqrt{3}\left(ab^2c^2 + a^2bcd\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/27*sqrt(3)*(sqrt(3)*(a*b^2*c^2 + a^2*b*c*d - 2*a^3*d^2 + (b^3*c
^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*log((-a^2*b)^(2/3)*x^2 + (-a^2
*b)^(1/3)*a*x + a^2) - 2*sqrt(3)*(a*b^2*c^2 + a^2*b*c*d - 2*a^3*d
^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*log((-a^2*b)^(1/3)*
x - a) + 6*(a*b^2*c^2 + a^2*b*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*
c*d - 2*a^2*b*d^2)*x^3)*arctan(1/3*(2*sqrt(3)*(-a^2*b)^(1/3)*x +
sqrt(3)*a)/a) + 3*sqrt(3)*(3*a*b*d^2*x^4 + (b^2*c^2 - 2*a*b*c*d +
 4*a^2*d^2)*x)*(-a^2*b)^(1/3))/((a*b^3*x^3 + a^2*b^2)*(-a^2*b)^(1
/3))
```

---

**Sympy [A]** time = 5.08011, size = 189, normalized size = 0.93

$$\frac{x\left(a^2d^2 - 2abcd + b^2c^2\right)}{3a^2b^2 + 3ab^3x^3}$$
$$+\mathrm{RootSum}\left(729t^3a^5b^7 + 64a^6d^6 - 96a^5bcd^5 - 48a^4b^2c^2d^4 + 88a^3b^3c^3d^3 + 24a^2b^4c^4d^2 - 24ab^5c^5d - 8b^6c^6, \left(t \mapsto t\log\left(-\frac{}{4a}\right.\right.\right.$$
$$+\frac{d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c)**2/(b*x**3+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*a**2*b**2 + 3*a*b**3*x**
3) + RootSum(729*_t**3*a**5*b**7 + 64*a**6*d**6 - 96*a**5*b*c*d**
5 - 48*a**4*b**2*c**2*d**4 + 88*a**3*b**3*c**3*d**3 + 24*a**2*b**
4*c**4*d**2 - 24*a*b**5*c**5*d - 8*b**6*c**6, Lambda(_t, _t*log(-
9*_t*a**2*b**2/(4*a**2*d**2 - 2*a*b*c*d - 2*b**2*c**2) + x))) + d
**2*x/b**2
```

---

**GIAC/XCAS [A]** time = 0.218488, size = 350, normalized size = 1.72

$$\frac{d^2x}{b^2} - \frac{2\left(b^2c^2 + abcd - 2a^2d^2\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^2b^2} + \frac{2\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c^2 + \left(-ab^2\right)^{\frac{1}{3}}abcd - 2\left(-ab^2\right)^{\frac{1}{3}}a^2d^2\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2b^3} + \frac{b^2c^2x - 2abcdx + a^2d^2x}{3(bx^3 + a)ab^2} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c^2 + \left(-ab^2\right)^{\frac{1}{3}}abcd - 2\left(-ab^2\right)^{\frac{1}{3}}a^2d^2\right)\ln\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 - 2/9*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*(-a/b)^(1/3)*ln(a
bs(x - (-a/b)^(1/3)))/(a^2*b^2) + 2/9*sqrt(3)*((-a*b^2)^(1/3)*b^2
*c^2 + (-a*b^2)^(1/3)*a*b*c*d - 2*(-a*b^2)^(1/3)*a^2*d^2)*arctan(
1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^3) + 1/3*(b
^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^3 + a)*a*b^2) + 1/9*((-
a*b^2)^(1/3)*b^2*c^2 + (-a*b^2)^(1/3)*a*b*c*d - 2*(-a*b^2)^(1/3)*
a^2*d^2)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3)
```

## 3.24 $\int \frac{c+dx^3}{(a+bx^3)^2}\,dx$

**Optimal**. Leaf size=169

$$-\frac{(ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}+\frac{(ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}}-\frac{(ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}}+\frac{x(bc-ad)}{3ab\left(a+bx^3\right)}$$

```
[Out] ((b*c - a*d)*x)/(3*a*b*(a + b*x^3)) - ((2*b*c + a*d)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(4/3)
) + ((2*b*c + a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(4/3))
- ((2*b*c + a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
(18*a^(5/3)*b^(4/3))
```

---

**Rubi [A]** time = 0.193477, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412

$$-\frac{(ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}+\frac{(ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}}-\frac{(ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}}+\frac{x(bc-ad)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(3*a*b*(a + b*x^3)) - ((2*b*c + a*d)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(4/3)
) + ((2*b*c + a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(4/3))
- ((2*b*c + a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
(18*a^(5/3)*b^(4/3))
```

---

**Rubi in Sympy [A]** time = 34.112, size = 155, normalized size = 0.92

$$-\frac{x\,(ad-bc)}{3ab\,(a+bx^3)}+\frac{(ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{5}{3}}b^{\frac{4}{3}}}-\frac{(ad+2bc)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{18a^{\frac{5}{3}}b^{\frac{4}{3}}}-\frac{\sqrt{3}\,(ad+2bc)\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{5}{3}}b^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -x*(a*d - b*c)/(3*a*b*(a + b*x**3)) + (a*d + 2*b*c)*log(a**(1/3)
+ b**(1/3)*x)/(9*a**(5/3)*b**(4/3)) - (a*d + 2*b*c)*log(a**(2/3)
- a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(18*a**(5/3)*b**(4/3)) - s
qrt(3)*(a*d + 2*b*c)*atan(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a
**(1/3))/(9*a**(5/3)*b**(4/3))
```

---

**Mathematica [A]** time = 0.173095, size = 145, normalized size = 0.86

$$\frac{-(ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{6a^{2/3}\sqrt[3]{b}x(ad-bc)}{a+bx^3}+2(ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}(ad+2bc)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{18a^{5/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a^(2/3)*b^(1/3)*(-(b*c) + a*d)*x)/(a + b*x^3) - 2*Sqrt[3]*(2
*b*c + a*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(2*b*
c + a*d)*Log[a^(1/3) + b^(1/3)*x] - (2*b*c + a*d)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(4/3))
```

---

**Maple [A]** time = 0.012, size = 221, normalized size = 1.3

$$-\frac{(ad-bc)\,x}{3\,ab\,(bx^3+a)}+\frac{d}{9\,b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2\,c}{9\,ab}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{d}{18\,b^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{9\,ab}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}d}{9\,b^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2\,\sqrt{3}c}{9\,ab}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] -1/3*(a*d-b*c)/a/b*x/(b*x^3+a)+1/9/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/
3))*d+2/9/b/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c-1/18/b^2/(a/b)^(2/3
)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d-1/9/b/a/(a/b)^(2/3)*ln(x^2-
x*(a/b)^(1/3)+(a/b)^(2/3))*c+1/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+2/9/b/a/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.216441, size = 277, normalized size = 1.64

$$\frac{\sqrt{3}\left(6\,\sqrt{3}\,(a^2b)^{\frac{1}{3}}\,(bc-ad)x-\sqrt{3}\left(\left(2\,b^2c+abd\right)x^3+2\,abc+a^2d\right)\log\left(\left(a^2b\right)^{\frac{2}{3}}x^2-\left(a^2b\right)^{\frac{1}{3}}ax+a^2\right)+2\,\sqrt{3}\left(\left(2\,b^2c+abd\right)x^3\right.\right.}{54\,(ab^2x^3+a^2b)\,(a^2b)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/54*sqrt(3)*(6*sqrt(3)*(a^2*b)^(1/3)*(b*c - a*d)*x - sqrt(3)*((2
*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*log((a^2*b)^(2/3)*x^2 - (a
^2*b)^(1/3)*a*x + a^2) + 2*sqrt(3)*((2*b^2*c + a*b*d)*x^3 + 2*a*b
*c + a^2*d)*log((a^2*b)^(1/3)*x + a) + 6*((2*b^2*c + a*b*d)*x^3 +
 2*a*b*c + a^2*d)*arctan(1/3*(2*sqrt(3)*(a^2*b)^(1/3)*x - sqrt(3)
*a)/a))/((a*b^2*x^3 + a^2*b)*(a^2*b)^(1/3))
```

---

**Sympy [A]** time = 2.68576, size = 97, normalized size = 0.57

$$-\frac{x\left(ad-bc\right)}{3a^2b+3ab^2x^3}+\operatorname{RootSum}\left(729t^3a^5b^4-a^3d^3-6a^2bcd^2-12ab^2c^2d-8b^3c^3,\left(t\mapsto t\log\left(\frac{9ta^2b}{ad+2bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -x*(a*d - b*c)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**
5*b**4 - a**3*d**3 - 6*a**2*b*c*d**2 - 12*a*b**2*c**2*d - 8*b**3*
c**3, Lambda(_t, _t*log(9*_t*a**2*b/(a*d + 2*b*c) + x)))
```

---

**GIAC/XCAS [A]** time = 0.217503, size = 246, normalized size = 1.46

$$-\frac{(2\,bc+ad)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b}+\frac{\sqrt{3}\left(2\,\left(-ab^2\right)^{\frac{1}{3}}bc+\left(-ab^2\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2}+\frac{bcx-adx}{3\,(bx^3+a)ab}+\frac{\left(2\,\left(-ab^2\right)^{\frac{1}{3}}bc+\left(-ab^2\right)^{\frac{1}{3}}ad\right)\ln\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*b*c + a*d)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^2*b)
 + 1/9*sqrt(3)*(2*(-a*b^2)^(1/3)*b*c + (-a*b^2)^(1/3)*a*d)*arctan
(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) + 1/3*(
b*c*x - a*d*x)/((b*x^3 + a)*a*b) + 1/18*(2*(-a*b^2)^(1/3)*b*c + (
-a*b^2)^(1/3)*a*d)*ln(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b
^2)
```

## 3.25 $\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx$

**Optimal**. Leaf size=346

$$-\frac{b^{2/3}(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}(bc-ad)^2}+\frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^2}-\frac{b^{2/3}(2bc-5ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^2}-\frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)^2}+\frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)^2}-\frac{d^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^2}+\frac{bx}{3a\left(a+bx^3\right)(bc-ad)}$$

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^3)) - (b^(2/3)*(2*b*c - 5*a*d)*Ar
cTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/
3)*(b*c - a*d)^2) - (d^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt
[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*(b*c - a*d)^2) + (b^(2/3)*(2*b*c
- 5*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*(b*c - a*d)^2) + (d
^(5/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c - a*d)^2) - (b^(
2/3)*(2*b*c - 5*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/(18*a^(5/3)*(b*c - a*d)^2) - (d^(5/3)*Log[c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*(b*c - a*d)^2)
```

---

**Rubi [A]** time = 0.589269, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$-\frac{b^{2/3}(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}(bc-ad)^2}+\frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^2}-\frac{b^{2/3}(2bc-5ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^2}-\frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)^2}+\frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)^2}-\frac{d^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^2}+\frac{bx}{3a\left(a+bx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^3)^2*(c + d*x^3)),x]
```

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^3)) - (b^(2/3)*(2*b*c - 5*a*d)*Ar
cTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/
3)*(b*c - a*d)^2) - (d^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt
[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*(b*c - a*d)^2) + (b^(2/3)*(2*b*c
- 5*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*(b*c - a*d)^2) + (d
```

```
^(5/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c - a*d)^2) - (b^(
2/3)*(2*b*c - 5*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/(18*a^(5/3)*(b*c - a*d)^2) - (d^(5/3)*Log[c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*(b*c - a*d)^2)
```

---

**Rubi in Sympy [A]** time = 109.102, size = 321, normalized size = 0.93

$$\frac{d^{\frac{5}{3}}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{\frac{2}{3}}(ad-bc)^2} - \frac{d^{\frac{5}{3}}\log\left(c^{\frac{2}{3}}-\sqrt[3]{c}\sqrt[3]{d}x+d^{\frac{2}{3}}x^2\right)}{6c^{\frac{2}{3}}(ad-bc)^2} - \frac{\sqrt{3}d^{\frac{5}{3}}\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3}-\frac{2\sqrt[3]{d}x}{3}\right)}{\sqrt[3]{c}}\right)}{3c^{\frac{2}{3}}(ad-bc)^2} - \frac{bx}{3a(a+bx^3)(ad-bc)} - \frac{b^{\frac{2}{3}}(5ad-2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{5}{3}}(ad-bc)^2} + \frac{b^{\frac{2}{3}}(5ad-2bc)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{18a^{\frac{5}{3}}(ad-bc)^2} + \frac{\sqrt{3}b^{\frac{2}{3}}(5ad-2bc)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{5}{3}}(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**3+a)**2/(d*x**3+c),x)
```

```
[Out] d**(5/3)*log(c**(1/3) + d**(1/3)*x)/(3*c**(2/3)*(a*d - b*c)**2) -
 d**(5/3)*log(c**(2/3) - c**(1/3)*d**(1/3)*x + d**(2/3)*x**2)/(6*
c**(2/3)*(a*d - b*c)**2) - sqrt(3)*d**(5/3)*atan(sqrt(3)*(c**(1/3
)/3 - 2*d**(1/3)*x/3)/c**(1/3))/(3*c**(2/3)*(a*d - b*c)**2) - b*x
/(3*a*(a + b*x**3)*(a*d - b*c)) - b**(2/3)*(5*a*d - 2*b*c)*log(a*
*(1/3) + b**(1/3)*x)/(9*a**(5/3)*(a*d - b*c)**2) + b**(2/3)*(5*a*
d - 2*b*c)*log(a**(2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(1
8*a**(5/3)*(a*d - b*c)**2) + sqrt(3)*b**(2/3)*(5*a*d - 2*b*c)*ata
n(sqrt(3)*(a**(1/3)/3 - 2*b**(1/3)*x/3)/a**(1/3))/(9*a**(5/3)*(a*
d - b*c)**2)
```

---

**Mathematica [A]** time = 0.360038, size = 337, normalized size = 0.97

$$-b^{2/3}c^{2/3}\left(a+bx^3\right)(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) - 3a^{5/3}d^{5/3}\left(a+bx^3\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right) + 6a^{2/3}bc$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^3)^2*(c + d*x^3)),x]
```

```
[Out] (6*a^(2/3)*b*c^(2/3)*(b*c - a*d)*x - 2*Sqrt[3]*b^(2/3)*c^(2/3)*(2
*b*c - 5*a*d)*(a + b*x^3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] - 6*Sqrt[3]*a^(5/3)*d^(5/3)*(a + b*x^3)*ArcTan[(1 - (2*d^(1/
3)*x)/c^(1/3))/Sqrt[3]] + 2*b^(2/3)*c^(2/3)*(2*b*c - 5*a*d)*(a +
b*x^3)*Log[a^(1/3) + b^(1/3)*x] + 6*a^(5/3)*d^(5/3)*(a + b*x^3)*L
og[c^(1/3) + d^(1/3)*x] - b^(2/3)*c^(2/3)*(2*b*c - 5*a*d)*(a + b*
x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 3*a^(5/3)*d
^(5/3)*(a + b*x^3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]
)/(18*a^(5/3)*c^(2/3)*(b*c - a*d)^2*(a + b*x^3))
```

---

**Maple [A]** time = 0.017, size = 406, normalized size = 1.2

$$
\begin{aligned}
&-\frac{bxd}{3\,(ad-bc)^2\,(bx^3+a)}+\frac{b^2xc}{3\,(ad-bc)^2\,a\,(bx^3+a)}-\frac{5\,d}{9\,(ad-bc)^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}\\
&+\frac{2\,bc}{9\,(ad-bc)^2\,a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{5\,d}{18\,(ad-bc)^2}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}\\
&-\frac{bc}{9\,(ad-bc)^2\,a}\ln\left(x^2-x\sqrt[3]{\frac{a}{b}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}\\
&-\frac{5\,\sqrt{3}d}{9\,(ad-bc)^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}\\
&+\frac{2\,b\sqrt{3}c}{9\,(ad-bc)^2\,a}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}\\
&+\frac{d}{3\,(ad-bc)^2}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{d}{6\,(ad-bc)^2}\ln\left(x^2-x\sqrt[3]{\frac{c}{d}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}\\
&+\frac{\sqrt{3}d}{3\,(ad-bc)^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{c}{d}}}-1\right)\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^3+a)^2/(d*x^3+c),x)
```

```
[Out] -1/3*b/(a*d-b*c)^2*x/(b*x^3+a)*d+1/3*b^2/(a*d-b*c)^2*x/a/(b*x^3+a
)*c-5/9/(a*d-b*c)^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d+2/9*b/(a*d-b*
c)^2/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c+5/18/(a*d-b*c)^2/(a/b)^(2/
3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d-1/9*b/(a*d-b*c)^2/a/(a/b)^
(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c-5/9/(a*d-b*c)^2/(a/b)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+2/9*b/(a*d
```

```
-b*c)^2/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))*c+1/3*d/(a*d-b*c)^2/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6*d/(a*d
-b*c)^2/(c/d)^(2/3)*ln(x^2-x*(c/d)^(1/3)+(c/d)^(2/3))+1/3*d/(a*d-
b*c)^2/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)
)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^3 + a)^2*(d*x^3 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 7.40408, size = 633, normalized size = 1.83

$$\frac{\sqrt{3}\left(\sqrt{3}\left(\left(2\,b^2c-5\,abd\right)x^3+2\,abc-5\,a^2d\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)-3\sqrt{3}\left(abdx^3+a^2d\right)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^3 + a)^2*(d*x^3 + c)),x, algorithm="fricas")
```

```
[Out] 1/54*sqrt(3)*(sqrt(3)*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*
d)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-
b^2/a^2)^(2/3)) - 3*sqrt(3)*(a*b*d*x^3 + a^2*d)*(d^2/c^2)^(1/3)*l
og(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) - 2*sqr
t(3)*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b^2/a^2)^(1/
3)*log(b*x - a*(-b^2/a^2)^(1/3)) + 6*sqrt(3)*(a*b*d*x^3 + a^2*d)*
(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) + 6*sqrt(3)*(b^2*c -
 a*b*d)*x + 6*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b^2
/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x + sqrt(3)*a*(-b^2/a^2)^(1/3
))/(a*(-b^2/a^2)^(1/3))) - 18*(a*b*d*x^3 + a^2*d)*(d^2/c^2)^(1/3)
*arctan(-1/3*(2*sqrt(3)*d*x - sqrt(3)*c*(d^2/c^2)^(1/3))/(c*(d^2/
c^2)^(1/3))))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 -
 2*a^2*b^2*c*d + a^3*b*d^2)*x^3)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**3+a)**2/(d*x**3+c),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [A]** time = 0.228681, size = 598, normalized size = 1.73

$$-\frac{d^{2}\left(-\frac{c}{d}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{2}c^{3}-2\,abc^{2}d+a^{2}cd^{2}\right)}+\frac{\left(-cd^{2}\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^{2}c^{3}-2\sqrt{3}abc^{2}d+\sqrt{3}a^{2}cd^{2}}+\frac{\left(-cd^{2}\right)^{\frac{1}{3}}d\ln\left(x^{2}+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^{2}c^{3}-2\,abc^{2}d+a^{2}cd^{2}\right)}-\frac{\left(2\,b^{2}c-5\,abd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\left(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}\right)}+\frac{\left(2\left(-ab^{2}\right)^{\frac{1}{3}}bc-5\left(-ab^{2}\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(\sqrt{3}a^{2}b^{2}c^{2}-2\sqrt{3}a^{3}bcd+\sqrt{3}a^{4}d^{2}\right)}+\frac{\left(2\left(-ab^{2}\right)^{\frac{1}{3}}bc-5\left(-ab^{2}\right)^{\frac{1}{3}}ad\right)\ln\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}\right)}+\frac{bx}{3\left(bx^{3}+a\right)\left(abc-a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^3 + a)^2*(d*x^3 + c)),x, algorithm="giac")
```

```
[Out] -1/3*d^2*(-c/d)^(1/3)*ln(abs(x - (-c/d)^(1/3)))/(b^2*c^3 - 2*a*b*
c^2*d + a^2*c*d^2) + (-c*d^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (
-c/d)^(1/3))/(-c/d)^(1/3))/(sqrt(3)*b^2*c^3 - 2*sqrt(3)*a*b*c^2*d
 + sqrt(3)*a^2*c*d^2) + 1/6*(-c*d^2)^(1/3)*d*ln(x^2 + x*(-c/d)^(1
/3) + (-c/d)^(2/3))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2) - 1/9*(2*
b^2*c - 5*a*b*d)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(a^2*b^2*
c^2 - 2*a^3*b*c*d + a^4*d^2) + 1/3*(2*(-a*b^2)^(1/3)*b*c - 5*(-a*
b^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/(sqrt(3)*a^2*b^2*c^2 - 2*sqrt(3)*a^3*b*c*d + sqrt(3)*a^4*d^2
) + 1/18*(2*(-a*b^2)^(1/3)*b*c - 5*(-a*b^2)^(1/3)*a*d)*ln(x^2 + x
*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^
2) + 1/3*b*x/((b*x^3 + a)*(a*b*c - a^2*d))
```

## 3.26 $\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx$

**Optimal**. Leaf size=419

$$-\frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}(bc-ad)^3}+\frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^3}-\frac{2b^{5/3}(bc-4ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^3}-\frac{d^{5/3}(4bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{9c^{5/3}(bc-ad)^3}+\frac{2d^{5/3}(4bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^3}-\frac{2d^{5/3}(4bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^3}+\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}+\frac{dx(ad+bc)}{3ac\left(c+dx^3\right)(bc-ad)^2}$$

```
[Out] (d*(b*c + a*d)*x)/(3*a*c*(b*c - a*d)^2*(c + d*x^3)) + (b*x)/(3*a*
(b*c - a*d)*(a + b*x^3)*(c + d*x^3)) - (2*b^(5/3)*(b*c - 4*a*d)*A
rcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5
/3)*(b*c - a*d)^3) - (2*d^(5/3)*(4*b*c - a*d)*ArcTan[(c^(1/3) - 2
*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(5/3)*(b*c - a*d)^3)
 + (2*b^(5/3)*(b*c - 4*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*
(b*c - a*d)^3) + (2*d^(5/3)*(4*b*c - a*d)*Log[c^(1/3) + d^(1/3)*x
])/(9*c^(5/3)*(b*c - a*d)^3) - (b^(5/3)*(b*c - 4*a*d)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*(b*c - a*d)^3) -
(d^(5/3)*(4*b*c - a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*
x^2])/(9*c^(5/3)*(b*c - a*d)^3)
```

---

**Rubi [A]** time = 1.07782, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$

$$-\frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}(bc-ad)^3}+\frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^3}-\frac{2b^{5/3}(bc-4ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^3}-\frac{d^{5/3}(4bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{9c^{5/3}(bc-ad)^3}+\frac{2d^{5/3}(4bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^3}-\frac{2d^{5/3}(4bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^3}+\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}+\frac{dx(ad+bc)}{3ac\left(c+dx^3\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^3)^2*(c + d*x^3)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(3*a*c*(b*c - a*d)^2*(c + d*x^3)) + (b*x)/(3*a*
(b*c - a*d)*(a + b*x^3)*(c + d*x^3)) - (2*b^(5/3)*(b*c - 4*a*d)*A
rcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5
/3)*(b*c - a*d)^3) - (2*d^(5/3)*(4*b*c - a*d)*ArcTan[(c^(1/3) - 2
*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(5/3)*(b*c - a*d)^3)
 + (2*b^(5/3)*(b*c - 4*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*
(b*c - a*d)^3) + (2*d^(5/3)*(4*b*c - a*d)*Log[c^(1/3) + d^(1/3)*x
])/(9*c^(5/3)*(b*c - a*d)^3) - (b^(5/3)*(b*c - 4*a*d)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*(b*c - a*d)^3) -
(d^(5/3)*(4*b*c - a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*
x^2])/(9*c^(5/3)*(b*c - a*d)^3)
```

---

**Rubi in Sympy [A]** time = 177.573, size = 386, normalized size = 0.92

$$\frac{dx}{3c\left(a+bx^3\right)\left(c+dx^3\right)\left(ad-bc\right)}+\frac{2d^{\frac{5}{3}}\left(ad-4bc\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{\frac{5}{3}}\left(ad-bc\right)^3}-\frac{d^{\frac{5}{3}}\left(ad-4bc\right)\log\left(c^{\frac{2}{3}}-\sqrt[3]{c}\sqrt[3]{d}x+d^{\frac{2}{3}}x^2\right)}{9c^{\frac{5}{3}}\left(ad-bc\right)^3}-\frac{2\sqrt{3}d^{\frac{5}{3}}\left(ad-4bc\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{c}}{3}-\frac{2\sqrt[3]{d}x}{3}\right)}{\sqrt[3]{c}}\right)}{9c^{\frac{5}{3}}\left(ad-bc\right)^3}+\frac{bx\left(ad+bc\right)}{3ac\left(a+bx^3\right)\left(ad-bc\right)^2}+\frac{2b^{\frac{5}{3}}\left(4ad-bc\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{\frac{5}{3}}\left(ad-bc\right)^3}-\frac{b^{\frac{5}{3}}\left(4ad-bc\right)\log\left(a^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{b}x+b^{\frac{2}{3}}x^2\right)}{9a^{\frac{5}{3}}\left(ad-bc\right)^3}-\frac{2\sqrt{3}b^{\frac{5}{3}}\left(4ad-bc\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}-\frac{2\sqrt[3]{b}x}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{5}{3}}\left(ad-bc\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**3+a)**2/(d*x**3+c)**2,x)
```

```
[Out] d*x/(3*c*(a + b*x**3)*(c + d*x**3)*(a*d - b*c)) + 2*d**(5/3)*(a*d
 - 4*b*c)*log(c**(1/3) + d**(1/3)*x)/(9*c**(5/3)*(a*d - b*c)**3)
- d**(5/3)*(a*d - 4*b*c)*log(c**(2/3) - c**(1/3)*d**(1/3)*x + d**
(2/3)*x**2)/(9*c**(5/3)*(a*d - b*c)**3) - 2*sqrt(3)*d**(5/3)*(a*d
 - 4*b*c)*atan(sqrt(3)*(c**(1/3)/3 - 2*d**(1/3)*x/3)/c**(1/3))/(9
*c**(5/3)*(a*d - b*c)**3) + b*x*(a*d + b*c)/(3*a*c*(a + b*x**3)*(
a*d - b*c)**2) + 2*b**(5/3)*(4*a*d - b*c)*log(a**(1/3) + b**(1/3)
*x)/(9*a**(5/3)*(a*d - b*c)**3) - b**(5/3)*(4*a*d - b*c)*log(a**(
2/3) - a**(1/3)*b**(1/3)*x + b**(2/3)*x**2)/(9*a**(5/3)*(a*d - b*
c)**3) - 2*sqrt(3)*b**(5/3)*(4*a*d - b*c)*atan(sqrt(3)*(a**(1/3)/
3 - 2*b**(1/3)*x/3)/a**(1/3))/(9*a**(5/3)*(a*d - b*c)**3)
```

---

**Mathematica [A]** time = 1.63819, size = 381, normalized size = 0.91

$$\frac{1}{9}\left(\frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}(ad-bc)^3}+\frac{2b^{5/3}(4ad-bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}(ad-bc)^3}+\frac{2\sqrt{3}b^{5/3}(bc-4ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}(ad-bc)^3}+\frac{3b^2x}{a\left(a+bx^3\right)(bc-ad)^2}+\frac{d^{5/3}(ad-4bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{5/3}(bc-ad)^3}+\frac{2d^{5/3}(4bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{5/3}(bc-ad)^3}+\frac{2\sqrt{3}d^{5/3}(ad-4bc)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}(bc-ad)^3}+\frac{3d^2x}{c\left(c+dx^3\right)(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^3)^2*(c + d*x^3)^2),x]
```

```
[Out] ((3*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^3)) + (3*d^2*x)/(c*(b*c - a*
d)^2*(c + d*x^3)) + (2*Sqrt[3]*b^(5/3)*(b*c - 4*a*d)*ArcTan[(1 -
(2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(a^(5/3)*(-(b*c) + a*d)^3) + (2*
Sqrt[3]*d^(5/3)*(-4*b*c + a*d)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))
/Sqrt[3]])/(c^(5/3)*(b*c - a*d)^3) + (2*b^(5/3)*(-(b*c) + 4*a*d)*
Log[a^(1/3) + b^(1/3)*x])/(a^(5/3)*(-(b*c) + a*d)^3) + (2*d^(5/3)
*(4*b*c - a*d)*Log[c^(1/3) + d^(1/3)*x])/(c^(5/3)*(b*c - a*d)^3)
+ (b^(5/3)*(b*c - 4*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(a^(5/3)*(-(b*c) + a*d)^3) + (d^(5/3)*(-4*b*c + a*d)*Log[
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(c^(5/3)*(b*c - a*d)^
3))/9
```

---

**Maple [A]** time = 0.023, size = 606, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^3+a)^2/(d*x^3+c)^2,x)
```

```
[Out] 1/3*b^2/(a*d-b*c)^3*x/(b*x^3+a)*d-1/3*b^3/(a*d-b*c)^3*x/a/(b*x^3+
a)*c+8/9*b/(a*d-b*c)^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d-2/9*b^2/(a
*d-b*c)^3/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c-4/9*b/(a*d-b*c)^3/(a/
b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*d+1/9*b^2/(a*d-b*c)^3/
a/(a/b)^(2/3)*ln(x^2-x*(a/b)^(1/3)+(a/b)^(2/3))*c+8/9*b/(a*d-b*c)
^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d-
2/9*b^2/(a*d-b*c)^3/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1))*c+1/3*d^3/(a*d-b*c)^3/c*x/(d*x^3+c)*a-1/3*d^2/(a
*d-b*c)^3*x/(d*x^3+c)*b+2/9*d^2/(a*d-b*c)^3/c/(c/d)^(2/3)*ln(x+(c
/d)^(1/3))*a-8/9*d/(a*d-b*c)^3/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*b-1/
9*d^2/(a*d-b*c)^3/c/(c/d)^(2/3)*ln(x^2-x*(c/d)^(1/3)+(c/d)^(2/3))
*a+4/9*d/(a*d-b*c)^3/(c/d)^(2/3)*ln(x^2-x*(c/d)^(1/3)+(c/d)^(2/3)
)*b+2/9*d^2/(a*d-b*c)^3/c/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*
(2/(c/d)^(1/3)*x-1))*a-8/9*d/(a*d-b*c)^3/(c/d)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 77.3424, size = 1250, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^2),x, algorithm="fricas")
```

```
[Out] -1/27*sqrt(3)*(sqrt(3)*((b^3*c^2*d - 4*a*b^2*c*d^2)*x^6 + a*b^2*c
^3 - 4*a^2*b*c^2*d + (b^3*c^3 - 3*a*b^2*c^2*d - 4*a^2*b*c*d^2)*x^
3)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2
/a^2)^(2/3)) + sqrt(3)*((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b
*c^2*d - a^3*c*d^2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^
3)*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c^2*(d^2
/c^2)^(2/3)) - 2*sqrt(3)*((b^3*c^2*d - 4*a*b^2*c*d^2)*x^6 + a*b^2
*c^3 - 4*a^2*b*c^2*d + (b^3*c^3 - 3*a*b^2*c^2*d - 4*a^2*b*c*d^2)*
x^3)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) - 2*sqrt(3)*((4
*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*
```

```
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*(d^2/c^2)^(1/3)*log(d*x
 + c*(d^2/c^2)^(1/3)) + 6*((b^3*c^2*d - 4*a*b^2*c*d^2)*x^6 + a*b^
2*c^3 - 4*a^2*b*c^2*d + (b^3*c^3 - 3*a*b^2*c^2*d - 4*a^2*b*c*d^2)
*x^3)*(b^2/a^2)^(1/3)*arctan(-1/3*(2*sqrt(3)*b*x - sqrt(3)*a*(b^2
/a^2)^(1/3))/(a*(b^2/a^2)^(1/3))) + 6*((4*a*b^2*c*d^2 - a^2*b*d^3
)*x^6 + 4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^
2 - a^3*d^3)*x^3)*(d^2/c^2)^(1/3)*arctan(-1/3*(2*sqrt(3)*d*x - sq
rt(3)*c*(d^2/c^2)^(1/3))/(c*(d^2/c^2)^(1/3))) - 3*sqrt(3)*((b^3*c
^2*d - a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^
3*d^3)*x))/(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5
*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 -
 a^4*b*c*d^4)*x^6 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^
3 - a^5*c*d^4)*x^3)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**3+a)**2/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

**GIAC/XCAS [A]** time = 0.230403, size = 896, normalized size = 2.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^2),x, algorithm="giac")
```

```
[Out] -2/9*(b^3*c - 4*a*b^2*d)*(-a/b)^(1/3)*ln(abs(x - (-a/b)^(1/3)))/(
a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) - 2/9*(4
*b*c*d^2 - a*d^3)*(-c/d)^(1/3)*ln(abs(x - (-c/d)^(1/3)))/(b^3*c^5
 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3) + 2/3*((-a*b^2)
^(1/3)*b^2*c - 4*(-a*b^2)^(1/3)*a*b*d)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b^3*c^3 - 3*sqrt(3)*a^3*
b^2*c^2*d + 3*sqrt(3)*a^4*b*c*d^2 - sqrt(3)*a^5*d^3) + 2/3*(4*(-c
*d^2)^(1/3)*b*c*d - (-c*d^2)^(1/3)*a*d^2)*arctan(1/3*sqrt(3)*(2*x
 + (-c/d)^(1/3))/(-c/d)^(1/3))/(sqrt(3)*b^3*c^5 - 3*sqrt(3)*a*b^2
*c^4*d + 3*sqrt(3)*a^2*b*c^3*d^2 - sqrt(3)*a^3*c^2*d^3) + 1/9*((-
a*b^2)^(1/3)*b^2*c - 4*(-a*b^2)^(1/3)*a*b*d)*ln(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^
2 - a^5*d^3) + 1/9*(4*(-c*d^2)^(1/3)*b*c*d - (-c*d^2)^(1/3)*a*d^2
)*ln(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b^3*c^5 - 3*a*b^2*c^4*
d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3) + 1/3*(b^2*c*d*x^4 + a*b*d^2*x
```

```
^4 + b^2*c^2*x + a^2*d^2*x)/((b*d*x^6 + b*c*x^3 + a*d*x^3 + a*c)*
(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2))
```

## 3.27 $\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx$

**Optimal**. Leaf size=109

$$\frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}}+\frac{27a^2x\left(a+bx^3\right)}{140c^3\left(c+dx^3\right)^{4/3}}+\frac{9ax\left(a+bx^3\right)^2}{70c^2\left(c+dx^3\right)^{7/3}}+\frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{10/3}}$$

```
[Out] (x*(a + b*x^3)^3)/(10*c*(c + d*x^3)^(10/3)) + (9*a*x*(a + b*x^3)^
2)/(70*c^2*(c + d*x^3)^(7/3)) + (27*a^2*x*(a + b*x^3))/(140*c^3*(
c + d*x^3)^(4/3)) + (81*a^3*x)/(140*c^4*(c + d*x^3)^(1/3))
```

---

**Rubi [A]** time = 0.115874, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}}+\frac{27a^2x\left(a+bx^3\right)}{140c^3\left(c+dx^3\right)^{4/3}}+\frac{9ax\left(a+bx^3\right)^2}{70c^2\left(c+dx^3\right)^{7/3}}+\frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^3/(c + d*x^3)^(13/3),x]
```

```
[Out] (x*(a + b*x^3)^3)/(10*c*(c + d*x^3)^(10/3)) + (9*a*x*(a + b*x^3)^
2)/(70*c^2*(c + d*x^3)^(7/3)) + (27*a^2*x*(a + b*x^3))/(140*c^3*(
c + d*x^3)^(4/3)) + (81*a^3*x)/(140*c^4*(c + d*x^3)^(1/3))
```

---

**Rubi in Sympy [A]** time = 17.977, size = 102, normalized size = 0.94

$$\frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}}+\frac{27a^2x\left(a+bx^3\right)}{140c^3\left(c+dx^3\right)^{\frac{4}{3}}}+\frac{9ax\left(a+bx^3\right)^2}{70c^2\left(c+dx^3\right)^{\frac{7}{3}}}+\frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**3/(d*x**3+c)**(13/3),x)
```

```
[Out] 81*a**3*x/(140*c**4*(c + d*x**3)**(1/3)) + 27*a**2*x*(a + b*x**3)
/(140*c**3*(c + d*x**3)**(4/3)) + 9*a*x*(a + b*x**3)**2/(70*c**2*
(c + d*x**3)**(7/3)) + x*(a + b*x**3)**3/(10*c*(c + d*x**3)**(10/
3))
```

---

**Mathematica [A]** time = 0.132955, size = 120, normalized size = 1.1

$$\frac{x\left(a^3\left(140c^3+315c^2dx^3+270cd^2x^6+81d^3x^9\right)+3a^2bcx^3\left(35c^2+30cdx^3+9d^2x^6\right)+6ab^2c^2x^6\left(10c+3dx^3\right)+14b^3c^3x^9\right)}{140c^4\left(c+dx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)^3/(c + d*x^3)^(13/3),x]
```

```
[Out] (x*(14*b^3*c^3*x^9 + 6*a*b^2*c^2*x^6*(10*c + 3*d*x^3) + 3*a^2*b*c
*x^3*(35*c^2 + 30*c*d*x^3 + 9*d^2*x^6) + a^3*(140*c^3 + 315*c^2*d
*x^3 + 270*c*d^2*x^6 + 81*d^3*x^9)))/(140*c^4*(c + d*x^3)^(10/3))
```

---

**Maple [A]** time = 0.01, size = 134, normalized size = 1.2

$$\frac{x\left(81\,a^3d^3x^9+27\,a^2bcd^2x^9+18\,ab^2c^2dx^9+14\,b^3c^3x^9+270\,a^3cd^2x^6+90\,a^2bc^2dx^6+60\,ab^2c^3x^6+315\,a^3c^2dx^3+105\,a^2bc^3x\right.}{140\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^3/(d*x^3+c)^(13/3),x)
```

```
[Out] 1/140*x*(81*a^3*d^3*x^9+27*a^2*b*c*d^2*x^9+18*a*b^2*c^2*d*x^9+14*
b^3*c^3*x^9+270*a^3*c*d^2*x^6+90*a^2*b*c^2*d*x^6+60*a*b^2*c^3*x^6
+315*a^3*c^2*d*x^3+105*a^2*b*c^3*x^3+140*a^3*c^3)/(d*x^3+c)^(10/3
)/c^4
```

---

**Maxima [A]** time = 1.52004, size = 246, normalized size = 2.26

$$\frac{b^3x^{10}}{10\,(dx^3+c)^{\frac{10}{3}}c}-\frac{3\,ab^2\left(7\,d-\frac{10\,(dx^3+c)}{x^3}\right)x^{10}}{70\,(dx^3+c)^{\frac{10}{3}}c^2}+\frac{3\left(14\,d^2-\frac{40\,(dx^3+c)\,d}{x^3}+\frac{35\,(dx^3+c)^2}{x^6}\right)a^2bx^{10}}{140\,(dx^3+c)^{\frac{10}{3}}c^3}-\frac{\left(14\,d^3-\frac{60\,(dx^3+c)\,d^2}{x^3}+\frac{105\,(dx^3+c)^2d}{x^6}-\frac{140\,(dx^3+c)^3}{x^9}\right)a^3x^{10}}{140\,(dx^3+c)^{\frac{10}{3}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^3/(d*x^3 + c)^(13/3),x, algorithm="maxima")
```

```
[Out] 1/10*b^3*x^10/((d*x^3 + c)^(10/3)*c) - 3/70*a*b^2*(7*d - 10*(d*x^
3 + c)/x^3)*x^10/((d*x^3 + c)^(10/3)*c^2) + 3/140*(14*d^2 - 40*(d
*x^3 + c)*d/x^3 + 35*(d*x^3 + c)^2/x^6)*a^2*b*x^10/((d*x^3 + c)^(
```

```
10/3)*c^3) - 1/140*(14*d^3 - 60*(d*x^3 + c)*d^2/x^3 + 105*(d*x^3
+ c)^2*d/x^6 - 140*(d*x^3 + c)^3/x^9)*a^3*x^10/((d*x^3 + c)^(10/3
)*c^4)
```

---

**Fricas [A]** time = 0.222484, size = 224, normalized size = 2.06

$$\frac{\left(\left(14\,b^{3}c^{3}+18\,ab^{2}c^{2}d+27\,a^{2}bcd^{2}+81\,a^{3}d^{3}\right)x^{10}+30\,\left(2\,ab^{2}c^{3}+3\,a^{2}bc^{2}d+9\,a^{3}cd^{2}\right)x^{7}+140\,a^{3}c^{3}x+105\,\left(a^{2}bc^{3}+3\,a^{3}c^{2}d\right)x}{140\left(c^{4}d^{4}x^{12}+4\,c^{5}d^{3}x^{9}+6\,c^{6}d^{2}x^{6}+4\,c^{7}dx^{3}+c^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^3/(d*x^3 + c)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*((14*b^3*c^3 + 18*a*b^2*c^2*d + 27*a^2*b*c*d^2 + 81*a^3*d^3
)*x^10 + 30*(2*a*b^2*c^3 + 3*a^2*b*c^2*d + 9*a^3*c*d^2)*x^7 + 140
*a^3*c^3*x + 105*(a^2*b*c^3 + 3*a^3*c^2*d)*x^4)*(d*x^3 + c)^(2/3)
/(c^4*d^4*x^12 + 4*c^5*d^3*x^9 + 6*c^6*d^2*x^6 + 4*c^7*d*x^3 + c^
8)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**3/(d*x**3+c)**(13/3),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{3}+a\right)^{3}}{\left(dx^{3}+c\right)^{\frac{13}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^3/(d*x^3 + c)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^3/(d*x^3 + c)^(13/3), x)
```

## 3.28 $\int \frac{(a+bx^3)^2}{(c+dx^3)^{10/3}} \, dx$

**Optimal**. Leaf size=78

$$\frac{9a^2x}{14c^3\sqrt[3]{c+dx^3}}+\frac{3ax\left(a+bx^3\right)}{14c^2\left(c+dx^3\right)^{4/3}}+\frac{x\left(a+bx^3\right)^2}{7c\left(c+dx^3\right)^{7/3}}$$

```
[Out] (x*(a + b*x^3)^2)/(7*c*(c + d*x^3)^(7/3)) + (3*a*x*(a + b*x^3))/(
14*c^2*(c + d*x^3)^(4/3)) + (9*a^2*x)/(14*c^3*(c + d*x^3)^(1/3))
```

---

**Rubi [A]** time = 0.0701422, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{9a^2x}{14c^3\sqrt[3]{c+dx^3}}+\frac{3ax\left(a+bx^3\right)}{14c^2\left(c+dx^3\right)^{4/3}}+\frac{x\left(a+bx^3\right)^2}{7c\left(c+dx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^2/(c + d*x^3)^(10/3),x]
```

```
[Out] (x*(a + b*x^3)^2)/(7*c*(c + d*x^3)^(7/3)) + (3*a*x*(a + b*x^3))/(
14*c^2*(c + d*x^3)^(4/3)) + (9*a^2*x)/(14*c^3*(c + d*x^3)^(1/3))
```

---

**Rubi in Sympy [A]** time = 11.2865, size = 71, normalized size = 0.91

$$\frac{9a^2x}{14c^3\sqrt[3]{c+dx^3}}+\frac{3ax\left(a+bx^3\right)}{14c^2\left(c+dx^3\right)^{\frac{4}{3}}}+\frac{x\left(a+bx^3\right)^2}{7c\left(c+dx^3\right)^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**2/(d*x**3+c)**(10/3),x)
```

```
[Out] 9*a**2*x/(14*c**3*(c + d*x**3)**(1/3)) + 3*a*x*(a + b*x**3)/(14*c
**2*(c + d*x**3)**(4/3)) + x*(a + b*x**3)**2/(7*c*(c + d*x**3)**(
7/3))
```

---

**Mathematica [A]** time = 0.0835213, size = 73, normalized size = 0.94

$$\frac{a^2\left(14c^2x+21cdx^4+9d^2x^7\right)+abcx^4\left(7c+3dx^3\right)+2b^2c^2x^7}{14c^3\left(c+dx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)^2/(c + d*x^3)^(10/3),x]
```

```
[Out] (2*b^2*c^2*x^7 + a*b*c*x^4*(7*c + 3*d*x^3) + a^2*(14*c^2*x + 21*c
*d*x^4 + 9*d^2*x^7))/(14*c^3*(c + d*x^3)^(7/3))
```

---

**Maple [A]** time = 0.01, size = 76, normalized size = 1.

$$\frac{x\left(9\,a^{2}d^{2}x^{6}+3\,abcdx^{6}+2\,b^{2}c^{2}x^{6}+21\,a^{2}cdx^{3}+7\,abc^{2}x^{3}+14\,a^{2}c^{2}\right)}{14\,c^{3}}\left(dx^{3}+c\right)^{-\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^2/(d*x^3+c)^(10/3),x)
```

```
[Out] 1/14*x*(9*a^2*d^2*x^6+3*a*b*c*d*x^6+2*b^2*c^2*x^6+21*a^2*c*d*x^3+
7*a*b*c^2*x^3+14*a^2*c^2)/(d*x^3+c)^(7/3)/c^3
```

---

**Maxima [A]** time = 1.38355, size = 147, normalized size = 1.88

$$\frac{b^{2}x^{7}}{7\,(dx^{3}+c)^{\frac{7}{3}}c}-\frac{ab\left(4\,d-\frac{7\,(dx^{3}+c)}{x^{3}}\right)x^{7}}{14\,(dx^{3}+c)^{\frac{7}{3}}c^{2}}+\frac{\left(2\,d^{2}-\frac{7\,(dx^{3}+c)\,d}{x^{3}}+\frac{14\,(dx^{3}+c)^{2}}{x^{6}}\right)a^{2}x^{7}}{14\,(dx^{3}+c)^{\frac{7}{3}}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2/(d*x^3 + c)^(10/3),x, algorithm="maxima")
```

```
[Out] 1/7*b^2*x^7/((d*x^3 + c)^(7/3)*c) - 1/14*a*b*(4*d - 7*(d*x^3 + c)
/x^3)*x^7/((d*x^3 + c)^(7/3)*c^2) + 1/14*(2*d^2 - 7*(d*x^3 + c)*d
/x^3 + 14*(d*x^3 + c)^2/x^6)*a^2*x^7/((d*x^3 + c)^(7/3)*c^3)
```

---

**Fricas [A]** time = 0.21906, size = 139, normalized size = 1.78

$$\frac{\left(\left(2\,b^{2}c^{2}+3\,abcd+9\,a^{2}d^{2}\right)x^{7}+14\,a^{2}c^{2}x+7\left(abc^{2}+3\,a^{2}cd\right)x^{4}\right)\left(dx^{3}+c\right)^{\frac{2}{3}}}{14\,(c^{3}d^{3}x^{9}+3\,c^{4}d^{2}x^{6}+3\,c^{5}dx^{3}+c^{6})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2/(d*x^3 + c)^(10/3),x, algorithm="fricas")
```

```
[Out] 1/14*((2*b^2*c^2 + 3*a*b*c*d + 9*a^2*d^2)*x^7 + 14*a^2*c^2*x + 7*
(a*b*c^2 + 3*a^2*c*d)*x^4)*(d*x^3 + c)^(2/3)/(c^3*d^3*x^9 + 3*c^4
*d^2*x^6 + 3*c^5*d*x^3 + c^6)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**2/(d*x**3+c)**(10/3),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3+a)^2}{(dx^3+c)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2/(d*x^3 + c)^(10/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^2/(d*x^3 + c)^(10/3), x)
```

## 3.29 $\int \frac{a+bx^3}{(c+dx^3)^{7/3}}\,dx$

**Optimal**. Leaf size=47

$$\frac{x\left(a+bx^3\right)}{4c\left(c+dx^3\right)^{4/3}}+\frac{3ax}{4c^2\sqrt[3]{c+dx^3}}$$

```
[Out] (x*(a + b*x^3))/(4*c*(c + d*x^3)^(4/3)) + (3*a*x)/(4*c^2*(c + d*x
^3)^(1/3))
```

---

**Rubi [A]** time = 0.0364384, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{x\left(a+bx^3\right)}{4c\left(c+dx^3\right)^{4/3}}+\frac{3ax}{4c^2\sqrt[3]{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)/(c + d*x^3)^(7/3),x]
```

```
[Out] (x*(a + b*x^3))/(4*c*(c + d*x^3)^(4/3)) + (3*a*x)/(4*c^2*(c + d*x
^3)^(1/3))
```

---

**Rubi in Sympy [A]** time = 5.49093, size = 41, normalized size = 0.87

$$\frac{3ax}{4c^2\sqrt[3]{c+dx^3}}+\frac{x\left(a+bx^3\right)}{4c\left(c+dx^3\right)^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)/(d*x**3+c)**(7/3),x)
```

```
[Out] 3*a*x/(4*c**2*(c + d*x**3)**(1/3)) + x*(a + b*x**3)/(4*c*(c + d*x
**3)**(4/3))
```

---

**Mathematica [A]** time = 0.044828, size = 37, normalized size = 0.79

$$\frac{x\left(4ac+3adx^3+bcx^3\right)}{4c^2\left(c+dx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)/(c + d*x^3)^(7/3),x]

[Out] (x*(4*a*c + b*c*x^3 + 3*a*d*x^3))/(4*c^2*(c + d*x^3)^(4/3))
```

---

**Maple [A]** time = 0.005, size = 34, normalized size = 0.7

$$\frac{x\left(3\,ad{x}^{3}+bc{x}^{3}+4\,ac\right)}{4\,{c}^{2}}\left(d{x}^{3}+c\right)^{-\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)/(d*x^3+c)^(7/3),x)

[Out] 1/4*x*(3*a*d*x^3+b*c*x^3+4*a*c)/(d*x^3+c)^(4/3)/c^2
```

---

**Maxima [A]** time = 1.36086, size = 69, normalized size = 1.47

$$\frac{bx^{4}}{4\,{\left(dx^{3}+c\right)}^{\frac{4}{3}}c}-\frac{a{\left(d-\frac{4\,{\left(dx^{3}+c\right)}}{x^{3}}\right)}x^{4}}{4\,{\left(dx^{3}+c\right)}^{\frac{4}{3}}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)/(d*x^3 + c)^(7/3),x, algorithm="maxima")

[Out] 1/4*b*x^4/((d*x^3 + c)^(4/3)*c) - 1/4*a*(d - 4*(d*x^3 + c)/x^3)*x
^4/((d*x^3 + c)^(4/3)*c^2)
```

---

**Fricas [A]** time = 0.217738, size = 73, normalized size = 1.55

$$\frac{{\left({\left(bc+3\,ad\right)}x^{4}+4\,acx\right)}{\left(dx^{3}+c\right)}^{\frac{2}{3}}}{4\,{\left(c^{2}d^{2}x^{6}+2\,c^{3}dx^{3}+c^{4}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)/(d*x^3 + c)^(7/3),x, algorithm="fricas")

[Out] 1/4*((b*c + 3*a*d)*x^4 + 4*a*c*x)*(d*x^3 + c)^(2/3)/(c^2*d^2*x^6
+ 2*c^3*d*x^3 + c^4)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)/(d*x**3+c)**(7/3),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^3 + a}{(dx^3 + c)^{\frac{7}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)/(d*x^3 + c)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)/(d*x^3 + c)^(7/3), x)
```

## 3.30 $\int \frac{1}{(c+dx^3)^{4/3}}\,dx$

**Optimal**. Leaf size=16

$$\frac{x}{c\sqrt[3]{c+dx^3}}$$

```
[Out] x/(c*(c + d*x^3)^(1/3))
```

---

**Rubi [A]** time = 0.00856531, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{x}{c\sqrt[3]{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3)^(-4/3),x]

[Out] x/(c*(c + d*x^3)^(1/3))
```

---

**Rubi in Sympy [A]** time = 1.25739, size = 12, normalized size = 0.75

$$\frac{x}{c\sqrt[3]{c+dx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x**3+c)**(4/3),x)

[Out] x/(c*(c + d*x**3)**(1/3))
```

---

**Mathematica [C]** time = 2.2023, size = 674, normalized size = 42.12

$$\frac{i\sqrt{\frac{\pi}{3}}\left(\frac{1}{3}\right)\left(\frac{(-1)^{2/3}\sqrt[3]{c}}{\sqrt[3]{d}}+x\right)\left(\frac{\sqrt[3]{c}+(-1)^{2/3}\sqrt[3]{d}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{c}}\right)^{4/3}\left(\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+1\right)\left(-36i\left(\left(\sqrt{3}-i\right)\sqrt[3]{c}-\left(\sqrt{3}+i\right)\sqrt[3]{d}x\right)\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2 {}_3F_2\left(2,2,\frac{7}{3};1,\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x^3)^(-4/3),x]
```

```
[Out] ((I/40)*Sqrt[Pi/3]*(((-1)^(2/3)*c^(1/3))/d^(1/3) + x)*((c^(1/3) +
 (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3)))^(4/3)*(1 + (d^
(1/3)*x)/c^(1/3))*Gamma[1/3]*(48*(4*c + 2*(2 - I*Sqrt[3])*c^(2/3)
*d^(1/3)*x + 2*(3 + I*Sqrt[3])*c^(1/3)*d^(2/3)*x^2 + 3*(1 + I*Sqr
t[3])*d*x^3)*Hypergeometric2F1[1, 4/3, 8/3, (6*((1 + I*Sqrt[3])*c
^(1/3) + (1 - I*Sqrt[3])*d^(1/3)*x))/((3*I + Sqrt[3])*((3*I + Sqr
t[3])*c^(1/3) - 2*Sqrt[3]*d^(1/3)*x))] - (12*I)*(c^(1/3) + d^(1/3
)*x)*((-3*I + 7*Sqrt[3])*c^(2/3) + 2*(-9*I + 2*Sqrt[3])*c^(1/3)*d
^(1/3)*x - 9*(I + Sqrt[3])*d^(2/3)*x^2)*Hypergeometric2F1[2, 7/3,
 11/3, (6*((1 + I*Sqrt[3])*c^(1/3) + (1 - I*Sqrt[3])*d^(1/3)*x))/
((3*I + Sqrt[3])*((3*I + Sqrt[3])*c^(1/3) - 2*Sqrt[3]*d^(1/3)*x))
] - (36*I)*(c^(1/3) + d^(1/3)*x)^2*((-I + Sqrt[3])*c^(1/3) - (I +
 Sqrt[3])*d^(1/3)*x)*HypergeometricPFQ[{2, 2, 7/3}, {1, 11/3}, (6
*((1 + I*Sqrt[3])*c^(1/3) + (1 - I*Sqrt[3])*d^(1/3)*x))/((3*I + S
qrt[3])*((3*I + Sqrt[3])*c^(1/3) - 2*Sqrt[3]*d^(1/3)*x))]))/(2^(1
/3)*(3*I + Sqrt[3])*c^(2/3)*((3*I + Sqrt[3])*c^(1/3) - 2*Sqrt[3]*
d^(1/3)*x)*(c + d*x^3)^(4/3)*(1 + (I*((-1)^(2/3)*c^(1/3) + d^(1/3
)*x))/(Sqrt[3]*c^(1/3)))^(4/3)*Gamma[2/3]*Gamma[7/6])
```

---

**Maple [A]** time = 0.003, size = 15, normalized size = 0.9

$$\frac{x}{c}\frac{1}{\sqrt[3]{dx^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x^3+c)^(4/3),x)
```

```
[Out] x/c/(d*x^3+c)^(1/3)
```

---

**Maxima [A]** time = 1.40159, size = 19, normalized size = 1.19

$$\frac{x}{(dx^3+c)^{\frac{1}{3}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^(-4/3),x, algorithm="maxima")
```

```
[Out] x/((d*x^3 + c)^(1/3)*c)
```

---

**Fricas [A]** time = 0.211339, size = 31, normalized size = 1.94

$$\frac{\left(dx^3+c\right)^{\frac{2}{3}}x}{cdx^3+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^(-4/3),x, algorithm="fricas")
```

```
[Out] (d*x^3 + c)^(2/3)*x/(c*d*x^3 + c^2)
```

---

**Sympy [A]** time = 1.95163, size = 29, normalized size = 1.81

$$\frac{x\,\Gamma\left(\frac{1}{3}\right)}{3c^{\frac{4}{3}}\sqrt[3]{1+\frac{dx^3}{c}}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x**3+c)**(4/3),x)
```

```
[Out] x*gamma(1/3)/(3*c**(4/3)*(1 + d*x**3/c)**(1/3)*gamma(4/3))
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(dx^3+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^(-4/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(-4/3), x)
```

## 3.31 $\int \frac{1}{(a+bx^3)\sqrt[3]{c+dx^3}}\,dx$

**Optimal**. Leaf size=208

$$\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}}+\sqrt[3]{a}\right)}{3a^{2/3}\sqrt[3]{bc-ad}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(a^{2/3}-\frac{\sqrt[3]{a}x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}}+\frac{x^2(bc-ad)^{2/3}}{(c+dx^3)^{2/3}}\right)}{6a^{2/3}\sqrt[3]{bc-ad}}$$

```
[Out] -(ArcTan[(a^(1/3) - (2*(b*c - a*d)^(1/3)*x)/(c + d*x^3)^(1/3))/(S
qrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*(b*c - a*d)^(1/3))) + Log[a^(1/
3) + ((b*c - a*d)^(1/3)*x)/(c + d*x^3)^(1/3)]/(3*a^(2/3)*(b*c - a
*d)^(1/3)) - Log[a^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(c + d*x^3)^(2
/3) - (a^(1/3)*(b*c - a*d)^(1/3)*x)/(c + d*x^3)^(1/3)]/(6*a^(2/3)
*(b*c - a*d)^(1/3))
```

---

**Rubi [A]** time = 0.457942, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}}+\sqrt[3]{a}\right)}{3a^{2/3}\sqrt[3]{bc-ad}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(a^{2/3}-\frac{\sqrt[3]{a}x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}}+\frac{x^2(bc-ad)^{2/3}}{(c+dx^3)^{2/3}}\right)}{6a^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^3)*(c + d*x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[(a^(1/3) - (2*(b*c - a*d)^(1/3)*x)/(c + d*x^3)^(1/3))/(S
qrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*(b*c - a*d)^(1/3))) + Log[a^(1/
3) + ((b*c - a*d)^(1/3)*x)/(c + d*x^3)^(1/3)]/(3*a^(2/3)*(b*c - a
*d)^(1/3)) - Log[a^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(c + d*x^3)^(2
/3) - (a^(1/3)*(b*c - a*d)^(1/3)*x)/(c + d*x^3)^(1/3)]/(6*a^(2/3)
*(b*c - a*d)^(1/3))
```

---

**Rubi in Sympy [A]** time = 34.6177, size = 185, normalized size = 0.89

$$-\frac{\log\left(\sqrt[3]{a}-\frac{x\sqrt[3]{ad-bc}}{\sqrt[3]{c+dx^3}}\right)}{3a^{\frac{2}{3}}\sqrt[3]{ad-bc}}+\frac{\log\left(a^{\frac{2}{3}}+\frac{\sqrt[3]{a}x\sqrt[3]{ad-bc}}{\sqrt[3]{c+dx^3}}+\frac{x^2(ad-bc)^{\frac{2}{3}}}{(c+dx^3)^{\frac{2}{3}}}\right)}{6a^{\frac{2}{3}}\sqrt[3]{ad-bc}}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}+\frac{2x\sqrt[3]{ad-bc}}{3\sqrt[3]{c+dx^3}}\right)}{\sqrt[3]{a}}\right)}{3a^{\frac{2}{3}}\sqrt[3]{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**3+a)/(d*x**3+c)**(1/3),x)
```

```
[Out] -log(a**(1/3) - x*(a*d - b*c)**(1/3)/(c + d*x**3)**(1/3))/(3*a**(
2/3)*(a*d - b*c)**(1/3)) + log(a**(2/3) + a**(1/3)*x*(a*d - b*c)*
*(1/3)/(c + d*x**3)**(1/3) + x**2*(a*d - b*c)**(2/3)/(c + d*x**3)
**(2/3))/(6*a**(2/3)*(a*d - b*c)**(1/3)) + sqrt(3)*atan(sqrt(3)*(
a**(1/3)/3 + 2*x*(a*d - b*c)**(1/3)/(3*(c + d*x**3)**(1/3)))/a**(
1/3))/(3*a**(2/3)*(a*d - b*c)**(1/3))
```

---

**Mathematica [A]** time = 0.344175, size = 168, normalized size = 0.81

$$\frac{\log\left(a^{2/3}+\frac{\sqrt[3]{a}x\sqrt[3]{ad-bc}}{\sqrt[3]{cx^3+d}}+\frac{x^2(ad-bc)^{2/3}}{(cx^3+d)^{2/3}}\right)-2\log\left(\sqrt[3]{a}-\frac{x\sqrt[3]{ad-bc}}{\sqrt[3]{cx^3+d}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{ad-bc}}{\sqrt[3]{a}\sqrt[3]{cx^3+d}}+1}{\sqrt{3}}\right)}{6a^{2/3}\sqrt[3]{ad-bc}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^3)*(c + d*x^3)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(-(b*c) + a*d)^(1/3)*x)/(a^(1/3)*(d + c
*x^3)^(1/3)))/Sqrt[3]] - 2*Log[a^(1/3) - ((-(b*c) + a*d)^(1/3)*x)
/(d + c*x^3)^(1/3)] + Log[a^(2/3) + ((-(b*c) + a*d)^(2/3)*x^2)/(d
 + c*x^3)^(2/3) + (a^(1/3)*(-(b*c) + a*d)^(1/3)*x)/(d + c*x^3)^(1
/3)])/(6*a^(2/3)*(-(b*c) + a*d)^(1/3))
```

---

**Maple [F]** time = 0.061, size = 0, normalized size = 0.

$$\int \frac{1}{bx^3+a}\frac{1}{\sqrt[3]{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^3+a)/(d*x^3+c)^(1/3),x)
```

```
[Out] int(1/(b*x^3+a)/(d*x^3+c)^(1/3),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^3 + a)*(d*x^3 + c)^(1/3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(1/3)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^3 + a)*(d*x^3 + c)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^3)\sqrt[3]{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**3+a)/(d*x**3+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x**3)*(c + d*x**3)**(1/3)), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^3 + a)*(d*x^3 + c)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(1/3)), x)
```

## 3.32 $\int \frac{(c+dx^3)^{2/3}}{(a+bx^3)^2}\,dx$

**Optimal**. Leaf size=241

$$\frac{2c\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}}+\sqrt[3]{a}\right)}{9a^{5/3}\sqrt[3]{bc-ad}}-\frac{2c\tan^{-1}\left(\frac{\sqrt[3]{a}-\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}\sqrt[3]{bc-ad}}-\frac{c\log\left(a^{2/3}-\frac{\sqrt[3]{a}x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}}+\frac{x^2(bc-ad)^{2/3}}{(c+dx^3)^{2/3}}\right)}{9a^{5/3}\sqrt[3]{bc-ad}}+\frac{x\left(c+dx^3\right)^{2/3}}{3a\left(a+bx^3\right)}$$

```
[Out] (x*(c + d*x^3)^(2/3))/(3*a*(a + b*x^3)) - (2*c*ArcTan[(a^(1/3) -
(2*(b*c - a*d)^(1/3)*x)/(c + d*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3
*Sqrt[3]*a^(5/3)*(b*c - a*d)^(1/3)) + (2*c*Log[a^(1/3) + ((b*c -
a*d)^(1/3)*x)/(c + d*x^3)^(1/3)])/(9*a^(5/3)*(b*c - a*d)^(1/3)) -
 (c*Log[a^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(c + d*x^3)^(2/3) - (a^
(1/3)*(b*c - a*d)^(1/3)*x)/(c + d*x^3)^(1/3)])/(9*a^(5/3)*(b*c -
a*d)^(1/3))
```

---

**Rubi [A]** time = 0.384953, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$\frac{2c\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}}+\sqrt[3]{a}\right)}{9a^{5/3}\sqrt[3]{bc-ad}}-\frac{2c\tan^{-1}\left(\frac{\sqrt[3]{a}-\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}\sqrt[3]{bc-ad}}-\frac{c\log\left(a^{2/3}-\frac{\sqrt[3]{a}x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}}+\frac{x^2(bc-ad)^{2/3}}{(c+dx^3)^{2/3}}\right)}{9a^{5/3}\sqrt[3]{bc-ad}}+\frac{x\left(c+dx^3\right)^{2/3}}{3a\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3)^(2/3)/(a + b*x^3)^2,x]
```


```
[Out] (x*(c + d*x^3)^(2/3))/(3*a*(a + b*x^3)) - (2*c*ArcTan[(a^(1/3) -
(2*(b*c - a*d)^(1/3)*x)/(c + d*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3
*Sqrt[3]*a^(5/3)*(b*c - a*d)^(1/3)) + (2*c*Log[a^(1/3) + ((b*c -
a*d)^(1/3)*x)/(c + d*x^3)^(1/3)])/(9*a^(5/3)*(b*c - a*d)^(1/3)) -
 (c*Log[a^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(c + d*x^3)^(2/3) - (a^
(1/3)*(b*c - a*d)^(1/3)*x)/(c + d*x^3)^(1/3)])/(9*a^(5/3)*(b*c -
a*d)^(1/3))
```

**Rubi in Sympy [A]** time = 44.9982, size = 216, normalized size = 0.9

$$\frac{x\left(c+dx^3\right)^{\frac{2}{3}}}{3a\left(a+bx^3\right)}-\frac{2c\log\left(\sqrt[3]{a}-\frac{x\sqrt[3]{ad-bc}}{\sqrt[3]{c+dx^3}}\right)}{9a^{\frac{5}{3}}\sqrt[3]{ad-bc}}+\frac{c\log\left(a^{\frac{2}{3}}+\frac{\sqrt[3]{a}x\sqrt[3]{ad-bc}}{\sqrt[3]{c+dx^3}}+\frac{x^2(ad-bc)^{\frac{2}{3}}}{(c+dx^3)^{\frac{2}{3}}}\right)}{9a^{\frac{5}{3}}\sqrt[3]{ad-bc}}+\frac{2\sqrt{3}c\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}+\frac{2x\sqrt[3]{ad-bc}}{3\sqrt[3]{c+dx^3}}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{5}{3}}\sqrt[3]{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c)**(2/3)/(b*x**3+a)**2,x)
```

```
[Out] x*(c + d*x**3)**(2/3)/(3*a*(a + b*x**3)) - 2*c*log(a**(1/3) - x*(
a*d - b*c)**(1/3)/(c + d*x**3)**(1/3))/(9*a**(5/3)*(a*d - b*c)**(
1/3)) + c*log(a**(2/3) + a**(1/3)*x*(a*d - b*c)**(1/3)/(c + d*x**
3)**(1/3) + x**2*(a*d - b*c)**(2/3)/(c + d*x**3)**(2/3))/(9*a**(5
/3)*(a*d - b*c)**(1/3)) + 2*sqrt(3)*c*atan(sqrt(3)*(a**(1/3)/3 +
2*x*(a*d - b*c)**(1/3)/(3*(c + d*x**3)**(1/3)))/a**(1/3))/(9*a**(
5/3)*(a*d - b*c)**(1/3))
```

**Mathematica [A]** time = 0.5817, size = 198, normalized size = 0.82

$$\frac{c\left(\log\left(a^{2/3}+\frac{\sqrt[3]{a}x\sqrt[3]{ad-bc}}{\sqrt[3]{cx^3+d}}+\frac{x^2(ad-bc)^{2/3}}{(cx^3+d)^{2/3}}\right)-2\log\left(\sqrt[3]{a}-\frac{x\sqrt[3]{ad-bc}}{\sqrt[3]{cx^3+d}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{ad-bc}}{\sqrt[3]{a}\sqrt[3]{cx^3+d}}+1}{\sqrt{3}}\right)\right)}{9a^{5/3}\sqrt[3]{ad-bc}}+\frac{x\left(c+dx^3\right)^{2/3}}{3a\left(a+bx^3\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x^3)^(2/3)/(a + b*x^3)^2,x]
```

```
[Out] (x*(c + d*x^3)^(2/3))/(3*a*(a + b*x^3)) + (c*(2*Sqrt[3]*ArcTan[(1
 + (2*(-(b*c) + a*d)^(1/3)*x)/(a^(1/3)*(d + c*x^3)^(1/3)))/Sqrt[3
]] - 2*Log[a^(1/3) - ((-(b*c) + a*d)^(1/3)*x)/(d + c*x^3)^(1/3)]
+ Log[a^(2/3) + ((-(b*c) + a*d)^(2/3)*x^2)/(d + c*x^3)^(2/3) + (a
^(1/3)*(-(b*c) + a*d)^(1/3)*x)/(d + c*x^3)^(1/3)]))/(9*a^(5/3)*(-
(b*c) + a*d)^(1/3))
```

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{3}+a\right)^{2}} \left(d x^{3}+c\right)^{\frac{2}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c)^(2/3)/(b*x^3+a)^2,x)

[Out] int((d*x^3+c)^(2/3)/(b*x^3+a)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d x^{3}+c\right)^{\frac{2}{3}}}{\left(b x^{3}+a\right)^{2}}\, d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^(2/3)/(b*x^3 + a)^2,x, algorithm="maxima")

[Out] integrate((d*x^3 + c)^(2/3)/(b*x^3 + a)^2, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^(2/3)/(b*x^3 + a)^2,x, algorithm="fricas")

[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c)**(2/3)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3 + c\right)^{\frac{2}{3}}}{\left(bx^3 + a\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^(2/3)/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(2/3)/(b*x^3 + a)^2, x)
```

## 3.33 $\int \frac{(c+dx^3)^{5/3}}{(a+bx^3)^3}\,dx$

**Optimal**. Leaf size=276

$$\frac{5c^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}}+\sqrt[3]{a}\right)}{27a^{8/3}\sqrt[3]{bc-ad}}-\frac{5c^2\tan^{-1}\left(\frac{\sqrt[3]{a}-\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}\sqrt[3]{bc-ad}}-\frac{5c^2\log\left(a^{2/3}-\frac{\sqrt[3]{a}x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}}+\frac{x^2(bc-ad)^{2/3}}{(c+dx^3)^{2/3}}\right)}{54a^{8/3}\sqrt[3]{bc-ad}}+\frac{5cx\left(c+dx^3\right)^{2/3}}{18a^2\left(a+bx^3\right)}+\frac{x\left(c+dx^3\right)^{5/3}}{6a\left(a+bx^3\right)^2}$$

```
[Out] (5*c*x*(c + d*x^3)^(2/3))/(18*a^2*(a + b*x^3)) + (x*(c + d*x^3)^(
5/3))/(6*a*(a + b*x^3)^2) - (5*c^2*ArcTan[(a^(1/3) - (2*(b*c - a*
d)^(1/3)*x)/(c + d*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(
8/3)*(b*c - a*d)^(1/3)) + (5*c^2*Log[a^(1/3) + ((b*c - a*d)^(1/3)
*x)/(c + d*x^3)^(1/3)])/(27*a^(8/3)*(b*c - a*d)^(1/3)) - (5*c^2*L
og[a^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(c + d*x^3)^(2/3) - (a^(1/3)
*(b*c - a*d)^(1/3)*x)/(c + d*x^3)^(1/3)])/(54*a^(8/3)*(b*c - a*d)
^(1/3))
```

---

**Rubi [A]** time = 0.457373, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381

$$\frac{5c^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}}+\sqrt[3]{a}\right)}{27a^{8/3}\sqrt[3]{bc-ad}}-\frac{5c^2\tan^{-1}\left(\frac{\sqrt[3]{a}-\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}\sqrt[3]{bc-ad}}-\frac{5c^2\log\left(a^{2/3}-\frac{\sqrt[3]{a}x\sqrt[3]{bc-ad}}{\sqrt[3]{c+dx^3}}+\frac{x^2(bc-ad)^{2/3}}{(c+dx^3)^{2/3}}\right)}{54a^{8/3}\sqrt[3]{bc-ad}}+\frac{5cx\left(c+dx^3\right)^{2/3}}{18a^2\left(a+bx^3\right)}+\frac{x\left(c+dx^3\right)^{5/3}}{6a\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3)^(5/3)/(a + b*x^3)^3,x]
```


```
[Out] (5*c*x*(c + d*x^3)^(2/3))/(18*a^2*(a + b*x^3)) + (x*(c + d*x^3)^(
5/3))/(6*a*(a + b*x^3)^2) - (5*c^2*ArcTan[(a^(1/3) - (2*(b*c - a*
d)^(1/3)*x)/(c + d*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(
8/3)*(b*c - a*d)^(1/3)) + (5*c^2*Log[a^(1/3) + ((b*c - a*d)^(1/3)
*x)/(c + d*x^3)^(1/3)])/(27*a^(8/3)*(b*c - a*d)^(1/3)) - (5*c^2*L
og[a^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(c + d*x^3)^(2/3) - (a^(1/3)
*(b*c - a*d)^(1/3)*x)/(c + d*x^3)^(1/3)])/(54*a^(8/3)*(b*c - a*d)
```

```
^(1/3))
```

---

**Rubi in Sympy [A]** time = 58.5826, size = 252, normalized size = 0.91

$$\frac{x\left(c+dx^3\right)^{\frac{5}{3}}}{6a\left(a+bx^3\right)^2}+\frac{5cx\left(c+dx^3\right)^{\frac{2}{3}}}{18a^2\left(a+bx^3\right)}-\frac{5c^2\log\left(\sqrt[3]{a}-\frac{x\sqrt[3]{ad-bc}}{\sqrt[3]{c+dx^3}}\right)}{27a^{\frac{8}{3}}\sqrt[3]{ad-bc}}+\frac{5c^2\log\left(a^{\frac{2}{3}}+\frac{\sqrt[3]{a}x\sqrt[3]{ad-bc}}{\sqrt[3]{c+dx^3}}+\frac{x^2(ad-bc)^{\frac{2}{3}}}{(c+dx^3)^{\frac{2}{3}}}\right)}{54a^{\frac{8}{3}}\sqrt[3]{ad-bc}}+\frac{5\sqrt{3}c^2\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3}+\frac{2x\sqrt[3]{ad-bc}}{3\sqrt[3]{c+dx^3}}\right)}{\sqrt[3]{a}}\right)}{27a^{\frac{8}{3}}\sqrt[3]{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c)**(5/3)/(b*x**3+a)**3,x)
```

```
[Out] x*(c + d*x**3)**(5/3)/(6*a*(a + b*x**3)**2) + 5*c*x*(c + d*x**3)*
*(2/3)/(18*a**2*(a + b*x**3)) - 5*c**2*log(a**(1/3) - x*(a*d - b*
c)**(1/3)/(c + d*x**3)**(1/3))/(27*a**(8/3)*(a*d - b*c)**(1/3)) +
 5*c**2*log(a**(2/3) + a**(1/3)*x*(a*d - b*c)**(1/3)/(c + d*x**3)
**(1/3) + x**2*(a*d - b*c)**(2/3)/(c + d*x**3)**(2/3))/(54*a**(8/
3)*(a*d - b*c)**(1/3)) + 5*sqrt(3)*c**2*atan(sqrt(3)*(a**(1/3)/3
+ 2*x*(a*d - b*c)**(1/3)/(3*(c + d*x**3)**(1/3)))/a**(1/3))/(27*a
**(8/3)*(a*d - b*c)**(1/3))
```

---

**Mathematica [A]** time = 0.727875, size = 219, normalized size = 0.79

$$\frac{5c^2\left(\log\left(a^{2/3}+\frac{\sqrt[3]{a}x\sqrt[3]{ad-bc}}{\sqrt[3]{cx^3+d}}+\frac{x^2(ad-bc)^{2/3}}{(cx^3+d)^{2/3}}\right)-2\log\left(\sqrt[3]{a}-\frac{x\sqrt[3]{ad-bc}}{\sqrt[3]{cx^3+d}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{ad-bc}}{\sqrt[3]{a}\sqrt[3]{cx^3+d}}+1}{\sqrt{3}}\right)\right)}{54a^{8/3}\sqrt[3]{ad-bc}}+\frac{x\left(c+dx^3\right)^{2/3}\left(8ac+3adx^3+5bcx^3\right)}{18a^2\left(a+bx^3\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x^3)^(5/3)/(a + b*x^3)^3,x]
```

```
[Out] (x*(c + d*x^3)^(2/3)*(8*a*c + 5*b*c*x^3 + 3*a*d*x^3))/(18*a^2*(a
+ b*x^3)^2) + (5*c^2*(2*Sqrt[3]*ArcTan[(1 + (2*(-(b*c) + a*d)^(1/
3)*x)/(a^(1/3)*(d + c*x^3)^(1/3)))/Sqrt[3]] - 2*Log[a^(1/3) - ((-
(b*c) + a*d)^(1/3)*x)/(d + c*x^3)^(1/3)] + Log[a^(2/3) + ((-(b*c)
```

```
 + a*d)^(2/3)*x^2)/(d + c*x^3)^(2/3) + (a^(1/3)*(-(b*c) + a*d)^(1
/3)*x)/(d + c*x^3)^(1/3)]))/(54*a^(8/3)*(-(b*c) + a*d)^(1/3))
```

---

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^3}\left(dx^3+c\right)^{\frac{5}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c)^(5/3)/(b*x^3+a)^3,x)

[Out] int((d*x^3+c)^(5/3)/(b*x^3+a)^3,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{5}{3}}}{\left(bx^3+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^(5/3)/(b*x^3 + a)^3,x, algorithm="maxima")

[Out] integrate((d*x^3 + c)^(5/3)/(b*x^3 + a)^3, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^(5/3)/(b*x^3 + a)^3,x, algorithm="fricas")

[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c)**(5/3)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{5}{3}}}{\left(bx^3+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^(5/3)/(b*x^3 + a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(5/3)/(b*x^3 + a)^3, x)
```

## 3.34 $\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p \, dx$

**Optimal**. Leaf size=79

$$x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(c+dx^3\right)^p\left(\frac{dx^3}{c}+1\right)^{-p}F_1\left(\frac{1}{3};-m,-p;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)$$

```
[Out] (x*(a + b*x^3)^m*(c + d*x^3)^p*AppellF1[1/3, -m, -p, 4/3, -((b*x^
3)/a), -((d*x^3)/c)])/((1 + (b*x^3)/a)^m*(1 + (d*x^3)/c)^p)
```

---

**Rubi [A]** time = 0.106559, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(c+dx^3\right)^p\left(\frac{dx^3}{c}+1\right)^{-p}F_1\left(\frac{1}{3};-m,-p;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^m*(c + d*x^3)^p,x]
```

```
[Out] (x*(a + b*x^3)^m*(c + d*x^3)^p*AppellF1[1/3, -m, -p, 4/3, -((b*x^
3)/a), -((d*x^3)/c)])/((1 + (b*x^3)/a)^m*(1 + (d*x^3)/c)^p)
```

---

**Rubi in Sympy [A]** time = 21.1481, size = 61, normalized size = 0.77

$$x\left(1+\frac{bx^3}{a}\right)^{-m}\left(1+\frac{dx^3}{c}\right)^{-p}\left(a+bx^3\right)^m\left(c+dx^3\right)^p \operatorname{appellf}_1\left(\frac{1}{3},-m,-p,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**m*(d*x**3+c)**p,x)
```

```
[Out] x*(1 + b*x**3/a)**(-m)*(1 + d*x**3/c)**(-p)*(a + b*x**3)**m*(c +
d*x**3)**p*appellf1(1/3, -m, -p, 4/3, -b*x**3/a, -d*x**3/c)
```

---

**Mathematica [B]** time = 0.355429, size = 172, normalized size = 2.18

$$\frac{4acx\left(a+bx^3\right)^m\left(c+dx^3\right)^pF_1\left(\frac{1}{3};-m,-p;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{3x^3\left(bcmF_1\left(\frac{4}{3};1-m,-p;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+adpF_1\left(\frac{4}{3};-m,1-p;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)+4acF_1\left(\frac{1}{3};-m,-p;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3)^m*(c + d*x^3)^p,x]
```

```
[Out] (4*a*c*x*(a + b*x^3)^m*(c + d*x^3)^p*AppellF1[1/3, -m, -p, 4/3, -
((b*x^3)/a), -((d*x^3)/c)])/(4*a*c*AppellF1[1/3, -m, -p, 4/3, -((
b*x^3)/a), -((d*x^3)/c)] + 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, -p,
7/3, -((b*x^3)/a), -((d*x^3)/c)] + a*d*p*AppellF1[4/3, -m, 1 - p,
 7/3, -((b*x^3)/a), -((d*x^3)/c)]))
```

---

**Maple [F]** time = 0.114, size = 0, normalized size = 0.

$$\int \left(b x^3+a\right)^m \left(d x^3+c\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^m*(d*x^3+c)^p,x)
```

```
[Out] int((b*x^3+a)^m*(d*x^3+c)^p,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(b x^3+a\right)^m \left(d x^3+c\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^m*(d*x^3 + c)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b x^3+a\right)^m \left(d x^3+c\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^m*(d*x^3 + c)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**m*(d*x**3+c)**p,x)
```

```
[Out] Timed out
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + a)^m (dx^3 + c)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^m*(d*x^3 + c)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

## 3.35 $\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^q \, dx$

**Optimal**. Leaf size=167

$$\frac{x\left(c+dx^3\right)^{q+1}\left(a^2d^2\left(9q^2+33q+28\right)-2abcd(3q+7)+4b^2c^2\right){}_2F_1\left(1,q+\frac{4}{3};\frac{4}{3};-\frac{dx^3}{c}\right)}{cd^2(3q+4)(3q+7)}-\frac{bx\left(c+dx^3\right)^{q+1}(4bc-ad(3q+10))}{d^2(3q+4)(3q+7)}+\frac{bx\left(a+bx^3\right)\left(c+dx^3\right)^{q+1}}{d(3q+7)}$$

```
[Out] -((b*(4*b*c - a*d*(10 + 3*q))*x*(c + d*x^3)^(1 + q))/(d^2*(4 + 3*
q)*(7 + 3*q))) + (b*x*(a + b*x^3)*(c + d*x^3)^(1 + q))/(d*(7 + 3*
q)) + ((4*b^2*c^2 - 2*a*b*c*d*(7 + 3*q) + a^2*d^2*(28 + 33*q + 9*
q^2))*x*(c + d*x^3)^(1 + q)*Hypergeometric2F1[1, 4/3 + q, 4/3, -(
(d*x^3)/c)])/(c*d^2*(4 + 3*q)*(7 + 3*q))
```

---

**Rubi [A]** time = 0.278101, antiderivative size = 176, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}\left(a^2d^2\left(9q^2+33q+28\right)-2abcd(3q+7)+4b^2c^2\right){}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};-\frac{dx^3}{c}\right)}{d^2(3q+4)(3q+7)}-\frac{bx\left(c+dx^3\right)^{q+1}(4bc-ad(3q+10))}{d^2(3q+4)(3q+7)}+\frac{bx\left(a+bx^3\right)\left(c+dx^3\right)^{q+1}}{d(3q+7)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^2*(c + d*x^3)^q,x]
```

```
[Out] -((b*(4*b*c - a*d*(10 + 3*q))*x*(c + d*x^3)^(1 + q))/(d^2*(4 + 3*
q)*(7 + 3*q))) + (b*x*(a + b*x^3)*(c + d*x^3)^(1 + q))/(d*(7 + 3*
q)) + ((4*b^2*c^2 - 2*a*b*c*d*(7 + 3*q) + a^2*d^2*(28 + 33*q + 9*
q^2))*x*(c + d*x^3)^q*Hypergeometric2F1[1/3, -q, 4/3, -((d*x^3)/c
)])/(d^2*(4 + 3*q)*(7 + 3*q)*(1 + (d*x^3)/c)^q)
```

---

**Rubi in Sympy [A]** time = 33.0944, size = 153, normalized size = 0.92

$$\frac{bx\left(a+bx^3\right)\left(c+dx^3\right)^{q+1}}{d\,(3q+7)}-\frac{bx\left(c+dx^3\right)^{q+1}\left(-ad\,(3q+10)+4bc\right)}{d^2\,(3q+4)\,(3q+7)}+\frac{x\left(1+\frac{dx^3}{c}\right)^{-q}\left(c+dx^3\right)^q\left(-ad\,(3q+4)\left(-ad\,(3q+7)+bc\right)+bc\left(-ad\,(3q+10)+4bc\right)\right){}_2F_1\left(\begin{matrix}-q,\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|-\frac{dx^3}{c}\right)}{d^2\,(3q+4)\,(3q+7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**2*(d*x**3+c)**q,x)
```

```
[Out] b*x*(a + b*x**3)*(c + d*x**3)**(q + 1)/(d*(3*q + 7)) - b*x*(c + d
*x**3)**(q + 1)*(-a*d*(3*q + 10) + 4*b*c)/(d**2*(3*q + 4)*(3*q +
7)) + x*(1 + d*x**3/c)**(-q)*(c + d*x**3)**q*(-a*d*(3*q + 4)*(-a*
d*(3*q + 7) + b*c) + b*c*(-a*d*(3*q + 10) + 4*b*c))*hyper((-q, 1/
3), (4/3,), -d*x**3/c)/(d**2*(3*q + 4)*(3*q + 7))
```

---

**Mathematica [A]** time = 0.0799186, size = 106, normalized size = 0.63

$$\frac{1}{14}x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}\left(14a^2\,{}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};-\frac{dx^3}{c}\right)+bx^3\left(7a\,{}_2F_1\left(\frac{4}{3},-q;\frac{7}{3};-\frac{dx^3}{c}\right)+2bx^3\,{}_2F_1\left(\frac{7}{3},-q;\frac{10}{3};-\frac{dx^3}{c}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)^2*(c + d*x^3)^q,x]
```

```
[Out] (x*(c + d*x^3)^q*(14*a^2*Hypergeometric2F1[1/3, -q, 4/3, -((d*x^3
)/c)] + b*x^3*(7*a*Hypergeometric2F1[4/3, -q, 7/3, -((d*x^3)/c)]
+ 2*b*x^3*Hypergeometric2F1[7/3, -q, 10/3, -((d*x^3)/c)])))/(14*(
1 + (d*x^3)/c)^q)
```

---

**Maple [F]** time = 0.082, size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^2\left(dx^3+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^2*(d*x^3+c)^q,x)
```

```
[Out] int((b*x^3+a)^2*(d*x^3+c)^q,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^2\left(dx^3+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(d*x^3 + c)^q,x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^2*(d*x^3 + c)^q, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^6 + 2\,abx^3 + a^2\right)\left(dx^3 + c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(d*x^3 + c)^q,x, algorithm="fricas")

[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x^3 + c)^q, x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**2*(d*x**3+c)**q,x)

[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3 + a\right)^2 \left(dx^3 + c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^2*(d*x^3 + c)^q,x, algorithm="giac")

[Out] integrate((b*x^3 + a)^2*(d*x^3 + c)^q, x)
```

## 3.36 $\int (a+bx^3) (c+dx^3)^q \, dx$

**Optimal**. Leaf size=84

$$\frac{bx(c+dx^3)^{q+1}}{d(3q+4)}-\frac{x(c+dx^3)^{q+1}(bc-ad(3q+4))\,{}_2F_1\left(1,q+\frac{4}{3};\frac{4}{3};-\frac{dx^3}{c}\right)}{cd(3q+4)}$$

```
[Out] (b*x*(c + d*x^3)^(1 + q))/(d*(4 + 3*q)) - ((b*c - a*d*(4 + 3*q))*
x*(c + d*x^3)^(1 + q)*Hypergeometric2F1[1, 4/3 + q, 4/3, -((d*x^3
)/c)])/(c*d*(4 + 3*q))
```

---

**Rubi [A]** time = 0.0984101, antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$x(c+dx^3)^q\left(\frac{dx^3}{c}+1\right)^{-q}\left(a-\frac{bc}{3dq+4d}\right)\,{}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};-\frac{dx^3}{c}\right)+\frac{bx(c+dx^3)^{q+1}}{d(3q+4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)*(c + d*x^3)^q,x]
```

```
[Out] (b*x*(c + d*x^3)^(1 + q))/(d*(4 + 3*q)) + ((a - (b*c)/(4*d + 3*d*
q))*x*(c + d*x^3)^q*Hypergeometric2F1[1/3, -q, 4/3, -((d*x^3)/c)]
)/(1 + (d*x^3)/c)^q
```

---

**Rubi in Sympy [A]** time = 11.049, size = 73, normalized size = 0.87

$$\frac{bx(c+dx^3)^{q+1}}{d(3q+4)}-\frac{x\left(1+\frac{dx^3}{c}\right)^{-q}(c+dx^3)^q(-ad(3q+4)+bc)\,{}_2F_1\left(\begin{matrix}-q,\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|-\frac{dx^3}{c}\right)}{d(3q+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)*(d*x**3+c)**q,x)
```

```
[Out] b*x*(c + d*x**3)**(q + 1)/(d*(3*q + 4)) - x*(1 + d*x**3/c)**(-q)*
(c + d*x**3)**q*(-a*d*(3*q + 4) + b*c)*hyper((-q, 1/3), (4/3,), -
d*x**3/c)/(d*(3*q + 4))
```

---

**Mathematica [A]** time = 0.0312364, size = 75, normalized size = 0.89

$$\frac{1}{4}x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}\left(4a\,{}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};-\frac{dx^3}{c}\right)+bx^3\,{}_2F_1\left(\frac{4}{3},-q;\frac{7}{3};-\frac{dx^3}{c}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)*(c + d*x^3)^q,x]
```

```
[Out] (x*(c + d*x^3)^q*(4*a*Hypergeometric2F1[1/3, -q, 4/3, -((d*x^3)/c
)] + b*x^3*Hypergeometric2F1[4/3, -q, 7/3, -((d*x^3)/c)]))/(4*(1
+ (d*x^3)/c)^q)
```

---

**Maple [F]** time = 0.048, size = 0, normalized size = 0.

$$\int\left(bx^3+a\right)\left(dx^3+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)*(d*x^3+c)^q,x)
```

```
[Out] int((b*x^3+a)*(d*x^3+c)^q,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\left(bx^3+a\right)\left(dx^3+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(d*x^3 + c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)*(d*x^3 + c)^q, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^3+a\right)\left(dx^3+c\right)^q,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(d*x^3 + c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)*(d*x^3 + c)^q, x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)*(d*x**3+c)**q,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + a) (dx^3 + c)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)*(d*x^3 + c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)*(d*x^3 + c)^q, x)
```

## 3.37 $\int \frac{(c+dx^3)^q}{a+bx^3} \, dx$

**Optimal**. Leaf size=57

$$\frac{x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}F_1\left(\frac{1}{3};1,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a}$$

```
[Out] (x*(c + d*x^3)^q*AppellF1[1/3, 1, -q, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a*(1 + (d*x^3)/c)^q)
```

---

**Rubi [A]** time = 0.0747915, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}F_1\left(\frac{1}{3};1,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3)^q/(a + b*x^3),x]
```

```
[Out] (x*(c + d*x^3)^q*AppellF1[1/3, 1, -q, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a*(1 + (d*x^3)/c)^q)
```

---

**Rubi in Sympy [A]** time = 21.6503, size = 42, normalized size = 0.74

$$\frac{x\left(1+\frac{dx^3}{c}\right)^{-q}\left(c+dx^3\right)^q \operatorname{appellf}_1\left(\frac{1}{3},1,-q,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c)**q/(b*x**3+a),x)
```

```
[Out] x*(1 + d*x**3/c)**(-q)*(c + d*x**3)**q*appellf1(1/3, 1, -q, 4/3,
-b*x**3/a, -d*x**3/c)/a
```

---

**Mathematica [B]** time = 0.284151, size = 162, normalized size = 2.84

$$\frac{4acx\left(c+dx^3\right)^q F_1\left(\frac{1}{3};-q,1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)\left(3x^3\left(adqF_1\left(\frac{4}{3};1-q,1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-bcF_1\left(\frac{4}{3};-q,2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)+4acF_1\left(\frac{1}{3};-q,1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x^3)^q/(a + b*x^3),x]
```

```
[Out] (4*a*c*x*(c + d*x^3)^q*AppellF1[1/3, -q, 1, 4/3, -((d*x^3)/c), -(
(b*x^3)/a)])/((a + b*x^3)*(4*a*c*AppellF1[1/3, -q, 1, 4/3, -((d*x
^3)/c), -((b*x^3)/a)] + 3*x^3*(a*d*q*AppellF1[4/3, 1 - q, 1, 7/3,
 -((d*x^3)/c), -((b*x^3)/a)] - b*c*AppellF1[4/3, -q, 2, 7/3, -((d
*x^3)/c), -((b*x^3)/a)])))
```

---

**Maple [F]** time = 0.069, size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^q}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c)^q/(b*x^3+a),x)
```

```
[Out] int((d*x^3+c)^q/(b*x^3+a),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^q}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^q/(b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(dx^3+c\right)^q}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^q/(b*x^3 + a),x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)^q/(b*x^3 + a), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c)**q/(b*x**3+a),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3 + c\right)^q}{bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^q/(b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a), x)
```

## 3.38 $\int \frac{(c+dx^3)^q}{(a+bx^3)^2}\,dx$

**Optimal**. Leaf size=57

$$\frac{x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}F_1\left(\frac{1}{3};2,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2}$$

```
[Out] (x*(c + d*x^3)^q*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a^2*(1 + (d*x^3)/c)^q)
```

---

**Rubi [A]** time = 0.0734547, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}F_1\left(\frac{1}{3};2,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^3)^q/(a + b*x^3)^2,x]
```

```
[Out] (x*(c + d*x^3)^q*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a^2*(1 + (d*x^3)/c)^q)
```

---

**Rubi in Sympy [A]** time = 20.4988, size = 44, normalized size = 0.77

$$\frac{x\left(1+\frac{dx^3}{c}\right)^{-q}\left(c+dx^3\right)^q\operatorname{appellf}_1\left(\frac{1}{3},2,-q,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**3+c)**q/(b*x**3+a)**2,x)
```

```
[Out] x*(1 + d*x**3/c)**(-q)*(c + d*x**3)**q*appellf1(1/3, 2, -q, 4/3,
-b*x**3/a, -d*x**3/c)/a**2
```

---

**Mathematica [B]** time = 0.296378, size = 162, normalized size = 2.84

$$\frac{4acx\left(c+dx^3\right)^qF_1\left(\frac{1}{3};2,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(a+bx^3\right)^2\left(3x^3\left(adqF_1\left(\frac{4}{3};2,1-q;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2bcF_1\left(\frac{4}{3};3,-q;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)+4acF_1\left(\frac{1}{3};2,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x^3)^q/(a + b*x^3)^2,x]
```

```
[Out] (4*a*c*x*(c + d*x^3)^q*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -(
(d*x^3)/c)])/((a + b*x^3)^2*(4*a*c*AppellF1[1/3, 2, -q, 4/3, -((b
*x^3)/a), -((d*x^3)/c)] + 3*x^3*(a*d*q*AppellF1[4/3, 2, 1 - q, 7/
3, -((b*x^3)/a), -((d*x^3)/c)] - 2*b*c*AppellF1[4/3, 3, -q, 7/3,
-((b*x^3)/a), -((d*x^3)/c)])))
```

---

**Maple [F]** time = 0.097, size = 0, normalized size = 0.

$$\int \frac{(dx^3+c)^q}{(bx^3+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^3+c)^q/(b*x^3+a)^2,x)
```

```
[Out] int((d*x^3+c)^q/(b*x^3+a)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3+c)^q}{(bx^3+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^q/(b*x^3 + a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a)^2, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^3+c)^q}{b^2x^6+2\,abx^3+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^q/(b*x^3 + a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)^q/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**3+c)**q/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3 + c\right)^q}{\left(bx^3 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^q/(b*x^3 + a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a)^2, x)
```

## 3.39 $\int (a+bx^3)^m (c+dx^3)^3 \, dx$

**Optimal**. Leaf size=298

$$\frac{dx\left(a+bx^3\right)^{m+1}\left(28a^2d^2-abcd(15m+92)+b^2c^2\left(9m^2+60m+118\right)\right)}{b^3(3m+4)(3m+7)(3m+10)}$$
$$-\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(28a^3d^3-12a^2bcd^2(3m+10)+3ab^2c^2d\left(9m^2+51m+70\right)-b^3c^3\left(27m^3+189m^2+414m+28\right.\right.}{b^3(3m+4)(3m+7)(3m+10)}$$
$$-\frac{dx\left(c+dx^3\right)\left(a+bx^3\right)^{m+1}(7ad-bc(3m+16))}{b^2(3m+7)(3m+10)}+\frac{dx\left(c+dx^3\right)^2\left(a+bx^3\right)^{m+1}}{b(3m+10)}$$

```
[Out] (d*(28*a^2*d^2 - a*b*c*d*(92 + 15*m) + b^2*c^2*(118 + 60*m + 9*m^
2))*x*(a + b*x^3)^(1 + m))/(b^3*(4 + 3*m)*(7 + 3*m)*(10 + 3*m)) -
 (d*(7*a*d - b*c*(16 + 3*m))*x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(
b^2*(7 + 3*m)*(10 + 3*m)) + (d*x*(a + b*x^3)^(1 + m)*(c + d*x^3)^
2)/(b*(10 + 3*m)) - ((28*a^3*d^3 - 12*a^2*b*c*d^2*(10 + 3*m) + 3*
a*b^2*c^2*d*(70 + 51*m + 9*m^2) - b^3*c^3*(280 + 414*m + 189*m^2
+ 27*m^3))*x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x
^3)/a)])/(b^3*(4 + 3*m)*(7 + 3*m)*(10 + 3*m)*(1 + (b*x^3)/a)^m)
```

---

**Rubi [A]** time = 0.689545, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$


$$\frac{dx\left(a+bx^3\right)^{m+1}\left(28a^2d^2-abcd(15m+92)+b^2c^2\left(9m^2+60m+118\right)\right)}{b^3(3m+4)(3m+7)(3m+10)}$$
$$-\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(28a^3d^3-12a^2bcd^2(3m+10)+3ab^2c^2d\left(9m^2+51m+70\right)-b^3c^3\left(27m^3+189m^2+414m+28\right.\right.}{b^3(3m+4)(3m+7)(3m+10)}$$
$$-\frac{dx\left(c+dx^3\right)\left(a+bx^3\right)^{m+1}(7ad-bc(3m+16))}{b^2(3m+7)(3m+10)}+\frac{dx\left(c+dx^3\right)^2\left(a+bx^3\right)^{m+1}}{b(3m+10)}$$


Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^m*(c + d*x^3)^3,x]
```


```
[Out] (d*(28*a^2*d^2 - a*b*c*d*(92 + 15*m) + b^2*c^2*(118 + 60*m + 9*m^
2))*x*(a + b*x^3)^(1 + m))/(b^3*(4 + 3*m)*(7 + 3*m)*(10 + 3*m)) -
 (d*(7*a*d - b*c*(16 + 3*m))*x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(
b^2*(7 + 3*m)*(10 + 3*m)) + (d*x*(a + b*x^3)^(1 + m)*(c + d*x^3)^
2)/(b*(10 + 3*m)) - ((28*a^3*d^3 - 12*a^2*b*c*d^2*(10 + 3*m) + 3*
a*b^2*c^2*d*(70 + 51*m + 9*m^2) - b^3*c^3*(280 + 414*m + 189*m^2
+ 27*m^3))*x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x
^3)/a)])/(b^3*(4 + 3*m)*(7 + 3*m)*(10 + 3*m)*(1 + (b*x^3)/a)^m)
```


---

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**m*(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

---

**Mathematica [A]** time = 0.103983, size = 137, normalized size = 0.46

$$\frac{1}{140}x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(140c^3\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)+dx^3\left(105c^2\,{}_2F_1\left(\frac{4}{3},-m;\frac{7}{3};-\frac{bx^3}{a}\right)+2dx^3\left(30c\,{}_2F_1\left(\frac{7}{3},-m;\frac{10}{3};-\frac{bx^3}{a}\right)+7dx^3\,{}_2F_1\left(\frac{10}{3},-m;\frac{13}{3};-\frac{bx^3}{a}\right)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)^m*(c + d*x^3)^3,x]
```

```
[Out] (x*(a + b*x^3)^m*(140*c^3*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^
3)/a)] + d*x^3*(105*c^2*Hypergeometric2F1[4/3, -m, 7/3, -((b*x^3)
/a)] + 2*d*x^3*(30*c*Hypergeometric2F1[7/3, -m, 10/3, -((b*x^3)/a
)] + 7*d*x^3*Hypergeometric2F1[10/3, -m, 13/3, -((b*x^3)/a)]))))/
(140*(1 + (b*x^3)/a)^m)
```

---

**Maple [F]** time = 0.07, size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^m \left(dx^3+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^m*(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^m*(d*x^3+c)^3,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(dx^3+c\right)^3\left(bx^3+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3*(b*x^3 + a)^m,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^3*(b*x^3 + a)^m, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^{3}x^{9} + 3\,cd^{2}x^{6} + 3\,c^{2}dx^{3} + c^{3}\right)\left(bx^{3} + a\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3*(b*x^3 + a)^m,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3)*(b*x^3 + a)^
m, x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**m*(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(dx^{3} + c\right)^{3}\left(bx^{3} + a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^3*(b*x^3 + a)^m,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^3*(b*x^3 + a)^m, x)
```

## 3.40 $\int \left(a+bx^3\right)^m \left(c+dx^3\right)^2\, dx$

**Optimal**. Leaf size=176

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(4a^2d^2-2abcd(3m+7)+b^2c^2\left(9m^2+33m+28\right)\right)\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)}{b^2(3m+4)(3m+7)}-\frac{dx\left(a+bx^3\right)^{m+1}(4ad-bc(3m+10))}{b^2(3m+4)(3m+7)}+\frac{dx\left(c+dx^3\right)\left(a+bx^3\right)^{m+1}}{b(3m+7)}$$

```
[Out] -((d*(4*a*d - b*c*(10 + 3*m))*x*(a + b*x^3)^(1 + m))/(b^2*(4 + 3*
m)*(7 + 3*m))) + (d*x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(b*(7 + 3*
m)) + ((4*a^2*d^2 - 2*a*b*c*d*(7 + 3*m) + b^2*c^2*(28 + 33*m + 9*
m^2))*x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a
)])/(b^2*(4 + 3*m)*(7 + 3*m)*(1 + (b*x^3)/a)^m)
```

---

**Rubi [A]** time = 0.28185, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(4a^2d^2-2abcd(3m+7)+b^2c^2\left(9m^2+33m+28\right)\right)\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)}{b^2(3m+4)(3m+7)}-\frac{dx\left(a+bx^3\right)^{m+1}(4ad-bc(3m+10))}{b^2(3m+4)(3m+7)}+\frac{dx\left(c+dx^3\right)\left(a+bx^3\right)^{m+1}}{b(3m+7)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^m*(c + d*x^3)^2,x]
```

```
[Out] -((d*(4*a*d - b*c*(10 + 3*m))*x*(a + b*x^3)^(1 + m))/(b^2*(4 + 3*
m)*(7 + 3*m))) + (d*x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(b*(7 + 3*
m)) + ((4*a^2*d^2 - 2*a*b*c*d*(7 + 3*m) + b^2*c^2*(28 + 33*m + 9*
m^2))*x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a
)])/(b^2*(4 + 3*m)*(7 + 3*m)*(1 + (b*x^3)/a)^m)
```

---

**Rubi in Sympy [A]** time = 32.2166, size = 153, normalized size = 0.87

$$\frac{dx\left(a+bx^3\right)^{m+1}\left(c+dx^3\right)}{b\,(3m+7)}-\frac{dx\left(a+bx^3\right)^{m+1}(4ad-bc\,(3m+10))}{b^2\,(3m+4)\,(3m+7)}+\frac{x\left(1+\frac{bx^3}{a}\right)^{-m}\left(a+bx^3\right)^m\left(ad\,(4ad-bc\,(3m+10))-bc\,(3m+4)\,(ad-bc\,(3m+7))\right){}_2F_1\left(\begin{matrix}-m,\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|-\frac{bx^3}{a}\right)}{b^2\,(3m+4)\,(3m+7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**m*(d*x**3+c)**2,x)
```

```
[Out] d*x*(a + b*x**3)**(m + 1)*(c + d*x**3)/(b*(3*m + 7)) - d*x*(a + b
*x**3)**(m + 1)*(4*a*d - b*c*(3*m + 10))/(b**2*(3*m + 4)*(3*m + 7
)) + x*(1 + b*x**3/a)**(-m)*(a + b*x**3)**m*(a*d*(4*a*d - b*c*(3*
m + 10)) - b*c*(3*m + 4)*(a*d - b*c*(3*m + 7)))*hyper((-m, 1/3),
(4/3,), -b*x**3/a)/(b**2*(3*m + 4)*(3*m + 7))
```

---

**Mathematica [A]** time = 0.0568683, size = 106, normalized size = 0.6

$$\frac{1}{14}x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(14c^2\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)+dx^3\left(7c\,{}_2F_1\left(\frac{4}{3},-m;\frac{7}{3};-\frac{bx^3}{a}\right)+2dx^3\,{}_2F_1\left(\frac{7}{3},-m;\frac{10}{3};-\frac{bx^3}{a}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)^m*(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^m*(14*c^2*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3
)/a)] + d*x^3*(7*c*Hypergeometric2F1[4/3, -m, 7/3, -((b*x^3)/a)]
+ 2*d*x^3*Hypergeometric2F1[7/3, -m, 10/3, -((b*x^3)/a)])))/(14*(
1 + (b*x^3)/a)^m)
```

---

**Maple [F]** time = 0.07, size = 0, normalized size = 0.

$$\int\left(bx^3+a\right)^m\left(dx^3+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^m*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^m*(d*x^3+c)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\left(dx^3+c\right)^2\left(bx^3+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2*(b*x^3 + a)^m,x, algorithm="maxima")

[Out] integrate((d*x^3 + c)^2*(b*x^3 + a)^m, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^2x^6 + 2\,cdx^3 + c^2\right)\left(bx^3 + a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2*(b*x^3 + a)^m,x, algorithm="fricas")

[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^m, x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**m*(d*x**3+c)**2,x)

[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int\left(dx^3 + c\right)^2\left(bx^3 + a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)^2*(b*x^3 + a)^m,x, algorithm="giac")

[Out] integrate((d*x^3 + c)^2*(b*x^3 + a)^m, x)
```

# 3.41 $\int \left(a+bx^3\right)^m \left(c+dx^3\right)\, dx$

**Optimal**. Leaf size=93

$$\frac{dx\left(a+bx^3\right)^{m+1}}{b(3m+4)}-\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}(ad-bc(3m+4))\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)}{b(3m+4)}$$

```
[Out] (d*x*(a + b*x^3)^(1 + m))/(b*(4 + 3*m)) - ((a*d - b*c*(4 + 3*m))*
x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(b
*(4 + 3*m)*(1 + (b*x^3)/a)^m)
```

---

**Rubi [A]** time = 0.099899, antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(c-\frac{ad}{3bm+4b}\right)\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)+\frac{dx\left(a+bx^3\right)^{m+1}}{b(3m+4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^m*(c + d*x^3),x]
```

```
[Out] (d*x*(a + b*x^3)^(1 + m))/(b*(4 + 3*m)) + ((c - (a*d)/(4*b + 3*b*
m))*x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)]
)/(1 + (b*x^3)/a)^m
```

---

**Rubi in Sympy [A]** time = 10.7156, size = 73, normalized size = 0.78

$$\frac{dx\left(a+bx^3\right)^{m+1}}{b\,(3m+4)}-\frac{x\left(1+\frac{bx^3}{a}\right)^{-m}\left(a+bx^3\right)^m(ad-bc\,(3m+4))\,{}_2F_1\left(\begin{matrix}-m,\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|-\frac{bx^3}{a}\right)}{b\,(3m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**m*(d*x**3+c),x)
```

```
[Out] d*x*(a + b*x**3)**(m + 1)/(b*(3*m + 4)) - x*(1 + b*x**3/a)**(-m)*
(a + b*x**3)**m*(a*d - b*c*(3*m + 4))*hyper((-m, 1/3), (4/3,), -b
*x**3/a)/(b*(3*m + 4))
```

---

**Mathematica [A]** time = 0.0298675, size = 75, normalized size = 0.81

$$\frac{1}{4}x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(4c\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)+dx^3\,{}_2F_1\left(\frac{4}{3},-m;\frac{7}{3};-\frac{bx^3}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3)^m*(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^m*(4*c*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a
)] + d*x^3*Hypergeometric2F1[4/3, -m, 7/3, -((b*x^3)/a)]))/(4*(1
+ (b*x^3)/a)^m)
```

---

**Maple [F]** time = 0.055, size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^m\left(dx^3+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^m*(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^m*(d*x^3+c),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(dx^3+c\right)\left(bx^3+a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)*(b*x^3 + a)^m,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)*(b*x^3 + a)^m, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(dx^3+c\right)\left(bx^3+a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)*(b*x^3 + a)^m,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)*(b*x^3 + a)^m, x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**m*(d*x**3+c),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^3 + c) (bx^3 + a)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^3 + c)*(b*x^3 + a)^m,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)*(b*x^3 + a)^m, x)
```

## 3.42 $\int \left(a+bx^3\right)^m dx$

**Optimal**. Leaf size=44

$$x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)$$

```
[Out] (x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(
1 + (b*x^3)/a)^m
```

---

**Rubi [A]** time = 0.0267679, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^m,x]
```

```
[Out] (x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(
1 + (b*x^3)/a)^m
```

---

**Rubi in Sympy [A]** time = 3.92162, size = 34, normalized size = 0.77

$$x\left(1+\frac{bx^3}{a}\right)^{-m}\left(a+bx^3\right)^m{}_2F_1\left(\left.\begin{matrix}-m,\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|-\frac{bx^3}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**m,x)
```

```
[Out] x*(1 + b*x**3/a)**(-m)*(a + b*x**3)**m*hyper((-m, 1/3), (4/3,), -
b*x**3/a)
```

---

**Mathematica [C]** time = 0.287412, size = 196, normalized size = 4.45

$$\frac{2^{-m}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{-m}\left(\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{\sqrt{3}+3i}\right)^{-m}\left(a+bx^3\right)^m F_1\left(m+1;-m,-m;m+2;-\frac{i\left(\sqrt[3]{b}x+(-1)^{2/3}\sqrt[3]{a}\right)}{\sqrt{3}\sqrt[3]{a}},\ldots\right.}{\sqrt[3]{b}(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3)^m,x]
```

```
[Out] (((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*(a + b*x^3)^m*AppellF1[1 + m, -
m, -m, 2 + m, ((-I)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(Sqrt[3]*a^
(1/3)), (I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3))/(3*I + Sqrt[3])
])/(2^m*b^(1/3)*(1 + m)*((a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (
-1)^(1/3))*a^(1/3)))^m*((I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt
[3]))^m)
```

---

**Maple [F]** time = 0.035, size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^m,x)
```

```
[Out] int((b*x^3+a)^m,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^m,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^m, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^3+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^m,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^m, x)
```

---

**Sympy [A]** time = 54.2667, size = 34, normalized size = 0.77

$$\frac{a^m x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, -m \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**m,x)
```

```
[Out] a**m*x*gamma(1/3)*hyper((1/3, -m), (4/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(4/3))
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3 + a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^m,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^m, x)
```

# 3.43 $\int \frac{(a+bx^3)^m}{c+dx^3} \, dx$

**Optimal**. Leaf size=57

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}F_1\left(\frac{1}{3};-m,1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}$$

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c*(1 + (b*x^3)/a)^m)
```

---

**Rubi [A]** time = 0.070779, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}F_1\left(\frac{1}{3};-m,1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^m/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c*(1 + (b*x^3)/a)^m)
```

---

**Rubi in Sympy [A]** time = 21.2229, size = 42, normalized size = 0.74

$$\frac{x\left(1+\frac{bx^3}{a}\right)^{-m}\left(a+bx^3\right)^m \operatorname{appellf}_1\left(\frac{1}{3},1,-m,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**m/(d*x**3+c),x)
```

```
[Out] x*(1 + b*x**3/a)**(-m)*(a + b*x**3)**m*appellf1(1/3, 1, -m, 4/3,
-d*x**3/c, -b*x**3/a)/c
```

---

**Mathematica [B]** time = 0.327331, size = 162, normalized size = 2.84

$$-\frac{4acx\left(a+bx^3\right)^m F_1\left(\frac{1}{3};-m,1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)\left(3x^3\left(adF_1\left(\frac{4}{3};-m,2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-bcmF_1\left(\frac{4}{3};1-m,1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};-m,1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3)^m/(c + d*x^3),x]
```

```
[Out] (-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -
((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, -m, 1, 4/3, -((b
*x^3)/a), -((d*x^3)/c)] + 3*x^3*(-(b*c*m*AppellF1[4/3, 1 - m, 1,
7/3, -((b*x^3)/a), -((d*x^3)/c)]) + a*d*AppellF1[4/3, -m, 2, 7/3,
 -((b*x^3)/a), -((d*x^3)/c)])))
```

---

**Maple [F]** time = 0.076, size = 0, normalized size = 0.

$$\int \frac{(bx^3 + a)^m}{dx^3 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^m/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^m/(d*x^3+c),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + a)^m}{dx^3 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^m/(d*x^3 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^3 + a)^m}{dx^3 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^m/(d*x^3 + c),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^m/(d*x^3 + c), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**m/(d*x**3+c),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^m}{dx^3 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^m/(d*x^3 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c), x)
```

## 3.44 $\int \frac{(a+bx^3)^m}{(c+dx^3)^2}\,dx$

**Optimal**. Leaf size=57

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}F_1\left(\frac{1}{3};-m,2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}$$

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c^2*(1 + (b*x^3)/a)^m)
```

---

**Rubi [A]** time = 0.0716967, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}F_1\left(\frac{1}{3};-m,2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^m/(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c^2*(1 + (b*x^3)/a)^m)
```

---

**Rubi in Sympy [A]** time = 19.708, size = 44, normalized size = 0.77

$$\frac{x\left(1+\frac{bx^3}{a}\right)^{-m}\left(a+bx^3\right)^m \operatorname{appellf}_1\left(\frac{1}{3},2,-m,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**m/(d*x**3+c)**2,x)
```

```
[Out] x*(1 + b*x**3/a)**(-m)*(a + b*x**3)**m*appellf1(1/3, 2, -m, 4/3,
-d*x**3/c, -b*x**3/a)/c**2
```

---

**Mathematica [B]** time = 0.297776, size = 162, normalized size = 2.84

$$-\frac{4acx\left(a+bx^3\right)^m F_1\left(\frac{1}{3};-m,2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)^2\left(-3x^3\left(bcmF_1\left(\frac{4}{3};1-m,2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2adF_1\left(\frac{4}{3};-m,3;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};-m,2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3)^m/(c + d*x^3)^2,x]
```

```
[Out] (-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -
((d*x^3)/c)])/((c + d*x^3)^2*(-4*a*c*AppellF1[1/3, -m, 2, 4/3, -(
(b*x^3)/a), -((d*x^3)/c)] - 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, 2,
7/3, -((b*x^3)/a), -((d*x^3)/c)] - 2*a*d*AppellF1[4/3, -m, 3, 7/3
, -((b*x^3)/a), -((d*x^3)/c)])))
```

---

**Maple [F]** time = 0.095, size = 0, normalized size = 0.

$$\int \frac{\left(b x^3 + a\right)^m}{\left(d x^3 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^m/(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^m/(d*x^3+c)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^3 + a\right)^m}{\left(d x^3 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^m/(d*x^3 + c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c)^2, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b x^3 + a\right)^m}{d^2 x^6 + 2\, c d x^3 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^m/(d*x^3 + c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^m/(d^2*x^6 + 2*c*d*x^3 + c^2), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**m/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3+a)^m}{(dx^3+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^m/(d*x^3 + c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c)^2, x)
```

## 3.45 $\int \frac{(a+bx^3)^m}{(c+dx^3)^3} dx$

**Optimal**. Leaf size=57

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}F_1\left(\frac{1}{3};-m,3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}$$

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c^3*(1 + (b*x^3)/a)^m)
```

---

**Rubi [A]** time = 0.0714109, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}F_1\left(\frac{1}{3};-m,3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^m/(c + d*x^3)^3,x]
```

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c^3*(1 + (b*x^3)/a)^m)
```

---

**Rubi in Sympy [A]** time = 19.8666, size = 44, normalized size = 0.77

$$\frac{x\left(1+\frac{bx^3}{a}\right)^{-m}\left(a+bx^3\right)^m \operatorname{appellf}_1\left(\frac{1}{3},3,-m,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**m/(d*x**3+c)**3,x)
```

```
[Out] x*(1 + b*x**3/a)**(-m)*(a + b*x**3)**m*appellf1(1/3, 3, -m, 4/3,
-d*x**3/c, -b*x**3/a)/c**3
```

---

**Mathematica [B]** time = 0.392116, size = 162, normalized size = 2.84

$$-\frac{4acx\left(a+bx^3\right)^m F_1\left(\frac{1}{3};-m,3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)^3\left(-3x^3\left(bcmF_1\left(\frac{4}{3};1-m,3;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3adF_1\left(\frac{4}{3};-m,4;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};-m,3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3)^m/(c + d*x^3)^3,x]
```

```
[Out] (-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -
((d*x^3)/c)])/((c + d*x^3)^3*(-4*a*c*AppellF1[1/3, -m, 3, 4/3, -(
(b*x^3)/a), -((d*x^3)/c)] - 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, 3,
7/3, -((b*x^3)/a), -((d*x^3)/c)] - 3*a*d*AppellF1[4/3, -m, 4, 7/3
, -((b*x^3)/a), -((d*x^3)/c)])))
```

---

**Maple [F]** time = 0.078, size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^m}{\left(dx^3 + c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^3+a)^m/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^m/(d*x^3+c)^3,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^m}{\left(dx^3 + c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^m/(d*x^3 + c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c)^3, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^3 + a\right)^m}{d^3x^9 + 3\,cd^2x^6 + 3\,c^2dx^3 + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^m/(d*x^3 + c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^m/(d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3
), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**3+a)**m/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + a)^m}{(dx^3 + c)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^3 + a)^m/(d*x^3 + c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c)^3, x)
```

## 3.46 $\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}} \, dx$

**Optimal**. Leaf size=53

$$\frac{x \left(a+bx^3\right)^{-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

```
[Out] (x*(c + d*x^3)^((a*d)/(3*b*c - 3*a*d)))/(a*c*(a + b*x^3)^((b*c)/(
3*b*c - 3*a*d)))
```

---

**Rubi [A]** time = 0.0608956, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.02

$$\frac{x \left(a+bx^3\right)^{-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3)^(-1 - (b*c)/(3*b*c - 3*a*d))*(c + d*x^3)^(-
1 + (a*d)/(3*b*c - 3*a*d)),x]
```

```
[Out] (x*(c + d*x^3)^((a*d)/(3*b*c - 3*a*d)))/(a*c*(a + b*x^3)^((b*c)/(
3*b*c - 3*a*d)))
```

---

**Rubi in Sympy [A]** time = 10.5525, size = 44, normalized size = 0.83

$$\frac{x \left(a+bx^3\right)^{\frac{bc}{3(ad-bc)}} \left(c+dx^3\right)^{-\frac{ad}{3(ad-bc)}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**3+a)**(-1-b*c/(-3*a*d+3*b*c))*(d*x**3+c)**(-
1+a*d/(-3*a*d+3*b*c)),x)
```

```
[Out] x*(a + b*x**3)**(b*c/(3*(a*d - b*c)))*(c + d*x**3)**(-a*d/(3*(a*d
 - b*c)))/(a*c)
```

---

**Mathematica [C]** time = 2.43632, size = 594, normalized size = 11.21

$$4acx\left(a+bx^3\right)^{\frac{bc}{3ad-3bc}}\left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}\left(\frac{bF_1\left(\frac{1}{3};\frac{bc}{3bc-3ad}+1,\frac{ad}{3ad-3bc};\frac{4}{3};-\frac{bx^3}{a},\right.}{\left(a+bx^3\right)\left(x^3\left(a^2d^2F_1\left(\frac{4}{3};\frac{bc}{3bc-3ad}+1,\frac{ad}{3ad-3bc}+1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bc(3ad-4bc)F_1\left(\frac{4}{3};\frac{bc}{3bc-3ad}+2,\frac{ad}{3ad-3bc}\right.\right.\right.}\right.$$
$$+\frac{dF_1\left(\frac{1}{3};\frac{bc}{3bc-3ad},\frac{ad}{3ad-3bc}+1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)\left(x^3\left(b^2c^2F_1\left(\frac{4}{3};\frac{bc}{3bc-3ad}+1,\frac{ad}{3ad-3bc}+1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+ad(3bc-4ad)F_1\left(\frac{4}{3};\frac{bc}{3bc-3ad},\frac{ad}{3ad-3bc}+2;\frac{7}{3};-\frac{bx^3}{a},-\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3)^(-1 - (b*c)/(3*b*c - 3*a*d))*(c + d*x^3)^(-
1 + (a*d)/(3*b*c - 3*a*d)),x]
```

```
[Out] 4*a*c*x*(a + b*x^3)^((b*c)/(-3*b*c + 3*a*d))*(c + d*x^3)^((a*d)/(
3*b*c - 3*a*d))*((d*AppellF1[1/3, (b*c)/(3*b*c - 3*a*d), 1 + (a*d
)/(-3*b*c + 3*a*d), 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3
)*(4*a*c*(-(b*c) + a*d)*AppellF1[1/3, (b*c)/(3*b*c - 3*a*d), 1 +
(a*d)/(-3*b*c + 3*a*d), 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(a
*d*(3*b*c - 4*a*d)*AppellF1[4/3, (b*c)/(3*b*c - 3*a*d), 2 + (a*d)
/(-3*b*c + 3*a*d), 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b^2*c^2*App
ellF1[4/3, 1 + (b*c)/(3*b*c - 3*a*d), 1 + (a*d)/(-3*b*c + 3*a*d),
 7/3, -((b*x^3)/a), -((d*x^3)/c)]))) + (b*AppellF1[1/3, 1 + (b*c)
/(3*b*c - 3*a*d), (a*d)/(-3*b*c + 3*a*d), 4/3, -((b*x^3)/a), -((d
*x^3)/c)])/((a + b*x^3)*(4*a*c*(b*c - a*d)*AppellF1[1/3, 1 + (b*c
)/(3*b*c - 3*a*d), (a*d)/(-3*b*c + 3*a*d), 4/3, -((b*x^3)/a), -((
d*x^3)/c)] + x^3*(a^2*d^2*AppellF1[4/3, 1 + (b*c)/(3*b*c - 3*a*d)
, 1 + (a*d)/(-3*b*c + 3*a*d), 7/3, -((b*x^3)/a), -((d*x^3)/c)] +
b*c*(-4*b*c + 3*a*d)*AppellF1[4/3, 2 + (b*c)/(3*b*c - 3*a*d), (a*
d)/(-3*b*c + 3*a*d), 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))
```

---

**Maple [A]** time = 0.006, size = 71, normalized size = 1.3

$$\frac{x}{ac}\left(bx^3+a\right)^{1-\frac{3ad-4bc}{3ad-3bc}}\left(dx^3+c\right)^{1-\frac{4ad-3bc}{3ad-3bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c)),x)
```

```
[Out] (b*x^3+a)^(1-1/3*(3*a*d-4*b*c)/(a*d-b*c))*(d*x^3+c)^(1-1/3*(4*a*d
-3*b*c)/(a*d-b*c))/a/c*x
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{-\frac{bc}{3(bc-ad)}-1} \left(dx^3+c\right)^{\frac{ad}{3(bc-ad)}-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3 + a)^(-1/3*b*c/(b*c - a*d) - 1)*(d*x^3 + c)^(1/3*a*d/(b*c - a*d)
```

```
[Out] integrate((b*x^3 + a)^(-1/3*b*c/(b*c - a*d) - 1)*(d*x^3 + c)^(1/3
*a*d/(b*c - a*d) - 1), x)
```

---

**Fricas [A]** time = 0.252313, size = 123, normalized size = 2.32

$$\frac{bdx^7 + (bc+ad)x^4 + acx}{\left(bx^3+a\right)^{\frac{4\,bc-3\,ad}{3(bc-ad)}} \left(dx^3+c\right)^{\frac{3\,bc-4\,ad}{3(bc-ad)}} ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3 + a)^(-1/3*b*c/(b*c - a*d) - 1)*(d*x^3 + c)^(1/3*a*d/(b*c - a*d)
```

```
[Out] (b*d*x^7 + (b*c + a*d)*x^4 + a*c*x)/((b*x^3 + a)^(1/3*(4*b*c - 3*
a*d)/(b*c - a*d))*(d*x^3 + c)^(1/3*(3*b*c - 4*a*d)/(b*c - a*d))*a
*c)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(-1-b*c/(-3*a*d+3*b*c))*(d*x**3+c)**(-1+a*d/(-
3*a*d+3*b*c)),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{-\frac{bc}{3(bc-ad)}-1} \left(dx^3+c\right)^{\frac{ad}{3(bc-ad)}-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3 + a)^(-1/3*b*c/(b*c - a*d) - 1)*(d*x^3 + c)^(1/3*a*d/(b*c - a*d)
```

```
[Out] integrate((b*x^3 + a)^(-1/3*b*c/(b*c - a*d) - 1)*(d*x^3 + c)^(1/3
*a*d/(b*c - a*d) - 1), x)
```

## 3.47 $\int \left(a+bx^4\right) \left(c+dx^4\right)^4 \, dx$

**Optimal**. Leaf size=94

$$\frac{1}{5}c^3x^5(4ad+bc)+\frac{2}{9}c^2dx^9(3ad+2bc)+\frac{1}{17}d^3x^{17}(ad+4bc)+\frac{2}{13}cd^2x^{13}(2ad+3bc)+ac^4x+\frac{1}{21}bd^4x^{21}$$

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^
9)/9 + (2*c*d^2*(3*b*c + 2*a*d)*x^13)/13 + (d^3*(4*b*c + a*d)*x^1
7)/17 + (b*d^4*x^21)/21
```

---

**Rubi [A]** time = 0.145413, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{1}{5}c^3x^5(4ad+bc)+\frac{2}{9}c^2dx^9(3ad+2bc)+\frac{1}{17}d^3x^{17}(ad+4bc)+\frac{2}{13}cd^2x^{13}(2ad+3bc)+ac^4x+\frac{1}{21}bd^4x^{21}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)*(c + d*x^4)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^
9)/9 + (2*c*d^2*(3*b*c + 2*a*d)*x^13)/13 + (d^3*(4*b*c + a*d)*x^1
7)/17 + (b*d^4*x^21)/21
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bd^4x^{21}}{21}+c^4\int a\,dx+\frac{c^3x^5\left(4ad+bc\right)}{5}+\frac{2c^2dx^9\left(3ad+2bc\right)}{9}+\frac{2cd^2x^{13}\left(2ad+3bc\right)}{13}+\frac{d^3x^{17}\left(ad+4bc\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)*(d*x**4+c)**4,x)
```

```
[Out] b*d**4*x**21/21 + c**4*Integral(a, x) + c**3*x**5*(4*a*d + b*c)/5
 + 2*c**2*d*x**9*(3*a*d + 2*b*c)/9 + 2*c*d**2*x**13*(2*a*d + 3*b*
c)/13 + d**3*x**17*(a*d + 4*b*c)/17
```

---

**Mathematica [A]** time = 0.0351537, size = 94, normalized size = 1.

$$\frac{1}{5}c^3x^5(4ad+bc)+\frac{2}{9}c^2dx^9(3ad+2bc)+\frac{1}{17}d^3x^{17}(ad+4bc)+\frac{2}{13}cd^2x^{13}(2ad+3bc)+ac^4x+\frac{1}{21}bd^4x^{21}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^4)*(c + d*x^4)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^
9)/9 + (2*c*d^2*(3*b*c + 2*a*d)*x^13)/13 + (d^3*(4*b*c + a*d)*x^1
7)/17 + (b*d^4*x^21)/21
```

---

**Maple [A]**    time = 0.001, size = 97, normalized size = 1.

$$\frac{bd^4x^{21}}{21}+\frac{\left(ad^4+4\,bcd^3\right)x^{17}}{17}+\frac{\left(4\,acd^3+6\,c^2d^2b\right)x^{13}}{13}+\frac{\left(6\,ac^2d^2+4\,c^3db\right)x^9}{9}+\frac{\left(4\,ac^3d+bc^4\right)x^5}{5}+ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)*(d*x^4+c)^4,x)
```

```
[Out] 1/21*b*d^4*x^21+1/17*(a*d^4+4*b*c*d^3)*x^17+1/13*(4*a*c*d^3+6*b*c
^2*d^2)*x^13+1/9*(6*a*c^2*d^2+4*b*c^3*d)*x^9+1/5*(4*a*c^3*d+b*c^4
)*x^5+a*c^4*x
```

---

**Maxima [A]**    time = 1.36215, size = 130, normalized size = 1.38

$$\frac{1}{21}\,bd^4x^{21}+\frac{1}{17}\left(4\,bcd^3+ad^4\right)x^{17}+\frac{2}{13}\left(3\,bc^2d^2+2\,acd^3\right)x^{13}+\frac{2}{9}\left(2\,bc^3d+3\,ac^2d^2\right)x^9+ac^4x+\frac{1}{5}\left(bc^4+4\,ac^3d\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)*(d*x^4 + c)^4,x, algorithm="maxima")
```

```
[Out] 1/21*b*d^4*x^21 + 1/17*(4*b*c*d^3 + a*d^4)*x^17 + 2/13*(3*b*c^2*d
^2 + 2*a*c*d^3)*x^13 + 2/9*(2*b*c^3*d + 3*a*c^2*d^2)*x^9 + a*c^4*
x + 1/5*(b*c^4 + 4*a*c^3*d)*x^5
```

---

**Fricas [A]**    time = 0.191658, size = 1, normalized size = 0.01

$$\frac{1}{21}x^{21}d^4b+\frac{4}{17}x^{17}d^3cb+\frac{1}{17}x^{17}d^4a+\frac{6}{13}x^{13}d^2c^2b+\frac{4}{13}x^{13}d^3ca+\frac{4}{9}x^9dc^3b+\frac{2}{3}x^9d^2c^2a+\frac{1}{5}x^5c^4b+\frac{4}{5}x^5dc^3a+xc^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)*(d*x^4 + c)^4,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*d^4*b + 4/17*x^17*d^3*c*b + 1/17*x^17*d^4*a + 6/13*x^13
*d^2*c^2*b + 4/13*x^13*d^3*c*a + 4/9*x^9*d*c^3*b + 2/3*x^9*d^2*c^
2*a + 1/5*x^5*c^4*b + 4/5*x^5*d*c^3*a + x*c^4*a
```

---

**Sympy [A]** time = 0.149023, size = 107, normalized size = 1.14

$$ac^4x + \frac{bd^4x^{21}}{21} + x^{17}\left(\frac{ad^4}{17} + \frac{4bcd^3}{17}\right) + x^{13}\left(\frac{4acd^3}{13} + \frac{6bc^2d^2}{13}\right) + x^9\left(\frac{2ac^2d^2}{3} + \frac{4bc^3d}{9}\right) + x^5\left(\frac{4ac^3d}{5} + \frac{bc^4}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)*(d*x**4+c)**4,x)
```

```
[Out] a*c**4*x + b*d**4*x**21/21 + x**17*(a*d**4/17 + 4*b*c*d**3/17) +
x**13*(4*a*c*d**3/13 + 6*b*c**2*d**2/13) + x**9*(2*a*c**2*d**2/3
+ 4*b*c**3*d/9) + x**5*(4*a*c**3*d/5 + b*c**4/5)
```

---

**GIAC/XCAS [A]** time = 0.212669, size = 132, normalized size = 1.4

$$\frac{1}{21}\,bd^4x^{21} + \frac{4}{17}\,bcd^3x^{17} + \frac{1}{17}\,ad^4x^{17} + \frac{6}{13}\,bc^2d^2x^{13} + \frac{4}{13}\,acd^3x^{13} + \frac{4}{9}\,bc^3dx^9 + \frac{2}{3}\,ac^2d^2x^9 + \frac{1}{5}\,bc^4x^5 + \frac{4}{5}\,ac^3dx^5 + ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)*(d*x^4 + c)^4,x, algorithm="giac")
```

```
[Out] 1/21*b*d^4*x^21 + 4/17*b*c*d^3*x^17 + 1/17*a*d^4*x^17 + 6/13*b*c^
2*d^2*x^13 + 4/13*a*c*d^3*x^13 + 4/9*b*c^3*d*x^9 + 2/3*a*c^2*d^2*
x^9 + 1/5*b*c^4*x^5 + 4/5*a*c^3*d*x^5 + a*c^4*x
```

## 3.48 $\int \left(a+bx^4\right) \left(c+dx^4\right)^3 \, dx$

**Optimal**. Leaf size=70

$$\frac{1}{5}c^2x^5(3ad+bc)+\frac{1}{13}d^2x^{13}(ad+3bc)+\frac{1}{3}cdx^9(ad+bc)+ac^3x+\frac{1}{17}bd^3x^{17}$$

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (
d^2*(3*b*c + a*d)*x^13)/13 + (b*d^3*x^17)/17
```

---

**Rubi [A]** time = 0.104301, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{1}{5}c^2x^5(3ad+bc)+\frac{1}{13}d^2x^{13}(ad+3bc)+\frac{1}{3}cdx^9(ad+bc)+ac^3x+\frac{1}{17}bd^3x^{17}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)*(c + d*x^4)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (
d^2*(3*b*c + a*d)*x^13)/13 + (b*d^3*x^17)/17
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bd^3x^{17}}{17}+c^3\int a\,dx+\frac{c^2x^5\,(3ad+bc)}{5}+\frac{cdx^9\,(ad+bc)}{3}+\frac{d^2x^{13}\,(ad+3bc)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)*(d*x**4+c)**3,x)
```

```
[Out] b*d**3*x**17/17 + c**3*Integral(a, x) + c**2*x**5*(3*a*d + b*c)/5
 + c*d*x**9*(a*d + b*c)/3 + d**2*x**13*(a*d + 3*b*c)/13
```

---

**Mathematica [A]** time = 0.0311638, size = 70, normalized size = 1.

$$\frac{1}{5}c^2x^5(3ad+bc)+\frac{1}{13}d^2x^{13}(ad+3bc)+\frac{1}{3}cdx^9(ad+bc)+ac^3x+\frac{1}{17}bd^3x^{17}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^4)*(c + d*x^4)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (
d^2*(3*b*c + a*d)*x^13)/13 + (b*d^3*x^17)/17
```

---

**Maple [A]** time = 0.001, size = 73, normalized size = 1.

$$\frac{bd^3x^{17}}{17}+\frac{\left(ad^3+3\,bcd^2\right)x^{13}}{13}+\frac{\left(3\,acd^2+3\,bc^2d\right)x^9}{9}+\frac{\left(3\,ac^2d+bc^3\right)x^5}{5}+ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)*(d*x^4+c)^3,x)
```

```
[Out] 1/17*b*d^3*x^17+1/13*(a*d^3+3*b*c*d^2)*x^13+1/9*(3*a*c*d^2+3*b*c^
2*d)*x^9+1/5*(3*a*c^2*d+b*c^3)*x^5+a*c^3*x
```

---

**Maxima [A]** time = 1.3555, size = 95, normalized size = 1.36

$$\frac{1}{17}\,bd^3x^{17}+\frac{1}{13}\left(3\,bcd^2+ad^3\right)x^{13}+\frac{1}{3}\left(bc^2d+acd^2\right)x^9+\frac{1}{5}\left(bc^3+3\,ac^2d\right)x^5+ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)*(d*x^4 + c)^3,x, algorithm="maxima")
```

```
[Out] 1/17*b*d^3*x^17 + 1/13*(3*b*c*d^2 + a*d^3)*x^13 + 1/3*(b*c^2*d +
a*c*d^2)*x^9 + 1/5*(b*c^3 + 3*a*c^2*d)*x^5 + a*c^3*x
```

---

**Fricas [A]** time = 0.191661, size = 1, normalized size = 0.01

$$\frac{1}{17}x^{17}d^3b+\frac{3}{13}x^{13}d^2cb+\frac{1}{13}x^{13}d^3a+\frac{1}{3}x^9dc^2b+\frac{1}{3}x^9d^2ca+\frac{1}{5}x^5c^3b+\frac{3}{5}x^5dc^2a+xc^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)*(d*x^4 + c)^3,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*d^3*b + 3/13*x^13*d^2*c*b + 1/13*x^13*d^3*a + 1/3*x^9*d
*c^2*b + 1/3*x^9*d^2*c*a + 1/5*x^5*c^3*b + 3/5*x^5*d*c^2*a + x*c^
3*a
```

---

**Sympy [A]** time = 0.122, size = 76, normalized size = 1.09

$$ac^3x + \frac{bd^3x^{17}}{17} + x^{13}\left(\frac{ad^3}{13} + \frac{3bcd^2}{13}\right) + x^9\left(\frac{acd^2}{3} + \frac{bc^2d}{3}\right) + x^5\left(\frac{3ac^2d}{5} + \frac{bc^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)*(d*x**4+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**17/17 + x**13*(a*d**3/13 + 3*b*c*d**2/13) +
x**9*(a*c*d**2/3 + b*c**2*d/3) + x**5*(3*a*c**2*d/5 + b*c**3/5)
```

---

**GIAC/XCAS [A]** time = 0.211977, size = 100, normalized size = 1.43

$$\frac{1}{17}\,bd^3x^{17} + \frac{3}{13}\,bcd^2x^{13} + \frac{1}{13}\,ad^3x^{13} + \frac{1}{3}\,bc^2dx^9 + \frac{1}{3}\,acd^2x^9 + \frac{1}{5}\,bc^3x^5 + \frac{3}{5}\,ac^2dx^5 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)*(d*x^4 + c)^3,x, algorithm="giac")
```

```
[Out] 1/17*b*d^3*x^17 + 3/13*b*c*d^2*x^13 + 1/13*a*d^3*x^13 + 1/3*b*c^2
*d*x^9 + 1/3*a*c*d^2*x^9 + 1/5*b*c^3*x^5 + 3/5*a*c^2*d*x^5 + a*c^
3*x
```

## 3.49 $\int (a+bx^4) (c+dx^4)^2 \, dx$

**Optimal**. Leaf size=50

$$\frac{1}{9}dx^9(ad+2bc)+\frac{1}{5}cx^5(2ad+bc)+ac^2x+\frac{1}{13}bd^2x^{13}$$

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*
d^2*x^13)/13
```

---

**Rubi [A]** time = 0.0740207, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{1}{9}dx^9(ad+2bc)+\frac{1}{5}cx^5(2ad+bc)+ac^2x+\frac{1}{13}bd^2x^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)*(c + d*x^4)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*
d^2*x^13)/13
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bd^2x^{13}}{13}+c^2\int a\,dx+\frac{cx^5\left(2ad+bc\right)}{5}+\frac{dx^9\left(ad+2bc\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)*(d*x**4+c)**2,x)
```

```
[Out] b*d**2*x**13/13 + c**2*Integral(a, x) + c*x**5*(2*a*d + b*c)/5 +
d*x**9*(a*d + 2*b*c)/9
```

---

**Mathematica [A]** time = 0.0200953, size = 50, normalized size = 1.

$$\frac{1}{9}dx^9(ad+2bc)+\frac{1}{5}cx^5(2ad+bc)+ac^2x+\frac{1}{13}bd^2x^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^4)*(c + d*x^4)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*
d^2*x^13)/13
```

---

**Maple [A]** time = 0.001, size = 49, normalized size = 1.

$$\frac{bd^2x^{13}}{13} + \frac{\left(ad^2 + 2\,bcd\right)x^9}{9} + \frac{\left(2\,acd + bc^2\right)x^5}{5} + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)*(d*x^4+c)^2,x)
```

```
[Out] 1/13*b*d^2*x^13+1/9*(a*d^2+2*b*c*d)*x^9+1/5*(2*a*c*d+b*c^2)*x^5+a
*c^2*x
```

---

**Maxima [A]** time = 1.41285, size = 65, normalized size = 1.3

$$\frac{1}{13}\,bd^2x^{13} + \frac{1}{9}\left(2\,bcd + ad^2\right)x^9 + \frac{1}{5}\left(bc^2 + 2\,acd\right)x^5 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)*(d*x^4 + c)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b*d^2*x^13 + 1/9*(2*b*c*d + a*d^2)*x^9 + 1/5*(b*c^2 + 2*a*c*
d)*x^5 + a*c^2*x
```

---

**Fricas [A]** time = 0.189588, size = 1, normalized size = 0.02

$$\frac{1}{13}x^{13}d^2b + \frac{2}{9}x^9dcb + \frac{1}{9}x^9d^2a + \frac{1}{5}x^5c^2b + \frac{2}{5}x^5dca + xc^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)*(d*x^4 + c)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^2*b + 2/9*x^9*d*c*b + 1/9*x^9*d^2*a + 1/5*x^5*c^2*b +
 2/5*x^5*d*c*a + x*c^2*a
```

---

**Sympy [A]** time = 0.104548, size = 53, normalized size = 1.06

$$ac^2x + \frac{bd^2x^{13}}{13} + x^9\left(\frac{ad^2}{9} + \frac{2bcd}{9}\right) + x^5\left(\frac{2acd}{5} + \frac{bc^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)*(d*x**4+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**13/13 + x**9*(a*d**2/9 + 2*b*c*d/9) + x**5*(
2*a*c*d/5 + b*c**2/5)
```

---

**GIAC/XCAS [A]** time = 0.211371, size = 68, normalized size = 1.36

$$\frac{1}{13}\,bd^2x^{13} + \frac{2}{9}\,bcdx^9 + \frac{1}{9}\,ad^2x^9 + \frac{1}{5}\,bc^2x^5 + \frac{2}{5}\,acdx^5 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)*(d*x^4 + c)^2,x, algorithm="giac")
```

```
[Out] 1/13*b*d^2*x^13 + 2/9*b*c*d*x^9 + 1/9*a*d^2*x^9 + 1/5*b*c^2*x^5 +
 2/5*a*c*d*x^5 + a*c^2*x
```

## 3.50 $\int \left(a+bx^4\right) \left(c+dx^4\right) \, dx$

**Optimal**. Leaf size=28

$$\frac{1}{5}x^5(ad+bc)+acx+\frac{1}{9}bdx^9$$

```
[Out] a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9
```

---

**Rubi [A]** time = 0.0381033, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{1}{5}x^5(ad+bc)+acx+\frac{1}{9}bdx^9$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)*(c + d*x^4),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bdx^9}{9}+c\int a\,dx+x^5\left(\frac{ad}{5}+\frac{bc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)*(d*x**4+c),x)
```

```
[Out] b*d*x**9/9 + c*Integral(a, x) + x**5*(a*d/5 + b*c/5)
```

---

**Mathematica [A]** time = 0.00912336, size = 28, normalized size = 1.

$$\frac{1}{5}x^5(ad+bc)+acx+\frac{1}{9}bdx^9$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^4)*(c + d*x^4),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9
```

---

**Maple [A]** time = 0.001, size = 25, normalized size = 0.9

$$acx + \frac{(ad + bc)\,x^5}{5} + \frac{bdx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)*(d*x^4+c),x)
```

```
[Out] a*c*x+1/5*(a*d+b*c)*x^5+1/9*b*d*x^9
```

---

**Maxima [A]** time = 1.36402, size = 32, normalized size = 1.14

$$\frac{1}{9}\,bdx^9 + \frac{1}{5}\,(bc + ad)x^5 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)*(d*x^4 + c),x, algorithm="maxima")
```

```
[Out] 1/9*b*d*x^9 + 1/5*(b*c + a*d)*x^5 + a*c*x
```

---

**Fricas [A]** time = 0.188586, size = 1, normalized size = 0.04

$$\frac{1}{9}x^9db + \frac{1}{5}x^5cb + \frac{1}{5}x^5da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)*(d*x^4 + c),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d*b + 1/5*x^5*c*b + 1/5*x^5*d*a + x*c*a
```

---

**Sympy [A]** time = 0.071019, size = 26, normalized size = 0.93

$$acx + \frac{bdx^9}{9} + x^5\left(\frac{ad}{5} + \frac{bc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)*(d*x**4+c),x)
```

```
[Out] a*c*x + b*d*x**9/9 + x**5*(a*d/5 + b*c/5)
```

---

**GIAC/XCAS [A]** time = 0.210788, size = 35, normalized size = 1.25

$$\frac{1}{9}\,bdx^9 + \frac{1}{5}\,bcx^5 + \frac{1}{5}\,adx^5 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)*(d*x^4 + c),x, algorithm="giac")
```

```
[Out] 1/9*b*d*x^9 + 1/5*b*c*x^5 + 1/5*a*d*x^5 + a*c*x
```

## 3.51 $\int \frac{a+bx^4}{c+dx^4}\,dx$

**Optimal**. Leaf size=223

$$\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}}-\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}}+\frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{3/4}d^{5/4}}+\frac{bx}{d}$$

```
[Out] (b*x)/d + ((b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(
2*Sqrt[2]*c^(3/4)*d^(5/4)) - ((b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(
1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*d^(5/4)) + ((b*c - a*d)*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^
(3/4)*d^(5/4)) - ((b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(5/4))
```

---

**Rubi [A]** time = 0.328354, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$

$$\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}}-\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}}+\frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{3/4}d^{5/4}}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)/(c + d*x^4),x]
```

```
[Out] (b*x)/d + ((b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(
2*Sqrt[2]*c^(3/4)*d^(5/4)) - ((b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(
1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*d^(5/4)) + ((b*c - a*d)*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^
(3/4)*d^(5/4)) - ((b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(5/4))
```

---

**Rubi in Sympy [A]** time = 57.1622, size = 204, normalized size = 0.91

$$\frac{bx}{d} - \frac{\sqrt{2}\,(ad-bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{8c^{\frac{3}{4}}d^{\frac{5}{4}}} + \frac{\sqrt{2}\,(ad-bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{8c^{\frac{3}{4}}d^{\frac{5}{4}}} - \frac{\sqrt{2}\,(ad-bc)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{4c^{\frac{3}{4}}d^{\frac{5}{4}}} + \frac{\sqrt{2}\,(ad-bc)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{4c^{\frac{3}{4}}d^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)/(d*x**4+c),x)
```

```
[Out] b*x/d - sqrt(2)*(a*d - b*c)*log(-sqrt(2)*c**(1/4)*d**(1/4)*x + sq
rt(c) + sqrt(d)*x**2)/(8*c**(3/4)*d**(5/4)) + sqrt(2)*(a*d - b*c)
*log(sqrt(2)*c**(1/4)*d**(1/4)*x + sqrt(c) + sqrt(d)*x**2)/(8*c**
(3/4)*d**(5/4)) - sqrt(2)*(a*d - b*c)*atan(1 - sqrt(2)*d**(1/4)*x
/c**(1/4))/(4*c**(3/4)*d**(5/4)) + sqrt(2)*(a*d - b*c)*atan(1 + s
qrt(2)*d**(1/4)*x/c**(1/4))/(4*c**(3/4)*d**(5/4))
```

---

**Mathematica [A]** time = 0.227625, size = 196, normalized size = 0.88

$$\frac{\sqrt{2}(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)-\sqrt{2}(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)+2\sqrt{2}(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right.}{8c^{3/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^4)/(c + d*x^4),x]
```

```
[Out] (8*b*c^(3/4)*d^(1/4)*x + 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 - (Sqrt[2
]*d^(1/4)*x)/c^(1/4)] - 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 + (Sqrt[2]
*d^(1/4)*x)/c^(1/4)] + Sqrt[2]*(b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*
c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] - Sqrt[2]*(b*c - a*d)*Log[Sqrt[c
] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(8*c^(3/4)*d^(5/4))
```

---

**Maple [A]** time = 0.01, size = 266, normalized size = 1.2

$$\frac{bx}{d}+\frac{\sqrt{2}a}{4c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)-\frac{\sqrt{2}b}{4d}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$
$$+\frac{\sqrt{2}a}{8c}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$-\frac{\sqrt{2}b}{8d}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$+\frac{\sqrt{2}a}{4c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)-\frac{\sqrt{2}b}{4d}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)/(d*x^4+c),x)
```

```
[Out] b*x/d+1/4*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a
-1/4/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*b+1/8*
(c/d)^(1/4)/c*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/
(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*a-1/8/d*(c/d)^(1/4)*2^(1
/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x
*2^(1/2)+(c/d)^(1/2)))*b+1/4*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)
/(c/d)^(1/4)*x+1)*a-1/4/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d
)^(1/4)*x+1)*b
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)/(d*x^4 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.239847, size = 743, normalized size = 3.33

$$-\frac{4\,d\left(-\frac{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}{c^{3}d^{5}}\right)^{\frac{1}{4}}\arctan\left(-\frac{cd\left(-\frac{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}{c^{3}d^{5}}\right)^{\frac{1}{4}}}{(bc-ad)x+(bc-ad)\sqrt{\frac{c^{2}d^{2}\sqrt{-\frac{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}{c^{3}d^{5}}}+\left(b^{2}c^{2}-2\,abcd+a^{2}\right.}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)/(d*x^4 + c),x, algorithm="fricas")
```

```
[Out] -1/4*(4*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4)*arctan(-c*d*(-(b^4*c^4 - 4*a*
b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5
))^(1/4)/((b*c - a*d)*x + (b*c - a*d)*sqrt((c^2*d^2*sqrt(-(b^4*c^
4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/
(c^3*d^5)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2)/(b^2*c^2 - 2*a*
b*c*d + a^2*d^2)))) - d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^
2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4)*log(c*d*(-(b^4*
c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4
)/(c^3*d^5))^(1/4) - (b*c - a*d)*x) + d*(-(b^4*c^4 - 4*a*b^3*c^3*
d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4)
*log(-c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4) - (b*c - a*d)*x) - 4*b*x)/d
```

---

**Sympy [A]** time = 2.23477, size = 87, normalized size = 0.39

$$\frac{bx}{d} + \operatorname{RootSum}\left(256t^{4}c^{3}d^{5} + a^{4}d^{4} - 4a^{3}bcd^{3} + 6a^{2}b^{2}c^{2}d^{2} - 4ab^{3}c^{3}d + b^{4}c^{4}, \left(t \mapsto t\log\left(\frac{4tcd}{ad - bc} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)/(d*x**4+c),x)
```

```
[Out] b*x/d + RootSum(256*_t**4*c**3*d**5 + a**4*d**4 - 4*a**3*b*c*d**3
 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4, Lambda(_t
, _t*log(4*_t*c*d/(a*d - b*c) + x)))
```

---

**GIAC/XCAS [A]** time = 0.220027, size = 331, normalized size = 1.48

$$\frac{bx}{d} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc - \left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\,cd^2} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc - \left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\,cd^2} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc - \left(cd^3\right)^{\frac{1}{4}}ad\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{8\,cd^2} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc - \left(cd^3\right)^{\frac{1}{4}}ad\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{8\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)/(d*x^4 + c),x, algorithm="giac")
```

```
[Out] b*x/d - 1/4*sqrt(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*arcta
n(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c*d^2) -
1/4*sqrt(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c*d^2) - 1/8*sqrt
(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*ln(x^2 + sqrt(2)*x*(c
/d)^(1/4) + sqrt(c/d))/(c*d^2) + 1/8*sqrt(2)*((c*d^3)^(1/4)*b*c -
 (c*d^3)^(1/4)*a*d)*ln(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(
c*d^2)
```

## 3.52 $\int \frac{a+bx^4}{(c+dx^4)^2} \, dx$

**Optimal**. Leaf size=245

$$-\frac{(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{5/4}}+\frac{(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{5/4}}-\frac{(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}}+\frac{(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}d^{5/4}}-\frac{x(bc-ad)}{4cd\,(c+dx^4)}$$

```
[Out] -((b*c - a*d)*x)/(4*c*d*(c + d*x^4)) - ((b*c + 3*a*d)*ArcTan[1 -
(Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(5/4)) + ((b*c
 + 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(
7/4)*d^(5/4)) - ((b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1
/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(5/4)) + ((b*c + 3*a*
d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sq
rt[2]*c^(7/4)*d^(5/4))
```

---

**Rubi [A]** time = 0.312072, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$

$$-\frac{(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{5/4}}+\frac{(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{5/4}}-\frac{(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}}+\frac{(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}d^{5/4}}-\frac{x(bc-ad)}{4cd\,(c+dx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)/(c + d*x^4)^2,x]
```

```
[Out] -((b*c - a*d)*x)/(4*c*d*(c + d*x^4)) - ((b*c + 3*a*d)*ArcTan[1 -
(Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(5/4)) + ((b*c
 + 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(
7/4)*d^(5/4)) - ((b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1
/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(5/4)) + ((b*c + 3*a*
d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sq
rt[2]*c^(7/4)*d^(5/4))
```

---

**Rubi in Sympy [A]** time = 59.1342, size = 226, normalized size = 0.92

$$\frac{x(ad-bc)}{4cd(c+dx^4)}-\frac{\sqrt{2}(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{32c^{\frac{7}{4}}d^{\frac{5}{4}}}+\frac{\sqrt{2}(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{32c^{\frac{7}{4}}d^{\frac{5}{4}}}-\frac{\sqrt{2}(3ad+bc)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{16c^{\frac{7}{4}}d^{\frac{5}{4}}}+\frac{\sqrt{2}(3ad+bc)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{16c^{\frac{7}{4}}d^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)/(d*x**4+c)**2,x)
```

```
[Out] x*(a*d - b*c)/(4*c*d*(c + d*x**4)) - sqrt(2)*(3*a*d + b*c)*log(-s
qrt(2)*c**(1/4)*d**(1/4)*x + sqrt(c) + sqrt(d)*x**2)/(32*c**(7/4)
*d**(5/4)) + sqrt(2)*(3*a*d + b*c)*log(sqrt(2)*c**(1/4)*d**(1/4)*
x + sqrt(c) + sqrt(d)*x**2)/(32*c**(7/4)*d**(5/4)) - sqrt(2)*(3*a
*d + b*c)*atan(1 - sqrt(2)*d**(1/4)*x/c**(1/4))/(16*c**(7/4)*d**(
5/4)) + sqrt(2)*(3*a*d + b*c)*atan(1 + sqrt(2)*d**(1/4)*x/c**(1/4
))/(16*c**(7/4)*d**(5/4))
```

---

**Mathematica [A]** time = 0.290427, size = 212, normalized size = 0.87

$$\frac{-\frac{8c^{3/4}\sqrt[4]{d}x(bc-ad)}{c+dx^4}-\sqrt{2}(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)+\sqrt{2}(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)-2\sqrt{2}(3ad}{32c^{7/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^4)/(c + d*x^4)^2,x]
```

```
[Out] ((-8*c^(3/4)*d^(1/4)*(b*c - a*d)*x)/(c + d*x^4) - 2*Sqrt[2]*(b*c
+ 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*Sqrt[2]*(b*c
 + 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - Sqrt[2]*(b*c
+ 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] +
 Sqrt[2]*(b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x +
Sqrt[d]*x^2])/(32*c^(7/4)*d^(5/4))
```

---

**Maple [A]** time = 0.013, size = 295, normalized size = 1.2

$$\frac{(ad-bc)\,x}{4\,cd\,(dx^4+c)}+\frac{3\,\sqrt{2}a}{16\,c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)+\frac{\sqrt{2}b}{16\,cd}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$
$$+\frac{3\,\sqrt{2}a}{32\,c^2}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$+\frac{\sqrt{2}b}{32\,cd}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$+\frac{3\,\sqrt{2}a}{16\,c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{\sqrt{2}b}{16\,cd}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)/(d*x^4+c)^2,x)
```

```
[Out] 1/4*(a*d-b*c)/c/d*x/(d*x^4+c)+3/16/c^2*(c/d)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(c/d)^(1/4)*x-1)*a+1/16/c/d*(c/d)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(c/d)^(1/4)*x-1)*b+3/32/c^2*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c
/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)
^(1/2)))*a+1/32/c/d*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(
1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*b+3/16
/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a+1/16/c
/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)/(d*x^4 + c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.240921, size = 838, normalized size = 3.42

$$-\frac{4\left(cd^2x^4+c^2d\right)\left(-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bcd^3+81\,a^4d^4}{c^7d^5}\right)^{\frac{1}{4}}\arctan\left(\frac{c^2d\left(-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bc}{c^7d^5}\right.}{(bc+3\,ad)x+(bc+3\,ad)\sqrt{\frac{c^4d^2\sqrt{-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3b}{c^7d^5}}}{b^2c^2+6\,abcd}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)/(d*x^4 + c)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*(c*d^2*x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2
*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*arc
tan(c^2*d*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*
a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)/((b*c + 3*a*d)*x + (b*
c + 3*a*d)*sqrt((c^4*d^2*sqrt(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2
*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5)) + (b^2*c^
2 + 6*a*b*c*d + 9*a^2*d^2)*x^2)/(b^2*c^2 + 6*a*b*c*d + 9*a^2*d^2)
))) - (c*d^2*x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^
2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*log(c^
2*d*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*
c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4) + (b*c + 3*a*d)*x) + (c*d^2*
x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 1
08*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*log(-c^2*d*(-(b^4*c
^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a
^4*d^4)/(c^7*d^5))^(1/4) + (b*c + 3*a*d)*x) + 4*(b*c - a*d)*x)/(c
*d^2*x^4 + c^2*d)
```

---

**Sympy [A]** time = 3.20672, size = 112, normalized size = 0.46

$$\frac{x\,(ad-bc)}{4c^2d+4cd^2x^4} + \operatorname{RootSum}\left(65536t^4c^7d^5+81a^4d^4+108a^3bcd^3+54a^2b^2c^2d^2+12ab^3c^3d+b^4c^4,\left(t\mapsto t\log\left(\frac{16tc^2d}{3ad+bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)/(d*x**4+c)**2,x)
```

```
[Out] x*(a*d - b*c)/(4*c**2*d + 4*c*d**2*x**4) + RootSum(65536*_t**4*c*
*7*d**5 + 81*a**4*d**4 + 108*a**3*b*c*d**3 + 54*a**2*b**2*c**2*d*
*2 + 12*a*b**3*c**3*d + b**4*c**4, Lambda(_t, _t*log(16*_t*c**2*d
/(3*a*d + b*c) + x)))
```

---

**GIAC/XCAS [A]** time = 0.219457, size = 359, normalized size = 1.47

$$\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}bc+3\,\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^{2}d^{2}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}bc+3\,\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^{2}d^{2}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}bc+3\,\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\ln\left(x^{2}+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32\,c^{2}d^{2}}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}bc+3\,\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\ln\left(x^{2}-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32\,c^{2}d^{2}}-\frac{bcx-adx}{4\left(dx^{4}+c\right)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)/(d*x^4 + c)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*arctan(1/2
*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^2*d^2) + 1/1
6*sqrt(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^2*d^2) + 1/32*s
qrt(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*ln(x^2 + sqrt(2)
*x*(c/d)^(1/4) + sqrt(c/d))/(c^2*d^2) - 1/32*sqrt(2)*((c*d^3)^(1/
4)*b*c + 3*(c*d^3)^(1/4)*a*d)*ln(x^2 - sqrt(2)*x*(c/d)^(1/4) + sq
rt(c/d))/(c^2*d^2) - 1/4*(b*c*x - a*d*x)/((d*x^4 + c)*c*d)
```

## 3.53 $\int \frac{a+bx^4}{(c+dx^4)^3} \, dx$

**Optimal**. Leaf size=273

$$-\frac{3(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}}+\frac{3(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}}-\frac{3(7ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{5/4}}+\frac{3(7ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{64\sqrt{2}c^{11/4}d^{5/4}}+\frac{x(7ad+bc)}{32c^2d\,(c+dx^4)}-\frac{x(bc-ad)}{8cd\,(c+dx^4)^2}$$

```
[Out] -((b*c - a*d)*x)/(8*c*d*(c + d*x^4)^2) + ((b*c + 7*a*d)*x)/(32*c^
2*d*(c + d*x^4)) - (3*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x
)/c^(1/4)])/(64*Sqrt[2]*c^(11/4)*d^(5/4)) + (3*(b*c + 7*a*d)*ArcT
an[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(64*Sqrt[2]*c^(11/4)*d^(5/4)
) - (3*(b*c + 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sq
rt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*d^(5/4)) + (3*(b*c + 7*a*d)*Log
[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]
*c^(11/4)*d^(5/4))
```

---

**Rubi [A]** time = 0.356832, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$

$$-\frac{3(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}}+\frac{3(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}}-\frac{3(7ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{5/4}}+\frac{3(7ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{64\sqrt{2}c^{11/4}d^{5/4}}+\frac{x(7ad+bc)}{32c^2d\,(c+dx^4)}-\frac{x(bc-ad)}{8cd\,(c+dx^4)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)/(c + d*x^4)^3,x]
```

```
[Out] -((b*c - a*d)*x)/(8*c*d*(c + d*x^4)^2) + ((b*c + 7*a*d)*x)/(32*c^
2*d*(c + d*x^4)) - (3*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x
)/c^(1/4)])/(64*Sqrt[2]*c^(11/4)*d^(5/4)) + (3*(b*c + 7*a*d)*ArcT
an[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(64*Sqrt[2]*c^(11/4)*d^(5/4)
) - (3*(b*c + 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sq
rt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*d^(5/4)) + (3*(b*c + 7*a*d)*Log
```

```
[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]
*c^(11/4)*d^(5/4))
```

---

**Rubi in Sympy [A]** time = 65.6386, size = 258, normalized size = 0.95

$$\frac{x(ad-bc)}{8cd(c+dx^4)^2}+\frac{x(7ad+bc)}{32c^2d(c+dx^4)}-\frac{3\sqrt{2}(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{256c^{\frac{11}{4}}d^{\frac{5}{4}}}+\frac{3\sqrt{2}(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{256c^{\frac{11}{4}}d^{\frac{5}{4}}}-\frac{3\sqrt{2}(7ad+bc)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{128c^{\frac{11}{4}}d^{\frac{5}{4}}}+\frac{3\sqrt{2}(7ad+bc)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{128c^{\frac{11}{4}}d^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)/(d*x**4+c)**3,x)
```

```
[Out] x*(a*d - b*c)/(8*c*d*(c + d*x**4)**2) + x*(7*a*d + b*c)/(32*c**2*
d*(c + d*x**4)) - 3*sqrt(2)*(7*a*d + b*c)*log(-sqrt(2)*c**(1/4)*d
**(1/4)*x + sqrt(c) + sqrt(d)*x**2)/(256*c**(11/4)*d**(5/4)) + 3*
sqrt(2)*(7*a*d + b*c)*log(sqrt(2)*c**(1/4)*d**(1/4)*x + sqrt(c) +
 sqrt(d)*x**2)/(256*c**(11/4)*d**(5/4)) - 3*sqrt(2)*(7*a*d + b*c)
*atan(1 - sqrt(2)*d**(1/4)*x/c**(1/4))/(128*c**(11/4)*d**(5/4)) +
 3*sqrt(2)*(7*a*d + b*c)*atan(1 + sqrt(2)*d**(1/4)*x/c**(1/4))/(1
28*c**(11/4)*d**(5/4))
```

---

**Mathematica [A]** time = 0.371676, size = 243, normalized size = 0.89

$$\frac{-\frac{32c^{7/4}\sqrt[4]{d}x(bc-ad)}{(c+dx^4)^2}+\frac{8c^{3/4}\sqrt[4]{d}x(7ad+bc)}{c+dx^4}-3\sqrt{2}(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)+3\sqrt{2}(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}\right.}{256c^{11/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^4)/(c + d*x^4)^3,x]
```

```
[Out] ((-32*c^(7/4)*d^(1/4)*(b*c - a*d)*x)/(c + d*x^4)^2 + (8*c^(3/4)*d
^(1/4)*(b*c + 7*a*d)*x)/(c + d*x^4) - 6*Sqrt[2]*(b*c + 7*a*d)*Arc
Tan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 6*Sqrt[2]*(b*c + 7*a*d)*Ar
cTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 3*Sqrt[2]*(b*c + 7*a*d)*L
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + 3*Sqrt[2]
*(b*c + 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*
x^2])/(256*c^(11/4)*d^(5/4))
```

**Maple [A]** time = 0.016, size = 314, normalized size = 1.2

$$\frac{1}{(dx^4+c)^2}\left(\frac{(7\,ad+bc)\,x^5}{32\,c^2}+\frac{(11\,ad-3\,bc)\,x}{32\,cd}\right)$$
$$+\frac{21\,\sqrt{2}a}{128\,c^3}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{3\,\sqrt{2}b}{128\,c^2d}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)$$
$$+\frac{21\,\sqrt{2}a}{128\,c^3}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)+\frac{3\,\sqrt{2}b}{128\,c^2d}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$
$$+\frac{21\,\sqrt{2}a}{256\,c^3}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$+\frac{3\,\sqrt{2}b}{256\,c^2d}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)/(d*x^4+c)^3,x)
```

```
[Out] (1/32*(7*a*d+b*c)/c^2*x^5+1/32*(11*a*d-3*b*c)/c/d*x)/(d*x^4+c)^2+
21/128/c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a+
3/128/c^2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b
+21/128/c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a
+3/128/c^2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*
b+21/256/c^3*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c
/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*a+3/256/c^2/d
*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(
x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*b
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)/(d*x^4 + c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 0.239178, size = 941, normalized size = 3.45

$$\frac{4\left(bcd+7\,ad^2\right)x^5-12\left(c^2d^3x^8+2\,c^3d^2x^4+c^4d\right)\left(-\frac{b^4c^4+28\,ab^3c^3d+294\,a^2b^2c^2d^2+1372\,a^3bcd^3+2401\,a^4d^4}{c^{11}d^5}\right)^{\frac{1}{4}}\arctan\left(\frac{}{(bc+7\,ad)x+(bc+7\,a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)/(d*x^4 + c)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(4*(b*c*d + 7*a*d^2)*x^5 - 12*(c^2*d^3*x^8 + 2*c^3*d^2*x^4
+ c^4*d)*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372
*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4)*arctan(c^3*d*(-(b^
4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 +
 2401*a^4*d^4)/(c^11*d^5))^(1/4)/((b*c + 7*a*d)*x + (b*c + 7*a*d)
*sqrt((c^6*d^2*sqrt(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*
d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5)) + (b^2*c^2 + 1
4*a*b*c*d + 49*a^2*d^2)*x^2)/(b^2*c^2 + 14*a*b*c*d + 49*a^2*d^2))
)) + 3*(c^2*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d)*(-(b^4*c^4 + 28*a*b^
3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/
(c^11*d^5))^(1/4)*log(3*c^3*d*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a
^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/
4) + 3*(b*c + 7*a*d)*x) - 3*(c^2*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d)
*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c
*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4)*log(-3*c^3*d*(-(b^4*c^4 +
28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^
4*d^4)/(c^11*d^5))^(1/4) + 3*(b*c + 7*a*d)*x) - 4*(3*b*c^2 - 11*a
*c*d)*x)/(c^2*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d)
```

---

**Sympy [A]** time = 6.69677, size = 151, normalized size = 0.55

$$\frac{x^5\left(7ad^2+bcd\right)+x\left(11acd-3bc^2\right)}{32c^4d+64c^3d^2x^4+32c^2d^3x^8}$$
$$+\mathrm{RootSum}\left(268435456t^4c^{11}d^5+194481a^4d^4+111132a^3bcd^3+23814a^2b^2c^2d^2+2268ab^3c^3d+81b^4c^4,\left(t\mapsto t\log\left(\frac{128tc^3}{21ad+}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)/(d*x**4+c)**3,x)
```

```
[Out] (x**5*(7*a*d**2 + b*c*d) + x*(11*a*c*d - 3*b*c**2))/(32*c**4*d +
64*c**3*d**2*x**4 + 32*c**2*d**3*x**8) + RootSum(268435456*_t**4*
c**11*d**5 + 194481*a**4*d**4 + 111132*a**3*b*c*d**3 + 23814*a**2
*b**2*c**2*d**2 + 2268*a*b**3*c**3*d + 81*b**4*c**4, Lambda(_t, _
t*log(128*_t*c**3*d/(21*a*d + 3*b*c) + x)))
```

**GIAC/XCAS [A]** time = 0.221907, size = 386, normalized size = 1.41

$$\frac{3\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}bc+7\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^{3}d^{2}}+\frac{3\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}bc+7\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^{3}d^{2}}+\frac{3\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}bc+7\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\ln\left(x^{2}+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{256\,c^{3}d^{2}}-\frac{3\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}bc+7\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\ln\left(x^{2}-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{256\,c^{3}d^{2}}+\frac{bcdx^{5}+7\,ad^{2}x^{5}-3\,bc^{2}x+11\,acdx}{32\left(dx^{4}+c\right)^{2}c^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)/(d*x^4 + c)^3,x, algorithm="giac")
```

```
[Out] 3/128*sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*arctan(1/
2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^3*d^2) + 3/
128*sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^3*d^2) + 3/25
6*sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*ln(x^2 + sqrt
(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^3*d^2) - 3/256*sqrt(2)*((c*d^3)
^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*ln(x^2 - sqrt(2)*x*(c/d)^(1/4)
+ sqrt(c/d))/(c^3*d^2) + 1/32*(b*c*d*x^5 + 7*a*d^2*x^5 - 3*b*c^2*
x + 11*a*c*d*x)/((d*x^4 + c)^2*c^2*d)
```

## 3.54 $\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^4 \, dx$

**Optimal**. Leaf size=154

$$\frac{1}{17}d^2x^{17}\left(a^2d^2+8abcd+6b^2c^2\right)+\frac{4}{13}cdx^{13}\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{9}c^2x^9\left(6a^2d^2+8abcd+b^2c^2\right)+a^2c^4x+\frac{2}{5}ac^3x^5(2ad+bc)+\frac{2}{21}bd^3x^{21}(ad+2bc)+\frac{1}{25}b^2d^4x^{25}$$

```
[Out] a^2*c^4*x + (2*a*c^3*(b*c + 2*a*d)*x^5)/5 + (c^2*(b^2*c^2 + 8*a*b
*c*d + 6*a^2*d^2)*x^9)/9 + (4*c*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)
*x^13)/13 + (d^2*(6*b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^17)/17 + (2*
b*d^3*(2*b*c + a*d)*x^21)/21 + (b^2*d^4*x^25)/25
```

---

**Rubi [A]** time = 0.235883, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{1}{17}d^2x^{17}\left(a^2d^2+8abcd+6b^2c^2\right)+\frac{4}{13}cdx^{13}\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{9}c^2x^9\left(6a^2d^2+8abcd+b^2c^2\right)+a^2c^4x+\frac{2}{5}ac^3x^5(2ad+bc)+\frac{2}{21}bd^3x^{21}(ad+2bc)+\frac{1}{25}b^2d^4x^{25}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^2*(c + d*x^4)^4,x]
```

```
[Out] a^2*c^4*x + (2*a*c^3*(b*c + 2*a*d)*x^5)/5 + (c^2*(b^2*c^2 + 8*a*b
*c*d + 6*a^2*d^2)*x^9)/9 + (4*c*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)
*x^13)/13 + (d^2*(6*b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^17)/17 + (2*
b*d^3*(2*b*c + a*d)*x^21)/21 + (b^2*d^4*x^25)/25
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2ac^3x^5\left(2ad+bc\right)}{5}+\frac{b^2d^4x^{25}}{25}+\frac{2bd^3x^{21}\left(ad+2bc\right)}{21}+c^4\int a^2\,dx+\frac{c^2x^9\left(6a^2d^2+8abcd+b^2c^2\right)}{9}+\frac{4cdx^{13}\left(a^2d^2+3abcd+b^2c^2\right)}{13}+\frac{d^2x^{17}\left(a^2d^2+8abcd+6b^2c^2\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**2*(d*x**4+c)**4,x)
```

```
[Out] 2*a*c**3*x**5*(2*a*d + b*c)/5 + b**2*d**4*x**25/25 + 2*b*d**3*x**
21*(a*d + 2*b*c)/21 + c**4*Integral(a**2, x) + c**2*x**9*(6*a**2*
```

```
d**2 + 8*a*b*c*d + b**2*c**2)/9 + 4*c*d*x**13*(a**2*d**2 + 3*a*b*
c*d + b**2*c**2)/13 + d**2*x**17*(a**2*d**2 + 8*a*b*c*d + 6*b**2*
c**2)/17
```

---

**Mathematica [A]** time = 0.0624063, size = 154, normalized size = 1.

$$\frac{1}{17}d^2x^{17}\left(a^2d^2+8abcd+6b^2c^2\right)+\frac{4}{13}cdx^{13}\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{9}c^2x^9\left(6a^2d^2+8abcd+b^2c^2\right)+a^2c^4x+\frac{2}{5}ac^3x^5(2ad+bc)+\frac{2}{21}bd^3x^{21}(ad+2bc)+\frac{1}{25}b^2d^4x^{25}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^4)^2*(c + d*x^4)^4,x]
```

```
[Out] a^2*c^4*x + (2*a*c^3*(b*c + 2*a*d)*x^5)/5 + (c^2*(b^2*c^2 + 8*a*b
*c*d + 6*a^2*d^2)*x^9)/9 + (4*c*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)
*x^13)/13 + (d^2*(6*b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^17)/17 + (2*
b*d^3*(2*b*c + a*d)*x^21)/21 + (b^2*d^4*x^25)/25
```

---

**Maple [A]** time = 0.001, size = 163, normalized size = 1.1

$$\frac{b^2d^4x^{25}}{25}+\frac{\left(2\,abd^4+4\,b^2cd^3\right)x^{21}}{21}+\frac{\left(a^2d^4+8\,abcd^3+6\,b^2c^2d^2\right)x^{17}}{17}+\frac{\left(4\,a^2cd^3+12\,abc^2d^2+4\,b^2c^3d\right)x^{13}}{13}+\frac{\left(6\,a^2c^2d^2+8\,abc^3d+b^2c^4\right)x^9}{9}+\frac{\left(4\,a^2c^3d+2\,abc^4\right)x^5}{5}+a^2c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^2*(d*x^4+c)^4,x)
```

```
[Out] 1/25*b^2*d^4*x^25+1/21*(2*a*b*d^4+4*b^2*c*d^3)*x^21+1/17*(a^2*d^4
+8*a*b*c*d^3+6*b^2*c^2*d^2)*x^17+1/13*(4*a^2*c*d^3+12*a*b*c^2*d^2
+4*b^2*c^3*d)*x^13+1/9*(6*a^2*c^2*d^2+8*a*b*c^3*d+b^2*c^4)*x^9+1/
5*(4*a^2*c^3*d+2*a*b*c^4)*x^5+a^2*c^4*x
```

---

**Maxima [A]** time = 1.37164, size = 213, normalized size = 1.38

$$\frac{1}{25}\,b^2d^4x^{25}+\frac{2}{21}\left(2\,b^2cd^3+abd^4\right)x^{21}+\frac{1}{17}\left(6\,b^2c^2d^2+8\,abcd^3+a^2d^4\right)x^{17}+\frac{4}{13}\left(b^2c^3d+3\,abc^2d^2+a^2cd^3\right)x^{13}+\frac{1}{9}\left(b^2c^4+8\,abc^3d+6\,a^2c^2d^2\right)x^9+a^2c^4x+\frac{2}{5}\left(abc^4+2\,a^2c^3d\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(d*x^4 + c)^4,x, algorithm="maxima")
```

```
[Out] 1/25*b^2*d^4*x^25 + 2/21*(2*b^2*c*d^3 + a*b*d^4)*x^21 + 1/17*(6*b
^2*c^2*d^2 + 8*a*b*c*d^3 + a^2*d^4)*x^17 + 4/13*(b^2*c^3*d + 3*a*
b*c^2*d^2 + a^2*c*d^3)*x^13 + 1/9*(b^2*c^4 + 8*a*b*c^3*d + 6*a^2*
c^2*d^2)*x^9 + a^2*c^4*x + 2/5*(a*b*c^4 + 2*a^2*c^3*d)*x^5
```

---

**Fricas [A]** time = 0.188992, size = 1, normalized size = 0.01

$$\frac{1}{25}x^{25}d^4b^2 + \frac{4}{21}x^{21}d^3cb^2 + \frac{2}{21}x^{21}d^4ba + \frac{6}{17}x^{17}d^2c^2b^2 + \frac{8}{17}x^{17}d^3cba + \frac{1}{17}x^{17}d^4a^2 + \frac{4}{13}x^{13}dc^3b^2 + \frac{12}{13}x^{13}d^2c^2ba + \frac{4}{13}x^{13}d^3ca^2 + \frac{1}{9}x^9c^4b^2 + \frac{8}{9}x^9dc^3ba + \frac{2}{3}x^9d^2c^2a^2 + \frac{2}{5}x^5c^4ba + \frac{4}{5}x^5dc^3a^2 + xc^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(d*x^4 + c)^4,x, algorithm="fricas")
```

```
[Out] 1/25*x^25*d^4*b^2 + 4/21*x^21*d^3*c*b^2 + 2/21*x^21*d^4*b*a + 6/1
7*x^17*d^2*c^2*b^2 + 8/17*x^17*d^3*c*b*a + 1/17*x^17*d^4*a^2 + 4/
13*x^13*d*c^3*b^2 + 12/13*x^13*d^2*c^2*b*a + 4/13*x^13*d^3*c*a^2
+ 1/9*x^9*c^4*b^2 + 8/9*x^9*d*c^3*b*a + 2/3*x^9*d^2*c^2*a^2 + 2/5
*x^5*c^4*b*a + 4/5*x^5*d*c^3*a^2 + x*c^4*a^2
```

---

**Sympy [A]** time = 0.182983, size = 185, normalized size = 1.2

$$a^2c^4x + \frac{b^2d^4x^{25}}{25} + x^{21}\left(\frac{2abd^4}{21} + \frac{4b^2cd^3}{21}\right) + x^{17}\left(\frac{a^2d^4}{17} + \frac{8abcd^3}{17} + \frac{6b^2c^2d^2}{17}\right) + x^{13}\left(\frac{4a^2cd^3}{13} + \frac{12abc^2d^2}{13} + \frac{4b^2c^3d}{13}\right) + x^9\left(\frac{2a^2c^2d^2}{3} + \frac{8abc^3d}{9} + \frac{b^2c^4}{9}\right) + x^5\left(\frac{4a^2c^3d}{5} + \frac{2abc^4}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**2*(d*x**4+c)**4,x)
```

```
[Out] a**2*c**4*x + b**2*d**4*x**25/25 + x**21*(2*a*b*d**4/21 + 4*b**2*
c*d**3/21) + x**17*(a**2*d**4/17 + 8*a*b*c*d**3/17 + 6*b**2*c**2*
d**2/17) + x**13*(4*a**2*c*d**3/13 + 12*a*b*c**2*d**2/13 + 4*b**2
*c**3*d/13) + x**9*(2*a**2*c**2*d**2/3 + 8*a*b*c**3*d/9 + b**2*c*
*4/9) + x**5*(4*a**2*c**3*d/5 + 2*a*b*c**4/5)
```

---

**GIAC/XCAS [A]** time = 0.209819, size = 234, normalized size = 1.52

$$\frac{1}{25}\,b^{2}d^{4}x^{25}+\frac{4}{21}\,b^{2}cd^{3}x^{21}+\frac{2}{21}\,abd^{4}x^{21}+\frac{6}{17}\,b^{2}c^{2}d^{2}x^{17}+\frac{8}{17}\,abcd^{3}x^{17}+\frac{1}{17}\,a^{2}d^{4}x^{17}+\frac{4}{13}\,b^{2}c^{3}dx^{13}+\frac{12}{13}\,abc^{2}d^{2}x^{13}+\frac{4}{13}\,a^{2}cd^{3}x^{13}+\frac{1}{9}\,b^{2}c^{4}x^{9}+\frac{8}{9}\,abc^{3}dx^{9}+\frac{2}{3}\,a^{2}c^{2}d^{2}x^{9}+\frac{2}{5}\,abc^{4}x^{5}+\frac{4}{5}\,a^{2}c^{3}dx^{5}+a^{2}c^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(d*x^4 + c)^4,x, algorithm="giac")
```

```
[Out] 1/25*b^2*d^4*x^25 + 4/21*b^2*c*d^3*x^21 + 2/21*a*b*d^4*x^21 + 6/1
7*b^2*c^2*d^2*x^17 + 8/17*a*b*c*d^3*x^17 + 1/17*a^2*d^4*x^17 + 4/
13*b^2*c^3*d*x^13 + 12/13*a*b*c^2*d^2*x^13 + 4/13*a^2*c*d^3*x^13
+ 1/9*b^2*c^4*x^9 + 8/9*a*b*c^3*d*x^9 + 2/3*a^2*c^2*d^2*x^9 + 2/5
*a*b*c^4*x^5 + 4/5*a^2*c^3*d*x^5 + a^2*c^4*x
```

## 3.55 $\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^3 \, dx$

**Optimal**. Leaf size=122

$$\frac{1}{13}dx^{13}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{17}bd^2x^{17}(2ad+3bc)+\frac{1}{21}b^2d^3x^{21}$$

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c
*d + 3*a^2*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^1
3)/13 + (b*d^2*(3*b*c + 2*a*d)*x^17)/17 + (b^2*d^3*x^21)/21
```

---

**Rubi [A]** time = 0.172716, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{1}{13}dx^{13}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{17}bd^2x^{17}(2ad+3bc)+\frac{1}{21}b^2d^3x^{21}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^2*(c + d*x^4)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c
*d + 3*a^2*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^1
3)/13 + (b*d^2*(3*b*c + 2*a*d)*x^17)/17 + (b^2*d^3*x^21)/21
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{ac^2x^5\left(3ad+2bc\right)}{5}+\frac{b^2d^3x^{21}}{21}+\frac{bd^2x^{17}\left(2ad+3bc\right)}{17}+c^3\int a^2\,dx+\frac{cx^9\left(3a^2d^2+6abcd+b^2c^2\right)}{9}+\frac{dx^{13}\left(a^2d^2+6abcd+3b^2c^2\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**2*(d*x**4+c)**3,x)
```

```
[Out] a*c**2*x**5*(3*a*d + 2*b*c)/5 + b**2*d**3*x**21/21 + b*d**2*x**17
*(2*a*d + 3*b*c)/17 + c**3*Integral(a**2, x) + c*x**9*(3*a**2*d**
2 + 6*a*b*c*d + b**2*c**2)/9 + d*x**13*(a**2*d**2 + 6*a*b*c*d + 3
*b**2*c**2)/13
```

**Mathematica [A]** time = 0.0428412, size = 122, normalized size = 1.

$$\frac{1}{13}dx^{13}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{17}bd^2x^{17}(2ad+3bc)+\frac{1}{21}b^2d^3x^{21}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^4)^2*(c + d*x^4)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c
*d + 3*a^2*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^1
3)/13 + (b*d^2*(3*b*c + 2*a*d)*x^17)/17 + (b^2*d^3*x^21)/21
```

**Maple [A]** time = 0.002, size = 125, normalized size = 1.

$$\frac{b^2d^3x^{21}}{21}+\frac{\left(2\,abd^3+3\,b^2cd^2\right)x^{17}}{17}+\frac{\left(a^2d^3+6\,abcd^2+3\,b^2c^2d\right)x^{13}}{13}+\frac{\left(3\,a^2cd^2+6\,abc^2d+b^2c^3\right)x^9}{9}+\frac{\left(3\,a^2c^2d+2\,abc^3\right)x^5}{5}+a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^2*(d*x^4+c)^3,x)
```

```
[Out] 1/21*b^2*d^3*x^21+1/17*(2*a*b*d^3+3*b^2*c*d^2)*x^17+1/13*(a^2*d^3
+6*a*b*c*d^2+3*b^2*c^2*d)*x^13+1/9*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c
^3)*x^9+1/5*(3*a^2*c^2*d+2*a*b*c^3)*x^5+a^2*c^3*x
```

**Maxima [A]** time = 1.35818, size = 167, normalized size = 1.37

$$\frac{1}{21}\,b^2d^3x^{21}+\frac{1}{17}\left(3\,b^2cd^2+2\,abd^3\right)x^{17}+\frac{1}{13}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{13}+\frac{1}{9}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^9+a^2c^3x+\frac{1}{5}\left(2\,abc^3+3\,a^2c^2d\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(d*x^4 + c)^3,x, algorithm="maxima")
```

```
[Out] 1/21*b^2*d^3*x^21 + 1/17*(3*b^2*c*d^2 + 2*a*b*d^3)*x^17 + 1/13*(3
*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^13 + 1/9*(b^2*c^3 + 6*a*b*c
```

```
^2*d + 3*a^2*c*d^2)*x^9 + a^2*c^3*x + 1/5*(2*a*b*c^3 + 3*a^2*c^2*
d)*x^5
```

---

**Fricas [A]** time = 0.191059, size = 1, normalized size = 0.01

$$\frac{1}{21}x^{21}d^3b^2 + \frac{3}{17}x^{17}d^2cb^2 + \frac{2}{17}x^{17}d^3ba + \frac{3}{13}x^{13}dc^2b^2 + \frac{6}{13}x^{13}d^2cba + \frac{1}{13}x^{13}d^3a^2 + \frac{1}{9}x^9c^3b^2 + \frac{2}{3}x^9dc^2ba + \frac{1}{3}x^9d^2ca^2 + \frac{2}{5}x^5c^3ba + \frac{3}{5}x^5dc^2a^2 + xc^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(d*x^4 + c)^3,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*d^3*b^2 + 3/17*x^17*d^2*c*b^2 + 2/17*x^17*d^3*b*a + 3/1
3*x^13*d*c^2*b^2 + 6/13*x^13*d^2*c*b*a + 1/13*x^13*d^3*a^2 + 1/9*
x^9*c^3*b^2 + 2/3*x^9*d*c^2*b*a + 1/3*x^9*d^2*c*a^2 + 2/5*x^5*c^3
*b*a + 3/5*x^5*d*c^2*a^2 + x*c^3*a^2
```

---

**Sympy [A]** time = 0.156153, size = 139, normalized size = 1.14

$$a^2c^3x + \frac{b^2d^3x^{21}}{21} + x^{17}\left(\frac{2abd^3}{17} + \frac{3b^2cd^2}{17}\right) + x^{13}\left(\frac{a^2d^3}{13} + \frac{6abcd^2}{13} + \frac{3b^2c^2d}{13}\right) + x^9\left(\frac{a^2cd^2}{3} + \frac{2abc^2d}{3} + \frac{b^2c^3}{9}\right) + x^5\left(\frac{3a^2c^2d}{5} + \frac{2abc^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**2*(d*x**4+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**21/21 + x**17*(2*a*b*d**3/17 + 3*b**2*
c*d**2/17) + x**13*(a**2*d**3/13 + 6*a*b*c*d**2/13 + 3*b**2*c**2*
d/13) + x**9*(a**2*c*d**2/3 + 2*a*b*c**2*d/3 + b**2*c**3/9) + x**
5*(3*a**2*c**2*d/5 + 2*a*b*c**3/5)
```

---

**GIAC/XCAS [A]** time = 0.212041, size = 178, normalized size = 1.46

$$\frac{1}{21}\,b^2d^3x^{21} + \frac{3}{17}\,b^2cd^2x^{17} + \frac{2}{17}\,abd^3x^{17} + \frac{3}{13}\,b^2c^2dx^{13} + \frac{6}{13}\,abcd^2x^{13} + \frac{1}{13}\,a^2d^3x^{13} + \frac{1}{9}\,b^2c^3x^9 + \frac{2}{3}\,abc^2dx^9 + \frac{1}{3}\,a^2cd^2x^9 + \frac{2}{5}\,abc^3x^5 + \frac{3}{5}\,a^2c^2dx^5 + a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(d*x^4 + c)^3,x, algorithm="giac")
```

```
[Out] 1/21*b^2*d^3*x^21 + 3/17*b^2*c*d^2*x^17 + 2/17*a*b*d^3*x^17 + 3/1
3*b^2*c^2*d*x^13 + 6/13*a*b*c*d^2*x^13 + 1/13*a^2*d^3*x^13 + 1/9*
b^2*c^3*x^9 + 2/3*a*b*c^2*d*x^9 + 1/3*a^2*c*d^2*x^9 + 2/5*a*b*c^3
*x^5 + 3/5*a^2*c^2*d*x^5 + a^2*c^3*x
```

## 3.56 $\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^2 \, dx$

**Optimal**. Leaf size=82

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{13}bdx^{13}(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{17}b^2d^2x^{17}$$

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a
^2*d^2)*x^9)/9 + (2*b*d*(b*c + a*d)*x^13)/13 + (b^2*d^2*x^17)/17
```

---

**Rubi [A]** time = 0.118699, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{13}bdx^{13}(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{17}b^2d^2x^{17}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^2*(c + d*x^4)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a
^2*d^2)*x^9)/9 + (2*b*d*(b*c + a*d)*x^13)/13 + (b^2*d^2*x^17)/17
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2acx^5(ad+bc)}{5}+\frac{b^2d^2x^{17}}{17}+\frac{2bdx^{13}(ad+bc)}{13}+c^2\int a^2\,dx+x^9\left(\frac{a^2d^2}{9}+\frac{4abcd}{9}+\frac{b^2c^2}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**2*(d*x**4+c)**2,x)
```

```
[Out] 2*a*c*x**5*(a*d + b*c)/5 + b**2*d**2*x**17/17 + 2*b*d*x**13*(a*d
+ b*c)/13 + c**2*Integral(a**2, x) + x**9*(a**2*d**2/9 + 4*a*b*c*
d/9 + b**2*c**2/9)
```

---

**Mathematica [A]** time = 0.0316722, size = 82, normalized size = 1.

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{13}bdx^{13}(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{17}b^2d^2x^{17}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^4)^2*(c + d*x^4)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a
^2*d^2)*x^9)/9 + (2*b*d*(b*c + a*d)*x^13)/13 + (b^2*d^2*x^17)/17
```

---

**Maple [A]** time = 0.001, size = 87, normalized size = 1.1

$$\frac{b^2d^2x^{17}}{17}+\frac{\left(2\,abd^2+2\,b^2cd\right)x^{13}}{13}+\frac{\left(a^2d^2+4\,cabd+b^2c^2\right)x^9}{9}+\frac{\left(2\,a^2cd+2\,abc^2\right)x^5}{5}+a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^2*(d*x^4+c)^2,x)
```

```
[Out] 1/17*b^2*d^2*x^17+1/13*(2*a*b*d^2+2*b^2*c*d)*x^13+1/9*(a^2*d^2+4*
a*b*c*d+b^2*c^2)*x^9+1/5*(2*a^2*c*d+2*a*b*c^2)*x^5+a^2*c^2*x
```

---

**Maxima [A]** time = 1.37829, size = 111, normalized size = 1.35

$$\frac{1}{17}\,b^2d^2x^{17}+\frac{2}{13}\left(b^2cd+abd^2\right)x^{13}+\frac{1}{9}\left(b^2c^2+4\,abcd+a^2d^2\right)x^9+\frac{2}{5}\left(abc^2+a^2cd\right)x^5+a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(d*x^4 + c)^2,x, algorithm="maxima")
```

```
[Out] 1/17*b^2*d^2*x^17 + 2/13*(b^2*c*d + a*b*d^2)*x^13 + 1/9*(b^2*c^2
+ 4*a*b*c*d + a^2*d^2)*x^9 + 2/5*(a*b*c^2 + a^2*c*d)*x^5 + a^2*c^
2*x
```

---

**Fricas [A]** time = 0.190083, size = 1, normalized size = 0.01

$$\frac{1}{17}x^{17}d^2b^2+\frac{2}{13}x^{13}dcb^2+\frac{2}{13}x^{13}d^2ba+\frac{1}{9}x^9c^2b^2+\frac{4}{9}x^9dcba+\frac{1}{9}x^9d^2a^2+\frac{2}{5}x^5c^2ba+\frac{2}{5}x^5dca^2+xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(d*x^4 + c)^2,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*d^2*b^2 + 2/13*x^13*d*c*b^2 + 2/13*x^13*d^2*b*a + 1/9*x
^9*c^2*b^2 + 4/9*x^9*d*c*b*a + 1/9*x^9*d^2*a^2 + 2/5*x^5*c^2*b*a
+ 2/5*x^5*d*c*a^2 + x*c^2*a^2
```

**Sympy [A]** time = 0.138062, size = 97, normalized size = 1.18

$$a^2c^2x + \frac{b^2d^2x^{17}}{17} + x^{13}\left(\frac{2abd^2}{13} + \frac{2b^2cd}{13}\right) + x^9\left(\frac{a^2d^2}{9} + \frac{4abcd}{9} + \frac{b^2c^2}{9}\right) + x^5\left(\frac{2a^2cd}{5} + \frac{2abc^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**2*(d*x**4+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**17/17 + x**13*(2*a*b*d**2/13 + 2*b**2*
c*d/13) + x**9*(a**2*d**2/9 + 4*a*b*c*d/9 + b**2*c**2/9) + x**5*(
2*a**2*c*d/5 + 2*a*b*c**2/5)
```

**GIAC/XCAS [A]** time = 0.209447, size = 123, normalized size = 1.5

$$\frac{1}{17}\,b^2d^2x^{17} + \frac{2}{13}\,b^2cdx^{13} + \frac{2}{13}\,abd^2x^{13} + \frac{1}{9}\,b^2c^2x^9 + \frac{4}{9}\,abcdx^9 + \frac{1}{9}\,a^2d^2x^9 + \frac{2}{5}\,abc^2x^5 + \frac{2}{5}\,a^2cdx^5 + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(d*x^4 + c)^2,x, algorithm="giac")
```

```
[Out] 1/17*b^2*d^2*x^17 + 2/13*b^2*c*d*x^13 + 2/13*a*b*d^2*x^13 + 1/9*b
^2*c^2*x^9 + 4/9*a*b*c*d*x^9 + 1/9*a^2*d^2*x^9 + 2/5*a*b*c^2*x^5
+ 2/5*a^2*c*d*x^5 + a^2*c^2*x
```

## 3.57 $\int \left(a+bx^4\right)^2 \left(c+dx^4\right) \, dx$

**Optimal**. Leaf size=50

$$a^2cx+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{13}b^2dx^{13}$$

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^
2*d*x^13)/13
```

---

**Rubi [A]** time = 0.0734854, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$a^2cx+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{13}b^2dx^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^2*(c + d*x^4),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^
2*d*x^13)/13
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$a^2\int c\,dx+\frac{ax^5\left(ad+2bc\right)}{5}+\frac{b^2dx^{13}}{13}+\frac{bx^9\left(2ad+bc\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**2*(d*x**4+c),x)
```

```
[Out] a**2*Integral(c, x) + a*x**5*(a*d + 2*b*c)/5 + b**2*d*x**13/13 +
b*x**9*(2*a*d + b*c)/9
```

---

**Mathematica [A]** time = 0.0136399, size = 50, normalized size = 1.

$$a^2cx+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{13}b^2dx^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^4)^2*(c + d*x^4),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^
2*d*x^13)/13
```

---

**Maple [A]** time = 0.002, size = 49, normalized size = 1.

$$\frac{b^2dx^{13}}{13} + \frac{\left(2\,abd + b^2c\right)x^9}{9} + \frac{\left(a^2d + 2\,abc\right)x^5}{5} + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^2*(d*x^4+c),x)
```

```
[Out] 1/13*b^2*d*x^13+1/9*(2*a*b*d+b^2*c)*x^9+1/5*(a^2*d+2*a*b*c)*x^5+a
^2*c*x
```

---

**Maxima [A]** time = 1.39505, size = 65, normalized size = 1.3

$$\frac{1}{13}\,b^2dx^{13} + \frac{1}{9}\left(b^2c + 2\,abd\right)x^9 + \frac{1}{5}\left(2\,abc + a^2d\right)x^5 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(d*x^4 + c),x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d*x^13 + 1/9*(b^2*c + 2*a*b*d)*x^9 + 1/5*(2*a*b*c + a^2*
d)*x^5 + a^2*c*x
```

---

**Fricas [A]** time = 0.192196, size = 1, normalized size = 0.02

$$\frac{1}{13}x^{13}db^2 + \frac{1}{9}x^9cb^2 + \frac{2}{9}x^9dba + \frac{2}{5}x^5cba + \frac{1}{5}x^5da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(d*x^4 + c),x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d*b^2 + 1/9*x^9*c*b^2 + 2/9*x^9*d*b*a + 2/5*x^5*c*b*a +
 1/5*x^5*d*a^2 + x*c*a^2
```

---

**Sympy [A]** time = 0.107578, size = 53, normalized size = 1.06

$$a^2cx + \frac{b^2dx^{13}}{13} + x^9\left(\frac{2abd}{9} + \frac{b^2c}{9}\right) + x^5\left(\frac{a^2d}{5} + \frac{2abc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**2*(d*x**4+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**13/13 + x**9*(2*a*b*d/9 + b**2*c/9) + x**5*(
a**2*d/5 + 2*a*b*c/5)
```

---

**GIAC/XCAS [A]** time = 0.214651, size = 68, normalized size = 1.36

$$\frac{1}{13}b^2dx^{13} + \frac{1}{9}b^2cx^9 + \frac{2}{9}abdx^9 + \frac{2}{5}abcx^5 + \frac{1}{5}a^2dx^5 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(d*x^4 + c),x, algorithm="giac")
```

```
[Out] 1/13*b^2*d*x^13 + 1/9*b^2*c*x^9 + 2/9*a*b*d*x^9 + 2/5*a*b*c*x^5 +
 1/5*a^2*d*x^5 + a^2*c*x
```

## 3.58 $\int \frac{(a+bx^4)^2}{c+dx^4} dx$

**Optimal**. Leaf size=253

$$-\frac{(bc-ad)^2 \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2 \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}} -\frac{(bc-ad)^2 \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{3/4}d^{9/4}}-\frac{bx(bc-2ad)}{d^2}+\frac{b^2x^5}{5d}$$

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^5)/(5*d) - ((b*c - a*d)^2*Arc
Tan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*d^(9/4))
 + ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqr
t[2]*c^(3/4)*d^(9/4)) - ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1
/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(9/4)) + ((b*c
 - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])
/(4*Sqrt[2]*c^(3/4)*d^(9/4))
```

---

**Rubi [A]** time = 0.414989, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368

$$-\frac{(bc-ad)^2 \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2 \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}} -\frac{(bc-ad)^2 \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{3/4}d^{9/4}}-\frac{bx(bc-2ad)}{d^2}+\frac{b^2x^5}{5d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^2/(c + d*x^4),x]
```


```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^5)/(5*d) - ((b*c - a*d)^2*Arc
Tan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*d^(9/4))
 + ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqr
t[2]*c^(3/4)*d^(9/4)) - ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1
/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(9/4)) + ((b*c
 - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])
/(4*Sqrt[2]*c^(3/4)*d^(9/4))
```


---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{b^2x^5}{5d}+\frac{(2ad-bc)\int b\,dx}{d^2}-\frac{\sqrt{2}\,(ad-bc)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{8c^{\frac{3}{4}}d^{\frac{9}{4}}}+\frac{\sqrt{2}\,(ad-bc)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{8c^{\frac{3}{4}}d^{\frac{9}{4}}}-\frac{\sqrt{2}\,(ad-bc)^2\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{4c^{\frac{3}{4}}d^{\frac{9}{4}}}+\frac{\sqrt{2}\,(ad-bc)^2\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{4c^{\frac{3}{4}}d^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**2/(d*x**4+c),x)
```

```
[Out] b**2*x**5/(5*d) + (2*a*d - b*c)*Integral(b, x)/d**2 - sqrt(2)*(a*
d - b*c)**2*log(-sqrt(2)*c**(1/4)*d**(1/4)*x + sqrt(c) + sqrt(d)*
x**2)/(8*c**(3/4)*d**(9/4)) + sqrt(2)*(a*d - b*c)**2*log(sqrt(2)*
c**(1/4)*d**(1/4)*x + sqrt(c) + sqrt(d)*x**2)/(8*c**(3/4)*d**(9/4
)) - sqrt(2)*(a*d - b*c)**2*atan(1 - sqrt(2)*d**(1/4)*x/c**(1/4))
/(4*c**(3/4)*d**(9/4)) + sqrt(2)*(a*d - b*c)**2*atan(1 + sqrt(2)*
d**(1/4)*x/c**(1/4))/(4*c**(3/4)*d**(9/4))
```

---

**Mathematica [A]** time = 0.192653, size = 231, normalized size = 0.91

$$\frac{-40bc^{3/4}\sqrt[4]{d}x(bc-2ad)-5\sqrt{2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)+5\sqrt{2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)-}{40c^{3/4}d^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^4)^2/(c + d*x^4),x]
```

```
[Out] (-40*b*c^(3/4)*d^(1/4)*(b*c - 2*a*d)*x + 8*b^2*c^(3/4)*d^(5/4)*x^
5 - 10*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/
4)] + 10*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(
1/4)] - 5*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(
1/4)*x + Sqrt[d]*x^2] + 5*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(40*c^(3/4)*d^(9/4))
```

---

**Maple [B]** time = 0.002, size = 436, normalized size = 1.7

$$\frac{b^2x^5}{5d}+2\frac{abx}{d}-\frac{b^2xc}{d^2}+\frac{\sqrt{2}a^2}{4c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)-\frac{\sqrt{2}ab}{2d}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{c\sqrt{2}b^2}{4d^2}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{\sqrt{2}a^2}{4c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)-\frac{\sqrt{2}ab}{2d}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)+\frac{c\sqrt{2}b^2}{4d^2}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)+\frac{\sqrt{2}a^2}{8c}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)-\frac{\sqrt{2}ab}{4d}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)+\frac{c\sqrt{2}b^2}{8d^2}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((b*x^4+a)^2/(d*x^4+c),x)
```

```
[Out] 1/5*b^2*x^5/d+2*b/d*a*x-b^2/d^2*x*c+1/4*(c/d)^(1/4)/c*2^(1/2)*arc
tan(2^(1/2)/(c/d)^(1/4)*x+1)*a^2-1/2/d*(c/d)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(c/d)^(1/4)*x+1)*a*b+1/4/d^2*(c/d)^(1/4)*c*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x+1)*b^2+1/4*(c/d)^(1/4)/c*2^(1/2)*arctan(2
^(1/2)/(c/d)^(1/4)*x-1)*a^2-1/2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(c/d)^(1/4)*x-1)*a*b+1/4/d^2*(c/d)^(1/4)*c*2^(1/2)*arctan(2^(
1/2)/(c/d)^(1/4)*x-1)*b^2+1/8*(c/d)^(1/4)/c*2^(1/2)*ln((x^2+(c/d)
^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1
/2)))*a^2-1/4/d*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)
+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*a*b+1/8/d^
2*(c/d)^(1/4)*c*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)
)/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*b^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2/(d*x^4 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.240962, size = 1457, normalized size = 5.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2/(d*x^4 + c),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^2*d*x^5 - 20*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^
6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*
c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9)
)^(1/4)*arctan(c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*
d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^
5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4
)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x + (b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*sqrt((c^2*d^4*sqrt(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c
^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3
*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9)) +
 (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*x^2)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)))) + 5*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^
2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*
b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*
d^9))^(1/4)*log(c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6
*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d
^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/
4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) - 5*d^2*(-(b^8*c^8 - 8*a*
b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*
c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7
 + a^8*d^8)/(c^3*d^9))^(1/4)*log(-c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*
d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4
- 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d
^8)/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) - 20*(b
^2*c - 2*a*b*d)*x)/d^2
```

---

**Sympy [A]** time = 3.58949, size = 187, normalized size = 0.74

$$\frac{b^2x^5}{5d}$$
$$+\mathrm{RootSum}\left(256t^4c^3d^9 + a^8d^8 - 8a^7bcd^7 + 28a^6b^2c^2d^6 - 56a^5b^3c^3d^5 + 70a^4b^4c^4d^4 - 56a^3b^5c^5d^3 + 28a^2b^6c^6d^2 - 8ab^7c^7d + b\right.$$
$$+\frac{x\left(2abd - b^2c\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**2/(d*x**4+c),x)
```

```
[Out] b**2*x**5/(5*d) + RootSum(256*_t**4*c**3*d**9 + a**8*d**8 - 8*a**
7*b*c*d**7 + 28*a**6*b**2*c**2*d**6 - 56*a**5*b**3*c**3*d**5 + 70
*a**4*b**4*c**4*d**4 - 56*a**3*b**5*c**5*d**3 + 28*a**2*b**6*c**6
*d**2 - 8*a*b**7*c**7*d + b**8*c**8, Lambda(_t, _t*log(4*_t*c*d**
2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x))) + x*(2*a*b*d - b**2*
c)/d**2
```

---

**GIAC/XCAS [A]** time = 0.220611, size = 477, normalized size = 1.89

$$\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}} b^2c^2 - 2\left(cd^3\right)^{\frac{1}{4}} abcd + \left(cd^3\right)^{\frac{1}{4}} a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\,cd^3} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}} b^2c^2 - 2\left(cd^3\right)^{\frac{1}{4}} abcd + \left(cd^3\right)^{\frac{1}{4}} a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\,cd^3} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}} b^2c^2 - 2\left(cd^3\right)^{\frac{1}{4}} abcd + \left(cd^3\right)^{\frac{1}{4}} a^2d^2\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{8\,cd^3} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}} b^2c^2 - 2\left(cd^3\right)^{\frac{1}{4}} abcd + \left(cd^3\right)^{\frac{1}{4}} a^2d^2\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{8\,cd^3} + \frac{b^2d^4x^5 - 5\,b^2cd^3x + 10\,abd^4x}{5\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2/(d*x^4 + c),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c
*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4
))/(c/d)^(1/4))/(c*d^3) + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*
(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c*d^3) + 1/8*sqrt(2)*(
(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a
^2*d^2)*ln(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^3) - 1/8
*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^
3)^(1/4)*a^2*d^2)*ln(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*
d^3) + 1/5*(b^2*d^4*x^5 - 5*b^2*c*d^3*x + 10*a*b*d^4*x)/d^5
```

## 3.59 $\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx$

**Optimal**. Leaf size=291

$$\frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}} - \frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}} + \frac{(bc-ad)(3ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}} - \frac{(bc-ad)(3ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}d^{9/4}} + \frac{x(bc-ad)^2}{4cd^2(c+dx^4)} + \frac{b^2x}{d^2}$$

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(4*c*d^2*(c + d*x^4)) + ((b*c - a
*d)*(5*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*S
qrt[2]*c^(7/4)*d^(9/4)) - ((b*c - a*d)*(5*b*c + 3*a*d)*ArcTan[1 +
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(9/4)) + ((b*
c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(9/4)) - ((b*c - a*d)*(5*b*
c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]
)/(16*Sqrt[2]*c^(7/4)*d^(9/4))
```

---

**Rubi [A]** time = 0.722594, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$\frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}} - \frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}} + \frac{(bc-ad)(3ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}} - \frac{(bc-ad)(3ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}d^{9/4}} + \frac{x(bc-ad)^2}{4cd^2(c+dx^4)} + \frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^2/(c + d*x^4)^2,x]
```


```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(4*c*d^2*(c + d*x^4)) + ((b*c - a
*d)*(5*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*S
qrt[2]*c^(7/4)*d^(9/4)) - ((b*c - a*d)*(5*b*c + 3*a*d)*ArcTan[1 +
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(9/4)) + ((b*
c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(9/4)) - ((b*c - a*d)*(5*b*
```

```
c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]
)/(16*Sqrt[2]*c^(7/4)*d^(9/4))
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int b^2\,dx}{d^2}+\frac{x\,(ad-bc)^2}{4cd^2\,(c+dx^4)}-\frac{\sqrt{2}\,(ad-bc)\,(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{32c^{\frac{7}{4}}d^{\frac{9}{4}}}+\frac{\sqrt{2}\,(ad-bc)\,(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{32c^{\frac{7}{4}}d^{\frac{9}{4}}}-\frac{\sqrt{2}\,(ad-bc)\,(3ad+5bc)\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{16c^{\frac{7}{4}}d^{\frac{9}{4}}}+\frac{\sqrt{2}\,(ad-bc)\,(3ad+5bc)\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{16c^{\frac{7}{4}}d^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**2/(d*x**4+c)**2,x)
```

```
[Out] Integral(b**2, x)/d**2 + x*(a*d - b*c)**2/(4*c*d**2*(c + d*x**4))
 - sqrt(2)*(a*d - b*c)*(3*a*d + 5*b*c)*log(-sqrt(2)*c**(1/4)*d**(
1/4)*x + sqrt(c) + sqrt(d)*x**2)/(32*c**(7/4)*d**(9/4)) + sqrt(2)
*(a*d - b*c)*(3*a*d + 5*b*c)*log(sqrt(2)*c**(1/4)*d**(1/4)*x + sq
rt(c) + sqrt(d)*x**2)/(32*c**(7/4)*d**(9/4)) - sqrt(2)*(a*d - b*c
)*(3*a*d + 5*b*c)*atan(1 - sqrt(2)*d**(1/4)*x/c**(1/4))/(16*c**(7
/4)*d**(9/4)) + sqrt(2)*(a*d - b*c)*(3*a*d + 5*b*c)*atan(1 + sqrt
(2)*d**(1/4)*x/c**(1/4))/(16*c**(7/4)*d**(9/4))
```

---

**Mathematica [A]** time = 0.282857, size = 298, normalized size = 1.02

$$\frac{\frac{\sqrt{2}(-3a^2d^2-2abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{c^{7/4}}-\frac{\sqrt{2}(-3a^2d^2-2abcd+5b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{c^{7/4}}+\frac{2\sqrt{2}(-3a^2d^2-2abcd+5b^2c^2)\tan^{-}}{c^{7/4}}}{32d^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^4)^2/(c + d*x^4)^2,x]
```

```
[Out] (32*b^2*d^(1/4)*x + (8*d^(1/4)*(b*c - a*d)^2*x)/(c*(c + d*x^4)) +
 (2*Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[
2]*d^(1/4)*x)/c^(1/4)])/c^(7/4) - (2*Sqrt[2]*(5*b^2*c^2 - 2*a*b*c
*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(7/4)
+ (Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt
[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4) - (Sqrt[2]*(5*b^2*c
^2 - 2*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)
```

```
*x + Sqrt[d]*x^2])/c^(7/4))/(32*d^(9/4))
```

---

**Maple [B]** time = 0.002, size = 475, normalized size = 1.6

$$\frac{b^2x}{d^2}+\frac{xa^2}{4\,c\,(dx^4+c)}-\frac{axb}{2\,d\,(dx^4+c)}+\frac{cxb^2}{4\,d^2\,(dx^4+c)}+\frac{3\,\sqrt{2}a^2}{16\,c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)+\frac{\sqrt{2}ab}{8\,cd}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)-\frac{5\,\sqrt{2}b^2}{16\,d^2}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)+\frac{3\,\sqrt{2}a^2}{32\,c^2}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)+\frac{\sqrt{2}ab}{16\,cd}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)-\frac{5\,\sqrt{2}b^2}{32\,d^2}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)+\frac{3\,\sqrt{2}a^2}{16\,c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{\sqrt{2}ab}{8\,cd}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)-\frac{5\,\sqrt{2}b^2}{16\,d^2}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^2/(d*x^4+c)^2,x)
```

```
[Out] b^2*x/d^2+1/4/c*x/(d*x^4+c)*a^2-1/2/d*x/(d*x^4+c)*a*b+1/4/d^2*c*x
/(d*x^4+c)*b^2+3/16/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^
(1/4)*x-1)*a^2+1/8/d/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(
1/4)*x-1)*a*b-5/16/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(
1/4)*x-1)*b^2+3/32/c^2*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*
2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*a^2
+1/16/d/c*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)
^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*a*b-5/32/d^2*(c/
d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-
(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*b^2+3/16/c^2*(c/d)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a^2+1/8/d/c*(c/d)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a*b-5/16/d^2*(c/d)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2/(d*x^4 + c)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.256329, size = 1592, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2/(d*x^4 + c)^2,x, algorithm="fricas")

[Out] 1/16*(16*b^2*c*d*x^5 - 4*(c*d^3*x^4 + c^2*d^2)*(-(625*b^8*c^8 - 1
000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 64
6*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 2
16*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*arctan(-c^2*d^2*(-(
625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b
^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*
b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)/((5*
b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*x + (5*b^2*c^2 - 2*a*b*c*d - 3*a
^2*d^2)*sqrt((c^4*d^4*sqrt(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900
*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 9
84*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a
^8*d^8)/(c^7*d^9)) + (25*b^4*c^4 - 20*a*b^3*c^3*d - 26*a^2*b^2*c^
2*d^2 + 12*a^3*b*c*d^3 + 9*a^4*d^4)*x^2)/(25*b^4*c^4 - 20*a*b^3*c
^3*d - 26*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + 9*a^4*d^4)))) + (c*d
^3*x^4 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6
*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b
^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/
(c^7*d^9))^(1/4)*log(c^2*d^2*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d -
900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4
- 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 8
1*a^8*d^8)/(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)
*x) - (c*d^3*x^4 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 9
00*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 -
 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81
*a^8*d^8)/(c^7*d^9))^(1/4)*log(-c^2*d^2*(-(625*b^8*c^8 - 1000*a*b
^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b
^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*
b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*a*b*c*d -
 3*a^2*d^2)*x) + 4*(5*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/(c*d^3*x^
4 + c^2*d^2)
```

**Sympy [A]** time = 6.90248, size = 219, normalized size = 0.75

$$\frac{b^2x}{d^2}+\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{4c^2d^2+4cd^3x^4}$$
$$+\mathrm{RootSum}\left(65536t^4c^7d^9+81a^8d^8+216a^7bcd^7-324a^6b^2c^2d^6-984a^5b^3c^3d^5+646a^4b^4c^4d^4+1640a^3b^5c^5d^3-900a^2b^6c^6d^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**2/(d*x**4+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(4*c**2*d**2
+ 4*c*d**3*x**4) + RootSum(65536*_t**4*c**7*d**9 + 81*a**8*d**8 +
 216*a**7*b*c*d**7 - 324*a**6*b**2*c**2*d**6 - 984*a**5*b**3*c**3
*d**5 + 646*a**4*b**4*c**4*d**4 + 1640*a**3*b**5*c**5*d**3 - 900*
a**2*b**6*c**6*d**2 - 1000*a*b**7*c**7*d + 625*b**8*c**8, Lambda(
_t, _t*log(16*_t*c**2*d**2/(3*a**2*d**2 + 2*a*b*c*d - 5*b**2*c**2
) + x)))
```

**GIAC/XCAS [A]** time = 0.222355, size = 508, normalized size = 1.75

$$\frac{b^2x}{d^2}-\frac{\sqrt{2}\left(5\left(cd^3\right)^{\frac{1}{4}}b^2c^2-2\left(cd^3\right)^{\frac{1}{4}}abcd-3\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16c^2d^3}$$
$$-\frac{\sqrt{2}\left(5\left(cd^3\right)^{\frac{1}{4}}b^2c^2-2\left(cd^3\right)^{\frac{1}{4}}abcd-3\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16c^2d^3}$$
$$-\frac{\sqrt{2}\left(5\left(cd^3\right)^{\frac{1}{4}}b^2c^2-2\left(cd^3\right)^{\frac{1}{4}}abcd-3\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\ln\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32c^2d^3}$$
$$+\frac{\sqrt{2}\left(5\left(cd^3\right)^{\frac{1}{4}}b^2c^2-2\left(cd^3\right)^{\frac{1}{4}}abcd-3\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\ln\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32c^2d^3}$$
$$+\frac{b^2c^2x-2abcdx+a^2d^2x}{4(dx^4+c)cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2/(d*x^4 + c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 - 1/16*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/
4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + s
```

```
qrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^2*d^3) - 1/16*sqrt(2)*(5*(c*d
^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2
*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))
/(c^2*d^3) - 1/32*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1
/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*ln(x^2 + sqrt(2)*x*(c/d)^(
1/4) + sqrt(c/d))/(c^2*d^3) + 1/32*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c
^2 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*ln(x^2 -
sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^2*d^3) + 1/4*(b^2*c^2*x - 2
*a*b*c*d*x + a^2*d^2*x)/((d*x^4 + c)*c*d^2)
```

## 3.60 $\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx$

**Optimal**. Leaf size=349

$$-\frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{9/4}}+\frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{9/4}}-\frac{(21a^2d^2+6abcd+5b^2c^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{9/4}}+\frac{(21a^2d^2+6abcd+5b^2c^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{64\sqrt{2}c^{11/4}d^{9/4}}-\frac{x(bc-ad)(7ad+5bc)}{32c^2d^2(c+dx^4)}-\frac{x(a+bx^4)(bc-ad)}{8cd(c+dx^4)^2}$$

```
[Out] -((b*c - a*d)*x*(a + b*x^4))/(8*c*d*(c + d*x^4)^2) - ((b*c - a*d)
*(5*b*c + 7*a*d)*x)/(32*c^2*d^2*(c + d*x^4)) - ((5*b^2*c^2 + 6*a*
b*c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(64*
Sqrt[2]*c^(11/4)*d^(9/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)
*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(64*Sqrt[2]*c^(11/4)*d^
(9/4)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] - Sqrt
[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*d^(9/
4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]
*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*d^(9/4))
```

---

**Rubi [A]** time = 0.549538, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$-\frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{9/4}}+\frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{9/4}}-\frac{(21a^2d^2+6abcd+5b^2c^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{9/4}}+\frac{(21a^2d^2+6abcd+5b^2c^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{64\sqrt{2}c^{11/4}d^{9/4}}-\frac{x(bc-ad)(7ad+5bc)}{32c^2d^2(c+dx^4)}-\frac{x(a+bx^4)(bc-ad)}{8cd(c+dx^4)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^2/(c + d*x^4)^3,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^4))/(8*c*d*(c + d*x^4)^2) - ((b*c - a*d)
*(5*b*c + 7*a*d)*x)/(32*c^2*d^2*(c + d*x^4)) - ((5*b^2*c^2 + 6*a*
b*c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(64*
Sqrt[2]*c^(11/4)*d^(9/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)
*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(64*Sqrt[2]*c^(11/4)*d^
(9/4)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] - Sqrt
[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*d^(9/
4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]
*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*d^(9/4))
```

---

**Rubi in Sympy [A]** time = 82.4633, size = 337, normalized size = 0.97

$$\frac{x\left(a+bx^4\right)(ad-bc)}{8cd\left(c+dx^4\right)^2}+\frac{x(ad-bc)(7ad+5bc)}{32c^2d^2\left(c+dx^4\right)} - \frac{\sqrt{2}\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{256c^{\frac{11}{4}}d^{\frac{9}{4}}} + \frac{\sqrt{2}\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{256c^{\frac{11}{4}}d^{\frac{9}{4}}} - \frac{\sqrt{2}\left(21a^2d^2+6abcd+5b^2c^2\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{128c^{\frac{11}{4}}d^{\frac{9}{4}}} + \frac{\sqrt{2}\left(21a^2d^2+6abcd+5b^2c^2\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{128c^{\frac{11}{4}}d^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**2/(d*x**4+c)**3,x)
```

```
[Out] x*(a + b*x**4)*(a*d - b*c)/(8*c*d*(c + d*x**4)**2) + x*(a*d - b*c
)*(7*a*d + 5*b*c)/(32*c**2*d**2*(c + d*x**4)) - sqrt(2)*(21*a**2*
d**2 + 6*a*b*c*d + 5*b**2*c**2)*log(-sqrt(2)*c**(1/4)*d**(1/4)*x
+ sqrt(c) + sqrt(d)*x**2)/(256*c**(11/4)*d**(9/4)) + sqrt(2)*(21*
a**2*d**2 + 6*a*b*c*d + 5*b**2*c**2)*log(sqrt(2)*c**(1/4)*d**(1/4
)*x + sqrt(c) + sqrt(d)*x**2)/(256*c**(11/4)*d**(9/4)) - sqrt(2)*
(21*a**2*d**2 + 6*a*b*c*d + 5*b**2*c**2)*atan(1 - sqrt(2)*d**(1/4
)*x/c**(1/4))/(128*c**(11/4)*d**(9/4)) + sqrt(2)*(21*a**2*d**2 +
6*a*b*c*d + 5*b**2*c**2)*atan(1 + sqrt(2)*d**(1/4)*x/c**(1/4))/(1
28*c**(11/4)*d**(9/4))
```

---

**Mathematica [A]** time = 0.320891, size = 319, normalized size = 0.91

$$-\sqrt{2}\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)+\sqrt{2}\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^4)^2/(c + d*x^4)^3,x]
```

```
[Out] ((32*c^(7/4)*d^(1/4)*(b*c - a*d)^2*x)/(c + d*x^4)^2 - (8*c^(3/4)*
d^(1/4)*(9*b^2*c^2 - 2*a*b*c*d - 7*a^2*d^2)*x)/(c + d*x^4) - 2*Sq
rt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^
(1/4)*x)/c^(1/4)] + 2*Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2
)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - Sqrt[2]*(5*b^2*c^2 +
6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x +
 Sqrt[d]*x^2] + Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(256*c^(11/4)
*d^(9/4))
```

---

**Maple [A]** time = 0.002, size = 499, normalized size = 1.4

$$\frac{1}{(dx^4+c)^2}\left(\frac{\left(7a^2d^2+2cabd-9b^2c^2\right)x^5}{32c^2d}+\frac{\left(11a^2d^2-6cabd-5b^2c^2\right)x}{32d^2c}\right)$$
$$+\frac{21\sqrt{2}a^2}{128c^3}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$
$$+\frac{3\sqrt{2}ab}{64c^2d}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)+\frac{5\sqrt{2}b^2}{128d^2c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$
$$+\frac{21\sqrt{2}a^2}{256c^3}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$+\frac{3\sqrt{2}ab}{128c^2d}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$+\frac{5\sqrt{2}b^2}{256d^2c}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$+\frac{21\sqrt{2}a^2}{128c^3}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)$$
$$+\frac{3\sqrt{2}ab}{64c^2d}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{5\sqrt{2}b^2}{128d^2c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^2/(d*x^4+c)^3,x)
```

```
[Out] (1/32*(7*a^2*d^2+2*a*b*c*d-9*b^2*c^2)/c^2/d*x^5+1/32*(11*a^2*d^2-
6*a*b*c*d-5*b^2*c^2)/d^2/c*x)/(d*x^4+c)^2+21/128/c^3*(c/d)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a^2+3/64/c^2/d*(c/d)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a*b+5/128/c/d^2*(c/d)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*b^2+21/256/c^3*(c/d)
^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c
/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*a^2+3/128/c^2/d*(c/d)^(1/4)*2^(
1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*
x*2^(1/2)+(c/d)^(1/2)))*a*b+5/256/c/d^2*(c/d)^(1/4)*2^(1/2)*ln((x
^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+
(c/d)^(1/2)))*b^2+21/128/c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
c/d)^(1/4)*x+1)*a^2+3/64/c^2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(c/d)^(1/4)*x+1)*a*b+5/128/c/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(c/d)^(1/4)*x+1)*b^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2/(d*x^4 + c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.258428, size = 1692, normalized size = 4.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2/(d*x^4 + c)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(4*(9*b^2*c^2*d - 2*a*b*c*d^2 - 7*a^2*d^3)*x^5 + 4*(c^2*d^
4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*
d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^
4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222
264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)*arctan(c^3*d^
2*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 421
20*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*
d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^
8)/(c^11*d^9))^(1/4)/((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*x + (5
*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*sqrt((c^6*d^4*sqrt(-(625*b^8*c
^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5
```

```
*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a
^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))
 + (25*b^4*c^4 + 60*a*b^3*c^3*d + 246*a^2*b^2*c^2*d^2 + 252*a^3*b
*c*d^3 + 441*a^4*d^4)*x^2)/(25*b^4*c^4 + 60*a*b^3*c^3*d + 246*a^2
*b^2*c^2*d^2 + 252*a^3*b*c*d^3 + 441*a^4*d^4)))) - (c^2*d^4*x^8 +
 2*c^3*d^3*x^4 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 159
00*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d
^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7
*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)*log(c^3*d^2*(-(625*b
^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5
*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 2804
76*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d
^9))^(1/4) + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*x) + (c^2*d^4*x
^8 + 2*c^3*d^3*x^4 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d +
 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c
^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264
*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)*log(-c^3*d^2*(-(
625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^
3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 +
 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c
^11*d^9))^(1/4) + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*x) + 4*(5*
b^2*c^3 + 6*a*b*c^2*d - 11*a^2*c*d^2)*x)/(c^2*d^4*x^8 + 2*c^3*d^3
*x^4 + c^4*d^2)
```

---

**Sympy [A]** time = 20.5634, size = 264, normalized size = 0.76

$$\frac{x^5\left(7a^2d^3+2abcd^2-9b^2c^2d\right)+x\left(11a^2cd^2-6abc^2d-5b^2c^3\right)}{32c^4d^2+64c^3d^3x^4+32c^2d^4x^8}$$
$$+\text{RootSum}\Big(268435456t^4c^{11}d^9+194481a^8d^8+222264a^7bcd^7+280476a^6b^2c^2d^6+176904a^5b^3c^3d^5+112806a^4b^4c^4d^4+4212$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**2/(d*x**4+c)**3,x)
```

```
[Out] (x**5*(7*a**2*d**3 + 2*a*b*c*d**2 - 9*b**2*c**2*d) + x*(11*a**2*c
*d**2 - 6*a*b*c**2*d - 5*b**2*c**3))/(32*c**4*d**2 + 64*c**3*d**3
*x**4 + 32*c**2*d**4*x**8) + RootSum(268435456*_t**4*c**11*d**9 +
 194481*a**8*d**8 + 222264*a**7*b*c*d**7 + 280476*a**6*b**2*c**2*
d**6 + 176904*a**5*b**3*c**3*d**5 + 112806*a**4*b**4*c**4*d**4 +
42120*a**3*b**5*c**5*d**3 + 15900*a**2*b**6*c**6*d**2 + 3000*a*b*
*7*c**7*d + 625*b**8*c**8, Lambda(_t, _t*log(128*_t*c**3*d**2/(21
*a**2*d**2 + 6*a*b*c*d + 5*b**2*c**2) + x)))
```

---

**GIAC/XCAS [A]** time = 0.223269, size = 549, normalized size = 1.57

$$\frac{\sqrt{2}\left(5\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\,\left(cd^3\right)^{\frac{1}{4}}abcd+21\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^3d^3}+\frac{\sqrt{2}\left(5\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\,\left(cd^3\right)^{\frac{1}{4}}abcd+21\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^3d^3}+\frac{\sqrt{2}\left(5\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\,\left(cd^3\right)^{\frac{1}{4}}abcd+21\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\ln\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{256\,c^3d^3}-\frac{\sqrt{2}\left(5\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\,\left(cd^3\right)^{\frac{1}{4}}abcd+21\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\ln\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{256\,c^3d^3}-\frac{9\,b^2c^2dx^5-2\,abcd^2x^5-7\,a^2d^3x^5+5\,b^2c^3x+6\,abc^2dx-11\,a^2cd^2x}{32\,(dx^4+c)^2c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2/(d*x^4 + c)^3,x, algorithm="giac")
```

```
[Out] 1/128*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d
+ 21*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/
d)^(1/4))/(c/d)^(1/4))/(c^3*d^3) + 1/128*sqrt(2)*(5*(c*d^3)^(1/4)
*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^3*d^
3) + 1/256*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b
*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*ln(x^2 + sqrt(2)*x*(c/d)^(1/4) +
 sqrt(c/d))/(c^3*d^3) - 1/256*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 +
6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*ln(x^2 - sqrt
(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^3*d^3) - 1/32*(9*b^2*c^2*d*x^5
- 2*a*b*c*d^2*x^5 - 7*a^2*d^3*x^5 + 5*b^2*c^3*x + 6*a*b*c^2*d*x -
 11*a^2*c*d^2*x)/((d*x^4 + c)^2*c^2*d^2)
```

## 3.61 $\int \frac{(c+dx^4)^4}{a+bx^4}\,dx$

**Optimal**. Leaf size=332

$$-\frac{(bc-ad)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{17/4}}+\frac{(bc-ad)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{17/4}}-\frac{(bc-ad)^4\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{17/4}}+\frac{(bc-ad)^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{17/4}}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}+\frac{d^2x^5\left(a^2d^2-4abcd+6b^2c^2\right)}{5b^3}+\frac{d^3x^9(4bc-ad)}{9b^2}+\frac{d^4x^{13}}{13b}$$

```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*
(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^3*(4*b*c - a*
d)*x^9)/(9*b^2) + (d^4*x^13)/(13*b) - ((b*c - a*d)^4*ArcTan[1 - (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c
 - a*d)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*b^(17/4)) - ((b*c - a*d)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c - a*d
)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sq
rt[2]*a^(3/4)*b^(17/4))
```

---

**Rubi [A]** time = 0.581279, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$-\frac{(bc-ad)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{17/4}}+\frac{(bc-ad)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{17/4}}-\frac{(bc-ad)^4\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{17/4}}+\frac{(bc-ad)^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{17/4}}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}+\frac{d^2x^5\left(a^2d^2-4abcd+6b^2c^2\right)}{5b^3}+\frac{d^3x^9(4bc-ad)}{9b^2}+\frac{d^4x^{13}}{13b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^4)^4/(a + b*x^4),x]
```



```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*
(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^3*(4*b*c - a*
d)*x^9)/(9*b^2) + (d^4*x^13)/(13*b) - ((b*c - a*d)^4*ArcTan[1 - (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c
 - a*d)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*b^(17/4)) - ((b*c - a*d)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c - a*d
)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sq
```

```
rt[2]*a^(3/4)*b^(17/4))
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{d^4x^{13}}{13b}-\frac{d^3x^9(ad-4bc)}{9b^2}+\frac{d^2x^5\left(a^2d^2-4abcd+6b^2c^2\right)}{5b^3}-\frac{(ad-2bc)\left(a^2d^2-2abcd+2b^2c^2\right)\int d\,dx}{b^4}-\frac{\sqrt{2}(ad-bc)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{8a^{\frac{3}{4}}b^{\frac{17}{4}}}+\frac{\sqrt{2}(ad-bc)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{8a^{\frac{3}{4}}b^{\frac{17}{4}}}-\frac{\sqrt{2}(ad-bc)^4\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{17}{4}}}+\frac{\sqrt{2}(ad-bc)^4\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{17}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**4+c)**4/(b*x**4+a),x)
```

```
[Out] d**4*x**13/(13*b) - d**3*x**9*(a*d - 4*b*c)/(9*b**2) + d**2*x**5*
(a**2*d**2 - 4*a*b*c*d + 6*b**2*c**2)/(5*b**3) - (a*d - 2*b*c)*(a
**2*d**2 - 2*a*b*c*d + 2*b**2*c**2)*Integral(d, x)/b**4 - sqrt(2)
*(a*d - b*c)**4*log(-sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt
(b)*x**2)/(8*a**(3/4)*b**(17/4)) + sqrt(2)*(a*d - b*c)**4*log(sqr
t(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(8*a**(3/4)*b*
*(17/4)) - sqrt(2)*(a*d - b*c)**4*atan(1 - sqrt(2)*b**(1/4)*x/a**
(1/4))/(4*a**(3/4)*b**(17/4)) + sqrt(2)*(a*d - b*c)**4*atan(1 + s
qrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(3/4)*b**(17/4))
```

---

**Mathematica [A]** time = 0.329157, size = 322, normalized size = 0.97

$$\frac{-\frac{585\sqrt{2}(bc-ad)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{3/4}}+\frac{585\sqrt{2}(bc-ad)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{3/4}}-\frac{1170\sqrt{2}(bc-ad)^4\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{1170\sqrt{2}(bc-ad}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^4)^4/(a + b*x^4),x]
```

```
[Out] (4680*b^(1/4)*d*(4*b^3*c^3 - 6*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*
d^3)*x + 936*b^(5/4)*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^5 +
520*b^(9/4)*d^3*(4*b*c - a*d)*x^9 + 360*b^(13/4)*d^4*x^13 - (1170
*Sqrt[2]*(b*c - a*d)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a
^(3/4) + (1170*Sqrt[2]*(b*c - a*d)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/a^(3/4) - (585*Sqrt[2]*(b*c - a*d)^4*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4) + (585*Sqrt[2]*(
b*c - a*d)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
```

```
2])/a^(3/4))/(4680*b^(17/4))
```

**Maple [B]** time = 0.008, size = 837, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^4+c)^4/(b*x^4+a),x)
```

```
[Out] 1/13*d^4*x^13/b+1/4*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x+1)*c^4-4/5*d^3/b^2*x^5*a*c+1/4*(a/b)^(1/4)/a*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x-1)*c^4-1/9*d^4/b^2*x^9*a+4/9*d^3/b*x^9*c+1
/5*d^4/b^3*x^5*a^2+6/5*d^2/b*x^5*c^2+4*d^3/b^3*a^2*c*x+3/2/b^2*(a
/b)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c^2*d^2-1/2/b
^3*(a/b)^(1/4)*a^2*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1
/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*c*d^3-6*d^2/b^2*a*c
^2*x+1/8*(a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b
)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*c^4-d^4/b^4*a^3
*x+4*d/b*c^3*x+3/4/b^2*(a/b)^(1/4)*a*2^(1/2)*ln((x^2+(a/b)^(1/4)*
x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*c
^2*d^2-1/b^3*(a/b)^(1/4)*a^2*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x
+1)*c*d^3+3/2/b^2*(a/b)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*x+1)*c^2*d^2-1/b^3*(a/b)^(1/4)*a^2*2^(1/2)*arctan(2^(1/2)/(a/b)
^(1/4)*x-1)*c*d^3+1/8/b^4*(a/b)^(1/4)*a^3*2^(1/2)*ln((x^2+(a/b)^(
1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2
)))*d^4-1/2/b*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(
a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*c^3*d-1/b*(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^3*d+1/4/b^4*(
a/b)^(1/4)*a^3*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d^4-1/b*(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c^3*d+1/4/b^4*(
a/b)^(1/4)*a^3*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*d^4
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^4/(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 0.258891, size = 2939, normalized size = 8.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^4/(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] 1/2340*(180*b^3*d^4*x^13 + 260*(4*b^3*c*d^3 - a*b^2*d^4)*x^9 + 46
8*(6*b^3*c^2*d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^5 - 2340*b^4*(-(b
^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^1
3*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 80
08*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*
d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*
b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 1
20*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^
(1/4)*arctan(a*b^4*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14
*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368
*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d
^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^
6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 56
0*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 +
a^16*d^16)/(a^3*b^17))^(1/4)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^
2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x + (b^4*c^4 - 4*a*b^3*c^3*d
 + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt((a^2*b^8*sqr
t(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^
3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5
 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8
*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*
a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^1
3 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^
17)) + (b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5
*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c
^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)*x^2)/(b^8*c^8 - 8*a*b^7*c^7*d +
 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 5
6*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
))) + 585*b^4*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14
*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*
b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 +
12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6
*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^1
3*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*
d^16)/(a^3*b^17))^(1/4)*log(a*b^4*(-(b^16*c^16 - 16*a*b^15*c^15*d
 + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*
c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 1144
0*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9
 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^
4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a
^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(1/4) + (b^4*c^4 - 4*a*b^3*
c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x) - 585*b^4
*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^
3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5
 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8
*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*
a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^1
3 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^
17))^(1/4)*log(-a*b^4*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b
^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4
368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^
```

```
9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10
*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 -
 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15
 + a^16*d^16)/(a^3*b^17))^(1/4) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x) + 2340*(4*b^3*c^3*d -
 6*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3 - a^3*d^4)*x)/b^4
```

---

**Sympy [A]** time = 9.62074, size = 430, normalized size = 1.3

$$\mathrm{RootSum}\left(256t^4a^3b^{17} + a^{16}d^{16} - 16a^{15}bcd^{15} + 120a^{14}b^2c^2d^{14} - 560a^{13}b^3c^3d^{13} + 1820a^{12}b^4c^4d^{12} - 4368a^{11}b^5c^5d^{11} + 8008a^1\right.$$
$$+ \frac{d^4x^{13}}{13b} - \frac{x^9\left(ad^4 - 4bcd^3\right)}{9b^2} + \frac{x^5\left(a^2d^4 - 4abcd^3 + 6b^2c^2d^2\right)}{5b^3} - \frac{x\left(a^3d^4 - 4a^2bcd^3 + 6ab^2c^2d^2 - 4b^3c^3d\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**4+c)**4/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**17 + a**16*d**16 - 16*a**15*b*c*d**15 +
 120*a**14*b**2*c**2*d**14 - 560*a**13*b**3*c**3*d**13 + 1820*a**
12*b**4*c**4*d**12 - 4368*a**11*b**5*c**5*d**11 + 8008*a**10*b**6
*c**6*d**10 - 11440*a**9*b**7*c**7*d**9 + 12870*a**8*b**8*c**8*d*
*8 - 11440*a**7*b**9*c**9*d**7 + 8008*a**6*b**10*c**10*d**6 - 436
8*a**5*b**11*c**11*d**5 + 1820*a**4*b**12*c**12*d**4 - 560*a**3*b
**13*c**13*d**3 + 120*a**2*b**14*c**14*d**2 - 16*a*b**15*c**15*d
+ b**16*c**16, Lambda(_t, _t*log(4*_t*a*b**4/(a**4*d**4 - 4*a**3*
b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4) +
 x))) + d**4*x**13/(13*b) - x**9*(a*d**4 - 4*b*c*d**3)/(9*b**2) +
 x**5*(a**2*d**4 - 4*a*b*c*d**3 + 6*b**2*c**2*d**2)/(5*b**3) - x*
(a**3*d**4 - 4*a**2*b*c*d**3 + 6*a*b**2*c**2*d**2 - 4*b**3*c**3*d
)/b**4
```

---

**GIAC/XCAS [A]** time = 0.219731, size = 833, normalized size = 2.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^4/(b*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d
+ 6*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 +
 (a*b^3)^(1/4)*a^4*d^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(
1/4))/(a/b)^(1/4))/(a*b^5) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 -
 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 -
4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d^4)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^5) + 1/8*sqr
```

```
t(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*
b^3)^(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3
)^(1/4)*a^4*d^4)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b
^5) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*
c^3*d + 6*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c
*d^3 + (a*b^3)^(1/4)*a^4*d^4)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sq
rt(a/b))/(a*b^5) + 1/585*(45*b^12*d^4*x^13 + 260*b^12*c*d^3*x^9 -
 65*a*b^11*d^4*x^9 + 702*b^12*c^2*d^2*x^5 - 468*a*b^11*c*d^3*x^5
+ 117*a^2*b^10*d^4*x^5 + 2340*b^12*c^3*d*x - 3510*a*b^11*c^2*d^2*
x + 2340*a^2*b^10*c*d^3*x - 585*a^3*b^9*d^4*x)/b^13
```

## 3.62 $\int \frac{(c+dx^4)^3}{a+bx^4}\,dx$

**Optimal**. Leaf size=288

$$-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{13/4}}+\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{d^2x^5(3bc-ad)}{5b^2}+\frac{d^3x^9}{9b}$$

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*
x^5)/(5*b^2) + (d^3*x^9)/(9*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[
2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*
d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
b^(13/4)) - ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*d)^3*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(13/4))
```

---

**Rubi [A]** time = 0.450354, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{13/4}}+\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{d^2x^5(3bc-ad)}{5b^2}+\frac{d^3x^9}{9b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^4)^3/(a + b*x^4),x]
```


```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*
x^5)/(5*b^2) + (d^3*x^9)/(9*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[
2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*
d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
b^(13/4)) - ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*d)^3*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(13/4))
```

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{d^3x^9}{9b} - \frac{d^2x^5\,(ad-3bc)}{5b^2} + \frac{\left(a^2d^2-3abcd+3b^2c^2\right)\int d\,dx}{b^3} + \frac{\sqrt{2}\,(ad-bc)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{8a^{\frac{3}{4}}b^{\frac{13}{4}}} - \frac{\sqrt{2}\,(ad-bc)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{8a^{\frac{3}{4}}b^{\frac{13}{4}}} + \frac{\sqrt{2}\,(ad-bc)^3\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{13}{4}}} - \frac{\sqrt{2}\,(ad-bc)^3\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{13}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**4+c)**3/(b*x**4+a),x)
```

```
[Out] d**3*x**9/(9*b) - d**2*x**5*(a*d - 3*b*c)/(5*b**2) + (a**2*d**2 -
 3*a*b*c*d + 3*b**2*c**2)*Integral(d, x)/b**3 + sqrt(2)*(a*d - b*
c)**3*log(-sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/
(8*a**(3/4)*b**(13/4)) - sqrt(2)*(a*d - b*c)**3*log(sqrt(2)*a**(1
/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(8*a**(3/4)*b**(13/4)) +
 sqrt(2)*(a*d - b*c)**3*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(4*
a**(3/4)*b**(13/4)) - sqrt(2)*(a*d - b*c)**3*atan(1 + sqrt(2)*b**
(1/4)*x/a**(1/4))/(4*a**(3/4)*b**(13/4))
```

**Mathematica [A]** time = 0.234732, size = 271, normalized size = 0.94

$$\frac{-72a^{3/4}b^{5/4}d^2x^5(ad-3bc)+40a^{3/4}b^{9/4}d^3x^9+360a^{3/4}\sqrt[4]{b}dx\left(a^2d^2-3abcd+3b^2c^2\right)-45\sqrt{2}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^4)^3/(a + b*x^4),x]
```

```
[Out] (360*a^(3/4)*b^(1/4)*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x - 72*a
^(3/4)*b^(5/4)*d^2*(-3*b*c + a*d)*x^5 + 40*a^(3/4)*b^(9/4)*d^3*x^
9 - 90*Sqrt[2]*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/
4)] + 90*Sqrt[2]*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(
1/4)] - 45*Sqrt[2]*(b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^
(1/4)*x + Sqrt[b]*x^2] + 45*Sqrt[2]*(b*c - a*d)^3*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(360*a^(3/4)*b^(13/4))
```

**Maple [B]** time = 0.002, size = 627, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^4+c)^3/(b*x^4+a),x)
```

```
[Out] 1/9*d^3*x^9/b-1/5*d^3/b^2*x^5*a+3/5*d^2/b*x^5*c+d^3/b^3*a^2*x-3*d
^2/b^2*a*c*x+3*d/b*c^2*x-1/4/b^3*(a/b)^(1/4)*a^2*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x+1)*d^3+3/4/b^2*(a/b)^(1/4)*a*2^(1/2)*arctan(
2^(1/2)/(a/b)^(1/4)*x+1)*c*d^2-3/4/b*(a/b)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x+1)*c^2*d+1/4*(a/b)^(1/4)/a*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x+1)*c^3-1/4/b^3*(a/b)^(1/4)*a^2*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x-1)*d^3+3/4/b^2*(a/b)^(1/4)*a*2^(1/2)*arcta
n(2^(1/2)/(a/b)^(1/4)*x-1)*c*d^2-3/4/b*(a/b)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x-1)*c^2*d+1/4*(a/b)^(1/4)/a*2^(1/2)*arctan(
2^(1/2)/(a/b)^(1/4)*x-1)*c^3-1/8/b^3*(a/b)^(1/4)*a^2*2^(1/2)*ln((
x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)
+(a/b)^(1/2)))*d^3+3/8/b^2*(a/b)^(1/4)*a*2^(1/2)*ln((x^2+(a/b)^(1
/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)
))*c*d^2-3/8/b*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+
(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*c^2*d+1/8*(
a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(
x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*c^3
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^3/(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.250542, size = 2198, normalized size = 7.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^3/(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] 1/180*(20*b^2*d^3*x^9 + 36*(3*b^2*c*d^2 - a*b*d^3)*x^5 + 180*b^3*
(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*
b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6
*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^
9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^
12)/(a^3*b^13))^(1/4)*arctan(-a*b^3*(-(b^12*c^12 - 12*a*b^11*c^11
```

```
*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8
*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^
5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c
^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)/((b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x + (b^3*c^3 - 3*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt((a^2*b^6*sqrt(-(b^12*c^
12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^
3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d
^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*
d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b
^13)) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^
3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*x^2)/(b
^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
+ 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)))) - 45*b^3*(-(b^
12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c
^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*
c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3
*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(
a^3*b^13))^(1/4)*log(a*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a
^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 79
2*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 4
95*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x) + 45*b^3*(-(b^12*c^12 -
12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 4
95*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))
^(1/4)*log(-a*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7
*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^
4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*
b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*
d + 3*a^2*b*c*d^2 - a^3*d^3)*x) + 180*(3*b^2*c^2*d - 3*a*b*c*d^2
+ a^2*d^3)*x)/b^3
```

---

**Sympy [A]** time = 6.1807, size = 301, normalized size = 1.05

$$\operatorname{RootSum}\left(256t^4a^3b^{13} + a^{12}d^{12} - 12a^{11}bcd^{11} + 66a^{10}b^2c^2d^{10} - 220a^9b^3c^3d^9 + 495a^8b^4c^4d^8 - 792a^7b^5c^5d^7 + 924a^6b^6c^6d^6 - \right.$$

$$+ \frac{d^3x^9}{9b} - \frac{x^5\left(ad^3 - 3bcd^2\right)}{5b^2} + \frac{x\left(a^2d^3 - 3abcd^2 + 3b^2c^2d\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**4+c)**3/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**13 + a**12*d**12 - 12*a**11*b*c*d**11 +
 66*a**10*b**2*c**2*d**10 - 220*a**9*b**3*c**3*d**9 + 495*a**8*b*
*4*c**4*d**8 - 792*a**7*b**5*c**5*d**7 + 924*a**6*b**6*c**6*d**6
- 792*a**5*b**7*c**7*d**5 + 495*a**4*b**8*c**8*d**4 - 220*a**3*b*
```

```
*9*c**9*d**3 + 66*a**2*b**10*c**10*d**2 - 12*a*b**11*c**11*d + b*
*12*c**12, Lambda(_t, _t*log(-4*_t*a*b**3/(a**3*d**3 - 3*a**2*b*c
*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x))) + d**3*x**9/(9*b) - x
**5*(a*d**3 - 3*b*c*d**2)/(5*b**2) + x*(a**2*d**3 - 3*a*b*c*d**2
+ 3*b**2*c**2*d)/b**3
```

---

**GIAC/XCAS [A]** time = 0.218108, size = 649, normalized size = 2.25

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^4}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^4}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^4}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^4}+\frac{5b^8d^3x^9+27b^8cd^2x^5-9ab^7d^3x^5+135b^8c^2dx-135ab^7cd^2x+45a^2b^6d^3x}{45b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^3/(b*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d
+ 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2
*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/4*s
qrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(
a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt
(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/8*sqrt(2
)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3
)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*ln(x^2 + sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(a*b^4) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*
c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 -
 (a*b^3)^(1/4)*a^3*d^3)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b
))/(a*b^4) + 1/45*(5*b^8*d^3*x^9 + 27*b^8*c*d^2*x^5 - 9*a*b^7*d^3
*x^5 + 135*b^8*c^2*d*x - 135*a*b^7*c*d^2*x + 45*a^2*b^6*d^3*x)/b^
9
```

## 3.63 $\int \frac{(c+dx^4)^2}{a+bx^4}\,dx$

**Optimal**. Leaf size=253

$$-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{9/4}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^5}{5b}$$

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^5)/(5*b) - ((b*c - a*d)^2*ArcTan
[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(9/4)) +
((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2
]*a^(3/4)*b^(9/4)) - ((b*c - a*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(9/4)) + ((b*c -
a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4
*Sqrt[2]*a^(3/4)*b^(9/4))
```

---

**Rubi [A]** time = 0.400891, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$


$$-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{9/4}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^5}{5b}$$


Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^4)^2/(a + b*x^4),x]
```



```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^5)/(5*b) - ((b*c - a*d)^2*ArcTan
[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(9/4)) +
((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2
]*a^(3/4)*b^(9/4)) - ((b*c - a*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(9/4)) + ((b*c -
a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4
*Sqrt[2]*a^(3/4)*b^(9/4))
```


---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{d^2x^5}{5b} - \frac{(ad-2bc)\int d\,dx}{b^2} - \frac{\sqrt{2}\,(ad-bc)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{8a^{\frac{3}{4}}b^{\frac{9}{4}}} + \frac{\sqrt{2}\,(ad-bc)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{8a^{\frac{3}{4}}b^{\frac{9}{4}}} - \frac{\sqrt{2}\,(ad-bc)^2\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{9}{4}}} + \frac{\sqrt{2}\,(ad-bc)^2\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**4+c)**2/(b*x**4+a),x)
```

```
[Out] d**2*x**5/(5*b) - (a*d - 2*b*c)*Integral(d, x)/b**2 - sqrt(2)*(a*
d - b*c)**2*log(-sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*
x**2)/(8*a**(3/4)*b**(9/4)) + sqrt(2)*(a*d - b*c)**2*log(sqrt(2)*
a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(8*a**(3/4)*b**(9/4
)) - sqrt(2)*(a*d - b*c)**2*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))
/(4*a**(3/4)*b**(9/4)) + sqrt(2)*(a*d - b*c)**2*atan(1 + sqrt(2)*
b**(1/4)*x/a**(1/4))/(4*a**(3/4)*b**(9/4))
```

---

**Mathematica [A]** time = 0.172543, size = 231, normalized size = 0.91

$$\frac{8a^{3/4}b^{5/4}d^2x^5 - 40a^{3/4}\sqrt[4]{b}dx(ad-2bc) - 5\sqrt{2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right) + 5\sqrt{2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\right.}{40a^{3/4}b^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^4)^2/(a + b*x^4),x]
```

```
[Out] (-40*a^(3/4)*b^(1/4)*d*(-2*b*c + a*d)*x + 8*a^(3/4)*b^(5/4)*d^2*x
^5 - 10*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1
/4)] + 10*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^
(1/4)] - 5*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^
(1/4)*x + Sqrt[b]*x^2] + 5*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(40*a^(3/4)*b^(9/4))
```

---

**Maple [B]** time = 0.003, size = 436, normalized size = 1.7

$$\frac{d^2x^5}{5b}-\frac{ad^2x}{b^2}+2\frac{dxc}{b}+\frac{a\sqrt{2}d^2}{4b^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$-\frac{\sqrt{2}cd}{2b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{\sqrt{2}c^2}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{a\sqrt{2}d^2}{8b^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$-\frac{\sqrt{2}cd}{4b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{\sqrt{2}c^2}{8a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{a\sqrt{2}d^2}{4b^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{\sqrt{2}cd}{2b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)$$
$$+\frac{\sqrt{2}c^2}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^4+c)^2/(b*x^4+a),x)
```

```
[Out] 1/5*d^2*x^5/b-d^2/b^2*a*x+2*d/b*x*c+1/4/b^2*(a/b)^(1/4)*a*2^(1/2)
*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*d^2-1/2/b*(a/b)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/b)^(1/4)*x-1)*c*d+1/4*(a/b)^(1/4)/a*2^(1/2)*arcta
n(2^(1/2)/(a/b)^(1/4)*x-1)*c^2+1/8/b^2*(a/b)^(1/4)*a*2^(1/2)*ln((
x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)
+(a/b)^(1/2)))*d^2-1/4/b*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*
x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*c
*d+1/8*(a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^
(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*c^2+1/4/b^2*(a/b)
^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d^2-1/2/b*(a/b)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c*d+1/4*(a/b)^(1/4)
/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^2/(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.243164, size = 1458, normalized size = 5.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^2/(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*d^2*x^5 - 20*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^
6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*
c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9)
)^(1/4)*arctan(a*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*
d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^
5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4
)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x + (b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*sqrt((a^2*b^4*sqrt(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c
^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3
*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9)) +
 (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*x^2)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)))) + 5*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^
2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*
b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*
b^9))^(1/4)*log(a*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6
*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d
^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/
4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) - 5*b^2*(-(b^8*c^8 - 8*a*
b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*
c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7
 + a^8*d^8)/(a^3*b^9))^(1/4)*log(-a*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*
d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4
- 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d
^8)/(a^3*b^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) + 20*(2
*b*c*d - a*d^2)*x)/b^2
```

---

**Sympy [A]** time = 3.88518, size = 187, normalized size = 0.74

$$\operatorname{RootSum}\left(256t^4a^3b^9 + a^8d^8 - 8a^7bcd^7 + 28a^6b^2c^2d^6 - 56a^5b^3c^3d^5 + 70a^4b^4c^4d^4 - 56a^3b^5c^5d^3 + 28a^2b^6c^6d^2 - 8ab^7c^7d + b^8\right.$$

$$+ \frac{d^2x^5}{5b} - \frac{x\left(ad^2 - 2bcd\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**4+c)**2/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**9 + a**8*d**8 - 8*a**7*b*c*d**7 + 28*a*
*6*b**2*c**2*d**6 - 56*a**5*b**3*c**3*d**5 + 70*a**4*b**4*c**4*d*
*4 - 56*a**3*b**5*c**5*d**3 + 28*a**2*b**6*c**6*d**2 - 8*a*b**7*c
**7*d + b**8*c**8, Lambda(_t, _t*log(4*_t*a*b**2/(a**2*d**2 - 2*a
*b*c*d + b**2*c**2) + x))) + d**2*x**5/(5*b) - x*(a*d**2 - 2*b*c*
d)/b**2
```

---

**GIAC/XCAS [A]** time = 0.218389, size = 477, normalized size = 1.89

$$\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(ab^{3}\right)^{\frac{1}{4}}abcd+\left(ab^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^{3}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(ab^{3}\right)^{\frac{1}{4}}abcd+\left(ab^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^{3}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(ab^{3}\right)^{\frac{1}{4}}abcd+\left(ab^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\ln\left(x^{2}+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^{3}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(ab^{3}\right)^{\frac{1}{4}}abcd+\left(ab^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\ln\left(x^{2}-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^{3}}+\frac{b^{4}d^{2}x^{5}+10b^{4}cdx-5ab^{3}d^{2}x}{5b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^2/(b*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a
*b^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4
))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*
(a*b^3)^(1/4)*a*b*c*d + (a*b^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*(
(a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*b^3)^(1/4)*a
^2*d^2)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8
*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*b^
3)^(1/4)*a^2*d^2)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*
b^3) + 1/5*(b^4*d^2*x^5 + 10*b^4*c*d*x - 5*a*b^3*d^2*x)/b^5
```

## 3.64 $\int \frac{c+dx^4}{a+bx^4}\,dx$

**Optimal**. Leaf size=223

$$-\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{dx}{b}$$

```
[Out] (d*x)/b - ((b*c - a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
2*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c - a*d)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - a*d)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^
(3/4)*b^(5/4)) + ((b*c - a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4))
```

---

**Rubi [A]** time = 0.298679, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$

$$-\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^4)/(a + b*x^4),x]
```

```
[Out] (d*x)/b - ((b*c - a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
2*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c - a*d)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - a*d)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^
(3/4)*b^(5/4)) + ((b*c - a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4))
```

---

**Rubi in Sympy [A]** time = 61.2056, size = 204, normalized size = 0.91

$$\frac{dx}{b} + \frac{\sqrt{2}\,(ad-bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{8a^{\frac{3}{4}}b^{\frac{5}{4}}} - \frac{\sqrt{2}\,(ad-bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{8a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\sqrt{2}\,(ad-bc)\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{5}{4}}} - \frac{\sqrt{2}\,(ad-bc)\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**4+c)/(b*x**4+a),x)
```

```
[Out] d*x/b + sqrt(2)*(a*d - b*c)*log(-sqrt(2)*a**(1/4)*b**(1/4)*x + sq
rt(a) + sqrt(b)*x**2)/(8*a**(3/4)*b**(5/4)) - sqrt(2)*(a*d - b*c)
*log(sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(8*a**
(3/4)*b**(5/4)) + sqrt(2)*(a*d - b*c)*atan(1 - sqrt(2)*b**(1/4)*x
/a**(1/4))/(4*a**(3/4)*b**(5/4)) - sqrt(2)*(a*d - b*c)*atan(1 + s
qrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(3/4)*b**(5/4))
```

---

**Mathematica [A]** time = 0.214753, size = 196, normalized size = 0.88

$$\frac{8a^{3/4}\sqrt[4]{b}dx-\sqrt{2}(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+\sqrt{2}(bc-ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-2\sqrt{2}(bc-ad)\tan}{8a^{3/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^4)/(a + b*x^4),x]
```

```
[Out] (8*a^(3/4)*b^(1/4)*d*x - 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 - (Sqrt[2
]*b^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*(b*c - a*d)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(b*c - a*d)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(8*a^(3/4)*b^(5/4))
```

---

**Maple [A]** time = 0.005, size = 266, normalized size = 1.2

$$\frac{dx}{b}-\frac{\sqrt{2}d}{4\,b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{\sqrt{2}c}{4\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$-\frac{\sqrt{2}d}{8\,b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{\sqrt{2}c}{8\,a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$-\frac{\sqrt{2}d}{4\,b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{\sqrt{2}c}{4\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^4+c)/(b*x^4+a),x)
```

```
[Out] d*x/b-1/4/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*d
+1/4*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c-1/8/
b*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/
(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*d+1/8*(a/b)^(1/4)/a*2^(1
/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x
*2^(1/2)+(a/b)^(1/2)))*c-1/4/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(a/b)^(1/4)*x+1)*d+1/4*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b
)^(1/4)*x+1)*c
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)/(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.238155, size = 743, normalized size = 3.33

$$\frac{4\,b\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{a^3b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{ab\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{a^3b^5}\right)^{\frac{1}{4}}}{(bc-ad)x+(bc-ad)\sqrt{\frac{a^2b^2\sqrt{-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{a^3b^5}}+\left(b^2c^2-2\,abcd+a^2d^2\right)}{b^2c^2-2\,abcd+a^2d^2}}}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)/(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4)*arctan(-a*b*(-(b^4*c^4 - 4*a*b
^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5)
)^(1/4)/((b*c - a*d)*x + (b*c - a*d)*sqrt((a^2*b^2*sqrt(-(b^4*c^4
 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(
a^3*b^5)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2)/(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)))) - b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2
*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4)*log(a*b*(-(b^4*c
^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
/(a^3*b^5))^(1/4) - (b*c - a*d)*x) + b*(-(b^4*c^4 - 4*a*b^3*c^3*d
 + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4)*
log(-a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4) - (b*c - a*d)*x) + 4*d*x)/b
```

---

**Sympy [A]** time = 2.37949, size = 87, normalized size = 0.39

$$\operatorname{RootSum}\left(256t^4a^3b^5+a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4,\left(t\mapsto t\log\left(-\frac{4tab}{ad-bc}+x\right)\right)\right)+\frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**4+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**5 + a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**
2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4, Lambda(_t, _t*log
(-4*_t*a*b/(a*d - b*c) + x))) + d*x/b
```

---

**GIAC/XCAS [A]** time = 0.217809, size = 331, normalized size = 1.48

$$\frac{dx}{b} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}bc - \left(ab^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}bc - \left(ab^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}bc - \left(ab^3\right)^{\frac{1}{4}}ad\right)\ln\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8\,ab^2} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}bc - \left(ab^3\right)^{\frac{1}{4}}ad\right)\ln\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{8\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)/(b*x^4 + a),x, algorithm="giac")
```

```
[Out] d*x/b + 1/4*sqrt(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*arcta
n(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2) +
1/4*sqrt(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2) + 1/8*sqrt
(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*ln(x^2 + sqrt(2)*x*(a
/b)^(1/4) + sqrt(a/b))/(a*b^2) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b*c -
 (a*b^3)^(1/4)*a*d)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(
a*b^2)
```

## 3.65 $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

**Optimal**. Leaf size=449

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)}-\frac{d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)}+\frac{d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)}-\frac{d^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{3/4}(bc-ad)}$$

```
[Out] -(b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*(b*c - a*d)) + (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/
4)])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (d^(3/4)*ArcTan[1 - (Sqrt[
2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4
)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*(b*
c - a*d)) - (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (b^(3/4)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*
(b*c - a*d)) + (d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x +
 Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*Log[Sqr
t[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/
4)*(b*c - a*d))
```

---

**Rubi [A]** time = 0.566816, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)}-\frac{d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)}+\frac{d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)}-\frac{d^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -(b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*(b*c - a*d)) + (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/
```

```
4)])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (d^(3/4)*ArcTan[1 - (Sqrt[
2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4
)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*(b*
c - a*d)) - (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (b^(3/4)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*
(b*c - a*d)) + (d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x +
 Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*Log[Sqr
t[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/
4)*(b*c - a*d))
```

---

**Rubi in Sympy [A]** time = 119.517, size = 400, normalized size = 0.89

$$-\frac{\sqrt{2}d^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{8c^{\frac{3}{4}}(ad-bc)}+\frac{\sqrt{2}d^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{8c^{\frac{3}{4}}(ad-bc)}-\frac{\sqrt{2}d^{\frac{3}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{4c^{\frac{3}{4}}(ad-bc)}+\frac{\sqrt{2}d^{\frac{3}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{4c^{\frac{3}{4}}(ad-bc)}+\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{8a^{\frac{3}{4}}(ad-bc)}-\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{8a^{\frac{3}{4}}(ad-bc)}+\frac{\sqrt{2}b^{\frac{3}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}(ad-bc)}-\frac{\sqrt{2}b^{\frac{3}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] -sqrt(2)*d**(3/4)*log(-sqrt(2)*c**(1/4)*d**(1/4)*x + sqrt(c) + sq
rt(d)*x**2)/(8*c**(3/4)*(a*d - b*c)) + sqrt(2)*d**(3/4)*log(sqrt(
2)*c**(1/4)*d**(1/4)*x + sqrt(c) + sqrt(d)*x**2)/(8*c**(3/4)*(a*d
 - b*c)) - sqrt(2)*d**(3/4)*atan(1 - sqrt(2)*d**(1/4)*x/c**(1/4))
/(4*c**(3/4)*(a*d - b*c)) + sqrt(2)*d**(3/4)*atan(1 + sqrt(2)*d**
(1/4)*x/c**(1/4))/(4*c**(3/4)*(a*d - b*c)) + sqrt(2)*b**(3/4)*log
(-sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(8*a**(3/
4)*(a*d - b*c)) - sqrt(2)*b**(3/4)*log(sqrt(2)*a**(1/4)*b**(1/4)*
x + sqrt(a) + sqrt(b)*x**2)/(8*a**(3/4)*(a*d - b*c)) + sqrt(2)*b*
*(3/4)*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(3/4)*(a*d - b
*c)) - sqrt(2)*b**(3/4)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(4*
a**(3/4)*(a*d - b*c))
```

---

**Mathematica [A]** time = 0.281602, size = 340, normalized size = 0.76

$$a^{3/4}d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)-a^{3/4}d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)+2a^{3/4}d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)-2a^{3/4}d$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (-2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b
^(3/4)*c^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(3/4
)*d^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(3/4)*d^(
3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - b^(3/4)*c^(3/4)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + b^(3/4)*c^
(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + a^
(3/4)*d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x
^2] - a^(3/4)*d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + S
qrt[d]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)*(b*c - a*d))
```

---

**Maple [A]** time = 0.002, size = 320, normalized size = 0.7

$$\frac{d\sqrt{2}}{(8\,ad-8\,bc)\,c}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$+\frac{d\sqrt{2}}{(4\,ad-4\,bc)\,c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{d\sqrt{2}}{(4\,ad-4\,bc)\,c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$
$$-\frac{b\sqrt{2}}{(8\,ad-8\,bc)\,a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$-\frac{b\sqrt{2}}{(4\,ad-4\,bc)\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{b\sqrt{2}}{(4\,ad-4\,bc)\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/8*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/
2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+1/4*d/(a
*d-b*c)*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+1/4
*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1
)-1/8*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(
1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))-1/4*b/
(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)-1
/4*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x
-1)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)*(d*x^4 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.395315, size = 1472, normalized size = 3.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)*(d*x^4 + c)),x, algorithm="fricas")
```

```
[Out] (-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*
b*c*d^3 + a^7*d^4))^(1/4)*arctan(-(a*b*c - a^2*d)*(-b^3/(a^3*b^4*
c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d
^4))^(1/4)/(b*x + b*sqrt((b^2*x^2 + (a^2*b^2*c^2 - 2*a^3*b*c*d +
a^4*d^2)*sqrt(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2
*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)))/b^2))) - (-d^3/(b^4*c^7 - 4*a*b
^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1
/4)*arctan(-(b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^
2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)/(d*x + d*sq
rt((d^2*x^2 + (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*sqrt(-d^3/(b^
4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4
*c^3*d^4)))/d^2))) + 1/4*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6
*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)*log(b*x + (a*b
*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*
d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-b^3/(a^3*b^4*c^4 -
 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^
(1/4)*log(b*x - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^
3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*
(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*
d^3 + a^4*c^3*d^4))^(1/4)*log(d*x + (b*c^2 - a*c*d)*(-d^3/(b^4*c^
7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3
*d^4))^(1/4)) + 1/4*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^
5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x - (b*c^2 -
a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3
*b*c^4*d^3 + a^4*c^3*d^4))^(1/4))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)(dx^4 + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)*(d*x^4 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)*(d*x^4 + c)), x)
```

## 3.66 $\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx$

**Optimal**. Leaf size=513

$$-\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2}+\frac{d^{3/4}(7bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^2}-\frac{d^{3/4}(7bc-3ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^2}+\frac{d^{3/4}(7bc-3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2}-\frac{d^{3/4}(7bc-3ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2}-\frac{dx}{4c\left(c+dx^4\right)(bc-ad)}$$

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(c + d*x^4)) - (b^(7/4)*ArcTan[1 - (Sqrt[
2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(
b*c - a*d)^2) + (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1
/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (d^(3/4)*(7*
b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*
c^(7/4)*(b*c - a*d)^2) - (b^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(
7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*S
qrt[2]*a^(3/4)*(b*c - a*d)^2) + (d^(3/4)*(7*b*c - 3*a*d)*Log[Sqrt
[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/
4)*(b*c - a*d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2
]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(b*c - a*
d)^2)
```

---

**Rubi [A]** time = 0.884336, antiderivative size = 513, normalized size of antiderivative = 1., number

of steps used = 20, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$-\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2}+\frac{d^{3/4}(7bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^2}-\frac{d^{3/4}(7bc-3ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^2}+\frac{d^{3/4}(7bc-3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2}-\frac{d^{3/4}(7bc-3ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2}-\frac{dx}{4c\left(c+dx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^4)*(c + d*x^4)^2),x]
```

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(c + d*x^4)) - (b^(7/4)*ArcTan[1 - (Sqrt[
2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(
b*c - a*d)^2) + (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1
/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (d^(3/4)*(7*
b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*
c^(7/4)*(b*c - a*d)^2) - (b^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(
7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*S
qrt[2]*a^(3/4)*(b*c - a*d)^2) + (d^(3/4)*(7*b*c - 3*a*d)*Log[Sqrt
[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/
4)*(b*c - a*d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2
]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(b*c - a*
d)^2)
```

**Rubi in Sympy [A]** time = 174.262, size = 474, normalized size = 0.92

$$\frac{dx}{4c\left(c+dx^4\right)\left(ad-bc\right)}-\frac{\sqrt{2}d^{\frac{3}{4}}\left(3ad-7bc\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{32c^{\frac{7}{4}}\left(ad-bc\right)^2}+\frac{\sqrt{2}d^{\frac{3}{4}}\left(3ad-7bc\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{32c^{\frac{7}{4}}\left(ad-bc\right)^2}-\frac{\sqrt{2}d^{\frac{3}{4}}\left(3ad-7bc\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{16c^{\frac{7}{4}}\left(ad-bc\right)^2}+\frac{\sqrt{2}d^{\frac{3}{4}}\left(3ad-7bc\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{16c^{\frac{7}{4}}\left(ad-bc\right)^2}-\frac{\sqrt{2}b^{\frac{7}{4}}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{8a^{\frac{3}{4}}\left(ad-bc\right)^2}+\frac{\sqrt{2}b^{\frac{7}{4}}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{8a^{\frac{3}{4}}\left(ad-bc\right)^2}-\frac{\sqrt{2}b^{\frac{7}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\left(ad-bc\right)^2}+\frac{\sqrt{2}b^{\frac{7}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{4a^{\frac{3}{4}}\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**4+a)/(d*x**4+c)**2,x)
```

```
[Out] d*x/(4*c*(c + d*x**4)*(a*d - b*c)) - sqrt(2)*d**(3/4)*(3*a*d - 7*
b*c)*log(-sqrt(2)*c**(1/4)*d**(1/4)*x + sqrt(c) + sqrt(d)*x**2)/(
32*c**(7/4)*(a*d - b*c)**2) + sqrt(2)*d**(3/4)*(3*a*d - 7*b*c)*lo
g(sqrt(2)*c**(1/4)*d**(1/4)*x + sqrt(c) + sqrt(d)*x**2)/(32*c**(7
/4)*(a*d - b*c)**2) - sqrt(2)*d**(3/4)*(3*a*d - 7*b*c)*atan(1 - s
qrt(2)*d**(1/4)*x/c**(1/4))/(16*c**(7/4)*(a*d - b*c)**2) + sqrt(2
)*d**(3/4)*(3*a*d - 7*b*c)*atan(1 + sqrt(2)*d**(1/4)*x/c**(1/4))/
(16*c**(7/4)*(a*d - b*c)**2) - sqrt(2)*b**(7/4)*log(-sqrt(2)*a**(
1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(8*a**(3/4)*(a*d - b*c)
**2) + sqrt(2)*b**(7/4)*log(sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a)
 + sqrt(b)*x**2)/(8*a**(3/4)*(a*d - b*c)**2) - sqrt(2)*b**(7/4)*a
tan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(3/4)*(a*d - b*c)**2)
+ sqrt(2)*b**(7/4)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(4*a**(3
/4)*(a*d - b*c)**2)
```

---

**Mathematica [A]** time = 0.561561, size = 498, normalized size = 0.97

$$\frac{8a^{3/4}c^{3/4}dx(ad-bc)-2\sqrt{2}a^{3/4}d^{3/4}\left(c+dx^4\right)\left(3ad-7bc\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+2\sqrt{2}a^{3/4}d^{3/4}\left(c+dx^4\right)\left(3ad-7bc\right)\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^4)*(c + d*x^4)^2),x]
```

```
[Out] (8*a^(3/4)*c^(3/4)*d*(-(b*c) + a*d)*x - 8*Sqrt[2]*b^(7/4)*c^(7/4)
*(c + d*x^4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 8*Sqrt[2]*
```

```
b^(7/4)*c^(7/4)*(c + d*x^4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)] - 2*Sqrt[2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c + d*x^4)*ArcTa
n[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*Sqrt[2]*a^(3/4)*d^(3/4)*(-
7*b*c + 3*a*d)*(c + d*x^4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
] - 4*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^4)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 4*Sqrt[2]*b^(7/4)*c^(7/4)*(c +
d*x^4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + S
qrt[2]*a^(3/4)*d^(3/4)*(7*b*c - 3*a*d)*(c + d*x^4)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + Sqrt[2]*a^(3/4)*d^(3/4
)*(-7*b*c + 3*a*d)*(c + d*x^4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1
/4)*x + Sqrt[d]*x^2])/(32*a^(3/4)*c^(7/4)*(b*c - a*d)^2*(c + d*x^
4))
```

---

**Maple [A]** time = 0.003, size = 550, normalized size = 1.1

$$\frac{d^2xa}{4\,(ad-bc)^2\,c\,(dx^4+c)}-\frac{dxb}{4\,(ad-bc)^2\,(dx^4+c)}+\frac{3\,d^2\sqrt{2}a}{16\,(ad-bc)^2\,c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)-\frac{7\,d\sqrt{2}b}{16\,(ad-bc)^2\,c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$
$$+\frac{3\,d^2\sqrt{2}a}{32\,(ad-bc)^2\,c^2}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$-\frac{7\,d\sqrt{2}b}{32\,(ad-bc)^2\,c}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$+\frac{3\,d^2\sqrt{2}a}{16\,(ad-bc)^2\,c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)-\frac{7\,d\sqrt{2}b}{16\,(ad-bc)^2\,c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)$$
$$+\frac{b^2\sqrt{2}}{8\,(ad-bc)^2\,a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{b^2\sqrt{2}}{4\,(ad-bc)^2\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{b^2\sqrt{2}}{4\,(ad-bc)^2\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^4+a)/(d*x^4+c)^2,x)
```

```
[Out] 1/4*d^2/(a*d-b*c)^2/c*x/(d*x^4+c)*a-1/4*d/(a*d-b*c)^2*x/(d*x^4+c)
*b+3/16*d^2/(a*d-b*c)^2/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c
/d)^(1/4)*x-1)*a-7/16*d/(a*d-b*c)^2/c*(c/d)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(c/d)^(1/4)*x-1)*b+3/32*d^2/(a*d-b*c)^2/c^2*(c/d)^(1/4)*2
^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4
```

```
)*x*2^(1/2)+(c/d)^(1/2)))*a-7/32*d/(a*d-b*c)^2/c*(c/d)^(1/4)*2^(1
/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x
*2^(1/2)+(c/d)^(1/2)))*b+3/16*d^2/(a*d-b*c)^2/c^2*(c/d)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a-7/16*d/(a*d-b*c)^2/c*(c/d)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b+1/8*b^2/(a*d-b*c
)^2*(a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/
2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4*b^2/(a*d-b*c)^2*
(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4*b^2/(a*
d-b*c)^2*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)*(d*x^4 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 21.891, size = 3665, normalized size = 7.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)*(d*x^4 + c)^2),x, algorithm="fricas")
```

```
[Out] -1/16*(16*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d
^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5
 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*((b*c^2
*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*arctan((-b^7/(a^3*b^8*c^8 -
8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^
7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*
b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)/(b
^2*x + b^2*sqrt((b^4*x^2 + (a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4
*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*sqrt(-b^7/(a^3*b^8*c^8 -
8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^
7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*
b*c*d^7 + a^11*d^8)))/b^4))) + 4*((b*c^2*d - a*c*d^2)*x^4 + b*c^3
 - a*c^2*d)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b
^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^
14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^1
1*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^
7 + a^8*c^7*d^8))^(1/4)*arctan(-(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*
d^2)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*
d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d +
```

```
28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 -
 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8
*c^7*d^8))^(1/4)/((7*b*c*d - 3*a*d^2)*x + (7*b*c*d - 3*a*d^2)*sqr
t(((49*b^2*c^2*d^2 - 42*a*b*c*d^3 + 9*a^2*d^4)*x^2 + (b^4*c^8 - 4
*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4)
*sqrt(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*
d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d +
28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 -
 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8
*c^7*d^8)))/(49*b^2*c^2*d^2 - 42*a*b*c*d^3 + 9*a^2*d^4)))) - 4*(-
b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*
b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^
2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*((b*c^2*d - a*c*d^2
)*x^4 + b*c^3 - a*c^2*d)*log(b^2*x + (-b^7/(a^3*b^8*c^8 - 8*a^4*b
^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c
^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7
 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)) + 4*(-b^
7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^
5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*
c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*((b*c^2*d - a*c*d^2)*
x^4 + b*c^3 - a*c^2*d)*log(b^2*x - (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7
*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4
*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 +
 a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)) - ((b*c^2*
d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*(-(2401*b^4*c^4*d^3 - 4116*a*
b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7
)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c
^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*
c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)*log(-(7*b*c*d - 3
*a*d^2)*x + (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(-(2401*b^4*c^4
*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^
6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2
- 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5
 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)) +
((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*(-(2401*b^4*c^4*d^3 -
 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81
*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a
^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*
a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)*log(-(7*b
*c*d - 3*a*d^2)*x - (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(-(2401
*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^
3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c
^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*
c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(
1/4)) + 4*d*x)/((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**4+a)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [A]** time = 0.228134, size = 900, normalized size = 1.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)*(d*x^4 + c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4)
)/(a/b)^(1/4))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)
*a^3*d^2) + 1/2*(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x - sqrt(2)
*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c
*d + sqrt(2)*a^3*d^2) + 1/4*(a*b^3)^(1/4)*b*ln(x^2 + sqrt(2)*x*(a
/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d +
 sqrt(2)*a^3*d^2) - 1/4*(a*b^3)^(1/4)*b*ln(x^2 - sqrt(2)*x*(a/b)^
(1/4) + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqr
t(2)*a^3*d^2) - 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*a
rctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(
2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/8*(7*
(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x
- sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*
a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/16*(7*(c*d^3)^(1/4)*b*c - 3*
(c*d^3)^(1/4)*a*d)*ln(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(s
qrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) + 1/1
6*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*ln(x^2 - sqrt(2)*x*
(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d +
 sqrt(2)*a^2*c^2*d^2) - 1/4*d*x/((d*x^4 + c)*(b*c^2 - a*c*d))
```

## 3.67 $\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx$

**Optimal**. Leaf size=407

$$-\frac{(bc-ad)^4(17ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}}+\frac{(bc-ad)^4(17ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}}-\frac{(bc-ad)^4(17ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{21/4}}+\frac{(bc-ad)^4(17ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}b^{21/4}}+\frac{d^3x^5\left(3a^2d^2-10abcd+10b^2c^2\right)}{5b^4}+\frac{d^2x\left(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3\right)}{b^5}+\frac{x(bc-ad)^5}{4ab^5\left(a+bx^4\right)}+\frac{d^4x^9(5bc-2ad)}{9b^3}+\frac{d^5x^{13}}{13b^2}$$

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x
)/b^5 + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^5)/(5*b^4) +
 (d^4*(5*b*c - 2*a*d)*x^9)/(9*b^3) + (d^5*x^13)/(13*b^2) + ((b*c
- a*d)^5*x)/(4*a*b^5*(a + b*x^4)) - ((b*c - a*d)^4*(3*b*c + 17*a*
d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^
(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(21/4)) - ((b*c - a*d)^4*(
3*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(16*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17
*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16
*Sqrt[2]*a^(7/4)*b^(21/4))
```

---

**Rubi [A]** time = 0.785902, antiderivative size = 407, normalized size of antiderivative = 1., number

of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$-\frac{(bc-ad)^4(17ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}}+\frac{(bc-ad)^4(17ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}}-\frac{(bc-ad)^4(17ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{21/4}}+\frac{(bc-ad)^4(17ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}b^{21/4}}+\frac{d^3x^5\left(3a^2d^2-10abcd+10b^2c^2\right)}{5b^4}+\frac{d^2x\left(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3\right)}{b^5}+\frac{x(bc-ad)^5}{4ab^5\left(a+bx^4\right)}+\frac{d^4x^9(5bc-2ad)}{9b^3}+\frac{d^5x^{13}}{13b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^4)^5/(a + b*x^4)^2,x]
```

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x
)/b^5 + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^5)/(5*b^4) +
 (d^4*(5*b*c - 2*a*d)*x^9)/(9*b^3) + (d^5*x^13)/(13*b^2) + ((b*c
- a*d)^5*x)/(4*a*b^5*(a + b*x^4)) - ((b*c - a*d)^4*(3*b*c + 17*a*
d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^
(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(21/4)) - ((b*c - a*d)^4*(
3*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(16*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17
*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16
*Sqrt[2]*a^(7/4)*b^(21/4))
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-d^2\left(4a^3d^3-15a^2bcd^2+20ab^2c^2d-10b^3c^3\right)\int\frac{1}{b^5}\,dx+\frac{d^5x^{13}}{13b^2}-\frac{d^4x^9\left(2ad-5bc\right)}{9b^3}+\frac{d^3x^5\left(3a^2d^2-10abcd+10b^2c^2\right)}{5b^4}-\frac{x\left(ad-bc\right)^5}{4ab^5\left(a+bx^4\right)}-\frac{\sqrt{2}\left(ad-bc\right)^4\left(17ad+3bc\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{32a^{\frac{7}{4}}b^{\frac{21}{4}}}+\frac{\sqrt{2}\left(ad-bc\right)^4\left(17ad+3bc\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{32a^{\frac{7}{4}}b^{\frac{21}{4}}}-\frac{\sqrt{2}\left(ad-bc\right)^4\left(17ad+3bc\right)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{21}{4}}}+\frac{\sqrt{2}\left(ad-bc\right)^4\left(17ad+3bc\right)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{21}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**4+c)**5/(b*x**4+a)**2,x)
```

```
[Out] -d**2*(4*a**3*d**3 - 15*a**2*b*c*d**2 + 20*a*b**2*c**2*d - 10*b**
3*c**3)*Integral(b**(-5), x) + d**5*x**13/(13*b**2) - d**4*x**9*(
2*a*d - 5*b*c)/(9*b**3) + d**3*x**5*(3*a**2*d**2 - 10*a*b*c*d + 1
0*b**2*c**2)/(5*b**4) - x*(a*d - b*c)**5/(4*a*b**5*(a + b*x**4))
- sqrt(2)*(a*d - b*c)**4*(17*a*d + 3*b*c)*log(-sqrt(2)*a**(1/4)*b
**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(32*a**(7/4)*b**(21/4)) + sqr
t(2)*(a*d - b*c)**4*(17*a*d + 3*b*c)*log(sqrt(2)*a**(1/4)*b**(1/4
)*x + sqrt(a) + sqrt(b)*x**2)/(32*a**(7/4)*b**(21/4)) - sqrt(2)*(
a*d - b*c)**4*(17*a*d + 3*b*c)*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/
4))/(16*a**(7/4)*b**(21/4)) + sqrt(2)*(a*d - b*c)**4*(17*a*d + 3*
b*c)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(16*a**(7/4)*b**(21/4)
)
```

---

**Mathematica [A]**  time = 0.673226, size = 391, normalized size = 0.96

$$-\frac{585\sqrt{2}(bc-ad)^4(17ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}+\frac{585\sqrt{2}(bc-ad)^4(17ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}-\frac{1170\sqrt{2}(bc-ad)^4(17ad+3bc)\tan}{a^{7/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^4)^5/(a + b*x^4)^2,x]
```

```
[Out] (18720*b^(1/4)*d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2
- 4*a^3*d^3)*x + 3744*b^(5/4)*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^
2*d^2)*x^5 + 2080*b^(9/4)*d^4*(5*b*c - 2*a*d)*x^9 + 1440*b^(13/4)
*d^5*x^13 + (4680*b^(1/4)*(b*c - a*d)^5*x)/(a*(a + b*x^4)) - (117
0*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1
/4)*x)/a^(1/4)])/a^(7/4) + (1170*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 1
7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) - (585*Sq
rt[2]*(b*c - a*d)^4*(3*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (585*Sqrt[2]*(b*c - a*d)^4*
(3*b*c + 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b
]*x^2])/a^(7/4))/(18720*b^(21/4))
```

---

**Maple [B]**  time = 0.018, size = 1118, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^4+c)^5/(b*x^4+a)^2,x)
```

```
[Out] 1/13*d^5*x^13/b^2-5/4/b*x/(b*x^4+a)*c^4*d-20*d^3/b^3*a*c^2*x+15*d
^4/b^4*a^2*c*x+10*d^2/b^2*c^3*x-4*d^5/b^5*a^3*x-2*d^4/b^3*x^5*a*c
+5/9*d^4/b^2*x^9*c+45/8/b^3*a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/b)^(1/4)*x-1)*c^2*d^3+5/16/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/b)^(1/4)*x-1)*c^4*d-65/32/b^4*a^2*(a/b)^(1/4)*2^(1/2)*ln((
x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)
+(a/b)^(1/2)))*c*d^4+45/16/b^3*a*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b
)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(
1/2)))*c^2*d^3+5/32/b/a*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x
*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*c^
4*d+5/4/b^4*x*a^3/(b*x^4+a)*c*d^4-5/2/b^3*x*a^2/(b*x^4+a)*c^2*d^3
+5/2/b^2*x*a/(b*x^4+a)*c^3*d^2+17/16/b^5*a^3*(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d^5-25/8/b^2*(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^3*d^2+17/16/b^5*a^3*(a/b)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*d^5-25/8/b^2*(a/b)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c^3*d^2+17/32/b^5*a^3*(a
/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2
-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*d^5-25/16/b^2*(a/b)^(1/4)*2^
(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)
*x*2^(1/2)+(a/b)^(1/2)))*c^3*d^2+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a/b)^(1/4)*x+1)*c^5+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a/b)^(1/4)*x-1)*c^5+3/32/a^2*(a/b)^(1/4)*2^(1/2)*ln(
(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2
)+(a/b)^(1/2)))*c^5-1/4/b^5*x*a^4/(b*x^4+a)*d^5-65/16/b^4*a^2*(a/
b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c*d^4+45/8/b^3*a
*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^2*d^3+5/16
/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^4*d-65
/16/b^4*a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c
*d^4-2/9*d^5/b^3*x^9*a+3/5*d^5/b^4*x^5*a^2+2*d^3/b^2*x^5*c^2+1/4*
x/a/(b*x^4+a)*c^5
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^5/(b*x^4 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.268173, size = 3853, normalized size = 9.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^5/(b*x^4 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/9360*(720*a*b^4*d^5*x^17 + 80*(65*a*b^4*c*d^4 - 17*a^2*b^3*d^5)
*x^13 + 208*(90*a*b^4*c^2*d^3 - 65*a^2*b^3*c*d^4 + 17*a^3*b^2*d^5
)*x^9 + 1872*(50*a*b^4*c^3*d^2 - 90*a^2*b^3*c^2*d^3 + 65*a^3*b^2*
c*d^4 - 17*a^4*b*d^5)*x^5 - 2340*(a*b^6*x^4 + a^2*b^5)*(-(81*b^20
*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^18*c^18*d^2 - 15780*a^3*b^
17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5*b^15*c^15*d^5
- 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 500110*
a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b
^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*
c^8*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d
^14 - 266040144*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 -
 39944900*a^17*b^3*c^3*d^17 + 9094830*a^18*b^2*c^2*d^18 - 1277380
*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))^(1/4)*arctan(a^2*b^
5*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^18*c^18*d^2 -
15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5*b^
15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d
^7 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 1748
73556*a^10*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 5175592
50*a^12*b^8*c^8*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^
14*b^6*c^6*d^14 - 266040144*a^15*b^5*c^5*d^15 + 120836285*a^16*b^
4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 9094830*a^18*b^2*c^2*d^
18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))^(1/4)/(
(3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*
d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*x + (3*b^5*c^5 + 5*a*b^4*c^4*d
 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*
a^5*d^5)*sqrt((a^4*b^10*sqrt(-(81*b^20*c^20 + 540*a*b^19*c^19*d -
 4050*a^2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^
16*c^16*d^4 - 13264*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6
 + 6137200*a^7*b^13*c^13*d^7 - 500110*a^8*b^12*c^12*d^8 - 4853004
0*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10*c^10*d^10 - 360900280*a
^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 548231440*a^13*b
^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5*c^
5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17
 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20
*d^20)/(a^7*b^21)) + (9*b^10*c^10 + 30*a*b^9*c^9*d - 275*a^2*b^8*
c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^
5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*
a^8*b^2*c^2*d^8 - 2210*a^9*b*c*d^9 + 289*a^10*d^10)*x^2)/(9*b^10*
c^10 + 30*a*b^9*c^9*d - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3
+ 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4
*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 - 2210*a^9*b*
c*d^9 + 289*a^10*d^10)))) + 585*(a*b^6*x^4 + a^2*b^5)*(-(81*b^20*
c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^18*c^18*d^2 - 15780*a^3*b^1
7*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5*b^15*c^15*d^5 -
 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 500110*a
^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^
10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c
^8*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^
14 - 266040144*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 -
39944900*a^17*b^3*c^3*d^17 + 9094830*a^18*b^2*c^2*d^18 - 1277380*
a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))^(1/4)*log(a^2*b^5*(-
(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^18*c^18*d^2 - 1578
0*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5*b^15*c
^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 -
 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 17487355
6*a^10*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a
```

```
^12*b^8*c^8*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b
^6*c^6*d^14 - 266040144*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^
4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 9094830*a^18*b^2*c^2*d^18 -
 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))^(1/4) + (3*
b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3
 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*x) - 585*(a*b^6*x^4 + a^2*b^5)*(-
(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^18*c^18*d^2 - 1578
0*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5*b^15*c
^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 -
 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 17487355
6*a^10*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a
^12*b^8*c^8*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b
^6*c^6*d^14 - 266040144*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^
4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 9094830*a^18*b^2*c^2*d^18 -
 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))^(1/4)*log(-
a^2*b^5*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^18*c^18*
d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*
a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*
c^13*d^7 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9
+ 174873556*a^10*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 5
17559250*a^12*b^8*c^8*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700
840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5*c^5*d^15 + 120836285*a
^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 9094830*a^18*b^2*
c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))^(
1/4) + (3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b
^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*x) + 2340*(b^5*c^5 - 5*
a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 - 90*a^3*b^2*c^2*d^3 + 65*a^4*b*
c*d^4 - 17*a^5*d^5)*x)/(a*b^6*x^4 + a^2*b^5)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**4+c)**5/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [A]** time = 0.221219, size = 1077, normalized size = 2.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^5/(b*x^4 + a)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4
*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*(a*b^3)^(1/4)*a^3*b^2*
c^2*d^3 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(1/4)*a^5*d^5
)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^
2*b^6) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^5*c^5 + 5*(a*b^3)^(1/4)*
a*b^4*c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*(a*b^3)^(1/4)
*a^3*b^2*c^2*d^3 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(1/4
)*a^5*d^5)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(a^2*b^6) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^5*c^5 + 5*(a*b^
3)^(1/4)*a*b^4*c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*(a*b
^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*
b^3)^(1/4)*a^5*d^5)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(
a^2*b^6) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^5*c^5 + 5*(a*b^3)^(1/4
)*a*b^4*c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*(a*b^3)^(1/
4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(1
/4)*a^5*d^5)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^6
) + 1/4*(b^5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*
a^3*b^2*c^2*d^3*x + 5*a^4*b*c*d^4*x - a^5*d^5*x)/((b*x^4 + a)*a*b
^5) + 1/585*(45*b^24*d^5*x^13 + 325*b^24*c*d^4*x^9 - 130*a*b^23*d
^5*x^9 + 1170*b^24*c^2*d^3*x^5 - 1170*a*b^23*c*d^4*x^5 + 351*a^2*
b^22*d^5*x^5 + 5850*b^24*c^3*d^2*x - 11700*a*b^23*c^2*d^3*x + 877
5*a^2*b^22*c*d^4*x - 2340*a^3*b^21*d^5*x)/b^26
```

## 3.68 $\int \frac{(c+dx^4)^4}{(a+bx^4)^2} \, dx$

**Optimal**. Leaf size=357

$$-\frac{(bc-ad)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}+\frac{(bc-ad)^3(13ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}-\frac{(bc-ad)^3(13ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{17/4}}+\frac{(bc-ad)^3(13ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}b^{17/4}}+\frac{d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)}{b^4}+\frac{x(bc-ad)^4}{4ab^4\left(a+bx^4\right)}+\frac{2d^3x^5(2bc-ad)}{5b^3}+\frac{d^4x^9}{9b^2}$$

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c -
 a*d)*x^5)/(5*b^3) + (d^4*x^9)/(9*b^2) + ((b*c - a*d)^4*x)/(4*a*b
^4*(a + b*x^4)) - ((b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(17/4)) + ((b*c -
a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/
(8*Sqrt[2]*a^(7/4)*b^(17/4)) - ((b*c - a*d)^3*(3*b*c + 13*a*d)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]
*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b
^(17/4))
```

---

**Rubi [A]** time = 0.718939, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$-\frac{(bc-ad)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}+\frac{(bc-ad)^3(13ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}-\frac{(bc-ad)^3(13ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{17/4}}+\frac{(bc-ad)^3(13ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}b^{17/4}}+\frac{d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)}{b^4}+\frac{x(bc-ad)^4}{4ab^4\left(a+bx^4\right)}+\frac{2d^3x^5(2bc-ad)}{5b^3}+\frac{d^4x^9}{9b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^4)^4/(a + b*x^4)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c -
 a*d)*x^5)/(5*b^3) + (d^4*x^9)/(9*b^2) + ((b*c - a*d)^4*x)/(4*a*b
^4*(a + b*x^4)) - ((b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(17/4)) + ((b*c -
a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/
(8*Sqrt[2]*a^(7/4)*b^(17/4)) - ((b*c - a*d)^3*(3*b*c + 13*a*d)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]
*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b
^(17/4))
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$d^2\left(3a^2d^2-8abcd+6b^2c^2\right)\int\frac{1}{b^4}\,dx+\frac{d^4x^9}{9b^2}-\frac{2d^3x^5\,(ad-2bc)}{5b^3}+\frac{x\,(ad-bc)^4}{4ab^4\,(a+bx^4)}+\frac{\sqrt{2}\,(ad-bc)^3\,(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{32a^{\frac{7}{4}}b^{\frac{17}{4}}}-\frac{\sqrt{2}\,(ad-bc)^3\,(13ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{32a^{\frac{7}{4}}b^{\frac{17}{4}}}+\frac{\sqrt{2}\,(ad-bc)^3\,(13ad+3bc)\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{17}{4}}}-\frac{\sqrt{2}\,(ad-bc)^3\,(13ad+3bc)\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{17}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**4+c)**4/(b*x**4+a)**2,x)
```

```
[Out] d**2*(3*a**2*d**2 - 8*a*b*c*d + 6*b**2*c**2)*Integral(b**(-4), x)
 + d**4*x**9/(9*b**2) - 2*d**3*x**5*(a*d - 2*b*c)/(5*b**3) + x*(a
*d - b*c)**4/(4*a*b**4*(a + b*x**4)) + sqrt(2)*(a*d - b*c)**3*(13
*a*d + 3*b*c)*log(-sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b
)*x**2)/(32*a**(7/4)*b**(17/4)) - sqrt(2)*(a*d - b*c)**3*(13*a*d
+ 3*b*c)*log(sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2
)/(32*a**(7/4)*b**(17/4)) + sqrt(2)*(a*d - b*c)**3*(13*a*d + 3*b*
c)*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(16*a**(7/4)*b**(17/4))
- sqrt(2)*(a*d - b*c)**3*(13*a*d + 3*b*c)*atan(1 + sqrt(2)*b**(1/
4)*x/a**(1/4))/(16*a**(7/4)*b**(17/4))
```

---

**Mathematica [A]** time = 0.501044, size = 341, normalized size = 0.96

$$\frac{\frac{45\sqrt{2}(ad-bc)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}+\frac{45\sqrt{2}(bc-ad)^3(13ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}+\frac{90\sqrt{2}(ad-bc)^3(13ad+3bc)\tan^{-1}\left(1-\right.}{a^{7/4}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^4)^4/(a + b*x^4)^2,x]
```

```
[Out] (1440*b^(1/4)*d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x + 576*b^(
5/4)*d^3*(2*b*c - a*d)*x^5 + 160*b^(9/4)*d^4*x^9 + (360*b^(1/4)*(
b*c - a*d)^4*x)/(a*(a + b*x^4)) + (90*Sqrt[2]*(-(b*c) + a*d)^3*(3
*b*c + 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) +
 (90*Sqrt[2]*(b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*x)/a^(1/4)])/a^(7/4) + (45*Sqrt[2]*(-(b*c) + a*d)^3*(3*b*c
 + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/a^(7/4) + (45*Sqrt[2]*(b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(1440*b^(1
7/4))
```

---

**Maple [B]** time = 0.017, size = 885, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^4+c)^4/(b*x^4+a)^2,x)
```

```
[Out] 1/9*d^4*x^9/b^2-8*d^3/b^3*a*c*x+3*d^4/b^4*a^2*x+6*d^2/b^2*c^2*x-1
/b^3*x*a^2/(b*x^4+a)*c*d^3+3/2/b^2*x*a/(b*x^4+a)*c^2*d^2-13/16/b^
4*a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d^4-15/
8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^2*d^2
-13/16/b^4*a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1
)*d^4-15/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1
)*c^2*d^2-13/32/b^4*a^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x
*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*d^
4-15/16/b^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/
b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*c^2*d^2-1/b*x/
(b*x^4+a)*c^3*d+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)
^(1/4)*x+1)*c^4+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)
^(1/4)*x-1)*c^4+3/32/a^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*
x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*c
^4+1/4/b^4*x*a^3/(b*x^4+a)*d^4+9/4/b^3*a*(a/b)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/b)^(1/4)*x+1)*c*d^3+1/4/b/a*(a/b)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a/b)^(1/4)*x+1)*c^3*d+9/4/b^3*a*(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c*d^3-2/5*d^4/b^3*x^5*a+4/5*d^3/b
^2*x^5*c+1/4*x/a/(b*x^4+a)*c^4+1/4/b/a*(a/b)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x-1)*c^3*d+9/8/b^3*a*(a/b)^(1/4)*2^(1/2)*ln(
(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2
)+(a/b)^(1/2)))*c*d^3+1/8/b/a*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(
1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2
)))*c^3*d
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^4/(b*x^4 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.2572, size = 3083, normalized size = 8.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^4/(b*x^4 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/720*(80*a*b^3*d^4*x^13 + 16*(36*a*b^3*c*d^3 - 13*a^2*b^2*d^4)*x
^9 + 144*(30*a*b^3*c^2*d^2 - 36*a^2*b^2*c*d^3 + 13*a^3*b*d^4)*x^5
 + 180*(a*b^5*x^4 + a^2*b^4)*(-(81*b^16*c^16 + 432*a*b^15*c^15*d
- 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^1
2*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 +
 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^
7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^1
1 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 15777
84*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a
^7*b^17))^(1/4)*arctan(-a^2*b^4*(-(81*b^16*c^16 + 432*a*b^15*c^15
*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*
b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^
6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9
*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*
d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 15
77784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)
/(a^7*b^17))^(1/4)/((3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*c^2*d
^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*x + (3*b^4*c^4 + 4*a*b^3*c^3*d
- 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*sqrt((a^4*b^8
*sqrt(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2
 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b
^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7
+ 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10
*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4
*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 3
16368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17)) + (9*b^8*c^8 +
 24*a*b^7*c^7*d - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110
*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 -
936*a^7*b*c*d^7 + 169*a^8*d^8)*x^2)/(9*b^8*c^8 + 24*a*b^7*c^7*d -
 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4
- 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 - 936*a^7*b*c*d^7 +
 169*a^8*d^8)))) - 45*(a*b^5*x^4 + a^2*b^4)*(-(81*b^16*c^16 + 432
*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3
+ 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*
b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 -
 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a
^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c
```

```
^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 2856
1*a^16*d^16)/(a^7*b^17))^(1/4)*log(a^2*b^4*(-(81*b^16*c^16 + 432*
a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 +
 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b
^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 -
4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^
11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^
3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561
*a^16*d^16)/(a^7*b^17))^(1/4) - (3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a
^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*x) + 45*(a*b^5*x^4
+ a^2*b^4)*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^
14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400
*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^
9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 972391
2*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b
^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^
14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(1/4)*lo
g(-a^2*b^4*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^
14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400
*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^
9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 972391
2*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b
^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^
14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(1/4) -
(3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3
- 13*a^4*d^4)*x) + 180*(b^4*c^4 - 4*a*b^3*c^3*d + 30*a^2*b^2*c^2*
d^2 - 36*a^3*b*c*d^3 + 13*a^4*d^4)*x)/(a*b^5*x^4 + a^2*b^4)
```

---

**Sympy [A]** time = 89.2013, size = 466, normalized size = 1.31

$$\frac{x\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)}{4a^2b^4+4ab^5x^4}$$
$$+\mathrm{RootSum}\Big(65536t^4a^7b^{17}+28561a^{16}d^{16}-316368a^{15}bcd^{15}+1577784a^{14}b^2c^2d^{14}-4651504a^{13}b^3c^3d^{13}+8923164a^{12}b^4c^4d^{12}$$
$$+\frac{d^4x^9}{9b^2}-\frac{x^5\left(2ad^4-4bcd^3\right)}{5b^3}+\frac{x\left(3a^2d^4-8abcd^3+6b^2c^2d^2\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**4+c)**4/(b*x**4+a)**2,x)
```

```
[Out] x*(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3
*c**3*d + b**4*c**4)/(4*a**2*b**4 + 4*a*b**5*x**4) + RootSum(6553
6*_t**4*a**7*b**17 + 28561*a**16*d**16 - 316368*a**15*b*c*d**15 +
 1577784*a**14*b**2*c**2*d**14 - 4651504*a**13*b**3*c**3*d**13 +
8923164*a**12*b**4*c**4*d**12 - 11486160*a**11*b**5*c**5*d**11 +
9723912*a**10*b**6*c**6*d**10 - 4810608*a**9*b**7*c**7*d**9 + 617
958*a**8*b**8*c**8*d**8 + 772112*a**7*b**9*c**9*d**7 - 434808*a**
6*b**10*c**10*d**6 + 20400*a**5*b**11*c**11*d**5 + 45724*a**4*b**
12*c**12*d**4 - 8304*a**3*b**13*c**13*d**3 - 2376*a**2*b**14*c**1
4*d**2 + 432*a*b**15*c**15*d + 81*b**16*c**16, Lambda(_t, _t*log(
```

```
-16*_t*a**2*b**4/(13*a**4*d**4 - 36*a**3*b*c*d**3 + 30*a**2*b**2*
c**2*d**2 - 4*a*b**3*c**3*d - 3*b**4*c**4) + x))) + d**4*x**9/(9*
b**2) - x**5*(2*a*d**4 - 4*b*c*d**3)/(5*b**3) + x*(3*a**2*d**4 -
8*a*b*c*d**3 + 6*b**2*c**2*d**2)/b**4
```

---

**GIAC/XCAS [A]** time = 0.220568, size = 867, normalized size = 2.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^4/(b*x^4 + a)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*c^3
*d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*
d^3 - 13*(a*b^3)^(1/4)*a^4*d^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)
*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^5) + 1/16*sqrt(2)*(3*(a*b^3)^(1
/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*c^3*d - 30*(a*b^3)^(1/4)*a^2*
b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 - 13*(a*b^3)^(1/4)*a^4
*d^4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))
/(a^2*b^5) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1
/4)*a*b^3*c^3*d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(
1/4)*a^3*b*c*d^3 - 13*(a*b^3)^(1/4)*a^4*d^4)*ln(x^2 + sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(a^2*b^5) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)
*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*c^3*d - 30*(a*b^3)^(1/4)*a^2*b^2
*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 - 13*(a*b^3)^(1/4)*a^4*d^
4)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^5) + 1/4*(b
^4*c^4*x - 4*a*b^3*c^3*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^3*
x + a^4*d^4*x)/((b*x^4 + a)*a*b^4) + 1/45*(5*b^16*d^4*x^9 + 36*b^
16*c*d^3*x^5 - 18*a*b^15*d^4*x^5 + 270*b^16*c^2*d^2*x - 360*a*b^1
5*c*d^3*x + 135*a^2*b^14*d^4*x)/b^18
```

## 3.69 $\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx$

**Optimal**. Leaf size=317

$$-\frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}-\frac{3(bc-ad)^2(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}b^{13/4}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{4ab^3(a+bx^4)}+\frac{d^3x^5}{5b^2}$$

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^5)/(5*b^2) + ((b*c - a*d)^3*
x)/(4*a*b^3*(a + b*x^4)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) +
(3*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(
1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) - (3*(b*c - a*d)^2*(b*c + 3*a
*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*S
qrt[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7
/4)*b^(13/4))
```

---

**Rubi [A]** time = 0.611067, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$-\frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}-\frac{3(bc-ad)^2(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}b^{13/4}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{4ab^3(a+bx^4)}+\frac{d^3x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^4)^3/(a + b*x^4)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^5)/(5*b^2) + ((b*c - a*d)^3*
x)/(4*a*b^3*(a + b*x^4)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) +
(3*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(
1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) - (3*(b*c - a*d)^2*(b*c + 3*a
```

```
*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*S
qrt[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7
/4)*b^(13/4))
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-d^2(2ad-3bc)\int\frac{1}{b^3}\,dx+\frac{d^3x^5}{5b^2}-\frac{x(ad-bc)^3}{4ab^3(a+bx^4)}-\frac{3\sqrt{2}(ad-bc)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{32a^{\frac{7}{4}}b^{\frac{13}{4}}}+\frac{3\sqrt{2}(ad-bc)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{32a^{\frac{7}{4}}b^{\frac{13}{4}}}-\frac{3\sqrt{2}(ad-bc)^2(3ad+bc)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{13}{4}}}+\frac{3\sqrt{2}(ad-bc)^2(3ad+bc)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{13}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**4+c)**3/(b*x**4+a)**2,x)
```

```
[Out] -d**2*(2*a*d - 3*b*c)*Integral(b**(-3), x) + d**3*x**5/(5*b**2) -
 x*(a*d - b*c)**3/(4*a*b**3*(a + b*x**4)) - 3*sqrt(2)*(a*d - b*c)
**2*(3*a*d + b*c)*log(-sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sq
rt(b)*x**2)/(32*a**(7/4)*b**(13/4)) + 3*sqrt(2)*(a*d - b*c)**2*(3
*a*d + b*c)*log(sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x
**2)/(32*a**(7/4)*b**(13/4)) - 3*sqrt(2)*(a*d - b*c)**2*(3*a*d +
b*c)*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(16*a**(7/4)*b**(13/4)
) + 3*sqrt(2)*(a*d - b*c)**2*(3*a*d + b*c)*atan(1 + sqrt(2)*b**(1
/4)*x/a**(1/4))/(16*a**(7/4)*b**(13/4))
```

---

**Mathematica [A]** time = 0.399996, size = 301, normalized size = 0.95

$$\frac{-\frac{15\sqrt{2}(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}+\frac{15\sqrt{2}(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}-\frac{30\sqrt{2}(bc-ad)^2(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}}}{160b^{13/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^4)^3/(a + b*x^4)^2,x]
```

```
[Out] (160*b^(1/4)*d^2*(3*b*c - 2*a*d)*x + 32*b^(5/4)*d^3*x^5 + (40*b^(
1/4)*(b*c - a*d)^3*x)/(a*(a + b*x^4)) - (30*Sqrt[2]*(b*c - a*d)^2
```

```
*(b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) +
 (30*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*x)/a^(1/4)])/a^(7/4) - (15*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4)
 + (15*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(160*b^(13/4))
```

---

**Maple [B]** time = 0.001, size = 669, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^4+c)^3/(b*x^4+a)^2,x)
```

```
[Out] 1/5*d^3*x^5/b^2-2*d^3/b^3*a*x+3*d^2/b^2*x*c-1/4/b^3*x*a^2/(b*x^4+
a)*d^3+3/4/b^2*x*a/(b*x^4+a)*c*d^2-3/4/b*x/(b*x^4+a)*c^2*d+1/4*x/
a/(b*x^4+a)*c^3+9/16/b^3*a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
b)^(1/4)*x-1)*d^3-15/16/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/b)^(1/4)*x-1)*c*d^2+3/16/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/b)^(1/4)*x-1)*c^2*d+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(a/b)^(1/4)*x-1)*c^3+9/32/b^3*a*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/
b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^
(1/2)))*d^3-15/32/b^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2
^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*c*d^
2+3/32/b/a*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b
)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*c^2*d+3/32/a^2*
(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x
^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*c^3+9/16/b^3*a*(a/b)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d^3-15/16/b^2*(a/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c*d^2+3/16/b/a*(a/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^2*d+3/16/a^2*(a/b)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^3
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^3/(b*x^4 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.248349, size = 2306, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^3/(b*x^4 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/80*(16*a*b^2*d^3*x^9 + 48*(5*a*b^2*c*d^2 - 3*a^2*b*d^3)*x^5 - 6
0*(a*b^4*x^4 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b
^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5
*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a
^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 -
540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4)*arctan(a^2*b^
3*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*
b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6
*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*
a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81
*a^12*d^12)/(a^7*b^13))^(1/4)/((b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c
*d^2 + 3*a^3*d^3)*x + (b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*
a^3*d^3)*sqrt((a^4*b^6*sqrt(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^
2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*
a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 103
9*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10
 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13)) + (b^6*c^6 + 2*a
*b^5*c^5*d - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c
^2*d^4 - 30*a^5*b*c*d^5 + 9*a^6*d^6)*x^2)/(b^6*c^6 + 2*a*b^5*c^5*
d - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 -
30*a^5*b*c*d^5 + 9*a^6*d^6)))) + 15*(a*b^4*x^4 + a^2*b^3)*(-(b^12
*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d
^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*
d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c
^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^1
2)/(a^7*b^13))^(1/4)*log(3*a^2*b^3*(-(b^12*c^12 + 4*a*b^11*c^11*d
 - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^
4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d
^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*
c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4) +
3*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*x) - 15*(a*
b^4*x^4 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c
^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*
c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^
4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a
^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4)*log(-3*a^2*b^3*(-(
b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c
^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*
c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b
^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12
*d^12)/(a^7*b^13))^(1/4) + 3*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d
^2 + 3*a^3*d^3)*x) + 20*(b^3*c^3 - 3*a*b^2*c^2*d + 15*a^2*b*c*d^2
 - 9*a^3*d^3)*x)/(a*b^4*x^4 + a^2*b^3)
```

**Sympy [A]** time = 20.5834, size = 335, normalized size = 1.06

$$-\frac{x\left(a^{3}d^{3}-3a^{2}bcd^{2}+3ab^{2}c^{2}d-b^{3}c^{3}\right)}{4a^{2}b^{3}+4ab^{4}x^{4}}+\operatorname{RootSum}\left(65536t^{4}a^{7}b^{13}+6561a^{12}d^{12}-43740a^{11}bcd^{11}+118098a^{10}b^{2}c^{2}d^{10}-156492a^{9}b^{3}c^{3}d^{9}+84159a^{8}b^{4}c^{4}d^{8}+26568a^{7}\right.$$

$$+\frac{d^{3}x^{5}}{5b^{2}}-\frac{x\left(2ad^{3}-3bcd^{2}\right)}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**4+c)**3/(b*x**4+a)**2,x)
```

```
[Out] -x*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(4
*a**2*b**3 + 4*a*b**4*x**4) + RootSum(65536*_t**4*a**7*b**13 + 65
61*a**12*d**12 - 43740*a**11*b*c*d**11 + 118098*a**10*b**2*c**2*d
**10 - 156492*a**9*b**3*c**3*d**9 + 84159*a**8*b**4*c**4*d**8 + 2
6568*a**7*b**5*c**5*d**7 - 52164*a**6*b**6*c**6*d**6 + 11016*a**5
*b**7*c**7*d**5 + 10287*a**4*b**8*c**8*d**4 - 3564*a**3*b**9*c**9
*d**3 - 1134*a**2*b**10*c**10*d**2 + 324*a*b**11*c**11*d + 81*b**
12*c**12, Lambda(_t, _t*log(16*_t*a**2*b**3/(9*a**3*d**3 - 15*a**
2*b*c*d**2 + 3*a*b**2*c**2*d + 3*b**3*c**3) + x))) + d**3*x**5/(5
*b**2) - x*(2*a*d**3 - 3*b*c*d**2)/b**3
```

---

**GIAC/XCAS [A]** time = 0.221806, size = 670, normalized size = 2.11

$$\frac{3\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}+\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d-5\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+3\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^{2}b^{4}}+\frac{3\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}+\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d-5\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+3\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^{2}b^{4}}+\frac{3\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}+\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d-5\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+3\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\ln\left(x^{2}+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^{2}b^{4}}-\frac{3\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}+\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d-5\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+3\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\ln\left(x^{2}-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^{2}b^{4}}+\frac{b^{3}c^{3}x-3ab^{2}c^{2}dx+3a^{2}bcd^{2}x-a^{3}d^{3}x}{4\left(bx^{4}+a\right)ab^{3}}+\frac{b^{8}d^{3}x^{5}+15b^{8}cd^{2}x-10ab^{7}d^{3}x}{5b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^3/(b*x^4 + a)^2,x, algorithm="giac")
```

```
[Out] 3/16*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d -
 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/
2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 3/
16*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5
*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 3/32
*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(
a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*ln(x^2 + sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4) - 3/32*sqrt(2)*((a*b^3)^
(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b
*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4)
+ sqrt(a/b))/(a^2*b^4) + 1/4*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2
*b*c*d^2*x - a^3*d^3*x)/((b*x^4 + a)*a*b^3) + 1/5*(b^8*d^3*x^5 +
15*b^8*c*d^2*x - 10*a*b^7*d^3*x)/b^10
```

# 3.70 $\int \frac{(c+dx^4)^2}{(a+bx^4)^2} dx$

**Optimal**. Leaf size=291

$$-\frac{(bc-ad)(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}}+\frac{(bc-ad)(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}}-\frac{(bc-ad)(5ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{9/4}}+\frac{(bc-ad)(5ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}b^{9/4}}+\frac{x(bc-ad)^2}{4ab^2(a+bx^4)}+\frac{d^2x}{b^2}$$

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(4*a*b^2*(a + b*x^4)) - ((b*c - a
*d)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*S
qrt[2]*a^(7/4)*b^(9/4)) + ((b*c - a*d)*(3*b*c + 5*a*d)*ArcTan[1 +
 (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(9/4)) - ((b*
c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x
+ Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4)) + ((b*c - a*d)*(3*b*
c + 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(16*Sqrt[2]*a^(7/4)*b^(9/4))
```

---

**Rubi [A]** time = 0.724132, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$-\frac{(bc-ad)(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}}+\frac{(bc-ad)(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}}-\frac{(bc-ad)(5ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{9/4}}+\frac{(bc-ad)(5ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}b^{9/4}}+\frac{x(bc-ad)^2}{4ab^2(a+bx^4)}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^4)^2/(a + b*x^4)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(4*a*b^2*(a + b*x^4)) - ((b*c - a
*d)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*S
qrt[2]*a^(7/4)*b^(9/4)) + ((b*c - a*d)*(3*b*c + 5*a*d)*ArcTan[1 +
 (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(9/4)) - ((b*
c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x
+ Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4)) + ((b*c - a*d)*(3*b*
```

```
c + 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(16*Sqrt[2]*a^(7/4)*b^(9/4))
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$d^2\int\frac{1}{b^2}\,dx+\frac{x\,(ad-bc)^2}{4ab^2\,(a+bx^4)}+\frac{\sqrt{2}\,(ad-bc)\,(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{32a^{\frac{7}{4}}b^{\frac{9}{4}}}-\frac{\sqrt{2}\,(ad-bc)\,(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{32a^{\frac{7}{4}}b^{\frac{9}{4}}}+\frac{\sqrt{2}\,(ad-bc)\,(5ad+3bc)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{9}{4}}}-\frac{\sqrt{2}\,(ad-bc)\,(5ad+3bc)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**4+c)**2/(b*x**4+a)**2,x)
```

```
[Out] d**2*Integral(b**(-2), x) + x*(a*d - b*c)**2/(4*a*b**2*(a + b*x**
4)) + sqrt(2)*(a*d - b*c)*(5*a*d + 3*b*c)*log(-sqrt(2)*a**(1/4)*b
**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(32*a**(7/4)*b**(9/4)) - sqrt
(2)*(a*d - b*c)*(5*a*d + 3*b*c)*log(sqrt(2)*a**(1/4)*b**(1/4)*x +
 sqrt(a) + sqrt(b)*x**2)/(32*a**(7/4)*b**(9/4)) + sqrt(2)*(a*d -
b*c)*(5*a*d + 3*b*c)*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(16*a*
*(7/4)*b**(9/4)) - sqrt(2)*(a*d - b*c)*(5*a*d + 3*b*c)*atan(1 + s
qrt(2)*b**(1/4)*x/a**(1/4))/(16*a**(7/4)*b**(9/4))
```

---

**Mathematica [A]** time = 0.299828, size = 297, normalized size = 1.02

$$\frac{\frac{\sqrt{2}(5a^2d^2-2abcd-3b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}+\frac{\sqrt{2}(-5a^2d^2+2abcd+3b^2c^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}+\frac{2\sqrt{2}(5a^2d^2-2abcd-3b^2c^2)\tan^{-1}\left(\right.}{a^{7/4}}}{32b^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^4)^2/(a + b*x^4)^2,x]
```

```
[Out] (32*b^(1/4)*d^2*x + (8*b^(1/4)*(b*c - a*d)^2*x)/(a*(a + b*x^4)) +
 (2*Sqrt[2]*(-3*b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (2*Sqrt[2]*(3*b^2*c^2 + 2*a*b*
c*d - 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4)
 + (Sqrt[2]*(-3*b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (Sqrt[2]*(3*b^2
*c^2 + 2*a*b*c*d - 5*a^2*d^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
```

```
4)*x + Sqrt[b]*x^2])/a^(7/4))/(32*b^(9/4))
```

**Maple [B]** time = 0.014, size = 475, normalized size = 1.6

$$\frac{d^2x}{b^2}+\frac{axd^2}{4\,b^2\,(bx^4+a)}-\frac{cxd}{2\,b\,(bx^4+a)}+\frac{xc^2}{4\,a\,(bx^4+a)}$$
$$-\frac{5\,\sqrt{2}d^2}{16\,b^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{\sqrt{2}cd}{8\,ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)$$
$$+\frac{3\,\sqrt{2}c^2}{16\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{5\,\sqrt{2}d^2}{16\,b^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$+\frac{\sqrt{2}cd}{8\,ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{3\,\sqrt{2}c^2}{16\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$-\frac{5\,\sqrt{2}d^2}{32\,b^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{\sqrt{2}cd}{16\,ab}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{3\,\sqrt{2}c^2}{32\,a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^4+c)^2/(b*x^4+a)^2,x)
```

```
[Out] d^2*x/b^2+1/4/b^2*x*a/(b*x^4+a)*d^2-1/2/b*x/(b*x^4+a)*c*d+1/4*x/a
/(b*x^4+a)*c^2-5/16/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^
(1/4)*x+1)*d^2+1/8/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(
1/4)*x+1)*c*d+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(
1/4)*x+1)*c^2-5/16/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(
1/4)*x-1)*d^2+1/8/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x-1)*c*d+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x-1)*c^2-5/32/b^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2
^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*d^2+
1/16/b/a*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^
(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*c*d+3/32/a^2*(a/b
)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(
a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*c^2
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^2/(b*x^4 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.245341, size = 1592, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^2/(b*x^4 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(16*a*b*d^2*x^5 + 4*(a*b^3*x^4 + a^2*b^2)*(-(81*b^8*c^8 + 21
6*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a
^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 100
0*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(1/4)*arctan(-a^2*b^2*(-(
81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*
c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^
2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(1/4)/((3*
b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*x + (3*b^2*c^2 + 2*a*b*c*d - 5*a
^2*d^2)*sqrt((a^4*b^4*sqrt(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a
^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640
*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a
^8*d^8)/(a^7*b^9)) + (9*b^4*c^4 + 12*a*b^3*c^3*d - 26*a^2*b^2*c^2
*d^2 - 20*a^3*b*c*d^3 + 25*a^4*d^4)*x^2)/(9*b^4*c^4 + 12*a*b^3*c^
3*d - 26*a^2*b^2*c^2*d^2 - 20*a^3*b*c*d^3 + 25*a^4*d^4)))) - (a*b
^3*x^4 + a^2*b^2)*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c
^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3
*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/
(a^7*b^9))^(1/4)*log(a^2*b^2*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 32
4*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1
640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 62
5*a^8*d^8)/(a^7*b^9))^(1/4) - (3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)
*x) + (a*b^3*x^4 + a^2*b^2)*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324
*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 16
40*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625
*a^8*d^8)/(a^7*b^9))^(1/4)*log(-a^2*b^2*(-(81*b^8*c^8 + 216*a*b^7
*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*
c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b
*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(1/4) - (3*b^2*c^2 + 2*a*b*c*d -
 5*a^2*d^2)*x) + 4*(b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*x)/(a*b^3*x^
4 + a^2*b^2)
```

---

**Sympy [A]** time = 7.23865, size = 219, normalized size = 0.75

$$\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{4a^2b^2+4ab^3x^4}$$
$$+\mathrm{RootSum}\left(65536t^4a^7b^9+625a^8d^8-1000a^7bcd^7-900a^6b^2c^2d^6+1640a^5b^3c^3d^5+646a^4b^4c^4d^4-984a^3b^5c^5d^3-324a^2b^6c^6\right.$$
$$+\frac{d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**4+c)**2/(b*x**4+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(4*a**2*b**2 + 4*a*b**3*x**
4) + RootSum(65536*_t**4*a**7*b**9 + 625*a**8*d**8 - 1000*a**7*b*
c*d**7 - 900*a**6*b**2*c**2*d**6 + 1640*a**5*b**3*c**3*d**5 + 646
*a**4*b**4*c**4*d**4 - 984*a**3*b**5*c**5*d**3 - 324*a**2*b**6*c*
*6*d**2 + 216*a*b**7*c**7*d + 81*b**8*c**8, Lambda(_t, _t*log(-16
*_t*a**2*b**2/(5*a**2*d**2 - 2*a*b*c*d - 3*b**2*c**2) + x))) + d*
*2*x/b**2
```

---

**GIAC/XCAS [A]** time = 0.222079, size = 508, normalized size = 1.75

$$\frac{d^2x}{b^2}+\frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac14}b^2c^2+2\left(ab^3\right)^{\frac14}abcd-5\left(ab^3\right)^{\frac14}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{16\,a^2b^3}$$
$$+\frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac14}b^2c^2+2\left(ab^3\right)^{\frac14}abcd-5\left(ab^3\right)^{\frac14}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{16\,a^2b^3}$$
$$+\frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac14}b^2c^2+2\left(ab^3\right)^{\frac14}abcd-5\left(ab^3\right)^{\frac14}a^2d^2\right)\ln\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$
$$-\frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac14}b^2c^2+2\left(ab^3\right)^{\frac14}abcd-5\left(ab^3\right)^{\frac14}a^2d^2\right)\ln\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$
$$+\frac{b^2c^2x-2\,abcdx+a^2d^2x}{4\left(bx^4+a\right)ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^2/(b*x^4 + a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 + 2*(a*b^3)^(1/
4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + s
```

```
qrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(3*(a*b
^3)^(1/4)*b^2*c^2 + 2*(a*b^3)^(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2
*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))
/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 + 2*(a*b^3)^(1
/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*ln(x^2 + sqrt(2)*x*(a/b)^(
1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c
^2 + 2*(a*b^3)^(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*ln(x^2 -
sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) + 1/4*(b^2*c^2*x - 2
*a*b*c*d*x + a^2*d^2*x)/((b*x^4 + a)*a*b^2)
```

## 3.71 $\int \frac{c+dx^4}{(a+bx^4)^2}\,dx$

**Optimal**. Leaf size=245

$$-\frac{(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}+\frac{(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}-\frac{(ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{x(bc-ad)}{4ab\,(a+bx^4)}$$

```
[Out] ((b*c - a*d)*x)/(4*a*b*(a + b*x^4)) - ((3*b*c + a*d)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*
c + a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7
/4)*b^(5/4)) - ((3*b*c + a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*c + a*d
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqr
t[2]*a^(7/4)*b^(5/4))
```

---

**Rubi [A]** time = 0.316828, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$

$$-\frac{(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}+\frac{(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}-\frac{(ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{x(bc-ad)}{4ab\,(a+bx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^4)/(a + b*x^4)^2,x]
```


```
[Out] ((b*c - a*d)*x)/(4*a*b*(a + b*x^4)) - ((3*b*c + a*d)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*
c + a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7
/4)*b^(5/4)) - ((3*b*c + a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*c + a*d
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqr
t[2]*a^(7/4)*b^(5/4))
```


---

**Rubi in Sympy [A]** time = 62.0599, size = 226, normalized size = 0.92

$$-\frac{x\,(ad-bc)}{4ab\,(a+bx^4)}-\frac{\sqrt{2}\,(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{32a^{\frac{7}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\,(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{32a^{\frac{7}{4}}b^{\frac{5}{4}}}-\frac{\sqrt{2}\,(ad+3bc)\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\,(ad+3bc)\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**4+c)/(b*x**4+a)**2,x)
```

```
[Out] -x*(a*d - b*c)/(4*a*b*(a + b*x**4)) - sqrt(2)*(a*d + 3*b*c)*log(-
sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(32*a**(7/4
)*b**(5/4)) + sqrt(2)*(a*d + 3*b*c)*log(sqrt(2)*a**(1/4)*b**(1/4)
*x + sqrt(a) + sqrt(b)*x**2)/(32*a**(7/4)*b**(5/4)) - sqrt(2)*(a*
d + 3*b*c)*atan(1 - sqrt(2)*b**(1/4)*x/a**(1/4))/(16*a**(7/4)*b**
(5/4)) + sqrt(2)*(a*d + 3*b*c)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/
4))/(16*a**(7/4)*b**(5/4))
```

---

**Mathematica [A]** time = 0.30402, size = 212, normalized size = 0.87

$$\frac{-\frac{8a^{3/4}\sqrt[4]{b}x(ad-bc)}{a+bx^4}-\sqrt{2}(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+\sqrt{2}(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-2\sqrt{2}(ad\ \cdots}{32a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^4)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*b^(1/4)*(-(b*c) + a*d)*x)/(a + b*x^4) - 2*Sqrt[2]*(3
*b*c + a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*(
3*b*c + a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*(3
*b*c + a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
] + Sqrt[2]*(3*b*c + a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2])/(32*a^(7/4)*b^(5/4))
```

---

**Maple [A]** time = 0.011, size = 295, normalized size = 1.2

$$-\frac{(ad-bc)x}{4ab(bx^4+a)}+\frac{\sqrt{2}d}{16ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{3\sqrt{2}c}{16a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{\sqrt{2}d}{16ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{3\sqrt{2}c}{16a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{\sqrt{2}d}{32ab}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{3\sqrt{2}c}{32a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^4+c)/(b*x^4+a)^2,x)
```

```
[Out] -1/4*(a*d-b*c)/a/b*x/(b*x^4+a)+1/16/a/b*(a/b)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(a/b)^(1/4)*x+1)*d+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/b)^(1/4)*x+1)*c+1/16/a/b*(a/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x-1)*d+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/b)^(1/4)*x-1)*c+1/32/a/b*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)
^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1
/2)))*d+3/32/a^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2
)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*c
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)/(b*x^4 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.229541, size = 838, normalized size = 3.42

$$-\frac{4\left(ab^2x^4+a^2b\right)\left(-\frac{81\,b^4c^4+108\,ab^3c^3d+54\,a^2b^2c^2d^2+12\,a^3bcd^3+a^4d^4}{a^7b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{a^2b\left(-\frac{81\,b^4c^4+108\,ab^3c^3d+54\,a^2b^2c^2d^2+12\,a^3b}{a^7b^5}\right.}{(3\,bc+ad)x+(3\,bc+ad)\sqrt{\frac{a^4b^2\sqrt{-\frac{81\,b^4c^4+108\,ab^3c^3d+54\,a^2b^2c^2d^2+12}{a^7b^5}}}{9\,b^2c^2+6\,abc}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)/(b*x^4 + a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*(a*b^2*x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54
*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*arc
tan(a^2*b*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 +
12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)/((3*b*c + a*d)*x + (3*
b*c + a*d)*sqrt((a^4*b^2*sqrt(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54
*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5)) + (9*b^2*
c^2 + 6*a*b*c*d + a^2*d^2)*x^2)/(9*b^2*c^2 + 6*a*b*c*d + a^2*d^2)
))) - (a*b^2*x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^
2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*log(a^
2*b*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3
*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4) + (3*b*c + a*d)*x) + (a*b^2*
x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2
 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*log(-a^2*b*(-(81*b^
4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a
^4*d^4)/(a^7*b^5))^(1/4) + (3*b*c + a*d)*x) - 4*(b*c - a*d)*x)/(a
*b^2*x^4 + a^2*b)
```

**Sympy [A]** time = 3.40875, size = 112, normalized size = 0.46

$$-\frac{x\left(ad-bc\right)}{4a^2b+4ab^2x^4}+\operatorname{RootSum}\left(65536t^4a^7b^5+a^4d^4+12a^3bcd^3+54a^2b^2c^2d^2+108ab^3c^3d+81b^4c^4,\left(t\mapsto t\log\left(\frac{16ta^2b}{ad+3bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**4+c)/(b*x**4+a)**2,x)
```

```
[Out] -x*(a*d - b*c)/(4*a**2*b + 4*a*b**2*x**4) + RootSum(65536*_t**4*a
**7*b**5 + a**4*d**4 + 12*a**3*b*c*d**3 + 54*a**2*b**2*c**2*d**2
+ 108*a*b**3*c**3*d + 81*b**4*c**4, Lambda(_t, _t*log(16*_t*a**2*
b/(a*d + 3*b*c) + x)))
```

**GIAC/XCAS [A]** time = 0.220218, size = 359, normalized size = 1.47

$$\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}bc+\left(ab^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^{2}b^{2}}+\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}bc+\left(ab^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^{2}b^{2}}+\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}bc+\left(ab^{3}\right)^{\frac{1}{4}}ad\right)\ln\left(x^{2}+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^{2}b^{2}}-\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}bc+\left(ab^{3}\right)^{\frac{1}{4}}ad\right)\ln\left(x^{2}-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^{2}b^{2}}+\frac{bcx-adx}{4\left(bx^{4}+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)/(b*x^4 + a)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*arctan(1/2
*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^2) + 1/1
6*sqrt(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^2) + 1/32*s
qrt(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*ln(x^2 + sqrt(2)
*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^2) - 1/32*sqrt(2)*(3*(a*b^3)^(
1/4)*b*c + (a*b^3)^(1/4)*a*d)*ln(x^2 - sqrt(2)*x*(a/b)^(1/4) + sq
rt(a/b))/(a^2*b^2) + 1/4*(b*c*x - a*d*x)/((b*x^4 + a)*a*b)
```

## 3.72 $\int \frac{1}{(a+bx^4)^2(c+dx^4)} \, dx$

**Optimal**. Leaf size=513

$$-\frac{b^{3/4}(3bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2}+\frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{b^{3/4}(3bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^2}+\frac{b^{3/4}(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{d^{7/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)^2}+\frac{d^{7/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)^2}-\frac{d^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)^2}+\frac{d^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{3/4}(bc-ad)^2}+\frac{bx}{4a\left(a+bx^4\right)(bc-ad)}$$

```
[Out] (b*x)/(4*a*(b*c - a*d)*(a + b*x^4)) - (b^(3/4)*(3*b*c - 7*a*d)*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c -
a*d)^2) + (b^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (d^(7/4)*ArcTan[1
- (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)^2)
 + (d^(7/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c
^(3/4)*(b*c - a*d)^2) - (b^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c
- a*d)^2) + (b^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) -
(d^(7/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/
(4*Sqrt[2]*c^(3/4)*(b*c - a*d)^2) + (d^(7/4)*Log[Sqrt[c] + Sqrt[2
]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d
)^2)
```

---

**Rubi [A]** time = 0.902967, antiderivative size = 513, normalized size of antiderivative = 1., number

of steps used = 20, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$-\frac{b^{3/4}(3bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2}+\frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{b^{3/4}(3bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^2}+\frac{b^{3/4}(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{d^{7/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)^2}+\frac{d^{7/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)^2}-\frac{d^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)^2}+\frac{d^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{3/4}(bc-ad)^2}+\frac{bx}{4a(a+bx^4)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^4)^2*(c + d*x^4)),x]
```

```
[Out] (b*x)/(4*a*(b*c - a*d)*(a + b*x^4)) - (b^(3/4)*(3*b*c - 7*a*d)*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c -
a*d)^2) + (b^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (d^(7/4)*ArcTan[1
- (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)^2)
 + (d^(7/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c
^(3/4)*(b*c - a*d)^2) - (b^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c
- a*d)^2) + (b^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) -
(d^(7/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/
(4*Sqrt[2]*c^(3/4)*(b*c - a*d)^2) + (d^(7/4)*Log[Sqrt[c] + Sqrt[2
]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d
)^2)
```

**Rubi in Sympy [A]** time = 168.513, size = 474, normalized size = 0.92

$$-\frac{\sqrt{2}d^{\frac{7}{4}}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^{2}\right)}{8c^{\frac{3}{4}}(ad-bc)^{2}}+\frac{\sqrt{2}d^{\frac{7}{4}}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^{2}\right)}{8c^{\frac{3}{4}}(ad-bc)^{2}}$$
$$-\frac{\sqrt{2}d^{\frac{7}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{4c^{\frac{3}{4}}(ad-bc)^{2}}+\frac{\sqrt{2}d^{\frac{7}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{4c^{\frac{3}{4}}(ad-bc)^{2}}$$
$$-\frac{bx}{4a(a+bx^{4})(ad-bc)}+\frac{\sqrt{2}b^{\frac{3}{4}}(7ad-3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^{2}\right)}{32a^{\frac{7}{4}}(ad-bc)^{2}}$$
$$-\frac{\sqrt{2}b^{\frac{3}{4}}(7ad-3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^{2}\right)}{32a^{\frac{7}{4}}(ad-bc)^{2}}$$
$$+\frac{\sqrt{2}b^{\frac{3}{4}}(7ad-3bc)\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}(ad-bc)^{2}}-\frac{\sqrt{2}b^{\frac{3}{4}}(7ad-3bc)\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{16a^{\frac{7}{4}}(ad-bc)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**4+a)**2/(d*x**4+c),x)
```

```
[Out] -sqrt(2)*d**(7/4)*log(-sqrt(2)*c**(1/4)*d**(1/4)*x + sqrt(c) + sq
rt(d)*x**2)/(8*c**(3/4)*(a*d - b*c)**2) + sqrt(2)*d**(7/4)*log(sq
rt(2)*c**(1/4)*d**(1/4)*x + sqrt(c) + sqrt(d)*x**2)/(8*c**(3/4)*(
a*d - b*c)**2) - sqrt(2)*d**(7/4)*atan(1 - sqrt(2)*d**(1/4)*x/c**
(1/4))/(4*c**(3/4)*(a*d - b*c)**2) + sqrt(2)*d**(7/4)*atan(1 + sq
rt(2)*d**(1/4)*x/c**(1/4))/(4*c**(3/4)*(a*d - b*c)**2) - b*x/(4*a
*(a + b*x**4)*(a*d - b*c)) + sqrt(2)*b**(3/4)*(7*a*d - 3*b*c)*log
(-sqrt(2)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(32*a**(7
/4)*(a*d - b*c)**2) - sqrt(2)*b**(3/4)*(7*a*d - 3*b*c)*log(sqrt(2
)*a**(1/4)*b**(1/4)*x + sqrt(a) + sqrt(b)*x**2)/(32*a**(7/4)*(a*d
 - b*c)**2) + sqrt(2)*b**(3/4)*(7*a*d - 3*b*c)*atan(1 - sqrt(2)*b
**(1/4)*x/a**(1/4))/(16*a**(7/4)*(a*d - b*c)**2) - sqrt(2)*b**(3/
4)*(7*a*d - 3*b*c)*atan(1 + sqrt(2)*b**(1/4)*x/a**(1/4))/(16*a**(
7/4)*(a*d - b*c)**2)
```

---

**Mathematica [A]** time = 0.544791, size = 499, normalized size = 0.97

$$8a^{3/4}bc^{3/4}x(bc-ad)-8\sqrt{2}a^{7/4}d^{7/4}\left(a+bx^{4}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+8\sqrt{2}a^{7/4}d^{7/4}\left(a+bx^{4}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)-4\sqrt{2}a^{7/4}d$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^4)^2*(c + d*x^4)),x]
```

```
[Out] (8*a^(3/4)*b*c^(3/4)*(b*c - a*d)*x - 2*Sqrt[2]*b^(3/4)*c^(3/4)*(3
*b*c - 7*a*d)*(a + b*x^4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
 + 2*Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a*d)*(a + b*x^4)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 8*Sqrt[2]*a^(7/4)*d^(7/4)*(a +
b*x^4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 8*Sqrt[2]*a^(7/4
)*d^(7/4)*(a + b*x^4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - S
qrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a*d)*(a + b*x^4)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(3/4)*c^(3/4
)*(3*b*c - 7*a*d)*(a + b*x^4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2] - 4*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*Log[S
qrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + 4*Sqrt[2]*a^(
7/4)*d^(7/4)*(a + b*x^4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2])/(32*a^(7/4)*c^(3/4)*(b*c - a*d)^2*(a + b*x^4))
```

---

**Maple [A]** time = 0.018, size = 550, normalized size = 1.1

$$\frac{d^2\sqrt{2}}{8\,(ad-bc)^2\,c}\sqrt[4]{\frac{c}{d}}\ln\left(1\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)$$
$$+\frac{d^2\sqrt{2}}{4\,(ad-bc)^2\,c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{d^2\sqrt{2}}{4\,(ad-bc)^2\,c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)$$
$$-\frac{bxd}{4\,(ad-bc)^2\,(bx^4+a)}+\frac{b^2xc}{4\,(ad-bc)^2\,a\,(bx^4+a)}$$
$$-\frac{7\,b\sqrt{2}d}{16\,(ad-bc)^2\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{3\,b^2\sqrt{2}c}{16\,(ad-bc)^2\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)$$
$$-\frac{7\,b\sqrt{2}d}{16\,(ad-bc)^2\,a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{3\,b^2\sqrt{2}c}{16\,(ad-bc)^2\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$
$$-\frac{7\,b\sqrt{2}d}{32\,(ad-bc)^2\,a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$
$$+\frac{3\,b^2\sqrt{2}c}{32\,(ad-bc)^2\,a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x^2+\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x^2-\sqrt[4]{\frac{a}{b}}x\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^4+a)^2/(d*x^4+c),x)
```

```
[Out] 1/8*d^2/(a*d-b*c)^2*(c/d)^(1/4)/c*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2
^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+1/4*
d^2/(a*d-b*c)^2*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*
x+1)+1/4*d^2/(a*d-b*c)^2*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/
```

```
d)^(1/4)*x-1)-1/4*b/(a*d-b*c)^2*x/(b*x^4+a)*d+1/4*b^2/(a*d-b*c)^2
*x/a/(b*x^4+a)*c-7/16*b/(a*d-b*c)^2/a*(a/b)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/b)^(1/4)*x+1)*d+3/16*b^2/(a*d-b*c)^2/a^2*(a/b)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c-7/16*b/(a*d-b*c)^2/a*(a/
b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*d+3/16*b^2/(a*d-
b*c)^2/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c-
7/32*b/(a*d-b*c)^2/a*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^
(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*d+3/3
2*b^2/(a*d-b*c)^2/a^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2
^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*c
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^2*(d*x^4 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 22.3349, size = 3665, normalized size = 7.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^2*(d*x^4 + c)),x, algorithm="fricas")
```

```
[Out] -1/16*(16*(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 -
 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 2
8*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2
*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*arctan((-d^7/(b^8*c^11 - 8*a
*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^
4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4
*d^7 + a^8*c^3*d^8))^(1/4)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)/(d
^2*x + d^2*sqrt((d^4*x^2 + (b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c
^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(-d^7/(b^8*c^11 - 8*a
*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^
4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4
*d^7 + a^8*c^3*d^8)))/d^4))) - 4*((a*b^2*c - a^2*b*d)*x^4 + a^2*b
*c - a^3*d)*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^
2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b
^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4
*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c
*d^7 + a^15*d^8))^(1/4)*arctan(-(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*
d^2)*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 411
```

```
6*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*
d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^
4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 +
a^15*d^8))^(1/4)/((3*b^2*c - 7*a*b*d)*x + (3*b^2*c - 7*a*b*d)*sqr
t(((9*b^4*c^2 - 42*a*b^3*c*d + 49*a^2*b^2*d^2)*x^2 + (a^4*b^4*c^4
 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4)
*sqrt(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 411
6*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*
d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^
4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 +
a^15*d^8)))/(9*b^4*c^2 - 42*a*b^3*c*d + 49*a^2*b^2*d^2)))) - 4*(-
d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*
c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^
5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d
)*x^4 + a^2*b*c - a^3*d)*log(d^2*x + (-d^7/(b^8*c^11 - 8*a*b^7*c^
10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d
^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 +
a^8*c^3*d^8))^(1/4)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)) + 4*(-d^
7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^
8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*
d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d)*
x^4 + a^2*b*c - a^3*d)*log(d^2*x - (-d^7/(b^8*c^11 - 8*a*b^7*c^10
*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4
 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^
8*c^3*d^8))^(1/4)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)) + ((a*b^2*
c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*(-(81*b^7*c^4 - 756*a*b^6*c^3
*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4
)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b
^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*
b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)*log(-(3*b^2*c - 7
*a*b*d)*x + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*(-(81*b^7*c^4 -
 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 24
01*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d
^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*
d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)) -
((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*(-(81*b^7*c^4 - 756*a
*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4
*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 5
6*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 +
28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)*log(-(3*b
^2*c - 7*a*b*d)*x - (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*(-(81*b
^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*
d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b
^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*
b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(
1/4)) - 4*b*x)/((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**4+a)**2/(d*x**4+c),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [A]** time = 0.227731, size = 900, normalized size = 1.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^2*(d*x^4 + c)),x, algorithm="giac")
```

```
[Out] 1/2*(c*d^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4)
)/(c/d)^(1/4))/(sqrt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(2)*a
^2*c*d^2) + 1/2*(c*d^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x - sqrt(2)
*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d
 + sqrt(2)*a^2*c*d^2) + 1/4*(c*d^3)^(1/4)*d*ln(x^2 + sqrt(2)*x*(c
/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + s
qrt(2)*a^2*c*d^2) - 1/4*(c*d^3)^(1/4)*d*ln(x^2 - sqrt(2)*x*(c/d)^
(1/4) + sqrt(c/d))/(sqrt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(
2)*a^2*c*d^2) + 1/8*(3*(a*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*a
rctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(
2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/8*(3*
(a*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x
- sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt
(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/16*(3*(a*b^3)^(1/4)*b*c - 7*
(a*b^3)^(1/4)*a*d)*ln(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(s
qrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - 1/1
6*(3*(a*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*ln(x^2 - sqrt(2)*x*
(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c
*d + sqrt(2)*a^4*d^2) + 1/4*b*x/((b*x^4 + a)*(a*b*c - a^2*d))
```

## 3.73 $\int \frac{1}{(a+bx^4)^2(c+dx^4)^2}\,dx$

**Optimal**. Leaf size=596

$$-\frac{b^{7/4}(3bc-11ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{b^{7/4}(3bc-11ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{d^{7/4}(11bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^3}+\frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^3}-\frac{d^{7/4}(11bc-3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3}+\frac{d^{7/4}(11bc-3ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3}+\frac{bx}{4a(a+bx^4)(c+dx^4)(bc-ad)}+\frac{dx(ad+bc)}{4ac(c+dx^4)(bc-ad)^2}$$

```
[Out] (d*(b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^4)) + (b*x)/(4*a*
(b*c - a*d)*(a + b*x^4)*(c + d*x^4)) - (b^(7/4)*(3*b*c - 11*a*d)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c
- a*d)^3) + (b^(7/4)*(3*b*c - 11*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(11*b*
c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^
(7/4)*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 + (Sqrt
[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) - (b^(
7/4)*(3*b*c - 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) + (b^(7/4)*(3*b*c
 - 11*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*
Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[
2]*c^(7/4)*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c]
 + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*
(b*c - a*d)^3)
```

---

**Rubi [A]** time = 1.46784, antiderivative size = 596, normalized size of antiderivative = 1., number of

steps used = 21, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$

$$-\frac{b^{7/4}(3bc-11ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{b^{7/4}(3bc-11ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{d^{7/4}(11bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^3}+\frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^3}-\frac{d^{7/4}(11bc-3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3}+\frac{d^{7/4}(11bc-3ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3}+\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}+\frac{dx(ad+bc)}{4ac\left(c+dx^4\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^4)^2*(c + d*x^4)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^4)) + (b*x)/(4*a*
(b*c - a*d)*(a + b*x^4)*(c + d*x^4)) - (b^(7/4)*(3*b*c - 11*a*d)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c
- a*d)^3) + (b^(7/4)*(3*b*c - 11*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(11*b*
c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^
(7/4)*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 + (Sqrt
[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) - (b^(
7/4)*(3*b*c - 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) + (b^(7/4)*(3*b*c
 - 11*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*
Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[
2]*c^(7/4)*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c]
 + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*
(b*c - a*d)^3)
```

---

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**4+a)**2/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

---

**Mathematica [A]** time = 3.49316, size = 561, normalized size = 0.94

$$\frac{1}{32}\left(\frac{\sqrt{2}b^{7/4}(11ad-3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}(bc-ad)^3}+\frac{\sqrt{2}b^{7/4}(11ad-3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}(ad-bc)^3}+\frac{2\sqrt{2}b^{7/4}(11ad-3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}(bc-ad)^3}+\frac{2\sqrt{2}b^{7/4}(11ad-3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{a^{7/4}(ad-bc)^3}+\frac{8b^2x}{a\left(a+bx^4\right)(bc-ad)^2}+\frac{\sqrt{2}d^{7/4}(11bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{c^{7/4}(ad-bc)^3}+\frac{\sqrt{2}d^{7/4}(11bc-3ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{c^{7/4}(bc-ad)^3}+\frac{2\sqrt{2}d^{7/4}(3ad-11bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{7/4}(bc-ad)^3}+\frac{2\sqrt{2}d^{7/4}(11bc-3ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{c^{7/4}(bc-ad)^3}+\frac{8d^2x}{c\left(c+dx^4\right)(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^4)^2*(c + d*x^4)^2),x]
```

```
[Out] ((8*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^4)) + (8*d^2*x)/(c*(b*c - a*
d)^2*(c + d*x^4)) + (2*Sqrt[2]*b^(7/4)*(-3*b*c + 11*a*d)*ArcTan[1
 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(a^(7/4)*(b*c - a*d)^3) + (2*Sqr
t[2]*b^(7/4)*(-3*b*c + 11*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(
1/4)])/(a^(7/4)*(-(b*c) + a*d)^3) + (2*Sqrt[2]*d^(7/4)*(-11*b*c +
 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(c^(7/4)*(b*c -
a*d)^3) + (2*Sqrt[2]*d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]
*d^(1/4)*x)/c^(1/4)])/(c^(7/4)*(b*c - a*d)^3) + (Sqrt[2]*b^(7/4)*
(-3*b*c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/(a^(7/4)*(b*c - a*d)^3) + (Sqrt[2]*b^(7/4)*(-3*b*c + 11*
a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(a^(
7/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(7/4)*(11*b*c - 3*a*d)*Log[Sq
rt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(c^(7/4)*(-(b*c
) + a*d)^3) + (Sqrt[2]*d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(c^(7/4)*(b*c - a*d)^3))/3
```

2

**Maple [A]** time = 0.003, size = 784, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^4+a)^2/(d*x^4+c)^2,x)
```

```
[Out] 1/4*d^3/(a*d-b*c)^3/c*x/(d*x^4+c)*a-1/4*d^2/(a*d-b*c)^3*x/(d*x^4+
c)*b+3/16*d^3/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(c/d)^(1/4)*x+1)*a-11/16*d^2/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(c/d)^(1/4)*x+1)*b+3/16*d^3/(a*d-b*c)^3/c^2*(c/d)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a-11/16*d^2/(a*d-b*c)
^3/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*b+3/32*d
^3/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1
/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*a-11/32
*d^2/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1
/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*b+1/4*b
^2/(a*d-b*c)^3*x/(b*x^4+a)*d-1/4*b^3/(a*d-b*c)^3*x/a/(b*x^4+a)*c+
11/16*b^2/(a*d-b*c)^3/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^
(1/4)*x-1)*d-3/16*b^3/(a*d-b*c)^3/a^2*(a/b)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/b)^(1/4)*x-1)*c+11/32*b^2/(a*d-b*c)^3/a*(a/b)^(1/4)*2^
(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)
*x*2^(1/2)+(a/b)^(1/2)))*d-3/32*b^3/(a*d-b*c)^3/a^2*(a/b)^(1/4)*2
^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4
)*x*2^(1/2)+(a/b)^(1/2)))*c+11/16*b^2/(a*d-b*c)^3/a*(a/b)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d-3/16*b^3/(a*d-b*c)^3/a^2
*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^2*(d*x^4 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^2*(d*x^4 + c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**4+a)**2/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^2(dx^4 + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^2*(d*x^4 + c)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^2*(d*x^4 + c)^2), x)
```

## 3.74 $\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx$

**Optimal**. Leaf size=321

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(47a^2d^2-56abcd+21b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{21d^3\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)^3\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^3\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)^3\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^3\sqrt{a-bx^4}}-\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{21d^2}+\frac{bx\left(a-bx^4\right)^{3/2}}{7d}$$

```
[Out] -(b*(7*b*c - 13*a*d)*x*Sqrt[a - b*x^4])/(21*d^2) + (b*x*(a - b*x^
4)^(3/2))/(7*d) + (a^(1/4)*b^(3/4)*(21*b^2*c^2 - 56*a*b*c*d + 47*
a^2*d^2)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)
], -1])/(21*d^3*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)^3*Sqrt[1
- (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), A
rcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*d^3*Sqrt[a - b*x^4]
) - (a^(1/4)*(b*c - a*d)^3*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a
]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(
2*b^(1/4)*c*d^3*Sqrt[a - b*x^4])
```

---

**Rubi [A]** time = 0.914184, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(47a^2d^2-56abcd+21b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{21d^3\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)^3\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^3\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)^3\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^3\sqrt{a-bx^4}}-\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{21d^2}+\frac{bx\left(a-bx^4\right)^{3/2}}{7d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b*x^4)^(5/2)/(c - d*x^4),x]
```

```
[Out] -(b*(7*b*c - 13*a*d)*x*Sqrt[a - b*x^4])/(21*d^2) + (b*x*(a - b*x^
4)^(3/2))/(7*d) + (a^(1/4)*b^(3/4)*(21*b^2*c^2 - 56*a*b*c*d + 47*
a^2*d^2)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)
], -1])/(21*d^3*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)^3*Sqrt[1
```

```
- (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), A
rcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*d^3*Sqrt[a - b*x^4]
) - (a^(1/4)*(b*c - a*d)^3*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a
]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(
2*b^(1/4)*c*d^3*Sqrt[a - b*x^4])
```

---

**Rubi in Sympy [A]** time = 151.844, size = 292, normalized size = 0.91

$$\frac{\sqrt[4]{a}b^{\frac{3}{4}}\sqrt{1-\frac{bx^4}{a}}\left(47a^2d^2-56abcd+21b^2c^2\right)F\left(\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{21d^3\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(ad-bc\right)^3\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^3\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(ad-bc\right)^3\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^3\sqrt{a-bx^4}} + \frac{bx\left(a-bx^4\right)^{\frac{3}{2}}}{7d} + \frac{bx\sqrt{a-bx^4}\left(13ad-7bc\right)}{21d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**4+a)**(5/2)/(-d*x**4+c),x)
```

```
[Out] a**(1/4)*b**(3/4)*sqrt(1 - b*x**4/a)*(47*a**2*d**2 - 56*a*b*c*d +
 21*b**2*c**2)*elliptic_f(asin(b**(1/4)*x/a**(1/4)), -1)/(21*d**3
*sqrt(a - b*x**4)) + a**(1/4)*sqrt(1 - b*x**4/a)*(a*d - b*c)**3*e
lliptic_pi(-sqrt(a)*sqrt(d)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/a*
*(1/4)), -1)/(2*b**(1/4)*c*d**3*sqrt(a - b*x**4)) + a**(1/4)*sqrt
(1 - b*x**4/a)*(a*d - b*c)**3*elliptic_pi(sqrt(a)*sqrt(d)/(sqrt(b
)*sqrt(c)), asin(b**(1/4)*x/a**(1/4)), -1)/(2*b**(1/4)*c*d**3*sqr
t(a - b*x**4)) + b*x*(a - b*x**4)**(3/2)/(7*d) + b*x*sqrt(a - b*x
**4)*(13*a*d - 7*b*c)/(21*d**2)
```

---

**Mathematica [C]** time = 1.72882, size = 385, normalized size = 1.2

$$\frac{x\left(-\frac{9abcx^4\left(47a^2d^2-56abcd+21b^2c^2\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{\left(c-dx^4\right)\left(2x^4\left(2adF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)}+\frac{25a^2c\left(21a^2d^2-16abcd+7b\right.}{\left(c-dx^4\right)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\right.\right.\right.}\right.}{105d^2\sqrt{a-bx^4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a - b*x^4)^(5/2)/(c - d*x^4),x]
```

```
[Out] (x*(5*b*(-a + b*x^4)*(7*b*c - 16*a*d + 3*b*d*x^4) + (25*a^2*c*(7*
b^2*c^2 - 16*a*b*c*d + 21*a^2*d^2)*AppellF1[1/4, 1/2, 1, 5/4, (b*
```

```
x^4)/a, (d*x^4)/c])/((c - d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4
, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4,
 (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a
, (d*x^4)/c]))) - (9*a*b*c*(21*b^2*c^2 - 56*a*b*c*d + 47*a^2*d^2)
*x^4*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/((c - d*x^
4)*(9*a*c*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + 2*x^
4*(2*a*d*AppellF1[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*
AppellF1[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c])))))/(105*d^2*S
qrt[a - b*x^4])
```

---

**Maple [C]** time = 0.069, size = 408, normalized size = 1.3

$$-\frac{b^2x^5}{7\,d}\sqrt{-bx^4+a}+\frac{x}{3\,b}\left(\frac{b^2\,(3\,ad-bc)}{d^2}-\frac{5\,ab^2}{7\,d}\right)\sqrt{-bx^4+a}$$
$$-1\left(-\frac{b\left(3\,a^2d^2-3\,cabd+b^2c^2\right)}{d^3}+\frac{a}{3\,b}\left(\frac{b^2\,(3\,ad-bc)}{d^2}-\frac{5\,ab^2}{7\,d}\right)\right)\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}},i\right)$$
$$+\frac{1}{8\,d^4}\sum_{\_alpha=RootOf\left(d\_Z^4-c\right)}\frac{-a^3d^3+3\,a^2cd^2b-3\,ac^2db^2+c^3b^3}{\_alpha^3}\left(-1Artanh\left(\frac{-2\,\_alpha^2bx^2+2\,a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{-bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-}{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^4+a)^(5/2)/(-d*x^4+c),x)
```

```
[Out] -1/7*b^2/d*x^5*(-b*x^4+a)^(1/2)+1/3*(b^2/d^2*(3*a*d-b*c)-5/7*b^2/
d*a)/b*x*(-b*x^4+a)^(1/2)-(-b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)/d^3+1
/3*(b^2/d^2*(3*a*d-b*c)-5/7*b^2/d*a)/b*a)/(1/a^(1/2)*b^(1/2))^(1/
2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-
b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)+1/8/d^4*s
um((-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/_alpha^3*(-1/((
a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/
d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d
/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-
b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(
1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^
(1/2))),_alpha=RootOf(_Z^4*d-c))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(-bx^4+a\right)^{\frac{5}{2}}}{dx^4-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(-b*x^4 + a)^(5/2)/(d*x^4 - c),x, algorithm="maxima")
```

```
[Out] -integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(-b*x^4 + a)^(5/2)/(d*x^4 - c),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{a^2\sqrt{a-bx^4}}{-c+dx^4}\,dx-\int \frac{b^2x^8\sqrt{a-bx^4}}{-c+dx^4}\,dx-\int \left(-\frac{2abx^4\sqrt{a-bx^4}}{-c+dx^4}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**4+a)**(5/2)/(-d*x**4+c),x)
```

```
[Out] -Integral(a**2*sqrt(a - b*x**4)/(-c + d*x**4), x) - Integral(b**2
*x**8*sqrt(a - b*x**4)/(-c + d*x**4), x) - Integral(-2*a*b*x**4*s
qrt(a - b*x**4)/(-c + d*x**4), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-bx^4+a\right)^{\frac{5}{2}}}{dx^4-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(-b*x^4 + a)^(5/2)/(d*x^4 - c),x, algorithm="giac")
```

```
[Out] integrate(-(-b*x^4 + a)^(5/2)/(d*x^4 - c), x)
```

## 3.75 $\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx$

**Optimal**. Leaf size=277

$$-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(3bc-5ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3d^2\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)^2\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^2\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)^2\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^2\sqrt{a-bx^4}}+\frac{bx\sqrt{a-bx^4}}{3d}$$

```
[Out] (b*x*Sqrt[a - b*x^4])/(3*d) - (a^(1/4)*b^(3/4)*(3*b*c - 5*a*d)*Sq
rt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(3*
d^2*Sqrt[a - b*x^4]) + (a^(1/4)*(b*c - a*d)^2*Sqrt[1 - (b*x^4)/a]
*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/
4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*d^2*Sqrt[a - b*x^4]) + (a^(1/4)
*(b*c - a*d)^2*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(
Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*
d^2*Sqrt[a - b*x^4])
```

---

**Rubi [A]** time = 0.679383, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304

$$-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(3bc-5ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3d^2\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)^2\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^2\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)^2\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^2\sqrt{a-bx^4}}+\frac{bx\sqrt{a-bx^4}}{3d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b*x^4)^(3/2)/(c - d*x^4),x]
```

```
[Out] (b*x*Sqrt[a - b*x^4])/(3*d) - (a^(1/4)*b^(3/4)*(3*b*c - 5*a*d)*Sq
rt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(3*
d^2*Sqrt[a - b*x^4]) + (a^(1/4)*(b*c - a*d)^2*Sqrt[1 - (b*x^4)/a]
*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/
4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*d^2*Sqrt[a - b*x^4]) + (a^(1/4)
```

```
*(b*c - a*d)^2*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(
Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*
d^2*Sqrt[a - b*x^4])
```

---

**Rubi in Sympy [A]** time = 110.42, size = 248, normalized size = 0.9

$$\frac{\sqrt[4]{a}b^{\frac{3}{4}}\sqrt{1-\frac{bx^4}{a}}\,(5ad-3bc)\,F\left(\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3d^2\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,(ad-bc)^2\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^2\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,(ad-bc)^2\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^2\sqrt{a-bx^4}}+\frac{bx\sqrt{a-bx^4}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**4+a)**(3/2)/(-d*x**4+c),x)
```

```
[Out] a**(1/4)*b**(3/4)*sqrt(1 - b*x**4/a)*(5*a*d - 3*b*c)*elliptic_f(a
sin(b**(1/4)*x/a**(1/4)), -1)/(3*d**2*sqrt(a - b*x**4)) + a**(1/4
)*sqrt(1 - b*x**4/a)*(a*d - b*c)**2*elliptic_pi(-sqrt(a)*sqrt(d)/
(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/a**(1/4)), -1)/(2*b**(1/4)*c*d
**2*sqrt(a - b*x**4)) + a**(1/4)*sqrt(1 - b*x**4/a)*(a*d - b*c)**
2*elliptic_pi(sqrt(a)*sqrt(d)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/
a**(1/4)), -1)/(2*b**(1/4)*c*d**2*sqrt(a - b*x**4)) + b*x*sqrt(a
- b*x**4)/(3*d)
```

---

**Mathematica [C]** time = 0.743145, size = 419, normalized size = 1.51

$$\frac{x\left(\frac{b\left(-10x^4(a-bx^4)(c-dx^4)\left(2adF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)-9ac\left(5a(c-2dx^4)-2bcx^4+5bdx^8\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)}{2x^4\left(2adF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}-\frac{\cdots}{2x^4(2\cdots}\right.}{15d\sqrt{a-bx^4}\,(dx^4-c)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a - b*x^4)^(3/2)/(c - d*x^4),x]
```

```
[Out] (x*((-25*a^2*c*(-(b*c) + 3*a*d)*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4
)/a, (d*x^4)/c])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*
x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x
^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])) +
 (b*(-9*a*c*(-2*b*c*x^4 + 5*b*d*x^8 + 5*a*(c - 2*d*x^4))*AppellF1
```

```
[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] - 10*x^4*(a - b*x^4)*(c
- d*x^4)*(2*a*d*AppellF1[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c]
 + b*c*AppellF1[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c])))/(9*a*
c*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d
*AppellF1[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1
[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c]))))/(15*d*Sqrt[a - b*x^
4]*(-c + d*x^4))
```

---

**Maple [C]** time = 0.027, size = 311, normalized size = 1.1

$$\frac{bx}{3\,d}\sqrt{-bx^4+a}$$
$$-1\left(-\frac{b\,(2\,ad-bc)}{d^2}+\frac{ab}{3\,d}\right)\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-bx^4+a}}$$
$$+\frac{1}{8\,d^3}\sum_{\_alpha=RootOf\left(\_Z^4d-c\right)}\frac{-a^2d^2+2\,cabd-b^2c^2}{\_alpha^3}\left(-1Artanh\left(\frac{-2\,\_alpha^2bx^2+2\,a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{-bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}-2\,\frac{\_al}{c\sqrt{-b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^4+a)^(3/2)/(-d*x^4+c),x)
```

```
[Out] 1/3*b*x*(-b*x^4+a)^(1/2)/d-(-b*(2*a*d-b*c)/d^2+1/3*b/d*a)/(1/a^(1
/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a
^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1
/2),I)+1/8/d^3*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/_alpha^3*(-1/((a*
d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)
^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c
*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*
x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/
2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1
/2))),_alpha=RootOf(_Z^4*d-c))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(-bx^4+a\right)^{\frac{3}{2}}}{dx^4-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(-b*x^4 + a)^(3/2)/(d*x^4 - c),x, algorithm="maxima")
```

```
[Out] -integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(-b*x^4 + a)^(3/2)/(d*x^4 - c),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{a\sqrt{a-bx^4}}{-c+dx^4}\,dx - \int \left(-\frac{bx^4\sqrt{a-bx^4}}{-c+dx^4}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**4+a)**(3/2)/(-d*x**4+c),x)
```

```
[Out] -Integral(a*sqrt(a - b*x**4)/(-c + d*x**4), x) - Integral(-b*x**4
*sqrt(a - b*x**4)/(-c + d*x**4), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-bx^4+a\right)^{\frac{3}{2}}}{dx^4-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(-b*x^4 + a)^(3/2)/(d*x^4 - c),x, algorithm="giac")
```

```
[Out] integrate(-(-b*x^4 + a)^(3/2)/(d*x^4 - c), x)
```

## 3.76 $\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx$

**Optimal**. Leaf size=240

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{d\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}}$$

```
[Out] (a^(1/4)*b^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)
/a^(1/4)], -1])/(d*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)*Sqrt[1
 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])),
ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*d*Sqrt[a - b*x^4])
 - (a^(1/4)*(b*c - a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*S
qrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b
^(1/4)*c*d*Sqrt[a - b*x^4])
```

---

**Rubi [A]** time = 0.448504, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{d\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a - b*x^4]/(c - d*x^4),x]
```


```
[Out] (a^(1/4)*b^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)
/a^(1/4)], -1])/(d*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)*Sqrt[1
 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])),
ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*d*Sqrt[a - b*x^4])
 - (a^(1/4)*(b*c - a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*S
qrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b
^(1/4)*c*d*Sqrt[a - b*x^4])
```


---

**Rubi in Sympy [A]** time = 73.1836, size = 211, normalized size = 0.88

$$\frac{\sqrt[4]{a}b^{\frac{3}{4}}\sqrt{1-\frac{bx^4}{a}}F\left(\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{d\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,(ad-bc)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,(ad-bc)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**4+a)**(1/2)/(-d*x**4+c),x)
```

```
[Out] a**(1/4)*b**(3/4)*sqrt(1 - b*x**4/a)*elliptic_f(asin(b**(1/4)*x/a
**(1/4)), -1)/(d*sqrt(a - b*x**4)) + a**(1/4)*sqrt(1 - b*x**4/a)*
(a*d - b*c)*elliptic_pi(-sqrt(a)*sqrt(d)/(sqrt(b)*sqrt(c)), asin(
b**(1/4)*x/a**(1/4)), -1)/(2*b**(1/4)*c*d*sqrt(a - b*x**4)) + a**
(1/4)*sqrt(1 - b*x**4/a)*(a*d - b*c)*elliptic_pi(sqrt(a)*sqrt(d)/
(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/a**(1/4)), -1)/(2*b**(1/4)*c*d
*sqrt(a - b*x**4))
```

---

**Mathematica [C]** time = 0.276908, size = 155, normalized size = 0.65

$$-\frac{5acx\sqrt{a-bx^4}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{(c-dx^4)\left(2x^4\left(bcF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)-2adF_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a - b*x^4]/(c - d*x^4),x]
```

```
[Out] (-5*a*c*x*Sqrt[a - b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a,
(d*x^4)/c])/((c - d*x^4)*(-5*a*c*AppellF1[1/4, -1/2, 1, 5/4, (b*x
^4)/a, (d*x^4)/c] + 2*x^4*(-2*a*d*AppellF1[5/4, -1/2, 2, 9/4, (b*
x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d
*x^4)/c])))
```

---

**Maple [C]** time = 0.025, size = 259, normalized size = 1.1

$$\frac{b}{d}\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-bx^4+a}}$$
$$+\frac{1}{8\,d^2}\sum_{\_alpha=RootOf(\_Z^4d-c)}\frac{-ad+bc}{\_alpha^3}\left(-1Artanh\left(\frac{-2\,\_alpha^2bx^2+2\,a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{-bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}-2\frac{\_alpha^3d}{c\sqrt{-bx^4+a}}\sqrt{1-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^4+a)^(1/2)/(-d*x^4+c),x)
```

```
[Out] b/d/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^
(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*
b^(1/2))^(1/2),I)+1/8/d^2*sum((-a*d+b*c)/_alpha^3*(-1/((a*d-b*c)/
d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/
(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-b^(
1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^
(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alp
ha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_
alpha=RootOf(_Z^4*d-c))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{-bx^4+a}}{dx^4-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(-b*x^4 + a)/(d*x^4 - c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-b*x^4 + a)/(d*x^4 - c), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(-b*x^4 + a)/(d*x^4 - c),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{a-bx^4}}{-c+dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**4+a)**(1/2)/(-d*x**4+c),x)

[Out] -Integral(sqrt(a - b*x**4)/(-c + d*x**4), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-bx^4+a}}{dx^4-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(-b*x^4 + a)/(d*x^4 - c),x, algorithm="giac")

[Out] integrate(-sqrt(-b*x^4 + a)/(d*x^4 - c), x)
```

## 3.77 $\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}\,dx$

**Optimal**. Leaf size=162

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}$$

```
[Out] (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt
[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqr
t[a - b*x^4]) + (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*
Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*
b^(1/4)*c*Sqrt[a - b*x^4])
```

---

**Rubi [A]** time = 0.326382, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a - b*x^4]*(c - d*x^4)),x]
```

```
[Out] (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt
[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqr
t[a - b*x^4]) + (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*
Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*
b^(1/4)*c*Sqrt[a - b*x^4])
```

---

**Rubi in Sympy [A]** time = 52.4592, size = 143, normalized size = 0.88

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**4+a)**(1/2)/(-d*x**4+c),x)
```

```
[Out] a**(1/4)*sqrt(1 - b*x**4/a)*elliptic_pi(-sqrt(a)*sqrt(d)/(sqrt(b)
*sqrt(c)), asin(b**(1/4)*x/a**(1/4)), -1)/(2*b**(1/4)*c*sqrt(a -
```

```
b*x**4)) + a**(1/4)*sqrt(1 - b*x**4/a)*elliptic_pi(sqrt(a)*sqrt(d
)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/a**(1/4)), -1)/(2*b**(1/4)*c
*sqrt(a - b*x**4))
```

---

**Mathematica [C]** time = 0.234785, size = 156, normalized size = 0.96

$$-\frac{5acxF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{\sqrt{a-bx^4}\,(dx^4-c)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(Sqrt[a - b*x^4]*(c - d*x^4)),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/(Sqrt
[a - b*x^4]*(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4
)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)
/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4
)/c])))
```

---

**Maple [C]** time = 0.023, size = 183, normalized size = 1.1

$$-\frac{1}{8\,d}\sum_{\_alpha=RootOf(\_Z^4d-c)}\frac{1}{\_alpha^3}\left(-1Artanh\left(\frac{-2\,\_alpha^2bx^2+2\,a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{-bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}-2\,\frac{d\_alpha^3}{c\sqrt{-bx^4+a}}\sqrt{1-\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x)
```

```
[Out] -1/8/d*sum(1/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_al
pha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/
2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b
^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2
)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2)
)^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{\sqrt{-bx^4+a}(dx^4-c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(-b*x^4 + a)*(d*x^4 - c)),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(-b*x^4 + a)*(d*x^4 - c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-c\sqrt{a-bx^4}+dx^4\sqrt{a-bx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**4+a)**(1/2)/(-d*x**4+c),x)
```

```
[Out] -Integral(1/(-c*sqrt(a - b*x**4) + d*x**4*sqrt(a - b*x**4)), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sqrt{-bx^4+a}\,(dx^4-c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sqrt(-b*x^4 + a)*(d*x^4 - c)),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(-b*x^4 + a)*(d*x^4 - c)), x)
```

## 3.78 $\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx$

**Optimal**. Leaf size=281

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{3/4}\sqrt{a-bx^4}(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}-\frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}(bc-ad)}-\frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}(bc-ad)}$$

```
[Out] (b*x)/(2*a*(b*c - a*d)*Sqrt[a - b*x^4]) + (b^(3/4)*Sqrt[1 - (b*x^
4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*a^(3/4)*(b*c
 - a*d)*Sqrt[a - b*x^4]) - (a^(1/4)*d*Sqrt[1 - (b*x^4)/a]*Ellipti
cPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^
(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)*Sqrt[a - b*x^4]) - (a^(1/4)
*d*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt
[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)*
Sqrt[a - b*x^4])
```

---

**Rubi [A]** time = 0.609153, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{3/4}\sqrt{a-bx^4}(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}-\frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}(bc-ad)}-\frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a - b*x^4)^(3/2)*(c - d*x^4)),x]
```

```
[Out] (b*x)/(2*a*(b*c - a*d)*Sqrt[a - b*x^4]) + (b^(3/4)*Sqrt[1 - (b*x^
4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*a^(3/4)*(b*c
 - a*d)*Sqrt[a - b*x^4]) - (a^(1/4)*d*Sqrt[1 - (b*x^4)/a]*Ellipti
cPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^
(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)*Sqrt[a - b*x^4]) - (a^(1/4)
*d*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt
[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)*
Sqrt[a - b*x^4])
```

---

**Rubi in Sympy [A]** time = 106.733, size = 241, normalized size = 0.86

$$\frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}\,(ad-bc)}+\frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}\,(ad-bc)}-\frac{bx}{2a\sqrt{a-bx^4}\,(ad-bc)}-\frac{b^{\frac{3}{4}}\sqrt{1-\frac{bx^4}{a}}F\left(\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{\frac{3}{4}}\sqrt{a-bx^4}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**4+a)**(3/2)/(-d*x**4+c),x)
```

```
[Out] a**(1/4)*d*sqrt(1 - b*x**4/a)*elliptic_pi(-sqrt(a)*sqrt(d)/(sqrt(
b)*sqrt(c)), asin(b**(1/4)*x/a**(1/4)), -1)/(2*b**(1/4)*c*sqrt(a
- b*x**4)*(a*d - b*c)) + a**(1/4)*d*sqrt(1 - b*x**4/a)*elliptic_p
i(sqrt(a)*sqrt(d)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/a**(1/4)), -
1)/(2*b**(1/4)*c*sqrt(a - b*x**4)*(a*d - b*c)) - b*x/(2*a*sqrt(a
- b*x**4)*(a*d - b*c)) - b**(3/4)*sqrt(1 - b*x**4/a)*elliptic_f(a
sin(b**(1/4)*x/a**(1/4)), -1)/(2*a**(3/4)*sqrt(a - b*x**4)*(a*d -
 b*c))
```

---

**Mathematica [C]** time = 0.603338, size = 329, normalized size = 1.17

$$\frac{x\left(\frac{9bcdx^4F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{(c-dx^4)\left(2x^4\left(2adF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)}-\frac{25c(bc-2ad)F_1\left(\frac{1}{4};\frac{1}{2}\right.}{(c-dx^4)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\right.\right.\right.}\right.}{10\sqrt{a-bx^4}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a - b*x^4)^(3/2)*(c - d*x^4)),x]
```

```
[Out] (x*((-5*b)/a - (25*c*(b*c - 2*a*d)*AppellF1[1/4, 1/2, 1, 5/4, (b*
x^4)/a, (d*x^4)/c])/((c - d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4
, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4,
 (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a
, (d*x^4)/c]))) + (9*b*c*d*x^4*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)
/a, (d*x^4)/c])/((c - d*x^4)*(9*a*c*AppellF1[5/4, 1/2, 1, 9/4, (b
*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[9/4, 1/2, 2, 13/4, (b
*x^4)/a, (d*x^4)/c] + b*c*AppellF1[9/4, 3/2, 1, 13/4, (b*x^4)/a,
(d*x^4)/c])))))/(10*(-(b*c) + a*d)*Sqrt[a - b*x^4])
```

---

**Maple [C]** time = 0.043, size = 301, normalized size = 1.1

$$-\frac{bx}{2a(ad-bc)}\frac{1}{\sqrt{-\left(x^4-\frac{a}{b}\right)b}}-\frac{b}{2a(ad-bc)}\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-bx^4+a}}-\frac{1}{8}\sum_{\_alpha=RootOf(\_Z^4d-c)}\frac{1}{(ad-bc)\_alpha^3}\left(-1Artanh\left(\frac{-2\_alpha^2bx^2+2a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{-bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}-2\frac{\_alpha^3d}{c\sqrt{-bx^4+a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x)
```

```
[Out] -1/2*b/a*x/(a*d-b*c)/(-(x^4-a/b)*b)^(1/2)-1/2*b/a/(a*d-b*c)/(1/a^
(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2
/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^
(1/2),I)-1/8*sum(1/(a*d-b*c)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arc
tanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(
1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-b^(1/2)*x^2/a^(
1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*Ellipt
icPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1
/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf
(_Z^4*d-c))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(-bx^4+a)^{\frac{3}{2}}(dx^4-c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-ac\sqrt{a-bx^4}+adx^4\sqrt{a-bx^4}+bcx^4\sqrt{a-bx^4}-bdx^8\sqrt{a-bx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**4+a)**(3/2)/(-d*x**4+c),x)
```

```
[Out] -Integral(1/(-a*c*sqrt(a - b*x**4) + a*d*x**4*sqrt(a - b*x**4) +
b*c*x**4*sqrt(a - b*x**4) - b*d*x**8*sqrt(a - b*x**4)), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(-bx^4+a)^{\frac{3}{2}}(dx^4-c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)),x, algorithm="giac")
```

```
[Out] integrate(-1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)), x)
```

## 3.79 $\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx$

**Optimal**. Leaf size=334

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}(5bc-11ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12a^{7/4}\sqrt{a-bx^4}(bc-ad)^2}+\frac{bx(5bc-11ad)}{12a^2\sqrt{a-bx^4}(bc-ad)^2}+\frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}(bc-ad)^2}+\frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}(bc-ad)^2}+\frac{bx}{6a\left(a-bx^4\right)^{3/2}(bc-ad)}$$

```
[Out] (b*x)/(6*a*(b*c - a*d)*(a - b*x^4)^(3/2)) + (b*(5*b*c - 11*a*d)*x
)/(12*a^2*(b*c - a*d)^2*Sqrt[a - b*x^4]) + (b^(3/4)*(5*b*c - 11*a
*d)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1
])/(12*a^(7/4)*(b*c - a*d)^2*Sqrt[a - b*x^4]) + (a^(1/4)*d^2*Sqrt
[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]))
, ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^2*Sq
rt[a - b*x^4]) + (a^(1/4)*d^2*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqr
t[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1]
)/(2*b^(1/4)*c*(b*c - a*d)^2*Sqrt[a - b*x^4])
```

---

**Rubi [A]** time = 0.961753, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}(5bc-11ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12a^{7/4}\sqrt{a-bx^4}(bc-ad)^2}+\frac{bx(5bc-11ad)}{12a^2\sqrt{a-bx^4}(bc-ad)^2}+\frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}(bc-ad)^2}+\frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}(bc-ad)^2}+\frac{bx}{6a\left(a-bx^4\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a - b*x^4)^(5/2)*(c - d*x^4)),x]
```

```
[Out] (b*x)/(6*a*(b*c - a*d)*(a - b*x^4)^(3/2)) + (b*(5*b*c - 11*a*d)*x
)/(12*a^2*(b*c - a*d)^2*Sqrt[a - b*x^4]) + (b^(3/4)*(5*b*c - 11*a
*d)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1
```

```
])/(12*a^(7/4)*(b*c - a*d)^2*Sqrt[a - b*x^4]) + (a^(1/4)*d^2*Sqrt
[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]))
, ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^2*Sq
rt[a - b*x^4]) + (a^(1/4)*d^2*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqr
t[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1]
)/(2*b^(1/4)*c*(b*c - a*d)^2*Sqrt[a - b*x^4])
```

---

**Rubi in Sympy [A]** time = 165.584, size = 298, normalized size = 0.89

$$\frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}\left(ad-bc\right)^2}+\frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}\left(ad-bc\right)^2}-\frac{bx}{6a\left(a-bx^4\right)^{\frac{3}{2}}\left(ad-bc\right)}-\frac{bx\left(11ad-5bc\right)}{12a^2\sqrt{a-bx^4}\left(ad-bc\right)^2}-\frac{b^{\frac{3}{4}}\sqrt{1-\frac{bx^4}{a}}\left(11ad-5bc\right)F\left(\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12a^{\frac{7}{4}}\sqrt{a-bx^4}\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**4+a)**(5/2)/(-d*x**4+c),x)
```

```
[Out] a**(1/4)*d**2*sqrt(1 - b*x**4/a)*elliptic_pi(-sqrt(a)*sqrt(d)/(sq
rt(b)*sqrt(c)), asin(b**(1/4)*x/a**(1/4)), -1)/(2*b**(1/4)*c*sqrt
(a - b*x**4)*(a*d - b*c)**2) + a**(1/4)*d**2*sqrt(1 - b*x**4/a)*e
lliptic_pi(sqrt(a)*sqrt(d)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/a**
(1/4)), -1)/(2*b**(1/4)*c*sqrt(a - b*x**4)*(a*d - b*c)**2) - b*x/
(6*a*(a - b*x**4)**(3/2)*(a*d - b*c)) - b*x*(11*a*d - 5*b*c)/(12*
a**2*sqrt(a - b*x**4)*(a*d - b*c)**2) - b**(3/4)*sqrt(1 - b*x**4/
a)*(11*a*d - 5*b*c)*elliptic_f(asin(b**(1/4)*x/a**(1/4)), -1)/(12
*a**(7/4)*sqrt(a - b*x**4)*(a*d - b*c)**2)
```

---

**Mathematica [C]** time = 1.56833, size = 396, normalized size = 1.19

$$\frac{x\left(\frac{25ac\left(12a^2d^2-11abcd+5b^2c^2\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{\left(c-dx^4\right)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)}+\frac{5b\left(13a^2d-ab\left(7c+11dx^4\right)+5b^2cx^4\right)}{bx^4-a}+\frac{\cdots}{\left(c-dx^4\right)\left(2x^4\cdots\right.}\right.}{60a^2\sqrt{a-bx^4}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a - b*x^4)^(5/2)*(c - d*x^4)),x]
```

```
[Out] (x*((5*b*(13*a^2*d + 5*b^2*c*x^4 - a*b*(7*c + 11*d*x^4)))/(-a + b
*x^4) + (25*a*c*(5*b^2*c^2 - 11*a*b*c*d + 12*a^2*d^2)*AppellF1[1/
```

```
4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/((c - d*x^4)*(5*a*c*Appell
F1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF
1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2
, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))) + (9*a*b*c*d*(-5*b*c + 11*a*d)
*x^4*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/((c - d*x^
4)*(9*a*c*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + 2*x^
4*(2*a*d*AppellF1[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*
AppellF1[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c])))))/(60*a^2*(b
*c - a*d)^2*Sqrt[a - b*x^4])
```

---

**Maple [C]** time = 0.047, size = 361, normalized size = 1.1

$$-\frac{x}{6\,ab\,(ad-bc)}\sqrt{-bx^4+a}\left(x^4-\frac{a}{b}\right)^{-2}-\frac{bx\,(11\,ad-5\,bc)}{12\,a^2\,(ad-bc)^2}\frac{1}{\sqrt{-\left(x^4-\frac{a}{b}\right)b}}$$
$$-\frac{b\,(11\,ad-5\,bc)}{12\,a^2\,(ad-bc)^2}\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-bx^4+a}}$$
$$-\frac{d}{8}\sum_{\_alpha=RootOf(\_Z^4d-c)}\frac{1}{(ad-bc)^2\,\_alpha^3}\left(-1Artanh\left(\frac{-2\,\_alpha^2bx^2+2\,a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{-bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}-2\,\frac{\_alpha^3d}{c\sqrt{-bx^4+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x)
```

```
[Out] -1/6/a*x/b/(a*d-b*c)*(-b*x^4+a)^(1/2)/(x^4-a/b)^2-1/12*b/a^2*x*(1
1*a*d-5*b*c)/(a*d-b*c)^2/(-(x^4-a/b)*b)^(1/2)-1/12*b/a^2*(11*a*d-
5*b*c)/(a*d-b*c)^2/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/
2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*Elliptic
F(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/8*d*sum(1/(a*d-b*c)^2/_alpha^3
*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*
d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_al
pha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(
1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1
/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^
(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{(-bx^4+a)^{\frac{5}{2}}(dx^4-c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(5/2)/(-d*x**4+c),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(-bx^4 + a)^{\frac{5}{2}}(dx^4 - c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)),x, algorithm="giac")
```

```
[Out] integrate(-1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)), x)
```

## 3.80 $\int \frac{(a+bx^4)^{3/2}}{c+dx^4} \, dx$

**Optimal**. Leaf size=973

result too large to display

```
[Out] (b*x*Sqrt[a + b*x^4])/(3*d) + ((b*c - a*d)^2*ArcTan[(Sqrt[-((Sqrt
[-c]*(b - (a*d)/c))/Sqrt[d])]*x)/Sqrt[a + b*x^4]])/(4*c*d^2*Sqrt[
(b*c - a*d)/(Sqrt[-c]*Sqrt[d])]) + ((b*c - a*d)^2*ArcTan[(Sqrt[(S
qrt[-c]*(b - (a*d)/c))/Sqrt[d]]*x)/Sqrt[a + b*x^4]])/(4*c*d^2*Sqr
t[-((b*c - a*d)/(Sqrt[-c]*Sqrt[d]))]) - (b^(3/4)*(3*b*c - 5*a*d)*
(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(1/4)*d^2*
Sqrt[a + b*x^4]) + (b^(1/4)*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c - Sqrt[a]*Sqrt[-
c]*Sqrt[d])*d^2*Sqrt[a + b*x^4]) + (b^(1/4)*(b*c - a*d)^2*(Sqrt[a
] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c
+ Sqrt[a]*Sqrt[-c]*Sqrt[d])*d^2*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqrt
[-c] + Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqr
t[-c] - Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]),
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[b
]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*d^2*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sq
rt[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sq
rt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]),
 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[
b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*d^2*Sqrt[a + b*x^4])
```

---

**Rubi [A]** time = 3.70666, antiderivative size = 973, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{\sqrt{-c}\left(b-\frac{ad}{c}\right)}{\sqrt{d}}}x}{\sqrt{bx^4+a}}\right)(bc-ad)^2}{4cd^2\sqrt{\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{-c}\left(b-\frac{ad}{c}\right)}{\sqrt{d}}}x}{\sqrt{bx^4+a}}\right)(bc-ad)^2}{4cd^2\sqrt{-\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}}$$
$$+\frac{\sqrt[4]{b}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)(bc-ad)^2}{4\sqrt[4]{a}\left(\sqrt{b}c-\sqrt{a}\sqrt{-c}\sqrt{d}\right)d^2\sqrt{bx^4+a}}$$
$$+\frac{\sqrt[4]{b}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)(bc-ad)^2}{4\sqrt[4]{a}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)d^2\sqrt{bx^4+a}}$$
$$-\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)(bc-ad)^2}{8\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)d^2\sqrt{bx^4+a}}$$
$$-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)(bc-ad)^2}{8\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)d^2\sqrt{bx^4+a}}$$
$$-\frac{b^{3/4}(3bc-5ad)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{a}d^2\sqrt{bx^4+a}}+\frac{bx\sqrt{bx^4+a}}{3d}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(a + b*x^4)^(3/2)/(c + d*x^4),x]
```

```
[Out] (b*x*Sqrt[a + b*x^4])/(3*d) + ((b*c - a*d)^2*ArcTan[(Sqrt[-((Sqrt
[-c]*(b - (a*d)/c))/Sqrt[d])]*x)/Sqrt[a + b*x^4]])/(4*c*d^2*Sqrt[
(b*c - a*d)/(Sqrt[-c]*Sqrt[d])]) + ((b*c - a*d)^2*ArcTan[(Sqrt[(S
qrt[-c]*(b - (a*d)/c))/Sqrt[d]]*x)/Sqrt[a + b*x^4]])/(4*c*d^2*Sqr
t[-((b*c - a*d)/(Sqrt[-c]*Sqrt[d]))]) - (b^(3/4)*(3*b*c - 5*a*d)*
(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(1/4)*d^2*
Sqrt[a + b*x^4]) + (b^(1/4)*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c - Sqrt[a]*Sqrt[-
c]*Sqrt[d])*d^2*Sqrt[a + b*x^4]) + (b^(1/4)*(b*c - a*d)^2*(Sqrt[a
] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c
+ Sqrt[a]*Sqrt[-c]*Sqrt[d])*d^2*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqrt
[-c] + Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqr
t[-c] - Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]),
```

```
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[b
]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*d^2*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sq
rt[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sq
rt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]),
 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[
b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*d^2*Sqrt[a + b*x^4])
```

---

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**(3/2)/(d*x**4+c),x)
```

```
[Out] Timed out
```

---

**Mathematica [C]** time = 1.14671, size = 435, normalized size = 0.45

$$\frac{x\left(\frac{25a^2c(3ad-bc)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}+\frac{b\left(10x^4\left(a+bx^4\right)\left(c+dx^4\right)\left(2adF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx}{c}\right.\right.\right.}{2x^4\left(2adF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};-\frac{bx^4}{a}\right.\right.}\right.}{15d\sqrt{a+bx^4}\left(c+dx^4\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^4)^(3/2)/(c + d*x^4),x]
```

```
[Out] (x*((25*a^2*c*(-(b*c) + 3*a*d)*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^
4)/a), -((d*x^4)/c)])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)
/a), -((d*x^4)/c)] - 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b
*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4
)/a), -((d*x^4)/c)])) + (b*(-9*a*c*(5*a*(c + 2*d*x^4) + b*x^4*(2*
c + 5*d*x^4))*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/
c)] + 10*x^4*(a + b*x^4)*(c + d*x^4)*(2*a*d*AppellF1[9/4, 1/2, 2,
 13/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[9/4, 3/2, 1, 13
/4, -((b*x^4)/a), -((d*x^4)/c)])))/(-9*a*c*AppellF1[5/4, 1/2, 1,
9/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*x^4*(2*a*d*AppellF1[9/4, 1/2
, 2, 13/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[9/4, 3/2, 1
, 13/4, -((b*x^4)/a), -((d*x^4)/c)]))))/(15*d*Sqrt[a + b*x^4]*(c
+ d*x^4))
```

---

**Maple [C]** time = 0.036, size = 322, normalized size = 0.3

$$\frac{bx}{3d}\sqrt{bx^4+a}$$
$$+1\left(\frac{b(2ad-bc)}{d^2}-\frac{ab}{3d}\right)\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$-\frac{1}{8d^3}\sum_{\_alpha=RootOf(d\_Z^4+c)}\frac{-a^2d^2+2cabd-b^2c^2}{\_alpha^3}\left(-1Artanh\left(\frac{2\_alpha^2bx^2+2a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}+2\frac{\_alpha^3}{c\sqrt{bx^4+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^(3/2)/(d*x^4+c),x)
```

```
[Out] 1/3*b*x*(b*x^4+a)^(1/2)/d+(b*(2*a*d-b*c)/d^2-1/3*b/d*a)/(I/a^(1/2
)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(
1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(
1/2),I)-1/8/d^3*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/_alpha^3*(-1/((a
*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)
^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*
(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(
b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b
^(1/2)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2)
)^(1/2))),_alpha=RootOf(_Z^4*d+c))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{(bx^4+a)^{\frac{3}{2}}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/2)/(d*x^4 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)/(d*x^4 + c), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/2)/(d*x^4 + c),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^4\right)^{\frac{3}{2}}}{c + dx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**(3/2)/(d*x**4+c),x)
```

```
[Out] Integral((a + b*x**4)**(3/2)/(c + d*x**4), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4 + a\right)^{\frac{3}{2}}}{dx^4 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/2)/(d*x^4 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/2)/(d*x^4 + c), x)
```

## 3.81 $\int \frac{\sqrt{a+bx^4}}{c+dx^4} \, dx$

**Optimal**. Leaf size=932

result too large to display

```
[Out] -((b*c - a*d)*ArcTan[(Sqrt[-((Sqrt[-c]*(b - (a*d)/c))/Sqrt[d])]*x
)/Sqrt[a + b*x^4]])/(4*c*d*Sqrt[(b*c - a*d)/(Sqrt[-c]*Sqrt[d])])
- ((b*c - a*d)*ArcTan[(Sqrt[(Sqrt[-c]*(b - (a*d)/c))/Sqrt[d]]*x)/
Sqrt[a + b*x^4]])/(4*c*d*Sqrt[-((b*c - a*d)/(Sqrt[-c]*Sqrt[d]))])
 + (b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*
a^(1/4)*d*Sqrt[a + b*x^4]) - (b^(1/4)*(b*c - a*d)*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*
ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c - Sqrt[a
]*Sqrt[-c]*Sqrt[d])*d*Sqrt[a + b*x^4]) - (b^(1/4)*(b*c - a*d)*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*
EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b
]*c + Sqrt[a]*Sqrt[-c]*Sqrt[d])*d*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sq
rt[-c] + Sqrt[a]*Sqrt[d])*(b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqr
t[-c] - Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]),
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[b
]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*d*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt
[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[
(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-
c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*A
rcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[b]*S
qrt[-c] + Sqrt[a]*Sqrt[d])*d*Sqrt[a + b*x^4])
```

---

**Rubi [A]** time = 1.76572, antiderivative size = 932, normalized size of antiderivative = 1., number of

steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{-\frac{\sqrt{-c}\left(b-\frac{ad}{c}\right)}{\sqrt{d}}}\,x}{\sqrt{bx^4+a}}\right)}{4cd\sqrt{\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{-c}\left(b-\frac{ad}{c}\right)}{\sqrt{d}}}\,x}{\sqrt{bx^4+a}}\right)}{4cd\sqrt{-\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}}$$
$$-\frac{\sqrt[4]{b}(bc-ad)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\left(\sqrt{b}c-\sqrt{a}\sqrt{-c}\sqrt{d}\right)d\sqrt{bx^4+a}}$$
$$-\frac{\sqrt[4]{b}(bc-ad)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)d\sqrt{bx^4+a}}$$
$$+\frac{b^{3/4}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{bx^4+a}}$$
$$+\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)(bc-ad)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)d\sqrt{bx^4+a}}$$
$$+\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(bc-ad)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)d\sqrt{bx^4+a}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[Sqrt[a + b*x^4]/(c + d*x^4),x]
```

```
[Out] -((b*c - a*d)*ArcTan[(Sqrt[-((Sqrt[-c]*(b - (a*d)/c))/Sqrt[d])]*x
)/Sqrt[a + b*x^4]])/(4*c*d*Sqrt[(b*c - a*d)/(Sqrt[-c]*Sqrt[d])])
- ((b*c - a*d)*ArcTan[(Sqrt[(Sqrt[-c]*(b - (a*d)/c))/Sqrt[d]]*x)/
Sqrt[a + b*x^4]])/(4*c*d*Sqrt[-((b*c - a*d)/(Sqrt[-c]*Sqrt[d]))])
 + (b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*
a^(1/4)*d*Sqrt[a + b*x^4]) - (b^(1/4)*(b*c - a*d)*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*
ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c - Sqrt[a
]*Sqrt[-c]*Sqrt[d])*d*Sqrt[a + b*x^4]) - (b^(1/4)*(b*c - a*d)*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*
EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b
]*c + Sqrt[a]*Sqrt[-c]*Sqrt[d])*d*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sq
rt[-c] + Sqrt[a]*Sqrt[d])*(b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqr
t[-c] - Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]),
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[b
```

```
]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*d*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt
[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[
(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-
c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*A
rcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[b]*S
qrt[-c] + Sqrt[a]*Sqrt[d])*d*Sqrt[a + b*x^4])
```

---

**Rubi in Sympy [A]** time = 149.324, size = 814, normalized size = 0.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**(1/2)/(d*x**4+c),x)
```

```
[Out] (a*d - b*c)*atan(x*sqrt(sqrt(-c)*(-a*d + b*c)/(c*sqrt(d)))/sqrt(a
 + b*x**4))/(4*c*d*sqrt(-sqrt(-c)*(a*d - b*c)/(c*sqrt(d)))) + (a*
d - b*c)*atan(x*sqrt(sqrt(-c)*(a*d - b*c)/(c*sqrt(d)))/sqrt(a + b
*x**4))/(4*c*d*sqrt(sqrt(-c)*(a*d - b*c)/(c*sqrt(d)))) + b**(3/4)
*sqrt((a + b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b
)*x**2)*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(2*a**(1/4)*
d*sqrt(a + b*x**4)) + b**(1/4)*sqrt((a + b*x**4)/(sqrt(a) + sqrt(
b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(a*d - b*c)*elliptic_f(2*at
an(b**(1/4)*x/a**(1/4)), 1/2)/(4*a**(1/4)*d*sqrt(a + b*x**4)*(sqr
t(a)*sqrt(d)*sqrt(-c) + sqrt(b)*c)) + b**(1/4)*sqrt((a + b*x**4)/
(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(a*d - b*c)
*elliptic_f(2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(4*a**(1/4)*d*sqrt(
a + b*x**4)*(-sqrt(a)*sqrt(d)*sqrt(-c) + sqrt(b)*c)) + sqrt((a +
b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(sq
rt(a)*sqrt(d) - sqrt(b)*sqrt(-c))*(a*d - b*c)*elliptic_pi((sqrt(a
)*sqrt(d) + sqrt(b)*sqrt(-c))**2/(4*sqrt(a)*sqrt(b)*sqrt(d)*sqrt(
-c)), 2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(8*a**(1/4)*b**(1/4)*c*d*
sqrt(a + b*x**4)*(sqrt(a)*sqrt(d) + sqrt(b)*sqrt(-c))) + sqrt((a
+ b*x**4)/(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(
sqrt(a)*sqrt(d) + sqrt(b)*sqrt(-c))*(a*d - b*c)*elliptic_pi(-(sqr
t(a)*sqrt(d) - sqrt(b)*sqrt(-c))**2/(4*sqrt(a)*sqrt(b)*sqrt(d)*sq
rt(-c)), 2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(8*a**(1/4)*b**(1/4)*c
*d*sqrt(a + b*x**4)*(sqrt(a)*sqrt(d) - sqrt(b)*sqrt(-c)))
```

---

**Mathematica [C]** time = 0.237976, size = 161, normalized size = 0.17

$$\frac{5acx\sqrt{a+bx^4}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(c+dx^4)\left(2x^4\left(bcF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-2adF_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a + b*x^4]/(c + d*x^4),x]
```

```
[Out] (5*a*c*x*Sqrt[a + b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a)
, -((d*x^4)/c)])/((c + d*x^4)*(5*a*c*AppellF1[1/4, -1/2, 1, 5/4,
-((b*x^4)/a), -((d*x^4)/c)] + 2*x^4*(-2*a*d*AppellF1[5/4, -1/2, 2
, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 1/2, 1, 9/
4, -((b*x^4)/a), -((d*x^4)/c)])))
```

---

**Maple [C]** time = 0.023, size = 273, normalized size = 0.3

$$\frac{b}{d}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$

$$-\frac{1}{8\,d^2}\sum_{\_alpha=RootOf\left(\_Z^4d+c\right)}\frac{-ad+bc}{\_alpha^3}\left(-1Artanh\left(\frac{2\,\_alpha^2bx^2+2\,a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}+2\frac{\_alpha^3d}{c\sqrt{bx^4+a}}\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^(1/2)/(d*x^4+c),x)
```

```
[Out] b/d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/
2)*b^(1/2))^(1/2),I)-1/8/d^2*sum((-a*d+b*c)/_alpha^3*(-1/((a*d-b*
c)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2
)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-I/
a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4
+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2
)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/
2))),_alpha=RootOf(_Z^4*d+c))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^4+a}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)/(d*x^4 + c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)/(d*x^4 + c), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)/(d*x^4 + c),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^4}}{c + dx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**(1/2)/(d*x**4+c),x)
```

```
[Out] Integral(sqrt(a + b*x**4)/(c + d*x**4), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4 + a}}{dx^4 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^4 + a)/(d*x^4 + c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)/(d*x^4 + c), x)
```

## 3.82 $\int \frac{1}{\sqrt{a+bx^4}(c+dx^4)}\,dx$

**Optimal**. Leaf size=775

$$\frac{\tan^{-1}\left(\frac{x\sqrt{-\frac{\sqrt{-c}\left(b-\frac{ad}{c}\right)}{\sqrt{d}}}}{\sqrt{a+bx^4}}\right)}{4c\sqrt{\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}}+\frac{\tan^{-1}\left(\frac{x\sqrt{\frac{\sqrt{-c}\left(b-\frac{ad}{c}\right)}{\sqrt{d}}}}{\sqrt{a+bx^4}}\right)}{4c\sqrt{-\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}}$$
$$+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{b}c-\sqrt{a}\sqrt{-c}\sqrt{d}\right)}$$
$$+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)}$$
$$-\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c\sqrt{a+bx^4}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)}$$
$$-\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c\sqrt{a+bx^4}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)}$$

```
[Out] ArcTan[(Sqrt[-((Sqrt[-c]*(b - (a*d)/c))/Sqrt[d])]*x)/Sqrt[a + b*x
^4]]/(4*c*Sqrt[(b*c - a*d)/(Sqrt[-c]*Sqrt[d])]) + ArcTan[(Sqrt[(S
qrt[-c]*(b - (a*d)/c))/Sqrt[d]]*x)/Sqrt[a + b*x^4]]/(4*c*Sqrt[-((
b*c - a*d)/(Sqrt[-c]*Sqrt[d]))]) + (b^(1/4)*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c - Sqrt[a]*Sqrt
[-c]*Sqrt[d])*Sqrt[a + b*x^4]) + (b^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-
c]*Sqrt[d])*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[
d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x
^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(4*Sqrt
[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*Sqrt
[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticP
i[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[
-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(
1/4)*c*(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*Sqrt[a + b*x^4])
```

**Rubi [A]** time = 1.23431, antiderivative size = 775, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\tan^{-1}\left(\frac{x\sqrt{-\frac{\sqrt{-c}\left(b-\frac{ad}{c}\right)}{\sqrt{d}}}}{\sqrt{a+bx^4}}\right)}{4c\sqrt{\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}}+\frac{\tan^{-1}\left(\frac{x\sqrt{\frac{\sqrt{-c}\left(b-\frac{ad}{c}\right)}{\sqrt{d}}}}{\sqrt{a+bx^4}}\right)}{4c\sqrt{-\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}}$$
$$+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{b}c-\sqrt{a}\sqrt{-c}\sqrt{d}\right)}$$
$$+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)}$$
$$-\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)\Pi\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c\sqrt{a+bx^4}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)}$$
$$-\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\Pi\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c\sqrt{a+bx^4}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/(Sqrt[a + b*x^4]*(c + d*x^4)),x]
```

```
[Out] ArcTan[(Sqrt[-((Sqrt[-c]*(b - (a*d)/c))/Sqrt[d])]*x)/Sqrt[a + b*x
^4]]/(4*c*Sqrt[(b*c - a*d)/(Sqrt[-c]*Sqrt[d])]) + ArcTan[(Sqrt[(S
qrt[-c]*(b - (a*d)/c))/Sqrt[d]]*x)/Sqrt[a + b*x^4]]/(4*c*Sqrt[-((
b*c - a*d)/(Sqrt[-c]*Sqrt[d]))]) + (b^(1/4)*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c - Sqrt[a]*Sqrt
[-c]*Sqrt[d])*Sqrt[a + b*x^4]) + (b^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-
c]*Sqrt[d])*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[
d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x
^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(4*Sqrt
[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*Sqrt
[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticP
i[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[
-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(
1/4)*c*(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*Sqrt[a + b*x^4])
```

**Rubi in Sympy [A]** time = 100.558, size = 690, normalized size = 0.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**4+a)**(1/2)/(d*x**4+c),x)
```

```
[Out] atan(x*sqrt(sqrt(-c)*(a*d - b*c)/(c*sqrt(d)))/sqrt(a + b*x**4))/(
4*c*sqrt(sqrt(-c)*(a*d - b*c)/(c*sqrt(d)))) + atan(x*sqrt(sqrt(-c
)*(-a*d + b*c)/(c*sqrt(d)))/sqrt(a + b*x**4))/(4*c*sqrt(sqrt(-c)*
(-a*d + b*c)/(c*sqrt(d)))) - b**(1/4)*sqrt((a + b*x**4)/(sqrt(a)
+ sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*elliptic_f(2*atan(b*
*(1/4)*x/a**(1/4)), 1/2)/(4*a**(1/4)*sqrt(-c)*sqrt(a + b*x**4)*(s
qrt(a)*sqrt(d) + sqrt(b)*sqrt(-c))) + b**(1/4)*sqrt((a + b*x**4)/
(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*elliptic_f(
2*atan(b**(1/4)*x/a**(1/4)), 1/2)/(4*a**(1/4)*sqrt(-c)*sqrt(a + b
*x**4)*(sqrt(a)*sqrt(d) - sqrt(b)*sqrt(-c))) + sqrt((a + b*x**4)/
(sqrt(a) + sqrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(sqrt(a)*sq
rt(d) - sqrt(b)*sqrt(-c))*elliptic_pi((sqrt(a)*sqrt(d) + sqrt(b)*
sqrt(-c))**2/(4*sqrt(a)*sqrt(b)*sqrt(d)*sqrt(-c)), 2*atan(b**(1/4
)*x/a**(1/4)), 1/2)/(8*a**(1/4)*b**(1/4)*c*sqrt(a + b*x**4)*(sqrt
(a)*sqrt(d) + sqrt(b)*sqrt(-c))) + sqrt((a + b*x**4)/(sqrt(a) + s
qrt(b)*x**2)**2)*(sqrt(a) + sqrt(b)*x**2)*(sqrt(a)*sqrt(d) + sqrt
(b)*sqrt(-c))*elliptic_pi(-(sqrt(a)*sqrt(d) - sqrt(b)*sqrt(-c))**
2/(4*sqrt(a)*sqrt(b)*sqrt(d)*sqrt(-c)), 2*atan(b**(1/4)*x/a**(1/4
)), 1/2)/(8*a**(1/4)*b**(1/4)*c*sqrt(a + b*x**4)*(sqrt(a)*sqrt(d)
 - sqrt(b)*sqrt(-c)))
```

**Mathematica [C]** time = 0.0810482, size = 161, normalized size = 0.21

$$-\frac{5acxF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\sqrt{a+bx^4}\left(c+dx^4\right)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(Sqrt[a + b*x^4]*(c + d*x^4)),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])
/(Sqrt[a + b*x^4]*(c + d*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4,
-((b*x^4)/a), -((d*x^4)/c)] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2,
9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4,
 -((b*x^4)/a), -((d*x^4)/c)])))
```

**Maple [C]** time = 0.021, size = 191, normalized size = 0.3

$$\frac{1}{8d}\sum_{\_alpha=RootOf(\_Z^4d+c)}\frac{1}{\_alpha^3}\left(-1Artanh\left(\frac{2\_alpha^2bx^2+2a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}+2\frac{d\_alpha^3}{c\sqrt{bx^4+a}}\sqrt{1-\frac{i\sqrt{b}x^2}{\sqrt{a}}}\sqrt{\phantom{x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(b*x^4+a)^(1/2)/(d*x^4+c),x)
```

```
[Out] 1/8/d*sum(1/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_alph
a^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*
b^(1/2))^(1/2)*_alpha^3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2
)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/
2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d+c))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4+a}(dx^4+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**4+a)**(1/2)/(d*x**4+c),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**4)*(c + d*x**4)), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + a}(dx^4 + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)), x)
```

## 3.83 $\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx$

**Optimal**. Leaf size=969

result too large to display

```
[Out] (b*x)/(2*a*(b*c - a*d)*Sqrt[a + b*x^4]) - (d*ArcTan[(Sqrt[-((Sqrt
[-c]*(b - (a*d)/c))/Sqrt[d])]*x)/Sqrt[a + b*x^4]])/(4*c*(b*c - a*
d)*Sqrt[(b*c - a*d)/(Sqrt[-c]*Sqrt[d])]) - (d*ArcTan[(Sqrt[(Sqrt[
-c]*(b - (a*d)/c))/Sqrt[d]]*x)/Sqrt[a + b*x^4]])/(4*c*(b*c - a*d)
*Sqrt[-((b*c - a*d)/(Sqrt[-c]*Sqrt[d]))]) + (b^(3/4)*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF
[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*(b*c - a*d)*Sqrt
[a + b*x^4]) - (b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4
)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/
4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c - Sqrt[a]*Sqrt[-c]*Sqrt[d])*(b*c
 - a*d)*Sqrt[a + b*x^4]) - (b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1
/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-c]*S
qrt[d])*(b*c - a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] + Sqrt[
a]*Sqrt[d])*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
 Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])
^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqr
t[d])*(b*c - a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] - Sqrt[a]
*Sqrt[d])*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/
(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d
])*(b*c - a*d)*Sqrt[a + b*x^4])
```

---

**Rubi [A]** time = 2.40571, antiderivative size = 969, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{bx}{2a(bc-ad)\sqrt{bx^4+a}} - \frac{d\tan^{-1}\left(\frac{\sqrt{-\frac{\sqrt{-c}\left(b-\frac{ad}{c}\right)}{\sqrt{d}}}\,x}{\sqrt{bx^4+a}}\right)}{4c(bc-ad)\sqrt{\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}} - \frac{d\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{-c}\left(b-\frac{ad}{c}\right)}{\sqrt{d}}}\,x}{\sqrt{bx^4+a}}\right)}{4c(bc-ad)\sqrt{-\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}}$$

$$- \frac{\sqrt[4]{b}d\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\left(\sqrt{b}c-\sqrt{a}\sqrt{-c}\sqrt{d}\right)(bc-ad)\sqrt{bx^4+a}}$$

$$- \frac{\sqrt[4]{b}d\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)(bc-ad)\sqrt{bx^4+a}}$$

$$+ \frac{b^{3/4}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}(bc-ad)\sqrt{bx^4+a}}$$

$$+ \frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)d\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(bc-ad)\sqrt{bx^4+a}}$$

$$+ \frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)d\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)(bc-ad)\sqrt{bx^4+a}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/((a + b*x^4)^(3/2)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(2*a*(b*c - a*d)*Sqrt[a + b*x^4]) - (d*ArcTan[(Sqrt[-((Sqrt
[-c]*(b - (a*d)/c))/Sqrt[d])]*x)/Sqrt[a + b*x^4]])/(4*c*(b*c - a*
d)*Sqrt[(b*c - a*d)/(Sqrt[-c]*Sqrt[d])]) - (d*ArcTan[(Sqrt[(Sqrt[
-c]*(b - (a*d)/c))/Sqrt[d]]*x)/Sqrt[a + b*x^4]])/(4*c*(b*c - a*d)
*Sqrt[-((b*c - a*d)/(Sqrt[-c]*Sqrt[d]))]) + (b^(3/4)*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF
[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*(b*c - a*d)*Sqrt
[a + b*x^4]) - (b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4
)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/
4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c - Sqrt[a]*Sqrt[-c]*Sqrt[d])*(b*c
 - a*d)*Sqrt[a + b*x^4]) - (b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1
/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-c]*S
qrt[d])*(b*c - a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] + Sqrt[
a]*Sqrt[d])*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
 Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])
^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(
```

```
1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqr
t[d])*(b*c - a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] - Sqrt[a]
*Sqrt[d])*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/
(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d
])*(b*c - a*d)*Sqrt[a + b*x^4])
```

---

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**4+a)**(3/2)/(d*x**4+c),x)
```

```
[Out] Timed out
```

---

**Mathematica [C]** time = 0.495144, size = 342, normalized size = 0.35

$$\frac{x\left(\frac{9bcdx^4F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(c+dx^4)\left(2x^4\left(2adF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}+\frac{25c(bc-}{(c+dx^4)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right.\right.\right.}\right.}{10\sqrt{a+bx^4}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^4)^(3/2)*(c + d*x^4)),x]
```

```
[Out] (x*((-5*b)/a + (25*c*(b*c - 2*a*d)*AppellF1[1/4, 1/2, 1, 5/4, -((
b*x^4)/a), -((d*x^4)/c)])/((c + d*x^4)*(-5*a*c*AppellF1[1/4, 1/2,
 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*x^4*(2*a*d*AppellF1[5/4,
 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2
, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))) + (9*b*c*d*x^4*AppellF1[
5/4, 1/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])/((c + d*x^4)*(-9*a
*c*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*x^4
*(2*a*d*AppellF1[9/4, 1/2, 2, 13/4, -((b*x^4)/a), -((d*x^4)/c)] +
 b*c*AppellF1[9/4, 3/2, 1, 13/4, -((b*x^4)/a), -((d*x^4)/c)]))))
/(10*(-(b*c) + a*d)*Sqrt[a + b*x^4])
```

---

**Maple [C]** time = 0.043, size = 313, normalized size = 0.3

$$-\frac{bx}{2a(ad-bc)}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}$$
$$-\frac{b}{2a(ad-bc)}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{1}{8}\sum_{\_alpha=RootOf(\_Z^4d+c)}\frac{1}{(ad-bc)\_alpha^3}\left(-1Artanh\left(\frac{2\_alpha^2bx^2+2a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}+2\frac{\_alpha^3d}{c\sqrt{bx^4+a}}\sqrt{1-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^4+a)^(3/2)/(d*x^4+c),x)
```

```
[Out] -1/2*b/a*x/(a*d-b*c)/((x^4+a/b)*b)^(1/2)-1/2*b/a/(a*d-b*c)/(I/a^(
1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2)
)^(1/2),I)+1/8*sum(1/(a*d-b*c)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*a
rctanh(1/2*(2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(
1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-I/a^(1/2)*b^(1/
2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ell
ipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2)*_alpha^2/c*
d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alpha=R
ootOf(_Z^4*d+c))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(bx^4+a)^{\frac{3}{2}}(dx^4+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^4\right)^{\frac{3}{2}}\left(c+dx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**4+a)**(3/2)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(3/2)*(c + d*x**4)), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{2}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)), x)
```

## 3.84 $\int \frac{1}{(a+bx^4)^{5/2}(c+dx^4)}\,dx$

**Optimal**. Leaf size=1029

result too large to display

```
[Out] (b*x)/(6*a*(b*c - a*d)*(a + b*x^4)^(3/2)) + (b*(5*b*c - 11*a*d)*x
)/(12*a^2*(b*c - a*d)^2*Sqrt[a + b*x^4]) + (d^2*ArcTan[(Sqrt[-((S
qrt[-c]*(b - (a*d)/c))/Sqrt[d])]*x)/Sqrt[a + b*x^4]])/(4*c*(b*c -
 a*d)^2*Sqrt[(b*c - a*d)/(Sqrt[-c]*Sqrt[d])]) + (d^2*ArcTan[(Sqrt
[(Sqrt[-c]*(b - (a*d)/c))/Sqrt[d]]*x)/Sqrt[a + b*x^4]])/(4*c*(b*c
 - a*d)^2*Sqrt[-((b*c - a*d)/(Sqrt[-c]*Sqrt[d]))]) + (b^(1/4)*d^2
*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)
^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sq
rt[b]*c - Sqrt[a]*Sqrt[-c]*Sqrt[d])*(b*c - a*d)^2*Sqrt[a + b*x^4]
) + (b^(1/4)*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]
)/(4*a^(1/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-c]*Sqrt[d])*(b*c - a*d)^2
*Sqrt[a + b*x^4]) + (b^(3/4)*(5*b*c - 11*a*d)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcT
an[(b^(1/4)*x)/a^(1/4)], 1/2])/(24*a^(9/4)*(b*c - a*d)^2*Sqrt[a +
 b*x^4]) - ((Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*d^2*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticP
i[-(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt
[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^
(1/4)*c*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*Sqrt[a
 + b*x^4]) - ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*d^2*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqr
t[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b
^(1/4)*c*(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*Sqrt[
a + b*x^4])
```

---

**Rubi [A]** time = 4.38356, antiderivative size = 1029, normalized size of antiderivative = 1., number

of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{\tan^{-1}\left(\frac{\sqrt{-\frac{\sqrt{-c}\left(b-\frac{ad}{c}\right)}{\sqrt{d}}}x}{\sqrt{bx^4+a}}\right)d^2}{4c(bc-ad)^2\sqrt{\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\frac{\sqrt{-c}\left(b-\frac{ad}{c}\right)}{\sqrt{d}}}x}{\sqrt{bx^4+a}}\right)d^2}{4c(bc-ad)^2\sqrt{-\frac{bc-ad}{\sqrt{-c}\sqrt{d}}}}$$
$$+\frac{\sqrt[4]{b}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)d^2}{4\sqrt[4]{a}\left(\sqrt{b}c-\sqrt{a}\sqrt{-c}\sqrt{d}\right)(bc-ad)^2\sqrt{bx^4+a}}$$
$$+\frac{\sqrt[4]{b}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)d^2}{4\sqrt[4]{a}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)(bc-ad)^2\sqrt{bx^4+a}}$$
$$-\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)d^2}{8\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(bc-ad)^2\sqrt{bx^4+a}}$$
$$-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)d^2}{8\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)(bc-ad)^2\sqrt{bx^4+a}}$$
$$+\frac{b^{3/4}(5bc-11ad)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{24a^{9/4}(bc-ad)^2\sqrt{bx^4+a}}$$
$$+\frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{bx^4+a}}+\frac{bx}{6a(bc-ad)\left(bx^4+a\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/((a + b*x^4)^(5/2)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(6*a*(b*c - a*d)*(a + b*x^4)^(3/2)) + (b*(5*b*c - 11*a*d)*x
)/(12*a^2*(b*c - a*d)^2*Sqrt[a + b*x^4]) + (d^2*ArcTan[(Sqrt[-((S
qrt[-c]*(b - (a*d)/c))/Sqrt[d])]*x)/Sqrt[a + b*x^4]])/(4*c*(b*c -
 a*d)^2*Sqrt[(b*c - a*d)/(Sqrt[-c]*Sqrt[d])]) + (d^2*ArcTan[(Sqrt
[(Sqrt[-c]*(b - (a*d)/c))/Sqrt[d]]*x)/Sqrt[a + b*x^4]])/(4*c*(b*c
 - a*d)^2*Sqrt[-((b*c - a*d)/(Sqrt[-c]*Sqrt[d]))]) + (b^(1/4)*d^2
*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)
^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(Sq
rt[b]*c - Sqrt[a]*Sqrt[-c]*Sqrt[d])*(b*c - a*d)^2*Sqrt[a + b*x^4]
) + (b^(1/4)*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]
)/(4*a^(1/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-c]*Sqrt[d])*(b*c - a*d)^2
*Sqrt[a + b*x^4]) + (b^(3/4)*(5*b*c - 11*a*d)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcT
an[(b^(1/4)*x)/a^(1/4)], 1/2])/(24*a^(9/4)*(b*c - a*d)^2*Sqrt[a +
```

```
 b*x^4]) - ((Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*d^2*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticP
i[-(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt
[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^
(1/4)*c*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*Sqrt[a
 + b*x^4]) - ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*d^2*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqr
t[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b
^(1/4)*c*(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*Sqrt[
a + b*x^4])
```

---

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**4+a)**(5/2)/(d*x**4+c),x)
```

```
[Out] Timed out
```

---

**Mathematica [C]** time = 1.64098, size = 406, normalized size = 0.39

$$\frac{x\left(\frac{25ac\left(12a^2d^2-11abcd+5b^2c^2\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(c+dx^4)\left(5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}+\frac{5b\left(-13a^2d+ab\left(7c-11dx^4\right)+5b^2cx^4\right)}{a+bx^4}+\cdots\right)}{60a^2\sqrt{a+bx^4}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^4)^(5/2)*(c + d*x^4)),x]
```

```
[Out] (x*((5*b*(-13*a^2*d + 5*b^2*c*x^4 + a*b*(7*c - 11*d*x^4)))/(a + b
*x^4) + (25*a*c*(5*b^2*c^2 - 11*a*b*c*d + 12*a^2*d^2)*AppellF1[1/
4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/((c + d*x^4)*(5*a*c*
AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 2*x^4*(2
*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c
*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))) + (9*a
*b*c*d*(-5*b*c + 11*a*d)*x^4*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)
/a), -((d*x^4)/c)])/((c + d*x^4)*(-9*a*c*AppellF1[5/4, 1/2, 1, 9/
4, -((b*x^4)/a), -((d*x^4)/c)] + 2*x^4*(2*a*d*AppellF1[9/4, 1/2,
2, 13/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[9/4, 3/2, 1,
13/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(60*a^2*(b*c - a*d)^2*Sqrt
[a + b*x^4])
```

---

**Maple [C]** time = 0.047, size = 371, normalized size = 0.4

$$-\frac{x}{6ab\,(ad-bc)}\sqrt{bx^4+a}\left(x^4+\frac{a}{b}\right)^{-2}-\frac{bx\,(11\,ad-5\,bc)}{12\,a^2\,(ad-bc)^2}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}$$
$$-\frac{b\,(11\,ad-5\,bc)}{12\,a^2\,(ad-bc)^2}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}$$
$$+\frac{d}{8}\sum_{\_alpha=RootOf(\_Z^4d+c)}\frac{1}{(ad-bc)^2\,\_alpha^3}\left(-1Artanh\left(\frac{2\,\_alpha^2bx^2+2\,a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}+2\,\frac{\_alpha^3d}{c\sqrt{bx^4+a}}\sqrt{1\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^4+a)^(5/2)/(d*x^4+c),x)
```

```
[Out] -1/6/a*x/b/(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^2-1/12*b/a^2*x*(11
*a*d-5*b*c)/(a*d-b*c)^2/((x^4+a/b)*b)^(1/2)-1/12*b/a^2*(11*a*d-5*
b*c)/(a*d-b*c)^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x
^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ellipti
cF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/8*d*sum(1/(a*d-b*c)^2/_alpha^
3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*b*x^2+2*a)/((a*
d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)*_alp
ha^3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2
)^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*
a^(1/2)/b^(1/2)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2
)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d+c))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(bx^4+a)^{\frac{5}{2}}(dx^4+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(5/2)*(d*x^4 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/2)*(d*x^4 + c)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(5/2)*(d*x^4 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^{4}\right)^{\frac{5}{2}}\left(c+dx^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**4+a)**(5/2)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(5/2)*(c + d*x**4)), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{4}+a\right)^{\frac{5}{2}}\left(dx^{4}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(5/2)*(d*x^4 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/2)*(d*x^4 + c)), x)
```

## 3.85 $\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx$

**Optimal**. Leaf size=426

$$-\frac{bx\sqrt{a-bx^4}\left(21a^2d^2-122abcd+77b^2c^2\right)}{84cd^3}+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{84cd^4\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(3ad+11bc)(bc-ad)^3\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^4\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(3ad+11bc)(bc-ad)^3\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^4\sqrt{a-bx^4}}+\frac{bx\left(a-bx^4\right)^{3/2}(11bc-7ad)}{28cd^2}-\frac{x\left(a-bx^4\right)^{5/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

```
[Out] -(b*(77*b^2*c^2 - 122*a*b*c*d + 21*a^2*d^2)*x*Sqrt[a - b*x^4])/(8
4*c*d^3) + (b*(11*b*c - 7*a*d)*x*(a - b*x^4)^(3/2))/(28*c*d^2) -
((b*c - a*d)*x*(a - b*x^4)^(5/2))/(4*c*d*(c - d*x^4)) + (a^(1/4)*
b^(3/4)*(231*b^3*c^3 - 553*a*b^2*c^2*d + 349*a^2*b*c*d^2 + 21*a^3
*d^3)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)],
-1])/(84*c*d^4*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)^3*(11*b*c
+ 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt
[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d
^4*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)^3*(11*b*c + 3*a*d)*Sqr
t[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]),
ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^4*Sqrt[a - b*x
^4])
```

---

**Rubi [A]** time = 1.39091, antiderivative size = 426, normalized size of antiderivative = 1., number of

steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$

$$-\frac{bx\sqrt{a-bx^4}\left(21a^2d^2-122abcd+77b^2c^2\right)}{84cd^3}+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{84cd^4\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(3ad+11bc)(bc-ad)^3\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^4\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(3ad+11bc)(bc-ad)^3\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^4\sqrt{a-bx^4}}+\frac{bx\left(a-bx^4\right)^{3/2}(11bc-7ad)}{28cd^2}-\frac{x\left(a-bx^4\right)^{5/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b*x^4)^(7/2)/(c - d*x^4)^2,x]
```

```
[Out] -(b*(77*b^2*c^2 - 122*a*b*c*d + 21*a^2*d^2)*x*Sqrt[a - b*x^4])/(8
4*c*d^3) + (b*(11*b*c - 7*a*d)*x*(a - b*x^4)^(3/2))/(28*c*d^2) -
((b*c - a*d)*x*(a - b*x^4)^(5/2))/(4*c*d*(c - d*x^4)) + (a^(1/4)*
b^(3/4)*(231*b^3*c^3 - 553*a*b^2*c^2*d + 349*a^2*b*c*d^2 + 21*a^3
*d^3)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)],
-1])/(84*c*d^4*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)^3*(11*b*c
+ 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt
[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d
^4*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)^3*(11*b*c + 3*a*d)*Sqr
t[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]),
ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^4*Sqrt[a - b*x
^4])
```

---

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**4+a)**(7/2)/(-d*x**4+c)**2,x)
```

```
[Out] Timed out
```

---

**Mathematica [C]** time = 1.82599, size = 580, normalized size = 1.36

$$\frac{x\left(\frac{25a^2(63a^3d^3+63a^2bcd^2-155ab^2c^2d+77b^3c^3)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}+\frac{9ac(105a^4d^3-63a^3bd^2(5c+2dx^4)+a^2b^2cd(775c-494dx^4)+ab^3}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a - b*x^4)^(7/2)/(c - d*x^4)^2,x]
```

```
[Out] (x*((25*a^2*(77*b^3*c^3 - 155*a*b^2*c^2*d + 63*a^2*b*c*d^2 + 63*a
^3*d^3)*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/(5*a*c*
AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*A
ppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/
4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])) + (9*a*c*(105*a^4*d^3 + a
^2*b^2*c*d*(775*c - 494*d*x^4) - 63*a^3*b*d^2*(5*c + 2*d*x^4) + 2
*b^4*c*x^4*(77*c^2 - 110*c*d*x^4 - 30*d^2*x^8) + a*b^3*c*(-385*c^
2 - 2*c*d*x^4 + 520*d^2*x^8))*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/
a, (d*x^4)/c] - 10*x^4*(-a + b*x^4)*(-63*a^2*b*c*d^2 + 21*a^3*d^3
 + a*b^2*c*d*(155*c - 92*d*x^4) + b^3*c*(-77*c^2 + 44*c*d*x^4 + 1
2*d^2*x^8))*(2*a*d*AppellF1[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)
/c] + b*c*AppellF1[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c]))/(c*
(9*a*c*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(
2*a*d*AppellF1[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*App
ellF1[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c])))))/(420*d^3*Sqrt
[a - b*x^4]*(c - d*x^4))
```

---

**Maple [C]** time = 0.041, size = 540, normalized size = 1.3

$$-\frac{(a^3d^3-3\,a^2cd^2b+3\,ac^2db^2-c^3b^3)\,x}{4\,cd^3\,(dx^4-c)}\sqrt{-bx^4+a}$$
$$-\frac{b^3x^5}{7\,d^2}\sqrt{-bx^4+a}-\frac{x}{3\,b}\left(-2\,\frac{b^3\,(2\,ad-bc)}{d^3}+\frac{5\,ab^3}{7\,d^2}\right)\sqrt{-bx^4+a}$$
$$+1\left(\frac{b^2\left(6\,a^2d^2-8\,cabd+3\,b^2c^2\right)}{d^4}+\frac{b\left(a^3d^3-3\,a^2cd^2b+3\,ac^2db^2-c^3b^3\right)}{4\,d^4c}+\frac{a}{3\,b}\left(-2\,\frac{b^3\,(2\,ad-bc)}{d^3}+\frac{5\,ab^3}{7\,d^2}\right)\right)\sqrt{1-x^2\sqrt{b}-}$$
$$-\frac{1}{32\,d^5c}\sum_{\_alpha=RootOf(\_Z^4d-c)}\frac{3\,a^4d^4+2\,a^3bd^3c-24\,a^2b^2c^2d^2+30\,ab^3c^3d-11\,b^4c^4}{\_alpha^3}\left(-1Artanh\left(\frac{-2\,\_alpha^2bx^2+2\,a}{2}\frac{1}{\sqrt{\frac{ad-}{d}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x)
```

```
[Out] -1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/c/d^3*x*(-b*x^
4+a)^(1/2)/(d*x^4-c)-1/7*b^3/d^2*x^5*(-b*x^4+a)^(1/2)-1/3*(-2*b^3
/d^3*(2*a*d-b*c)+5/7*b^3/d^2*a)/b*x*(-b*x^4+a)^(1/2)+(b^2*(6*a^2*
d^2-8*a*b*c*d+3*b^2*c^2)/d^4+1/4*b/d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a
*b^2*c^2*d-b^3*c^3)/c+1/3*(-2*b^3/d^3*(2*a*d-b*c)+5/7*b^3/d^2*a)/
b*a)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b
^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)
*b^(1/2))^(1/2),I)-1/32/d^5/c*sum((3*a^4*d^4+2*a^3*b*c*d^3-24*a^2
*b^2*c^2*d^2+30*a*b^3*c^3*d-11*b^4*c^4)/_alpha^3*(-1/((a*d-b*c)/d
)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(
-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-b^(1
/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(
1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alph
a^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_a
lpha=RootOf(_Z^4*d-c))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^4+a\right)^{\frac{7}{2}}}{\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**4+a)**(7/2)/(-d*x**4+c)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^4+a\right)^{\frac{7}{2}}}{\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2, x)
```

## 3.86 $\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx$

**Optimal**. Leaf size=365

$$-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-26abcd+21b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12cd^3\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(3ad+7bc)(bc-ad)^2\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(3ad+7bc)(bc-ad)^2\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}}+\frac{bx\sqrt{a-bx^4}(7bc-3ad)}{12cd^2}-\frac{x\left(a-bx^4\right)^{3/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

```
[Out] (b*(7*b*c - 3*a*d)*x*Sqrt[a - b*x^4])/(12*c*d^2) - ((b*c - a*d)*x
*(a - b*x^4)^(3/2))/(4*c*d*(c - d*x^4)) - (a^(1/4)*b^(3/4)*(21*b^
2*c^2 - 26*a*b*c*d - 3*a^2*d^2)*Sqrt[1 - (b*x^4)/a]*EllipticF[Arc
Sin[(b^(1/4)*x)/a^(1/4)], -1])/(12*c*d^3*Sqrt[a - b*x^4]) + (a^(1
/4)*(b*c - a*d)^2*(7*b*c + 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[
-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4
)], -1])/(8*b^(1/4)*c^2*d^3*Sqrt[a - b*x^4]) + (a^(1/4)*(b*c - a*
d)^2*(7*b*c + 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt
[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1
/4)*c^2*d^3*Sqrt[a - b*x^4])
```

---

**Rubi [A]** time = 1.04876, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$

$$-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-26abcd+21b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12cd^3\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(3ad+7bc)(bc-ad)^2\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(3ad+7bc)(bc-ad)^2\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}}+\frac{bx\sqrt{a-bx^4}(7bc-3ad)}{12cd^2}-\frac{x\left(a-bx^4\right)^{3/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b*x^4)^(5/2)/(c - d*x^4)^2,x]
```

```
[Out] (b*(7*b*c - 3*a*d)*x*Sqrt[a - b*x^4])/(12*c*d^2) - ((b*c - a*d)*x
*(a - b*x^4)^(3/2))/(4*c*d*(c - d*x^4)) - (a^(1/4)*b^(3/4)*(21*b^
2*c^2 - 26*a*b*c*d - 3*a^2*d^2)*Sqrt[1 - (b*x^4)/a]*EllipticF[Arc
Sin[(b^(1/4)*x)/a^(1/4)], -1])/(12*c*d^3*Sqrt[a - b*x^4]) + (a^(1
/4)*(b*c - a*d)^2*(7*b*c + 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[
-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4
)], -1])/(8*b^(1/4)*c^2*d^3*Sqrt[a - b*x^4]) + (a^(1/4)*(b*c - a*
d)^2*(7*b*c + 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt
[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1
/4)*c^2*d^3*Sqrt[a - b*x^4])
```

---

**Rubi in Sympy [A]** time = 168.627, size = 333, normalized size = 0.91

$$\frac{\sqrt[4]{a}b^{\frac{3}{4}}\sqrt{1-\frac{bx^4}{a}}\left(3a^2d^2+26abcd-21b^2c^2\right)F\left(\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12cd^3\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(ad-bc\right)^2\left(3ad+7bc\right)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(ad-bc\right)^2\left(3ad+7bc\right)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}}-\frac{bx\sqrt{a-bx^4}\left(3ad-7bc\right)}{12cd^2}+\frac{x\left(a-bx^4\right)^{\frac{3}{2}}\left(ad-bc\right)}{4cd\left(c-dx^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**4+a)**(5/2)/(-d*x**4+c)**2,x)
```

```
[Out] a**(1/4)*b**(3/4)*sqrt(1 - b*x**4/a)*(3*a**2*d**2 + 26*a*b*c*d -
21*b**2*c**2)*elliptic_f(asin(b**(1/4)*x/a**(1/4)), -1)/(12*c*d**
3*sqrt(a - b*x**4)) + a**(1/4)*sqrt(1 - b*x**4/a)*(a*d - b*c)**2*
(3*a*d + 7*b*c)*elliptic_pi(-sqrt(a)*sqrt(d)/(sqrt(b)*sqrt(c)), a
sin(b**(1/4)*x/a**(1/4)), -1)/(8*b**(1/4)*c**2*d**3*sqrt(a - b*x*
*4)) + a**(1/4)*sqrt(1 - b*x**4/a)*(a*d - b*c)**2*(3*a*d + 7*b*c)
*elliptic_pi(sqrt(a)*sqrt(d)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/a
**(1/4)), -1)/(8*b**(1/4)*c**2*d**3*sqrt(a - b*x**4)) - b*x*sqrt(
a - b*x**4)*(3*a*d - 7*b*c)/(12*c*d**2) + x*(a - b*x**4)**(3/2)*(
a*d - b*c)/(4*c*d*(c - d*x**4))
```

---

**Mathematica [C]** time = 1.3619, size = 491, normalized size = 1.35

$$\frac{x\left(\frac{-10x^4(a-bx^4)(3a^2d^2-6abcd+b^2c(7c-4dx^4))\left(2adF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)-9ac(15a^3d^2-6a^2bd(5c+3dx^4)+ab^2c(35c-16dx}{c\left(2x^4\left(2adF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)}\right.}{60d^2\sqrt{a-bx^4}\,(dx^4}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a - b*x^4)^(5/2)/(c - d*x^4)^2,x]
```

```
[Out] (x*((-25*a^2*(-7*b^2*c^2 + 6*a*b*c*d + 9*a^2*d^2)*AppellF1[1/4, 1
/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/(5*a*c*AppellF1[1/4, 1/2, 1, 5
/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/
4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)
/a, (d*x^4)/c])) + (-9*a*c*(15*a^3*d^2 + a*b^2*c*(35*c - 16*d*x^4
) - 6*a^2*b*d*(5*c + 3*d*x^4) + 2*b^3*c*x^4*(-7*c + 10*d*x^4))*Ap
pellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] - 10*x^4*(a - b*x^
4)*(-6*a*b*c*d + 3*a^2*d^2 + b^2*c*(7*c - 4*d*x^4))*(2*a*d*Appell
F1[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[9/4, 3
/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c]))/(c*(9*a*c*AppellF1[5/4, 1/2,
 1, 9/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[9/4, 1/2,
2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[9/4, 3/2, 1, 13/4,
(b*x^4)/a, (d*x^4)/c])))))/(60*d^2*Sqrt[a - b*x^4]*(-c + d*x^4))
```

---

**Maple [C]** time = 0.04, size = 412, normalized size = 1.1

$$-\frac{(a^2d^2-2\,cabd+b^2c^2)\,x}{4\,cd^2\,(dx^4-c)}\sqrt{-bx^4+a}+\frac{b^2x}{3\,d^2}\sqrt{-bx^4+a}$$

$$+1\left(\frac{b^2\,(3\,ad-2\,bc)}{d^3}+\frac{b\,(a^2d^2-2\,cabd+b^2c^2)}{4\,cd^3}-\frac{ab^2}{3\,d^2}\right)\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}\,EllipticF\left(x\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}}$$

$$-\frac{1}{32\,cd^4}\sum_{\_alpha=RootOf(\_Z^4d-c)}\frac{3\,a^3d^3+a^2cd^2b-11\,ac^2db^2+7\,c^3b^3}{\_alpha^3}\left(-1Artanh\left(\frac{-2\,\_alpha^2bx^2+2\,a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{-bx^4+a}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x)
```

```
[Out] -1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c/d^2*x*(-b*x^4+a)^(1/2)/(d*x^4-
c)+1/3*b^2/d^2*x*(-b*x^4+a)^(1/2)+(b^2*(3*a*d-2*b*c)/d^3+1/4*b/d^
3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c-1/3*b^2/d^2*a)/(1/a^(1/2)*b^(1/2)
)^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/
2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32
/c/d^4*sum((3*a^3*d^3+a^2*b*c*d^2-11*a*b^2*c^2*d+7*b^3*c^3)/_alph
```

```
a^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/(
(a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*
_alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2)
)^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a
^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)
*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^4+a\right)^{\frac{5}{2}}}{\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c)^2,x, algorithm="maxima")

[Out] integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c)^2, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c)^2,x, algorithm="fricas")

[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**4+a)**(5/2)/(-d*x**4+c)**2,x)

[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^4 + a\right)^{\frac{5}{2}}}{\left(dx^4 - c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c)^2,x, algorithm="giac")

[Out] integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c)^2, x)
```

## 3.87 $\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx$

**Optimal**. Leaf size=309

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+3bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2\sqrt{a-bx^4}} - \frac{3\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)(ad+bc)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^2\sqrt{a-bx^4}} - \frac{3\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)(ad+bc)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^2\sqrt{a-bx^4}} - \frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\,(c-dx^4)}$$

```
[Out] -((b*c - a*d)*x*Sqrt[a - b*x^4])/(4*c*d*(c - d*x^4)) + (a^(1/4)*b
^(3/4)*(3*b*c + a*d)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4
)*x)/a^(1/4)], -1])/(4*c*d^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*(b*c -
 a*d)*(b*c + a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[
d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1
/4)*c^2*d^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*(b*c - a*d)*(b*c + a*d)
*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c
]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^2*Sqrt[a -
 b*x^4])
```

---

**Rubi [A]** time = 0.745897, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+3bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2\sqrt{a-bx^4}} - \frac{3\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)(ad+bc)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^2\sqrt{a-bx^4}} - \frac{3\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)(ad+bc)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^2\sqrt{a-bx^4}} - \frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\,(c-dx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b*x^4)^(3/2)/(c - d*x^4)^2,x]
```

```
[Out] -((b*c - a*d)*x*Sqrt[a - b*x^4])/(4*c*d*(c - d*x^4)) + (a^(1/4)*b
^(3/4)*(3*b*c + a*d)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4
)*x)/a^(1/4)], -1])/(4*c*d^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*(b*c -
```

```
 a*d)*(b*c + a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[
d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1
/4)*c^2*d^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*(b*c - a*d)*(b*c + a*d)
*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c
]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^2*Sqrt[a -
 b*x^4])
```

---

**Rubi in Sympy [A]** time = 107.566, size = 279, normalized size = 0.9

$$\frac{\sqrt[4]{a}b^{\frac{3}{4}}\sqrt{1-\frac{bx^4}{a}}\,(ad+3bc)\,F\left(\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2\sqrt{a-bx^4}} + \frac{3\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,(ad-bc)\,(ad+bc)\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^2\sqrt{a-bx^4}} + \frac{3\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,(ad-bc)\,(ad+bc)\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^2\sqrt{a-bx^4}} + \frac{x\sqrt{a-bx^4}\,(ad-bc)}{4cd\,(c-dx^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**4+a)**(3/2)/(-d*x**4+c)**2,x)
```

```
[Out] a**(1/4)*b**(3/4)*sqrt(1 - b*x**4/a)*(a*d + 3*b*c)*elliptic_f(asi
n(b**(1/4)*x/a**(1/4)), -1)/(4*c*d**2*sqrt(a - b*x**4)) + 3*a**(1
/4)*sqrt(1 - b*x**4/a)*(a*d - b*c)*(a*d + b*c)*elliptic_pi(-sqrt(
a)*sqrt(d)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/a**(1/4)), -1)/(8*b
**(1/4)*c**2*d**2*sqrt(a - b*x**4)) + 3*a**(1/4)*sqrt(1 - b*x**4/
a)*(a*d - b*c)*(a*d + b*c)*elliptic_pi(sqrt(a)*sqrt(d)/(sqrt(b)*s
qrt(c)), asin(b**(1/4)*x/a**(1/4)), -1)/(8*b**(1/4)*c**2*d**2*sqr
t(a - b*x**4)) + x*sqrt(a - b*x**4)*(a*d - b*c)/(4*c*d*(c - d*x**
4))
```

---

**Mathematica [C]** time = 0.593966, size = 423, normalized size = 1.37

$$\frac{x\left(\frac{-9ac\left(5a^2d-ab\left(5c+6dx^4\right)+2b^2cx^4\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)-10x^4\left(a-bx^4\right)(ad-bc)\left(2adF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)}{c\left(2x^4\left(2adF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)} - \frac{\cdots}{2x^4\left(2adF\cdots\right.}\right.}{20d\sqrt{a-bx^4}\,(dx^4-c)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a - b*x^4)^(3/2)/(c - d*x^4)^2,x]
```

```
[Out] (x*((-25*a^2*(b*c + 3*a*d)*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a,
(d*x^4)/c])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/
```

```
c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c
] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])) + (-9*
a*c*(5*a^2*d + 2*b^2*c*x^4 - a*b*(5*c + 6*d*x^4))*AppellF1[5/4, 1
/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] - 10*(-(b*c) + a*d)*x^4*(a - b*
x^4)*(2*a*d*AppellF1[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b
*c*AppellF1[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c]))/(c*(9*a*c*
AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*A
ppellF1[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[9
/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c])))))/(20*d*Sqrt[a - b*x^4
]*(-c + d*x^4))
```

---

**Maple [C]** time = 0.036, size = 329, normalized size = 1.1

$$-\frac{(ad-bc)\,x}{4\,cd\,(dx^4-c)}\sqrt{-bx^4+a}$$
$$+1\left(\frac{b^2}{d^2}+\frac{(ad-bc)\,b}{4\,cd^2}\right)\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-bx^4+a}}$$
$$-\frac{3}{32\,cd^3}\sum_{\_alpha=RootOf(\_Z^4d-c)}\frac{a^2d^2-b^2c^2}{\_alpha^3}\left(-1Artanh\left(\frac{-2\,\_alpha^2bx^2+2\,a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{-bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}-2\,\frac{\_alpha^3d}{c\sqrt{-bx^4+a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x)
```

```
[Out] -1/4*(a*d-b*c)/c/d*x*(-b*x^4+a)^(1/2)/(d*x^4-c)+(b^2/d^2+1/4*b/d^
2*(a*d-b*c)/c)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^
(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*
(1/a^(1/2)*b^(1/2))^(1/2),I)-3/32/c/d^3*sum((a^2*d^2-b^2*c^2)/_al
pha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)
/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2
)*_alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/
2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2)
,a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/
2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-bx^4+a\right)^{\frac{3}{2}}}{\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2,x, algorithm="maxima")

[Out] integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2,x, algorithm="fricas")

[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**4+a)**(3/2)/(-d*x**4+c)**2,x)

[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^4 + a\right)^{\frac{3}{2}}}{\left(dx^4 - c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2,x, algorithm="giac")

[Out] integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2, x)
```

## 3.88 $\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2} \, dx$

**Optimal**. Leaf size=276

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-3ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-3ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}}+\frac{x\sqrt{a-bx^4}}{4c(c-dx^4)}$$

```
[Out] (x*Sqrt[a - b*x^4])/(4*c*(c - d*x^4)) + (a^(1/4)*b^(3/4)*Sqrt[1 -
 (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(4*c*d*Sq
rt[a - b*x^4]) - (a^(1/4)*(b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*Ellip
ticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/
a^(1/4)], -1])/(8*b^(1/4)*c^2*d*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c
- 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b
]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d*Sq
rt[a - b*x^4])
```

---

**Rubi [A]** time = 0.620553, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-3ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-3ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}}+\frac{x\sqrt{a-bx^4}}{4c(c-dx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a - b*x^4]/(c - d*x^4)^2,x]
```


```
[Out] (x*Sqrt[a - b*x^4])/(4*c*(c - d*x^4)) + (a^(1/4)*b^(3/4)*Sqrt[1 -
 (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(4*c*d*Sq
rt[a - b*x^4]) - (a^(1/4)*(b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*Ellip
ticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/
a^(1/4)], -1])/(8*b^(1/4)*c^2*d*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c
- 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b
]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d*Sq
rt[a - b*x^4])
```


---

**Rubi in Sympy [A]** time = 97.1465, size = 243, normalized size = 0.88

$$\frac{\sqrt[4]{a}b^{\frac{3}{4}}\sqrt{1-\frac{bx^4}{a}}F\left(\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,(3ad-bc)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,(3ad-bc)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}}+\frac{x\sqrt{a-bx^4}}{4c\,(c-dx^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**4+a)**(1/2)/(-d*x**4+c)**2,x)
```

```
[Out] a**(1/4)*b**(3/4)*sqrt(1 - b*x**4/a)*elliptic_f(asin(b**(1/4)*x/a
**(1/4)), -1)/(4*c*d*sqrt(a - b*x**4)) + a**(1/4)*sqrt(1 - b*x**4
/a)*(3*a*d - b*c)*elliptic_pi(-sqrt(a)*sqrt(d)/(sqrt(b)*sqrt(c)),
 asin(b**(1/4)*x/a**(1/4)), -1)/(8*b**(1/4)*c**2*d*sqrt(a - b*x**
4)) + a**(1/4)*sqrt(1 - b*x**4/a)*(3*a*d - b*c)*elliptic_pi(sqrt(
a)*sqrt(d)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/a**(1/4)), -1)/(8*b
**(1/4)*c**2*d*sqrt(a - b*x**4)) + x*sqrt(a - b*x**4)/(4*c*(c - d
*x**4))
```

---

**Mathematica [C]** time = 0.269328, size = 310, normalized size = 1.12

$$\frac{x\left(-\frac{75a^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}+\frac{9abx^4F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{2x^4\left(2adF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\cdots}\right.}{20\sqrt{a-bx^4}\,(dx^4-c)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a - b*x^4]/(c - d*x^4)^2,x]
```

```
[Out] (x*((-5*(a - b*x^4))/c - (75*a^2*AppellF1[1/4, 1/2, 1, 5/4, (b*x^
4)/a, (d*x^4)/c])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d
*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*
x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))
+ (9*a*b*x^4*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/(9
*a*c*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*
a*d*AppellF1[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*Appel
lF1[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c]))))/(20*Sqrt[a - b*x
^4]*(-c + d*x^4))
```

---

**Maple [C]** time = 0.033, size = 294, normalized size = 1.1

$$-\frac{x}{4c(dx^4-c)}\sqrt{-bx^4+a} + \frac{b}{4cd}\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-bx^4+a}} - \frac{1}{32cd^2}\sum_{\_alpha=RootOf(\_Z^4d-c)}\frac{3ad-bc}{\_alpha^3}\left(-1Artanh\left(\frac{-2\_alpha^2bx^2+2a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{-bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}} - 2\frac{\_alpha^3d}{c\sqrt{-bx^4+a}}\sqrt{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x)
```

```
[Out] -1/4/c*x*(-b*x^4+a)^(1/2)/(d*x^4-c)+1/4*b/c/d/(1/a^(1/2)*b^(1/2))
^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2
)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32/
c/d^2*sum((3*a*d-b*c)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/
2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2
/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(
1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*
(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2
)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d
-c))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^4+a}}{(dx^4-c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**4+a)**(1/2)/(-d*x**4+c)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^4 + a}}{(dx^4 - c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2, x)
```

## 3.89 $\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)^2}\,dx$

**Optimal**. Leaf size=310

$$-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c\sqrt{a-bx^4}(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(5bc-3ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(5bc-3ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)}-\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

```
[Out] -(d*x*Sqrt[a - b*x^4])/(4*c*(b*c - a*d)*(c - d*x^4)) - (a^(1/4)*b
^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)],
 -1])/(4*c*(b*c - a*d)*Sqrt[a - b*x^4]) + (a^(1/4)*(5*b*c - 3*a*d
)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqr
t[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a
*d)*Sqrt[a - b*x^4]) + (a^(1/4)*(5*b*c - 3*a*d)*Sqrt[1 - (b*x^4)/
a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4
)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)*Sqrt[a - b*x^4])
```

---

**Rubi [A]** time = 0.688348, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304

$$-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c\sqrt{a-bx^4}(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(5bc-3ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(5bc-3ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)}-\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a - b*x^4]*(c - d*x^4)^2),x]
```


```
[Out] -(d*x*Sqrt[a - b*x^4])/(4*c*(b*c - a*d)*(c - d*x^4)) - (a^(1/4)*b
^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)],
 -1])/(4*c*(b*c - a*d)*Sqrt[a - b*x^4]) + (a^(1/4)*(5*b*c - 3*a*d
)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqr
t[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a
*d)*Sqrt[a - b*x^4]) + (a^(1/4)*(5*b*c - 3*a*d)*Sqrt[1 - (b*x^4)/
a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4
)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)*Sqrt[a - b*x^4])
```


---

**Rubi in Sympy [A]** time = 109.539, size = 270, normalized size = 0.87

$$\frac{\sqrt[4]{a}b^{\frac{3}{4}}\sqrt{1-\frac{bx^4}{a}}F\left(\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c\sqrt{a-bx^4}\,(ad-bc)}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,(3ad-5bc)\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}\,(ad-bc)}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,(3ad-5bc)\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}\,(ad-bc)}+\frac{dx\sqrt{a-bx^4}}{4c\,(c-dx^4)\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**4+a)**(1/2)/(-d*x**4+c)**2,x)
```

```
[Out] a**(1/4)*b**(3/4)*sqrt(1 - b*x**4/a)*elliptic_f(asin(b**(1/4)*x/a
**(1/4)), -1)/(4*c*sqrt(a - b*x**4)*(a*d - b*c)) + a**(1/4)*sqrt(
1 - b*x**4/a)*(3*a*d - 5*b*c)*elliptic_pi(-sqrt(a)*sqrt(d)/(sqrt(
b)*sqrt(c)), asin(b**(1/4)*x/a**(1/4)), -1)/(8*b**(1/4)*c**2*sqrt
(a - b*x**4)*(a*d - b*c)) + a**(1/4)*sqrt(1 - b*x**4/a)*(3*a*d -
5*b*c)*elliptic_pi(sqrt(a)*sqrt(d)/(sqrt(b)*sqrt(c)), asin(b**(1/
4)*x/a**(1/4)), -1)/(8*b**(1/4)*c**2*sqrt(a - b*x**4)*(a*d - b*c)
) + d*x*sqrt(a - b*x**4)/(4*c*(c - d*x**4)*(a*d - b*c))
```

---

**Mathematica [C]** time = 0.991384, size = 349, normalized size = 1.13

$$\frac{x\left(\frac{9abdx^4F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{(ad-bc)\left(2x^4\left(2adF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{9}{4};\frac{3}{2},1;\frac{13}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+9acF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)}+\frac{25a(3ad-4bc)F_1\left(\frac{1}{4};\right.}{(bc-ad)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\right.\right.\right.}\right.}{20\sqrt{a-bx^4}\,(dx^4-c)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(Sqrt[a - b*x^4]*(c - d*x^4)^2),x]
```

```
[Out] (x*((5*d*(a - b*x^4))/(c*(b*c - a*d)) + (25*a*(-4*b*c + 3*a*d)*Ap
pellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/((b*c - a*d)*(5*a
*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*
d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1
[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))) + (9*a*b*d*x^4*Appell
F1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/((-(b*c) + a*d)*(9*a*
c*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d
*AppellF1[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1
[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/c]))))/(20*Sqrt[a - b*x^4
]*(-c + d*x^4))
```

---

**Maple [C]** time = 0.035, size = 322, normalized size = 1.

$$-\frac{dx}{(4\,ad-4\,bc)\,c\,(dx^4-c)}\sqrt{-bx^4+a}+\frac{b}{(4\,ad-4\,bc)\,c}\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-bx^4+a}}-\frac{1}{32\,cd}\sum_{\_alpha=RootOf(\_Z^4d-c)}\frac{3\,ad-5\,bc}{(ad-bc)\,\_alpha^3}\left(-1Artanh\left(\frac{-2\,\_alpha^2bx^2+2\,a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{-bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}-2\frac{\_alpha}{c\sqrt{-bx^4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x)
```

```
[Out] -1/4*d/(a*d-b*c)/c*x*(-b*x^4+a)^(1/2)/(d*x^4-c)+1/4*b/(a*d-b*c)/c
/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/
2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(
1/2))^(1/2),I)-1/32/c/d*sum((3*a*d-5*b*c)/(a*d-b*c)/_alpha^3*(-1/
((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c
)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3
*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/
(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b
^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2)
)^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-bx^4+a}(dx^4-c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**4+a)**(1/2)/(-d*x**4+c)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-bx^4 + a}(dx^4 - c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2), x)
```

## 3.90 $\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx$

**Optimal**. Leaf size=362

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+2bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4a^{3/4}c\sqrt{a-bx^4}(bc-ad)^2} - \frac{3\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}(3bc-ad)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)^2} - \frac{3\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}(3bc-ad)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)^2} + \frac{bx(ad+2bc)}{4ac\sqrt{a-bx^4}(bc-ad)^2} - \frac{dx}{4c\sqrt{a-bx^4}\,(c-dx^4)\,(bc-ad)}$$

```
[Out] (b*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*Sqrt[a - b*x^4]) - (d*x)
/(4*c*(b*c - a*d)*Sqrt[a - b*x^4]*(c - d*x^4)) + (b^(3/4)*(2*b*c
+ a*d)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)],
 -1])/(4*a^(3/4)*c*(b*c - a*d)^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*d*
(3*b*c - a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/
(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*
c^2*(b*c - a*d)^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*d*(3*b*c - a*d)*S
qrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])
, ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)^2*
Sqrt[a - b*x^4])
```

---

**Rubi [A]** time = 1.07374, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+2bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4a^{3/4}c\sqrt{a-bx^4}(bc-ad)^2} - \frac{3\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}(3bc-ad)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)^2} - \frac{3\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}(3bc-ad)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)^2} + \frac{bx(ad+2bc)}{4ac\sqrt{a-bx^4}(bc-ad)^2} - \frac{dx}{4c\sqrt{a-bx^4}\,(c-dx^4)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a - b*x^4)^(3/2)*(c - d*x^4)^2),x]
```

```
[Out] (b*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*Sqrt[a - b*x^4]) - (d*x)
/(4*c*(b*c - a*d)*Sqrt[a - b*x^4]*(c - d*x^4)) + (b^(3/4)*(2*b*c
+ a*d)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)],
 -1])/(4*a^(3/4)*c*(b*c - a*d)^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*d*
(3*b*c - a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/
(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*
c^2*(b*c - a*d)^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*d*(3*b*c - a*d)*S
qrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])
, ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)^2*
Sqrt[a - b*x^4])
```

---

**Rubi in Sympy [A]** time = 175.193, size = 323, normalized size = 0.89

$$\frac{3\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\,(ad-3bc)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}\,(ad-bc)^2}+\frac{3\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\,(ad-3bc)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}\,(ad-bc)^2}+\frac{dx}{4c\sqrt{a-bx^4}\,(c-dx^4)\,(ad-bc)}+\frac{bx\,(ad+2bc)}{4ac\sqrt{a-bx^4}\,(ad-bc)^2}+\frac{b^{\frac{3}{4}}\sqrt{1-\frac{bx^4}{a}}\,(ad+2bc)\,F\left(\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4a^{\frac{3}{4}}c\sqrt{a-bx^4}\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**4+a)**(3/2)/(-d*x**4+c)**2,x)
```

```
[Out] 3*a**(1/4)*d*sqrt(1 - b*x**4/a)*(a*d - 3*b*c)*elliptic_pi(-sqrt(a
)*sqrt(d)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/a**(1/4)), -1)/(8*b*
*(1/4)*c**2*sqrt(a - b*x**4)*(a*d - b*c)**2) + 3*a**(1/4)*d*sqrt(
1 - b*x**4/a)*(a*d - 3*b*c)*elliptic_pi(sqrt(a)*sqrt(d)/(sqrt(b)*
sqrt(c)), asin(b**(1/4)*x/a**(1/4)), -1)/(8*b**(1/4)*c**2*sqrt(a
- b*x**4)*(a*d - b*c)**2) + d*x/(4*c*sqrt(a - b*x**4)*(c - d*x**4
)*(a*d - b*c)) + b*x*(a*d + 2*b*c)/(4*a*c*sqrt(a - b*x**4)*(a*d -
 b*c)**2) + b**(3/4)*sqrt(1 - b*x**4/a)*(a*d + 2*b*c)*elliptic_f(
asin(b**(1/4)*x/a**(1/4)), -1)/(4*a**(3/4)*c*sqrt(a - b*x**4)*(a*
d - b*c)**2)
```

---

**Mathematica [C]** time = 1.18553, size = 465, normalized size = 1.28

$$\frac{x\left(\frac{25\left(3a^2d^2-8abcd+2b^2c^2\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}+\frac{9ac\left(5a^2d^2-6abd^2x^4+2b^2c\left(5c-6dx^4\right)\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)-\cdots}{ac\left(2x^4\left(2adF_1\left(\frac{9}{4};\frac{1}{2},2;\frac{13}{4};\frac{bx^4}{a},\cdots\right.\right.\right.}\right.}{20\sqrt{a-bx^4}\,(c-dx^4)\,(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a - b*x^4)^(3/2)*(c - d*x^4)^2),x]
```

```
[Out] (x*((25*(2*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*AppellF1[1/4, 1/2, 1,
 5/4, (b*x^4)/a, (d*x^4)/c])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b
*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*
x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d
*x^4)/c])) + (9*a*c*(5*a^2*d^2 - 6*a*b*d^2*x^4 + 2*b^2*c*(5*c - 6
*d*x^4))*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] - 10*x^
4*(-(a^2*d^2) + a*b*d^2*x^4 - 2*b^2*c*(c - d*x^4))*(2*a*d*AppellF
1[9/4, 1/2, 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[9/4, 3/
2, 1, 13/4, (b*x^4)/a, (d*x^4)/c]))/(a*c*(9*a*c*AppellF1[5/4, 1/2
, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[9/4, 1/2,
 2, 13/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[9/4, 3/2, 1, 13/4,
 (b*x^4)/a, (d*x^4)/c])))))/(20*(b*c - a*d)^2*Sqrt[a - b*x^4]*(c
- d*x^4))
```

---

**Maple [C]** time = 0.058, size = 375, normalized size = 1.

$$-\frac{d^2x}{4\,(ad-bc)^2\,c\,(dx^4-c)}\sqrt{-bx^4+a}+\frac{b^2x}{2\,a\,(ad-bc)^2}\frac{1}{\sqrt{-\left(x^4-\frac{a}{b}\right)b}}$$
$$+1\left(\frac{bd}{4\,(ad-bc)^2\,c}+\frac{b^2}{2\,a\,(ad-bc)^2}\right)\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-bx^4+a}}$$
$$-\frac{3}{32\,c}\sum_{\_alpha=RootOf\left(\_Z^4d-c\right)}\frac{ad-3\,bc}{(ad-bc)^2\,\_alpha^3}\left(-1Artanh\left(\frac{-2\,\_alpha^2bx^2+2\,a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{-bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}-2\,\frac{\_alpha}{c\sqrt{-bx^4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x)
```

```
[Out] -1/4*d^2/(a*d-b*c)^2/c*x*(-b*x^4+a)^(1/2)/(d*x^4-c)+1/2*b^2/a*x/(
a*d-b*c)^2/(-(x^4-a/b)*b)^(1/2)+(1/4*d*b/(a*d-b*c)^2/c+1/2*b^2/a/
(a*d-b*c)^2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1
/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1
/a^(1/2)*b^(1/2))^(1/2),I)-3/32/c*sum((a*d-3*b*c)/(a*d-b*c)^2/_al
pha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)
/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2
)*_alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/
2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2)
,a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/
2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^4 + a)^{\frac{3}{2}}(dx^4 - c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)^2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(3/2)/(-d*x**4+c)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^4 + a)^{\frac{3}{2}}(dx^4 - c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)^2), x)
```

## 3.91 $\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx$

**Optimal**. Leaf size=439

$$\frac{bx\left(-3a^2d^2-17abcd+5b^2c^2\right)}{12a^2c\sqrt{a-bx^4}(bc-ad)^3}+\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-17abcd+5b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12a^{7/4}c\sqrt{a-bx^4}(bc-ad)^3}+\frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}(13bc-3ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)^3}+\frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}(13bc-3ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)^3}-\frac{dx}{4c\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)(bc-ad)}+\frac{bx(3ad+2bc)}{12ac\left(a-bx^4\right)^{3/2}(bc-ad)^2}$$

```
[Out] (b*(2*b*c + 3*a*d)*x)/(12*a*c*(b*c - a*d)^2*(a - b*x^4)^(3/2)) +
(b*(5*b^2*c^2 - 17*a*b*c*d - 3*a^2*d^2)*x)/(12*a^2*c*(b*c - a*d)^
3*Sqrt[a - b*x^4]) - (d*x)/(4*c*(b*c - a*d)*(a - b*x^4)^(3/2)*(c
- d*x^4)) + (b^(3/4)*(5*b^2*c^2 - 17*a*b*c*d - 3*a^2*d^2)*Sqrt[1
- (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(12*a^(7
/4)*c*(b*c - a*d)^3*Sqrt[a - b*x^4]) + (a^(1/4)*d^2*(13*b*c - 3*a
*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*S
qrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c -
 a*d)^3*Sqrt[a - b*x^4]) + (a^(1/4)*d^2*(13*b*c - 3*a*d)*Sqrt[1 -
 (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSi
n[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)^3*Sqrt[a
- b*x^4])
```

---

**Rubi [A]** time = 1.43859, antiderivative size = 439, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$

$$\frac{bx\left(-3a^2d^2-17abcd+5b^2c^2\right)}{12a^2c\sqrt{a-bx^4}(bc-ad)^3}+\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-17abcd+5b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12a^{7/4}c\sqrt{a-bx^4}(bc-ad)^3}+\frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}(13bc-3ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)^3}+\frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}(13bc-3ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)^3}-\frac{dx}{4c\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)(bc-ad)}+\frac{bx(3ad+2bc)}{12ac\left(a-bx^4\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a - b*x^4)^(5/2)*(c - d*x^4)^2),x]
```

```
[Out] (b*(2*b*c + 3*a*d)*x)/(12*a*c*(b*c - a*d)^2*(a - b*x^4)^(3/2)) +
(b*(5*b^2*c^2 - 17*a*b*c*d - 3*a^2*d^2)*x)/(12*a^2*c*(b*c - a*d)^
3*Sqrt[a - b*x^4]) - (d*x)/(4*c*(b*c - a*d)*(a - b*x^4)^(3/2)*(c
- d*x^4)) + (b^(3/4)*(5*b^2*c^2 - 17*a*b*c*d - 3*a^2*d^2)*Sqrt[1
- (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(12*a^(7
/4)*c*(b*c - a*d)^3*Sqrt[a - b*x^4]) + (a^(1/4)*d^2*(13*b*c - 3*a
*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*S
qrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c -
 a*d)^3*Sqrt[a - b*x^4]) + (a^(1/4)*d^2*(13*b*c - 3*a*d)*Sqrt[1 -
 (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSi
n[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)^3*Sqrt[a
- b*x^4])
```

---

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**4+a)**(5/2)/(-d*x**4+c)**2,x)
```

```
[Out] Timed out
```

---

**Mathematica [C]** time = 2.18446, size = 617, normalized size = 1.41

$$x\left(\frac{25a\left(9a^3d^3-36a^2bcd^2+17ab^2c^2d-5b^3c^3\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}+\frac{10x^4\left(3a^4d^3-6a^3bd^3x^4+a^2b^2d\left(19c^2-19cdx^4+3d^2x^8\right)+ab^3c\left(-\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a - b*x^4)^(5/2)*(c - d*x^4)^2),x]
```

```
[Out] (x*((25*a*(-5*b^3*c^3 + 17*a*b^2*c^2*d - 36*a^2*b*c*d^2 + 9*a^3*d
^3)*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/(5*a*c*Appe
llF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*Appel
lF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3
/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])) + (9*a*c*(15*a^4*d^3 - 33*a^3
*b*d^3*x^4 + 5*b^4*c^2*x^4*(5*c - 6*d*x^4) + a^2*b^2*d*(95*c^2 -
112*c*d*x^4 + 18*d^2*x^8) + a*b^3*c*(-35*c^2 - 45*c*d*x^4 + 102*d
^2*x^8))*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + 10*x^
4*(3*a^4*d^3 - 6*a^3*b*d^3*x^4 + 5*b^4*c^2*x^4*(c - d*x^4) + a^2*
b^2*d*(19*c^2 - 19*c*d*x^4 + 3*d^2*x^8) + a*b^3*c*(-7*c^2 - 10*c*
```

```
d*x^4 + 17*d^2*x^8))*(2*a*d*AppellF1[9/4, 1/2, 2, 13/4, (b*x^4)/a
, (d*x^4)/c] + b*c*AppellF1[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)
/c]))/(c*(a - b*x^4)*(9*a*c*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a,
 (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[9/4, 1/2, 2, 13/4, (b*x^4)/a,
 (d*x^4)/c] + b*c*AppellF1[9/4, 3/2, 1, 13/4, (b*x^4)/a, (d*x^4)/
c])))))/(60*a^2*(-(b*c) + a*d)^3*Sqrt[a - b*x^4]*(c - d*x^4))
```

---

**Maple [C]** time = 0.057, size = 484, normalized size = 1.1

$$-\frac{bd^3x}{(4a^2d^2-8cabd+4b^2c^2)(ad-bc)c(bdx^4-bc)}\sqrt{-bx^4+a}$$
$$+\frac{x}{6(ad-bc)^2a}\sqrt{-bx^4+a}\left(x^4-\frac{a}{b}\right)^{-2}+\frac{b^2x(17ad-5bc)}{12a^2(ad-bc)^3}\frac{1}{\sqrt{-\left(x^4-\frac{a}{b}\right)b}}$$
$$+1\left(\frac{d^2b}{(4a^2d^2-8cabd+4b^2c^2)(ad-bc)c}+\frac{b^2(17ad-5bc)}{12a^2(ad-bc)^3}\right)\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{1\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{1\sqrt{}}}$$
$$-\frac{d}{32c}\sum_{\_alpha=RootOf(\_Z^4d-c)}\frac{3ad-13bc}{(ad-bc)^3\_alpha^3}\left(-1Artanh\left(\frac{-2\_alpha^2bx^2+2a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{-bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}-2\frac{\_alpha}{c\sqrt{-bx^4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x)
```

```
[Out] -1/4*b*d^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/c*x*(-b*x^4+a)^(
1/2)/(b*d*x^4-b*c)+1/6/(a*d-b*c)^2/a*x*(-b*x^4+a)^(1/2)/(x^4-a/b)
^2+1/12*b^2/a^2*x*(17*a*d-5*b*c)/(a*d-b*c)^3/(-(x^4-a/b)*b)^(1/2)
+(1/4*b*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/c+1/12*b^2/a^2*
(17*a*d-5*b*c)/(a*d-b*c)^3)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*
x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)
*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32*d/c*sum((3*a*d-13*
b*c)/(a*d-b*c)^3/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2
*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a
^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*
(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^
(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(
1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^4+a)^{\frac{5}{2}}(dx^4-c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)^2),x, algorithm="maxima")

[Out] integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)^2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)^2),x, algorithm="fricas")

[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(5/2)/(-d*x**4+c)**2,x)

[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^4 + a)^{\frac{5}{2}}(dx^4 - c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)^2),x, algorithm="giac")

[Out] integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)^2), x)
```

## 3.92 $\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx$

**Optimal**. Leaf size=103

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}$$

```
[Out] ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]]/(2*Sqrt[2]*a^
(1/4)*b^(1/4)*c) + ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b
*x^4]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*c)
```

---

**Rubi [A]** time = 0.141018, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^4]/(a*c - b*c*x^4),x]
```

```
[Out] ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]]/(2*Sqrt[2]*a^
(1/4)*b^(1/4)*c) + ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b
*x^4]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*c)
```

---

**Rubi in Sympy [A]** time = 22.7375, size = 94, normalized size = 0.91

$$\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{4\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{4\sqrt[4]{a}\sqrt[4]{b}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**(1/2)/(-b*c*x**4+a*c),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*a**(1/4)*b**(1/4)*x/sqrt(a + b*x**4))/(4*a**
(1/4)*b**(1/4)*c) + sqrt(2)*atanh(sqrt(2)*a**(1/4)*b**(1/4)*x/sqr
t(a + b*x**4))/(4*a**(1/4)*b**(1/4)*c)
```

---

**Mathematica [C]** time = 0.242136, size = 155, normalized size = 1.5

$$\frac{5ax\sqrt{a+bx^4}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},\frac{bx^4}{a}\right)}{c\left(a-bx^4\right)\left(2bx^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},\frac{bx^4}{a}\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{bx^4}{a},\frac{bx^4}{a}\right)\right)+5aF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},\frac{bx^4}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a + b*x^4]/(a*c - b*c*x^4),x]
```

```
[Out] (5*a*x*Sqrt[a + b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a),
(b*x^4)/a])/(c*(a - b*x^4)*(5*a*AppellF1[1/4, -1/2, 1, 5/4, -((b*
x^4)/a), (b*x^4)/a] + 2*b*x^4*(2*AppellF1[5/4, -1/2, 2, 9/4, -((b
*x^4)/a), (b*x^4)/a] + AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a), (
b*x^4)/a])))
```

---

**Maple [A]** time = 0.024, size = 103, normalized size = 1.

$$-\frac{\sqrt{2}}{4\,c}\arctan\left(\frac{\sqrt{2}}{2\,x}\sqrt{bx^4+a}\frac{1}{\sqrt[4]{ab}}\right)\frac{1}{\sqrt[4]{ab}}+\frac{\sqrt{2}}{8\,c}\ln\left(1\left(\frac{\sqrt{2}}{2\,x}\sqrt{bx^4+a}+\sqrt[4]{ab}\right)\left(\frac{\sqrt{2}}{2\,x}\sqrt{bx^4+a}-\sqrt[4]{ab}\right)^{-1}\right)\frac{1}{\sqrt[4]{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x)
```

```
[Out] -1/4/c*2^(1/2)/(a*b)^(1/4)*arctan(1/2*(b*x^4+a)^(1/2)*2^(1/2)/x/(
a*b)^(1/4))+1/8/c*2^(1/2)/(a*b)^(1/4)*ln((1/2*(b*x^4+a)^(1/2)*2^(
1/2)/x+(a*b)^(1/4))/(1/2*(b*x^4+a)^(1/2)*2^(1/2)/x-(a*b)^(1/4)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{bx^4+a}}{bcx^4-ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(b*x^4 + a)/(b*c*x^4 - a*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(b*x^4 + a)/(b*c*x^4 - a*c), x)
```

---

**Fricas [A]** time = 0.592124, size = 450, normalized size = 4.37

$$-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(2\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\right)}{\sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}}-x^2\right)-\frac{bx^4-a}{\sqrt{b}}}\right)$$
$$+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+2\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}+\sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}}+x^2\right)}{bx^4-a}\right)$$
$$-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(-\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+2\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}-\sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}}+x^2\right)}{bx^4-a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(b*x^4 + a)/(b*c*x^4 - a*c),x, algorithm="fricas")
```

```
[Out] -(1/4)^(1/4)*(1/(a*b*c^4))^(1/4)*arctan(2*(2*(1/4)^(3/4)*a*b*c^3*
x^3*(1/(a*b*c^4))^(3/4) - (1/4)^(1/4)*a*c*x*(1/(a*b*c^4))^(1/4))/
(sqrt(b*x^4 + a)*(a*c^2*sqrt(1/(a*b*c^4)) - x^2) - (b*x^4 - a)/sq
rt(b))) + 1/4*(1/4)^(1/4)*(1/(a*b*c^4))^(1/4)*log((4*(1/4)^(3/4)*
a*b*c^3*x^3*(1/(a*b*c^4))^(3/4) + 2*(1/4)^(1/4)*a*c*x*(1/(a*b*c^4
))^(1/4) + sqrt(b*x^4 + a)*(a*c^2*sqrt(1/(a*b*c^4)) + x^2))/(b*x^
4 - a)) - 1/4*(1/4)^(1/4)*(1/(a*b*c^4))^(1/4)*log(-(4*(1/4)^(3/4)
*a*b*c^3*x^3*(1/(a*b*c^4))^(3/4) + 2*(1/4)^(1/4)*a*c*x*(1/(a*b*c^
4))^(1/4) - sqrt(b*x^4 + a)*(a*c^2*sqrt(1/(a*b*c^4)) + x^2))/(b*x
^4 - a))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt{a+bx^4}}{-a+bx^4}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**(1/2)/(-b*c*x**4+a*c),x)
```

```
[Out] -Integral(sqrt(a + b*x**4)/(-a + b*x**4), x)/c
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{bx^4+a}}{bcx^4-ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(b*x^4 + a)/(b*c*x^4 - a*c),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(b*x^4 + a)/(b*c*x^4 - a*c), x)
```

## 3.93 $\int \frac{\sqrt{a-bx^4}}{ac+bcx^4} dx$

**Optimal**. Leaf size=116

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}-\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}$$

```
[Out] ArcTan[(b^(1/4)*x*(Sqrt[a] + Sqrt[b]*x^2))/(a^(1/4)*Sqrt[a - b*x^
4])]/(2*a^(1/4)*b^(1/4)*c) + ArcTanh[(b^(1/4)*x*(Sqrt[a] - Sqrt[b
]*x^2))/(a^(1/4)*Sqrt[a - b*x^4])]/(2*a^(1/4)*b^(1/4)*c)
```

---

**Rubi [A]** time = 0.0790384, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}-\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a - b*x^4]/(a*c + b*c*x^4),x]
```

```
[Out] ArcTan[(b^(1/4)*x*(Sqrt[a] + Sqrt[b]*x^2))/(a^(1/4)*Sqrt[a - b*x^
4])]/(2*a^(1/4)*b^(1/4)*c) + ArcTanh[(b^(1/4)*x*(Sqrt[a] - Sqrt[b
]*x^2))/(a^(1/4)*Sqrt[a - b*x^4])]/(2*a^(1/4)*b^(1/4)*c)
```

---

**Rubi in Sympy [A]** time = 12.6542, size = 100, normalized size = 0.86

$$\frac{\operatorname{atan}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\operatorname{atanh}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}-\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**4+a)**(1/2)/(b*c*x**4+a*c),x)
```

```
[Out] atan(b**(1/4)*x*(sqrt(a) + sqrt(b)*x**2)/(a**(1/4)*sqrt(a - b*x**
4)))/(2*a**(1/4)*b**(1/4)*c) + atanh(b**(1/4)*x*(sqrt(a) - sqrt(b
)*x**2)/(a**(1/4)*sqrt(a - b*x**4)))/(2*a**(1/4)*b**(1/4)*c)
```

**Mathematica [C]** time = 0.271918, size = 155, normalized size = 1.34

$$\frac{5ax\sqrt{a-bx^4}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},-\frac{bx^4}{a}\right)}{c\left(a+bx^4\right)\left(5aF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},-\frac{bx^4}{a}\right)-2bx^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},-\frac{bx^4}{a}\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},-\frac{bx^4}{a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a - b*x^4]/(a*c + b*c*x^4),x]
```

```
[Out] (5*a*x*Sqrt[a - b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, -((
b*x^4)/a)])/(c*(a + b*x^4)*(5*a*AppellF1[1/4, -1/2, 1, 5/4, (b*x^
4)/a, -((b*x^4)/a)] - 2*b*x^4*(2*AppellF1[5/4, -1/2, 2, 9/4, (b*x
^4)/a, -((b*x^4)/a)] + AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, -((b
*x^4)/a)])))
```

**Maple [A]** time = 0.025, size = 158, normalized size = 1.4

$$-\frac{1}{8c}\ln\left(1\left(\frac{-bx^4+a}{2x^2}-\frac{1}{x}\sqrt[4]{ab}\sqrt{-bx^4+a}+\sqrt{ab}\right)\left(\frac{-bx^4+a}{2x^2}+\frac{1}{x}\sqrt[4]{ab}\sqrt{-bx^4+a}+\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt[4]{ab}}-\frac{1}{4c}\arctan\left(\frac{1}{x}\sqrt{-bx^4+a}\frac{1}{\sqrt[4]{ab}}+1\right)\frac{1}{\sqrt[4]{ab}}+\frac{1}{4c}\arctan\left(-\frac{1}{x}\sqrt{-bx^4+a}\frac{1}{\sqrt[4]{ab}}+1\right)\frac{1}{\sqrt[4]{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x)
```

```
[Out] -1/8/c/(a*b)^(1/4)*ln((1/2*(-b*x^4+a)/x^2-(a*b)^(1/4)*(-b*x^4+a)^
(1/2)/x+(a*b)^(1/2))/(1/2*(-b*x^4+a)/x^2+(a*b)^(1/4)*(-b*x^4+a)^(
1/2)/x+(a*b)^(1/2)))-1/4/c/(a*b)^(1/4)*arctan(1/(a*b)^(1/4)*(-b*x
^4+a)^(1/2)/x+1)+1/4/c/(a*b)^(1/4)*arctan(-1/(a*b)^(1/4)*(-b*x^4+
a)^(1/2)/x+1)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^4+a}}{bcx^4+ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^4 + a)/(b*c*x^4 + a*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^4 + a)/(b*c*x^4 + a*c), x)
```

---

**Fricas [A]** time = 0.586726, size = 500, normalized size = 4.31

$$\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(2\,\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\right)}{\sqrt{-bx^4+a}ac^2\sqrt{-\frac{1}{abc^4}}-\sqrt{-bx^4+a}x^2+(bx^4+a)\sqrt{-\frac{1}{b}}}\right)-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(-\frac{4\,\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}+\sqrt{-bx^4+a}ac^2\sqrt{-\frac{1}{abc^4}}-2\,\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}+\sqrt{-bx^4+a}x^2}{bx^4+a}\right)+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{4\,\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}-\sqrt{-bx^4+a}ac^2\sqrt{-\frac{1}{abc^4}}-2\,\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}-\sqrt{-bx^4+a}x^2}{bx^4+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^4 + a)/(b*c*x^4 + a*c),x, algorithm="fricas")
```

```
[Out] (1/4)^(1/4)*(-1/(a*b*c^4))^(1/4)*arctan(2*(2*(1/4)^(3/4)*a*b*c^3*
x^3*(-1/(a*b*c^4))^(3/4) + (1/4)^(1/4)*a*c*x*(-1/(a*b*c^4))^(1/4)
)/(sqrt(-b*x^4 + a)*a*c^2*sqrt(-1/(a*b*c^4)) - sqrt(-b*x^4 + a)*x
^2 + (b*x^4 + a)*sqrt(-1/b))) - 1/4*(1/4)^(1/4)*(-1/(a*b*c^4))^(1
/4)*log(-(4*(1/4)^(3/4)*a*b*c^3*x^3*(-1/(a*b*c^4))^(3/4) + sqrt(-
b*x^4 + a)*a*c^2*sqrt(-1/(a*b*c^4)) - 2*(1/4)^(1/4)*a*c*x*(-1/(a*
b*c^4))^(1/4) + sqrt(-b*x^4 + a)*x^2)/(b*x^4 + a)) + 1/4*(1/4)^(1
/4)*(-1/(a*b*c^4))^(1/4)*log((4*(1/4)^(3/4)*a*b*c^3*x^3*(-1/(a*b*
c^4))^(3/4) - sqrt(-b*x^4 + a)*a*c^2*sqrt(-1/(a*b*c^4)) - 2*(1/4)
^(1/4)*a*c*x*(-1/(a*b*c^4))^(1/4) - sqrt(-b*x^4 + a)*x^2)/(b*x^4
+ a))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{a-bx^4}}{a+bx^4}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**4+a)**(1/2)/(b*c*x**4+a*c),x)
```

```
[Out] Integral(sqrt(a - b*x**4)/(a + b*x**4), x)/c
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^4 + a}}{bcx^4 + ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^4 + a)/(b*c*x^4 + a*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^4 + a)/(b*c*x^4 + a*c), x)
```

## 3.94 $\int \frac{(a+bx^4)^{7/4}}{c+dx^4} \, dx$

**Optimal**. Leaf size=211

$$-\frac{b^{3/4}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}-\frac{b^{3/4}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}+\frac{(bc-ad)^{7/4}\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2}+\frac{(bc-ad)^{7/4}\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2}+\frac{bx\left(a+bx^4\right)^{3/4}}{4d}$$

```
[Out] (b*x*(a + b*x^4)^(3/4))/(4*d) - (b^(3/4)*(4*b*c - 7*a*d)*ArcTan[(
b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^2) + ((b*c - a*d)^(7/4)*ArcTa
n[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*
d^2) - (b^(3/4)*(4*b*c - 7*a*d)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(
1/4)])/(8*d^2) + ((b*c - a*d)^(7/4)*ArcTanh[((b*c - a*d)^(1/4)*x)
/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d^2)
```

---

**Rubi [A]** time = 0.523448, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$-\frac{b^{3/4}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}-\frac{b^{3/4}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}+\frac{(bc-ad)^{7/4}\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2}+\frac{(bc-ad)^{7/4}\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2}+\frac{bx\left(a+bx^4\right)^{3/4}}{4d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^(7/4)/(c + d*x^4),x]
```


```
[Out] (b*x*(a + b*x^4)^(3/4))/(4*d) - (b^(3/4)*(4*b*c - 7*a*d)*ArcTan[(
b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^2) + ((b*c - a*d)^(7/4)*ArcTa
n[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*
d^2) - (b^(3/4)*(4*b*c - 7*a*d)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(
1/4)])/(8*d^2) + ((b*c - a*d)^(7/4)*ArcTanh[((b*c - a*d)^(1/4)*x)
/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d^2)
```


---

**Rubi in Sympy [A]** time = 62.9504, size = 192, normalized size = 0.91

$$\frac{b^{\frac{3}{4}}(7ad-4bc)\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}+\frac{b^{\frac{3}{4}}(7ad-4bc)\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}+\frac{bx\left(a+bx^4\right)^{\frac{3}{4}}}{4d}+\frac{(-ad+bc)^{\frac{7}{4}}\operatorname{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{\frac{3}{4}}d^2}+\frac{(-ad+bc)^{\frac{7}{4}}\operatorname{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{\frac{3}{4}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**(7/4)/(d*x**4+c),x)
```

```
[Out] b**(3/4)*(7*a*d - 4*b*c)*atan(b**(1/4)*x/(a + b*x**4)**(1/4))/(8*
d**2) + b**(3/4)*(7*a*d - 4*b*c)*atanh(b**(1/4)*x/(a + b*x**4)**(
1/4))/(8*d**2) + b*x*(a + b*x**4)**(3/4)/(4*d) + (-a*d + b*c)**(7
/4)*atan(x*(-a*d + b*c)**(1/4)/(c**(1/4)*(a + b*x**4)**(1/4)))/(2
*c**(3/4)*d**2) + (-a*d + b*c)**(7/4)*atanh(x*(-a*d + b*c)**(1/4)
/(c**(1/4)*(a + b*x**4)**(1/4)))/(2*c**(3/4)*d**2)
```

---

**Mathematica [C]** time = 1.80635, size = 396, normalized size = 1.88

$$\frac{1}{80}\left(\frac{5\left(4a^2d\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+4bc^{3/4}x\left(a+bx^4\right)^{3/4}\sqrt[4]{bc-ad}+a(bc-4ad)\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)-abc\log\left(\frac{x\sqrt[4]{bc}}{\sqrt[4]{ax}}\right.\right.}{c^{3/4}d\sqrt[4]{bc-ad}}\right.$$

$$-\frac{36abcx^5(7ad-4bc)F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{d\sqrt[4]{a+bx^4}\left(c+dx^4\right)\left(x^4\left(4adF_1\left(\frac{9}{4};\frac{1}{4},2;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{9}{4};\frac{5}{4},1;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-9acF_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^4)^(7/4)/(c + d*x^4),x]
```

```
[Out] ((-36*a*b*c*(-4*b*c + 7*a*d)*x^5*AppellF1[5/4, 1/4, 1, 9/4, -((b*
x^4)/a), -((d*x^4)/c)])/(d*(a + b*x^4)^(1/4)*(c + d*x^4)*(-9*a*c*
AppellF1[5/4, 1/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a
*d*AppellF1[9/4, 1/4, 2, 13/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*
AppellF1[9/4, 5/4, 1, 13/4, -((b*x^4)/a), -((d*x^4)/c)]))) + (5*(
4*b*c^(3/4)*(b*c - a*d)^(1/4)*x*(a + b*x^4)^(3/4) + 2*a*(-(b*c) +
 4*a*d)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))]
 + a*(b*c - 4*a*d)*Log[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4
)^(1/4)] - a*b*c*Log[c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^
(1/4)] + 4*a^2*d*Log[c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^
(1/4)]))/(c^(3/4)*d*(b*c - a*d)^(1/4)))/80
```

**Maple [F]** time = 0.084, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4+c}\left(bx^4+a\right)^{\frac{7}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^(7/4)/(d*x^4+c),x)
```

```
[Out] int((b*x^4+a)^(7/4)/(d*x^4+c),x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{7}{4}}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(7/4)/(d*x^4 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c), x)
```

**Fricas [A]** time = 2.72271, size = 2776, normalized size = 13.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(7/4)/(d*x^4 + c),x, algorithm="fricas")
```

```
[Out] 1/16*(4*(b*x^4 + a)^(3/4)*b*x - 16*d*((b^7*c^7 - 7*a*b^6*c^6*d +
21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21
*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(1/4)*arct
an(c^2*d^6*x*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*
a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6
*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4)/(x*sqrt(((b^7*c^8*d^4 - 7*a*
b^6*c^7*d^5 + 21*a^2*b^5*c^6*d^6 - 35*a^3*b^4*c^5*d^7 + 35*a^4*b^
3*c^4*d^8 - 21*a^5*b^2*c^3*d^9 + 7*a^6*b*c^2*d^10 - a^7*c*d^11)*x
^2*sqrt((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^
4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d
^6 - a^7*d^7)/(c^3*d^8)) + (b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b
^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*
b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*
```

```
b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10*d^10)*sqrt(b*x^4 + a))/x^2) -
 (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d
^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(b*x^4 + a)^(1/4))) + 4*d*((256*b^7
*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d
^3 + 2401*a^4*b^3*d^4)/d^8)^(1/4)*arctan(d^6*x*((256*b^7*c^4 - 17
92*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401
*a^4*b^3*d^4)/d^8)^(3/4)/(x*sqrt(((256*b^7*c^4*d^4 - 1792*a*b^6*c
^3*d^5 + 4704*a^2*b^5*c^2*d^6 - 5488*a^3*b^4*c*d^7 + 2401*a^4*b^3
*d^8)*x^2*sqrt((256*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2
*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8) + (4096*b^10*c
^6 - 43008*a*b^9*c^5*d + 188160*a^2*b^8*c^4*d^2 - 439040*a^3*b^7*
c^3*d^3 + 576240*a^4*b^6*c^2*d^4 - 403368*a^5*b^5*c*d^5 + 117649*
a^6*b^4*d^6)*sqrt(b*x^4 + a))/x^2) - (64*b^5*c^3 - 336*a*b^4*c^2*
d + 588*a^2*b^3*c*d^2 - 343*a^3*b^2*d^3)*(b*x^4 + a)^(1/4))) + 4*
d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4
*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 -
a^7*d^7)/(c^3*d^8))^(1/4)*log(-(c^2*d^6*x*((b^7*c^7 - 7*a*b^6*c^6
*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4
 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4)
 + (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2
*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(b*x^4 + a)^(1/4))/x) - 4*d*((b^7
*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 +
35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7
)/(c^3*d^8))^(1/4)*log((c^2*d^6*x*((b^7*c^7 - 7*a*b^6*c^6*d + 21*
a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^
5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4) - (b^5*
c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5
*a^4*b*c*d^4 - a^5*d^5)*(b*x^4 + a)^(1/4))/x) - d*((256*b^7*c^4 -
 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2
401*a^4*b^3*d^4)/d^8)^(1/4)*log(-(d^6*x*((256*b^7*c^4 - 1792*a*b^
6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^
3*d^4)/d^8)^(3/4) + (64*b^5*c^3 - 336*a*b^4*c^2*d + 588*a^2*b^3*c
*d^2 - 343*a^3*b^2*d^3)*(b*x^4 + a)^(1/4))/x) + d*((256*b^7*c^4 -
 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2
401*a^4*b^3*d^4)/d^8)^(1/4)*log((d^6*x*((256*b^7*c^4 - 1792*a*b^6
*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3
*d^4)/d^8)^(3/4) - (64*b^5*c^3 - 336*a*b^4*c^2*d + 588*a^2*b^3*c*
d^2 - 343*a^3*b^2*d^3)*(b*x^4 + a)^(1/4))/x))/d
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**(7/4)/(d*x**4+c),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4 + a\right)^{\frac{7}{4}}}{dx^4 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(7/4)/(d*x^4 + c),x, algorithm="giac")

[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c), x)
```

## 3.95 $\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx$

**Optimal**. Leaf size=173

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d}-\frac{(bc-ad)^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}-\frac{(bc-ad)^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}$$

```
[Out] (b^(3/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d) - ((b*c - a
*d)^(3/4)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4)
)])/(2*c^(3/4)*d) + (b^(3/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4
)])/(2*d) - ((b*c - a*d)^(3/4)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(
1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d)
```

---

**Rubi [A]** time = 0.239638, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d}-\frac{(bc-ad)^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}-\frac{(bc-ad)^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^(3/4)/(c + d*x^4),x]
```

```
[Out] (b^(3/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d) - ((b*c - a
*d)^(3/4)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4)
)])/(2*c^(3/4)*d) + (b^(3/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4
)])/(2*d) - ((b*c - a*d)^(3/4)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(
1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d)
```

---

**Rubi in Sympy [A]** time = 37.9971, size = 148, normalized size = 0.86

$$\frac{b^{\frac{3}{4}}\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d}+\frac{b^{\frac{3}{4}}\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d}-\frac{(-ad+bc)^{\frac{3}{4}}\operatorname{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{\frac{3}{4}}d}-\frac{(-ad+bc)^{\frac{3}{4}}\operatorname{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{\frac{3}{4}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**(3/4)/(d*x**4+c),x)
```

```
[Out] b**(3/4)*atan(b**(1/4)*x/(a + b*x**4)**(1/4))/(2*d) + b**(3/4)*at
anh(b**(1/4)*x/(a + b*x**4)**(1/4))/(2*d) - (-a*d + b*c)**(3/4)*a
tan(x*(-a*d + b*c)**(1/4)/(c**(1/4)*(a + b*x**4)**(1/4)))/(2*c**(
3/4)*d) - (-a*d + b*c)**(3/4)*atanh(x*(-a*d + b*c)**(1/4)/(c**(1/
4)*(a + b*x**4)**(1/4)))/(2*c**(3/4)*d)
```

---

**Mathematica [C]** time = 0.25475, size = 161, normalized size = 0.93

$$\frac{5acx\left(a+bx^4\right)^{3/4}F_1\left(\frac{1}{4};-\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\left(c+dx^4\right)\left(x^4\left(3bcF_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-4adF_1\left(\frac{5}{4};-\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};-\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^4)^(3/4)/(c + d*x^4),x]
```

```
[Out] (5*a*c*x*(a + b*x^4)^(3/4)*AppellF1[1/4, -3/4, 1, 5/4, -((b*x^4)/
a), -((d*x^4)/c)])/((c + d*x^4)*(5*a*c*AppellF1[1/4, -3/4, 1, 5/4
, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(-4*a*d*AppellF1[5/4, -3/4, 2
, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 1/4, 1,
9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

---

**Maple [F]** time = 0.055, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4+c}\left(bx^4+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^(3/4)/(d*x^4+c),x)
```

```
[Out] int((b*x^4+a)^(3/4)/(d*x^4+c),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/4)/(d*x^4 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c), x)
```

---

**Fricas [A]** time = 0.29378, size = 1006, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/4)/(d*x^4 + c),x, algorithm="fricas")
```

```
[Out] -((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^
(1/4)*arctan(c^2*d^3*x*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)/(c^3*d^4))^(3/4)/(x*sqrt(((b^3*c^4*d^2 - 3*a*b^2*c^3*d
^3 + 3*a^2*b*c^2*d^4 - a^3*c*d^5)*x^2*sqrt((b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4)) + (b^4*c^4 - 4*a*b^3*c^3
*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(b*x^4 + a)
)/x^2) + (b*x^4 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))) + (b
^3/d^4)^(1/4)*arctan(d^3*x*(b^3/d^4)^(3/4)/((b*x^4 + a)^(1/4)*b^2
 + x*sqrt((b^3*d^2*x^2*sqrt(b^3/d^4) + sqrt(b*x^4 + a)*b^4)/x^2))
) - 1/4*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3
*d^4))^(1/4)*log((c^2*d^3*x*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)/(c^3*d^4))^(3/4) + (b*x^4 + a)^(1/4)*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2))/x) + 1/4*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b
*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*log(-(c^2*d^3*x*((b^3*c^3 - 3*
a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(3/4) - (b*x^4
+ a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) + 1/4*(b^3/d^4)^(1
/4)*log((d^3*x*(b^3/d^4)^(3/4) + (b*x^4 + a)^(1/4)*b^2)/x) - 1/4*
(b^3/d^4)^(1/4)*log(-(d^3*x*(b^3/d^4)^(3/4) - (b*x^4 + a)^(1/4)*b
^2)/x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^4\right)^{\frac{3}{4}}}{c + dx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**(3/4)/(d*x**4+c),x)
```

```
[Out] Integral((a + b*x**4)**(3/4)/(c + d*x**4), x)
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4 + a\right)^{\frac{3}{4}}}{dx^4 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/4)/(d*x^4 + c),x, algorithm="giac")

[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c), x)
```

## 3.96 $\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)}\,dx$

**Optimal**. Leaf size=105

$$\frac{\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

```
[Out] ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3
/4)*(b*c - a*d)^(1/4)) + ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(
a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4))
```

---

**Rubi [A]** time = 0.135754, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$\frac{\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^4)^(1/4)*(c + d*x^4)),x]
```

```
[Out] ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3
/4)*(b*c - a*d)^(1/4)) + ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(
a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4))
```

---

**Rubi in Sympy [A]** time = 23.604, size = 90, normalized size = 0.86

$$\frac{\operatorname{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{\frac{3}{4}}\sqrt[4]{-ad+bc}}+\frac{\operatorname{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{\frac{3}{4}}\sqrt[4]{-ad+bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**4+a)**(1/4)/(d*x**4+c),x)
```

```
[Out] atan(x*(-a*d + b*c)**(1/4)/(c**(1/4)*(a + b*x**4)**(1/4)))/(2*c**
(3/4)*(-a*d + b*c)**(1/4)) + atanh(x*(-a*d + b*c)**(1/4)/(c**(1/4
)*(a + b*x**4)**(1/4)))/(2*c**(3/4)*(-a*d + b*c)**(1/4))
```

---

**Mathematica [A]** time = 0.123935, size = 122, normalized size = 1.16

$$\frac{-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)}{4c^{3/4}\sqrt[4]{bc-ad}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^4)^(1/4)*(c + d*x^4)),x]
```

```
[Out] (2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))] - Lo
g[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4
) + ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)])/(4*c^(3/4)*(b*c - a
*d)^(1/4))
```

---

**Maple [F]** time = 0.05, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4+c}\frac{1}{\sqrt[4]{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^4+a)^(1/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(1/4)/(d*x^4+c),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)^{\frac{1}{4}}(dx^4+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**4+a)**(1/4)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(1/4)*(c + d*x**4)), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)^{\frac{1}{4}}(dx^4+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x)
```

## 3.97 $\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx$

**Optimal**. Leaf size=134

$$-\frac{d\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}-\frac{d\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}+\frac{bx}{a\sqrt[4]{a+bx^4}(bc-ad)}$$

```
[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^4)^(1/4)) - (d*ArcTan[((b*c - a*d)^
(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(5/
4)) - (d*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4)
)])/(2*c^(3/4)*(b*c - a*d)^(5/4))
```

---

**Rubi [A]** time = 0.235548, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$-\frac{d\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}-\frac{d\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}+\frac{bx}{a\sqrt[4]{a+bx^4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^4)^(5/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^4)^(1/4)) - (d*ArcTan[((b*c - a*d)^
(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(5/
4)) - (d*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4)
)])/(2*c^(3/4)*(b*c - a*d)^(5/4))
```

---

**Rubi in Sympy [A]** time = 34.0446, size = 117, normalized size = 0.87

$$-\frac{d\,\mathrm{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{\frac{3}{4}}\left(-ad+bc\right)^{\frac{5}{4}}}-\frac{d\,\mathrm{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{\frac{3}{4}}\left(-ad+bc\right)^{\frac{5}{4}}}-\frac{bx}{a\sqrt[4]{a+bx^4}\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c),x)
```

```
[Out] -d*atan(x*(-a*d + b*c)**(1/4)/(c**(1/4)*(a + b*x**4)**(1/4)))/(2*
c**(3/4)*(-a*d + b*c)**(5/4)) - d*atanh(x*(-a*d + b*c)**(1/4)/(c*
*(1/4)*(a + b*x**4)**(1/4)))/(2*c**(3/4)*(-a*d + b*c)**(5/4)) - b
```

```
*x/(a*(a + b*x**4)**(1/4)*(a*d - b*c))
```

---

**Mathematica [A]** time = 0.367768, size = 153, normalized size = 1.14

$$\frac{\frac{d\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)\right)}{c^{3/4}\sqrt[4]{bc-ad}}-\frac{4bx}{a\sqrt[4]{a+bx^4}}}{4(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^4)^(5/4)*(c + d*x^4)),x]
```

```
[Out] ((-4*b*x)/(a*(a + b*x^4)^(1/4)) + (d*(2*ArcTan[((b*c - a*d)^(1/4)
*x)/(c^(1/4)*(b + a*x^4)^(1/4))] - Log[c^(1/4) - ((b*c - a*d)^(1/
4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b
 + a*x^4)^(1/4)]))/(c^(3/4)*(b*c - a*d)^(1/4)))/(4*(-(b*c) + a*d)
)
```

---

**Maple [F]** time = 0.055, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4+c}\left(bx^4+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^4+a)^(5/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(5/4)/(d*x^4+c),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)^{\frac{5}{4}}(dx^4+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)),x, algorithm="fricas")

[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^4)^{\frac{5}{4}}\,(c+dx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c),x)

[Out] Integral(1/((a + b*x**4)**(5/4)*(c + d*x**4)), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)^{\frac{5}{4}}(dx^4+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)),x, algorithm="giac")

[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)), x)
```

## 3.98 $\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx$

**Optimal**. Leaf size=180

$$\frac{bx(4bc-9ad)}{5a^2\sqrt[4]{a+bx^4}(bc-ad)^2}+\frac{d^2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}+\frac{d^2\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

```
[Out] (b*x)/(5*a*(b*c - a*d)*(a + b*x^4)^(5/4)) + (b*(4*b*c - 9*a*d)*x)
/(5*a^2*(b*c - a*d)^2*(a + b*x^4)^(1/4)) + (d^2*ArcTan[((b*c - a*
d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^
(9/4)) + (d^2*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^
(1/4))])/(2*c^(3/4)*(b*c - a*d)^(9/4))
```

---

**Rubi [A]** time = 0.489959, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{bx(4bc-9ad)}{5a^2\sqrt[4]{a+bx^4}(bc-ad)^2}+\frac{d^2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}+\frac{d^2\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^4)^(9/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(5*a*(b*c - a*d)*(a + b*x^4)^(5/4)) + (b*(4*b*c - 9*a*d)*x)
/(5*a^2*(b*c - a*d)^2*(a + b*x^4)^(1/4)) + (d^2*ArcTan[((b*c - a*
d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^
(9/4)) + (d^2*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^
(1/4))])/(2*c^(3/4)*(b*c - a*d)^(9/4))
```

---

**Rubi in Sympy [A]** time = 80.6157, size = 158, normalized size = 0.88

$$\frac{d^2\operatorname{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{\frac{3}{4}}\left(-ad+bc\right)^{\frac{9}{4}}}+\frac{d^2\operatorname{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{\frac{3}{4}}\left(-ad+bc\right)^{\frac{9}{4}}}-\frac{bx}{5a\left(a+bx^4\right)^{\frac{5}{4}}(ad-bc)}-\frac{bx\left(9ad-4bc\right)}{5a^2\sqrt[4]{a+bx^4}\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**4+a)**(9/4)/(d*x**4+c),x)
```

```
[Out] d**2*atan(x*(-a*d + b*c)**(1/4)/(c**(1/4)*(a + b*x**4)**(1/4)))/(
2*c**(3/4)*(-a*d + b*c)**(9/4)) + d**2*atanh(x*(-a*d + b*c)**(1/4
```

```
)/(c**(1/4)*(a + b*x**4)**(1/4)))/(2*c**(3/4)*(-a*d + b*c)**(9/4)
) - b*x/(5*a*(a + b*x**4)**(5/4)*(a*d - b*c)) - b*x*(9*a*d - 4*b*
c)/(5*a**2*(a + b*x**4)**(1/4)*(a*d - b*c)**2)
```

---

**Mathematica [A]** time = 0.563055, size = 184, normalized size = 1.02

$$\frac{bx\left(\left(a+bx^4\right)(4bc-9ad)+a(bc-ad)\right)}{5a^2\left(a+bx^4\right)^{5/4}(bc-ad)^2}+\frac{d^2\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)\right)}{4c^{3/4}(bc-ad)^{9/4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^4)^(9/4)*(c + d*x^4)),x]
```

```
[Out] (b*x*(a*(b*c - a*d) + (4*b*c - 9*a*d)*(a + b*x^4)))/(5*a^2*(b*c -
 a*d)^2*(a + b*x^4)^(5/4)) + (d^2*(2*ArcTan[((b*c - a*d)^(1/4)*x)
/(c^(1/4)*(b + a*x^4)^(1/4))] - Log[c^(1/4) - ((b*c - a*d)^(1/4)*
x)/(b + a*x^4)^(1/4)] + Log[c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b +
a*x^4)^(1/4)]))/(4*c^(3/4)*(b*c - a*d)^(9/4))
```

---

**Maple [F]** time = 0.057, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4+c}\left(bx^4+a\right)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^4+a)^(9/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(9/4)/(d*x^4+c),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{9}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**4+a)**(9/4)/(d*x**4+c),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)^{\frac{9}{4}}(dx^4+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)), x)
```

## 3.99 $\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx$

**Optimal**. Leaf size=233

$$\frac{bx(8bc-17ad)}{45a^2\left(a+bx^4\right)^{5/4}(bc-ad)^2}+\frac{bx\left(113a^2d^2-100abcd+32b^2c^2\right)}{45a^3\sqrt[4]{a+bx^4}(bc-ad)^3}-\frac{d^3\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}-\frac{d^3\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}+\frac{bx}{9a\left(a+bx^4\right)^{9/4}(bc-ad)}$$

```
[Out] (b*x)/(9*a*(b*c - a*d)*(a + b*x^4)^(9/4)) + (b*(8*b*c - 17*a*d)*x
)/(45*a^2*(b*c - a*d)^2*(a + b*x^4)^(5/4)) + (b*(32*b^2*c^2 - 100
*a*b*c*d + 113*a^2*d^2)*x)/(45*a^3*(b*c - a*d)^3*(a + b*x^4)^(1/4
)) - (d^3*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4)
)])/(2*c^(3/4)*(b*c - a*d)^(13/4)) - (d^3*ArcTanh[((b*c - a*d)^(1
/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(13/4
))
```

---

**Rubi [A]** time = 0.730128, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{bx(8bc-17ad)}{45a^2\left(a+bx^4\right)^{5/4}(bc-ad)^2}+\frac{bx\left(113a^2d^2-100abcd+32b^2c^2\right)}{45a^3\sqrt[4]{a+bx^4}(bc-ad)^3}-\frac{d^3\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}-\frac{d^3\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}+\frac{bx}{9a\left(a+bx^4\right)^{9/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^4)^(13/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(9*a*(b*c - a*d)*(a + b*x^4)^(9/4)) + (b*(8*b*c - 17*a*d)*x
)/(45*a^2*(b*c - a*d)^2*(a + b*x^4)^(5/4)) + (b*(32*b^2*c^2 - 100
*a*b*c*d + 113*a^2*d^2)*x)/(45*a^3*(b*c - a*d)^3*(a + b*x^4)^(1/4
)) - (d^3*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4)
)])/(2*c^(3/4)*(b*c - a*d)^(13/4)) - (d^3*ArcTanh[((b*c - a*d)^(1
/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(13/4
))
```

---

**Rubi in Sympy [A]** time = 133.593, size = 212, normalized size = 0.91

$$-\frac{d^3\operatorname{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{\frac{3}{4}}\left(-ad+bc\right)^{\frac{13}{4}}}-\frac{d^3\operatorname{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{\frac{3}{4}}\left(-ad+bc\right)^{\frac{13}{4}}}-\frac{bx}{9a\left(a+bx^4\right)^{\frac{9}{4}}\left(ad-bc\right)}-\frac{bx\left(17ad-8bc\right)}{45a^2\left(a+bx^4\right)^{\frac{5}{4}}\left(ad-bc\right)^2}-\frac{bx\left(113a^2d^2-100abcd+32b^2c^2\right)}{45a^3\sqrt[4]{a+bx^4}\left(ad-bc\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**4+a)**(13/4)/(d*x**4+c),x)
```

```
[Out] -d**3*atan(x*(-a*d + b*c)**(1/4)/(c**(1/4)*(a + b*x**4)**(1/4)))/
(2*c**(3/4)*(-a*d + b*c)**(13/4)) - d**3*atanh(x*(-a*d + b*c)**(1
/4)/(c**(1/4)*(a + b*x**4)**(1/4)))/(2*c**(3/4)*(-a*d + b*c)**(13
/4)) - b*x/(9*a*(a + b*x**4)**(9/4)*(a*d - b*c)) - b*x*(17*a*d -
8*b*c)/(45*a**2*(a + b*x**4)**(5/4)*(a*d - b*c)**2) - b*x*(113*a*
*2*d**2 - 100*a*b*c*d + 32*b**2*c**2)/(45*a**3*(a + b*x**4)**(1/4
)*(a*d - b*c)**3)
```

---

**Mathematica [A]** time = 0.683711, size = 231, normalized size = 0.99

$$\frac{bx\left(\left(a+bx^4\right)^2\left(113a^2d^2-100abcd+32b^2c^2\right)+5a^2(bc-ad)^2+a\left(a+bx^4\right)(ad-bc)(17ad-8bc)\right)}{45a^3\left(a+bx^4\right)^{9/4}(bc-ad)^3}-\frac{d^3\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)\right)}{4c^{3/4}(bc-ad)^{13/4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^4)^(13/4)*(c + d*x^4)),x]
```

```
[Out] (b*x*(5*a^2*(b*c - a*d)^2 + a*(-(b*c) + a*d)*(-8*b*c + 17*a*d)*(a
 + b*x^4) + (32*b^2*c^2 - 100*a*b*c*d + 113*a^2*d^2)*(a + b*x^4)^
2))/(45*a^3*(b*c - a*d)^3*(a + b*x^4)^(9/4)) - (d^3*(2*ArcTan[((b
*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))] - Log[c^(1/4) - (
(b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4) + ((b*c - a
*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(4*c^(3/4)*(b*c - a*d)^(13/4))
```

---

**Maple [F]** time = 0.061, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4+c}\left(bx^4+a\right)^{-\frac{13}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^4+a)^(13/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(13/4)/(d*x^4+c),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{13}{4}}(dx^4 + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**4+a)**(13/4)/(d*x**4+c),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)^{\frac{13}{4}}(dx^4+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)), x)
```

## 3.100 $\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx$

**Optimal**. Leaf size=316

$$\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(6bc-11ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{12d^2\left(a+bx^4\right)^{3/4}}-\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{12d^2}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)^2\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^2}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)^2\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^2}+\frac{bx\left(a+bx^4\right)^{5/4}}{6d}$$

```
[Out] -(b*(6*b*c - 11*a*d)*x*(a + b*x^4)^(1/4))/(12*d^2) + (b*x*(a + b*
x^4)^(5/4))/(6*d) + (Sqrt[a]*b^(3/2)*(6*b*c - 11*a*d)*(1 + a/(b*x
^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(12
*d^2*(a + b*x^4)^(3/4)) + ((b*c - a*d)^2*Sqrt[a/(a + b*x^4)]*Sqrt
[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcS
in[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*d^2) + ((b*c
 - a*d)^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c
 - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)],
 -1])/(2*b^(1/4)*c*d^2)
```

---

**Rubi [A]** time = 0.921635, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476

$$\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(6bc-11ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{12d^2\left(a+bx^4\right)^{3/4}}-\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{12d^2}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)^2\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^2}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)^2\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^2}+\frac{bx\left(a+bx^4\right)^{5/4}}{6d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^(9/4)/(c + d*x^4),x]
```

```
[Out] -(b*(6*b*c - 11*a*d)*x*(a + b*x^4)^(1/4))/(12*d^2) + (b*x*(a + b*
x^4)^(5/4))/(6*d) + (Sqrt[a]*b^(3/2)*(6*b*c - 11*a*d)*(1 + a/(b*x
^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(12
*d^2*(a + b*x^4)^(3/4)) + ((b*c - a*d)^2*Sqrt[a/(a + b*x^4)]*Sqrt
```

```
[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcS
in[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*d^2) + ((b*c
 - a*d)^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c
 - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)],
 -1])/(2*b^(1/4)*c*d^2)
```

---

**Rubi in Sympy [A]** time = 112.517, size = 282, normalized size = 0.89

$$-\frac{\sqrt{a}b^{\frac{3}{2}}x^{3}\left(11ad-6bc\right)\left(\frac{a}{bx^{4}}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x^{2}}\right)}{2}\middle|2\right)}{12d^{2}\left(a+bx^{4}\right)^{\frac{3}{4}}}+\frac{bx\left(a+bx^{4}\right)^{\frac{5}{4}}}{6d}+\frac{bx\sqrt[4]{a+bx^{4}}\left(11ad-6bc\right)}{12d^{2}}+\frac{\sqrt{\frac{a}{a+bx^{4}}}\sqrt{a+bx^{4}}\left(ad-bc\right)^{2}\left(-\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^{4}}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^{2}}+\frac{\sqrt{\frac{a}{a+bx^{4}}}\sqrt{a+bx^{4}}\left(ad-bc\right)^{2}\left(\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^{4}}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**(9/4)/(d*x**4+c),x)
```

```
[Out] -sqrt(a)*b**(3/2)*x**3*(11*a*d - 6*b*c)*(a/(b*x**4) + 1)**(3/4)*e
lliptic_f(atan(sqrt(a)/(sqrt(b)*x**2))/2, 2)/(12*d**2*(a + b*x**4
)**(3/4)) + b*x*(a + b*x**4)**(5/4)/(6*d) + b*x*(a + b*x**4)**(1/
4)*(11*a*d - 6*b*c)/(12*d**2) + sqrt(a/(a + b*x**4))*sqrt(a + b*x
**4)*(a*d - b*c)**2*elliptic_pi(-sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c
)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(2*b**(1/4)*c*d**2)
 + sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*(a*d - b*c)**2*elliptic_
pi(sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**
4)**(1/4)), -1)/(2*b**(1/4)*c*d**2)
```

---

**Mathematica [C]** time = 1.72467, size = 396, normalized size = 1.25

$$\frac{x\left(-\frac{9abcx^{4}\left(23a^{2}d^{2}-30abcd+12b^{2}c^{2}\right)F_{1}\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^{4}}{a},-\frac{dx^{4}}{c}\right)}{\left(c+dx^{4}\right)\left(x^{4}\left(4adF_{1}\left(\frac{9}{4};\frac{3}{4},2;\frac{13}{4};-\frac{bx^{4}}{a},-\frac{dx^{4}}{c}\right)+3bcF_{1}\left(\frac{9}{4};\frac{7}{4},1;\frac{13}{4};-\frac{bx^{4}}{a},-\frac{dx^{4}}{c}\right)\right)-9acF_{1}\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^{4}}{a},-\frac{dx^{4}}{c}\right)\right)}-\frac{25a^{2}c\left(12a^{2}d^{2}-\right.}{\left(c+dx^{4}\right)\left(x^{4}\left(4adF_{1}\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^{4}}{a},-\frac{dx}{c}\right.\right.\right.}\right.}{60d^{2}\left(a+bx^{4}\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^4)^(9/4)/(c + d*x^4),x]
```

```
[Out] (x*(5*b*(a + b*x^4)*(-6*b*c + 13*a*d + 2*b*d*x^4) - (25*a^2*c*(6*
b^2*c^2 - 13*a*b*c*d + 12*a^2*d^2)*AppellF1[1/4, 3/4, 1, 5/4, -((
```

```
b*x^4)/a), -((d*x^4)/c)])/((c + d*x^4)*(-5*a*c*AppellF1[1/4, 3/4,
 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3
/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4
, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))) - (9*a*b*c*(12*b^2*c^2 -
 30*a*b*c*d + 23*a^2*d^2)*x^4*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4
)/a), -((d*x^4)/c)])/((c + d*x^4)*(-9*a*c*AppellF1[5/4, 3/4, 1, 9
/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[9/4, 3/4, 2
, 13/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[9/4, 7/4, 1,
 13/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(60*d^2*(a + b*x^4)^(3/4)
)
```

---

**Maple [F]** time = 0.098, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4 + c} \left(bx^4 + a\right)^{\frac{9}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^(9/4)/(d*x^4+c),x)
```

```
[Out] int((b*x^4+a)^(9/4)/(d*x^4+c),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4 + a\right)^{\frac{9}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(9/4)/(d*x^4 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(9/4)/(d*x^4 + c), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(9/4)/(d*x^4 + c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**(9/4)/(d*x**4+c),x)
```

```
[Out] Timed out
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{9}{4}}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(9/4)/(d*x^4 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(9/4)/(d*x^4 + c), x)
```

## 3.101 $\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx$

**Optimal**. Leaf size=274

$$-\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2d\,(a+bx^4)^{3/4}}-\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd}-\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd}+\frac{bx\sqrt[4]{a+bx^4}}{2d}$$

```
[Out] (b*x*(a + b*x^4)^(1/4))/(2*d) - (Sqrt[a]*b^(3/2)*(1 + a/(b*x^4))^
(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*d*(a
+ b*x^4)^(3/4)) - ((b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4
]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4
)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*d) - ((b*c - a*d)*Sqrt
[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[
b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/
4)*c*d)
```

---

**Rubi [A]** time = 0.49314, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$-\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2d\,(a+bx^4)^{3/4}}-\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd}-\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd}+\frac{bx\sqrt[4]{a+bx^4}}{2d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^(5/4)/(c + d*x^4),x]
```

```
[Out] (b*x*(a + b*x^4)^(1/4))/(2*d) - (Sqrt[a]*b^(3/2)*(1 + a/(b*x^4))^
(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*d*(a
+ b*x^4)^(3/4)) - ((b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4
]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4
)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*d) - ((b*c - a*d)*Sqrt
```

```
[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[
b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/
4)*c*d)
```

---

**Rubi in Sympy [A]** time = 62.6497, size = 235, normalized size = 0.86

$$-\frac{\sqrt{a}b^{\frac{3}{2}}x^3\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}\middle|2\right)}{2d\left(a+bx^4\right)^{\frac{3}{4}}}+\frac{bx\sqrt[4]{a+bx^4}}{2d}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(ad-bc\right)\left(-\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(ad-bc\right)\left(\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**(5/4)/(d*x**4+c),x)
```

```
[Out] -sqrt(a)*b**(3/2)*x**3*(a/(b*x**4) + 1)**(3/4)*elliptic_f(atan(sq
rt(a)/(sqrt(b)*x**2))/2, 2)/(2*d*(a + b*x**4)**(3/4)) + b*x*(a +
b*x**4)**(1/4)/(2*d) + sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*(a*d
 - b*c)*elliptic_pi(-sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asin(b**
(1/4)*x/(a + b*x**4)**(1/4)), -1)/(2*b**(1/4)*c*d) + sqrt(a/(a +
b*x**4))*sqrt(a + b*x**4)*(a*d - b*c)*elliptic_pi(sqrt(-a*d + b*c
)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(2
*b**(1/4)*c*d)
```

---

**Mathematica [C]** time = 0.71372, size = 435, normalized size = 1.59

$$\frac{x\left(\frac{b\left(5x^4\left(a+bx^4\right)\left(c+dx^4\right)\left(4adF_1\left(\frac{9}{4};\frac{3}{4},2;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{9}{4};\frac{7}{4},1;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-9ac\left(5ac+8adx^4+3bcx^4+5bdx^8\right)F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}{x^4\left(4adF_1\left(\frac{9}{4};\frac{3}{4},2;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{9}{4};\frac{7}{4},1;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-9acF_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}-\cdots\right)}{10d\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^4)^(5/4)/(c + d*x^4),x]
```

```
[Out] (x*((-25*a^2*c*(-(b*c) + 2*a*d)*AppellF1[1/4, 3/4, 1, 5/4, -((b*x
^4)/a), -((d*x^4)/c)])/(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^
4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b
*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x
```

```
^4)/a), -((d*x^4)/c)])) + (b*(-9*a*c*(5*a*c + 3*b*c*x^4 + 8*a*d*x
^4 + 5*b*d*x^8)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4
)/c)] + 5*x^4*(a + b*x^4)*(c + d*x^4)*(4*a*d*AppellF1[9/4, 3/4, 2
, 13/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[9/4, 7/4, 1,
 13/4, -((b*x^4)/a), -((d*x^4)/c)])))/(-9*a*c*AppellF1[5/4, 3/4,
1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[9/4, 3/
4, 2, 13/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[9/4, 7/4
, 1, 13/4, -((b*x^4)/a), -((d*x^4)/c)]))))/(10*d*(a + b*x^4)^(3/4
)*(c + d*x^4))
```

---

**Maple [F]** time = 0.094, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4+c}\left(bx^4+a\right)^{\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^(5/4)/(d*x^4+c),x)
```

```
[Out] int((b*x^4+a)^(5/4)/(d*x^4+c),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(5/4)/(d*x^4 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/(d*x^4 + c), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(5/4)/(d*x^4 + c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^4\right)^{\frac{5}{4}}}{c + dx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**(5/4)/(d*x**4+c),x)
```

```
[Out] Integral((a + b*x**4)**(5/4)/(c + d*x**4), x)
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4 + a\right)^{\frac{5}{4}}}{dx^4 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(5/4)/(d*x^4 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/(d*x^4 + c), x)
```

## 3.102 $\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx$

**Optimal**. Leaf size=166

$$\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c}$$

```
[Out] (Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]
/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/
(2*b^(1/4)*c) + (Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[S
qrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^
(1/4)], -1])/(2*b^(1/4)*c)
```

---

**Rubi [A]** time = 0.29281, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^(1/4)/(c + d*x^4),x]
```

```
[Out] (Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]
/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/
(2*b^(1/4)*c) + (Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[S
qrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^
(1/4)], -1])/(2*b^(1/4)*c)
```

---

**Rubi in Sympy [A]** time = 41.3291, size = 143, normalized size = 0.86

$$\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(-\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**(1/4)/(d*x**4+c),x)
```

```
[Out] sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*elliptic_pi(-sqrt(-a*d + b*
c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(
```

```
2*b**(1/4)*c) + sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*elliptic_pi
(sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)
**(1/4)), -1)/(2*b**(1/4)*c)
```

---

**Mathematica [C]** time = 0.246518, size = 160, normalized size = 0.96

$$\frac{5acx\sqrt[4]{a+bx^4}F_1\left(\frac{1}{4};-\frac{1}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(c+dx^4)\left(x^4\left(bcF_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-4adF_1\left(\frac{5}{4};-\frac{1}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};-\frac{1}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^4)^(1/4)/(c + d*x^4),x]
```

```
[Out] (5*a*c*x*(a + b*x^4)^(1/4)*AppellF1[1/4, -1/4, 1, 5/4, -((b*x^4)/
a), -((d*x^4)/c)])/((c + d*x^4)*(5*a*c*AppellF1[1/4, -1/4, 1, 5/4
, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(-4*a*d*AppellF1[5/4, -1/4, 2
, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/4, 1, 9/
4, -((b*x^4)/a), -((d*x^4)/c)])))
```

---

**Maple [F]** time = 0.055, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4+c}\sqrt[4]{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^(1/4)/(d*x^4+c),x)
```

```
[Out] int((b*x^4+a)^(1/4)/(d*x^4+c),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4+a)^{\frac{1}{4}}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(1/4)/(d*x^4 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c), x)
```

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(1/4)/(d*x^4 + c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{a + bx^4}}{c + dx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**(1/4)/(d*x**4+c),x)
```

```
[Out] Integral((a + b*x**4)**(1/4)/(c + d*x**4), x)
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4 + a\right)^{\frac{1}{4}}}{dx^4 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(1/4)/(d*x^4 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c), x)
```

## 3.103 $\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx$

**Optimal**. Leaf size=259

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}(bc-ad)}-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)}-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)}$$

```
[Out] -((b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^
2)/Sqrt[a]]/2, 2])/(Sqrt[a]*(b*c - a*d)*(a + b*x^4)^(3/4))) - (d*
Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/
(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(
2*b^(1/4)*c*(b*c - a*d)) - (d*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]
*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)
/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d))
```

---

**Rubi [A]** time = 0.471444, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}(bc-ad)}-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)}-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^4)^(3/4)*(c + d*x^4)),x]
```


```
[Out] -((b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^
2)/Sqrt[a]]/2, 2])/(Sqrt[a]*(b*c - a*d)*(a + b*x^4)^(3/4))) - (d*
Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/
(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(
2*b^(1/4)*c*(b*c - a*d)) - (d*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]
*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)
```

```
/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d))
```

---

**Rubi in Sympy [A]** time = 62.6977, size = 221, normalized size = 0.85

$$\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(-\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\,(ad-bc)}+\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\,(ad-bc)}+\frac{b^{\frac{3}{2}}x^3\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}\middle|2\right)}{\sqrt{a}\,(a+bx^4)^{\frac{3}{4}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**4+a)**(3/4)/(d*x**4+c),x)
```

```
[Out] d*sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*elliptic_pi(-sqrt(-a*d +
b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)
/(2*b**(1/4)*c*(a*d - b*c)) + d*sqrt(a/(a + b*x**4))*sqrt(a + b*x
**4)*elliptic_pi(sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4
)*x/(a + b*x**4)**(1/4)), -1)/(2*b**(1/4)*c*(a*d - b*c)) + b**(3/
2)*x**3*(a/(b*x**4) + 1)**(3/4)*elliptic_f(atan(sqrt(a)/(sqrt(b)*
x**2))/2, 2)/(sqrt(a)*(a + b*x**4)**(3/4)*(a*d - b*c))
```

---

**Mathematica [C]** time = 0.0867231, size = 161, normalized size = 0.62

$$-\frac{5acxF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(a+bx^4)^{3/4}(c+dx^4)\left(x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^4)^(3/4)*(c + d*x^4)),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])
/((a + b*x^4)^(3/4)*(c + d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4
, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2,
9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/
4, -((b*x^4)/a), -((d*x^4)/c)])))
```

---

**Maple [F]** time = 0.052, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4+c}\left(bx^4+a\right)^{-\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^4+a)^(3/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(3/4)/(d*x^4+c),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{3}{4}}(dx^4 + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^4)^{\frac{3}{4}}\,(c + dx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**4+a)**(3/4)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(3/4)*(c + d*x**4)), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)^{\frac{3}{4}}(dx^4+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)), x)
```

## 3.104 $\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)} \, dx$

**Optimal**. Leaf size=304

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-5ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}(bc-ad)^2}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^2}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^2}+\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^4)^(3/4)) - (b^(3/2)*(2*b*c - 5*a
*d)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt
[a]]/2, 2])/(3*a^(3/2)*(b*c - a*d)^2*(a + b*x^4)^(3/4)) + (d^2*Sq
rt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(S
qrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*
b^(1/4)*c*(b*c - a*d)^2) + (d^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^
4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*
x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^2)
```

---

**Rubi [A]** time = 0.69783, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-5ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}(bc-ad)^2}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^2}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^2}+\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^4)^(7/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^4)^(3/4)) - (b^(3/2)*(2*b*c - 5*a
*d)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt
[a]]/2, 2])/(3*a^(3/2)*(b*c - a*d)^2*(a + b*x^4)^(3/4)) + (d^2*Sq
rt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(S
qrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*
```

```
b^(1/4)*c*(b*c - a*d)^2) + (d^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^
4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*
x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^2)
```

---

**Rubi in Sympy [A]** time = 99.6203, size = 265, normalized size = 0.87

$$\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(-\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\left(ad-bc\right)^2}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\left(ad-bc\right)^2}-\frac{bx}{3a\left(a+bx^4\right)^{\frac{3}{4}}\left(ad-bc\right)}+\frac{b^{\frac{3}{2}}x^3\left(5ad-2bc\right)\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}\middle|2\right)}{3a^{\frac{3}{2}}\left(a+bx^4\right)^{\frac{3}{4}}\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**4+a)**(7/4)/(d*x**4+c),x)
```

```
[Out] d**2*sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*elliptic_pi(-sqrt(-a*d
 + b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)),
-1)/(2*b**(1/4)*c*(a*d - b*c)**2) + d**2*sqrt(a/(a + b*x**4))*sqr
t(a + b*x**4)*elliptic_pi(sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asi
n(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(2*b**(1/4)*c*(a*d - b*c)*
*2) - b*x/(3*a*(a + b*x**4)**(3/4)*(a*d - b*c)) + b**(3/2)*x**3*(
5*a*d - 2*b*c)*(a/(b*x**4) + 1)**(3/4)*elliptic_f(atan(sqrt(a)/(s
qrt(b)*x**2))/2, 2)/(3*a**(3/2)*(a + b*x**4)**(3/4)*(a*d - b*c)**
2)
```

---

**Mathematica [C]** time = 0.536398, size = 343, normalized size = 1.13

$$\frac{x\left(\frac{18bcdx^4F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\left(c+dx^4\right)\left(x^4\left(4adF_1\left(\frac{9}{4};\frac{3}{4},2;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{9}{4};\frac{7}{4},1;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-9acF_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}+\frac{25c(2bc\cdots}{\left(c+dx^4\right)\left(x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\cdots\right.\right.\right.}\right.}{15\left(a+bx^4\right)^{3/4}\left(ad-bc\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^4)^(7/4)*(c + d*x^4)),x]
```

```
[Out] (x*((-5*b)/a + (25*c*(2*b*c - 3*a*d)*AppellF1[1/4, 3/4, 1, 5/4, -
((b*x^4)/a), -((d*x^4)/c)])/((c + d*x^4)*(-5*a*c*AppellF1[1/4, 3/
4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4,
```

```
 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7
/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))) + (18*b*c*d*x^4*Appell
F1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])/((c + d*x^4)*(-
9*a*c*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + x^
4*(4*a*d*AppellF1[9/4, 3/4, 2, 13/4, -((b*x^4)/a), -((d*x^4)/c)]
+ 3*b*c*AppellF1[9/4, 7/4, 1, 13/4, -((b*x^4)/a), -((d*x^4)/c)]))
)))/(15*(-(b*c) + a*d)*(a + b*x^4)^(3/4))
```

---

**Maple [F]** time = 0.054, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4+c}\left(bx^4+a\right)^{-\frac{7}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^4+a)^(7/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(7/4)/(d*x^4+c),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)^{\frac{7}{4}}(dx^4+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^4)^{\frac{7}{4}}\,(c+dx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**4+a)**(7/4)/(d*x**4+c),x)

[Out] Integral(1/((a + b*x**4)**(7/4)*(c + d*x**4)), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)^{\frac{7}{4}}(dx^4+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)),x, algorithm="giac")

[Out] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)), x)
```

## 3.105 $\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx$

**Optimal**. Leaf size=357

$$\frac{bx(6bc-13ad)}{21a^2\left(a+bx^4\right)^{3/4}(bc-ad)^2}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(47a^2d^2-38abcd+12b^2c^2\right)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{21a^{5/2}\left(a+bx^4\right)^{3/4}(bc-ad)^3}-\frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^3}-\frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^3}+\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

```
[Out] (b*x)/(7*a*(b*c - a*d)*(a + b*x^4)^(7/4)) + (b*(6*b*c - 13*a*d)*x
)/(21*a^2*(b*c - a*d)^2*(a + b*x^4)^(3/4)) - (b^(3/2)*(12*b^2*c^2
 - 38*a*b*c*d + 47*a^2*d^2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[A
rcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(21*a^(5/2)*(b*c - a*d)^3*(a
+ b*x^4)^(3/4)) - (d^3*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*Ellipt
icPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a
+ b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^3) - (d^3*Sqrt[a/(
a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*S
qrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c
*(b*c - a*d)^3)
```

---

**Rubi [A]** time = 1.07988, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$

$$\frac{bx(6bc-13ad)}{21a^2\left(a+bx^4\right)^{3/4}(bc-ad)^2}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(47a^2d^2-38abcd+12b^2c^2\right)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{21a^{5/2}\left(a+bx^4\right)^{3/4}(bc-ad)^3}-\frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^3}-\frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^3}+\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^4)^(11/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(7*a*(b*c - a*d)*(a + b*x^4)^(7/4)) + (b*(6*b*c - 13*a*d)*x
)/(21*a^2*(b*c - a*d)^2*(a + b*x^4)^(3/4)) - (b^(3/2)*(12*b^2*c^2
 - 38*a*b*c*d + 47*a^2*d^2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[A
rcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(21*a^(5/2)*(b*c - a*d)^3*(a
+ b*x^4)^(3/4)) - (d^3*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*Ellipt
icPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a
+ b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^3) - (d^3*Sqrt[a/(
a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*S
qrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c
*(b*c - a*d)^3)
```

---

**Rubi in Sympy [A]** time = 161.555, size = 318, normalized size = 0.89

$$\frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(-\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\,(ad-bc)^3}+\frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\,(ad-bc)^3}-\frac{bx}{7a\,(a+bx^4)^{\frac{7}{4}}\,(ad-bc)}-\frac{bx\,(13ad-6bc)}{21a^2\,(a+bx^4)^{\frac{3}{4}}\,(ad-bc)^2}+\frac{b^{\frac{3}{2}}x^3\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\left(47a^2d^2-38abcd+12b^2c^2\right)F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}\middle|2\right)}{21a^{\frac{5}{2}}\,(a+bx^4)^{\frac{3}{4}}\,(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**4+a)**(11/4)/(d*x**4+c),x)
```

```
[Out] d**3*sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*elliptic_pi(-sqrt(-a*d
 + b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)),
-1)/(2*b**(1/4)*c*(a*d - b*c)**3) + d**3*sqrt(a/(a + b*x**4))*sqr
t(a + b*x**4)*elliptic_pi(sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asi
n(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(2*b**(1/4)*c*(a*d - b*c)*
*3) - b*x/(7*a*(a + b*x**4)**(7/4)*(a*d - b*c)) - b*x*(13*a*d - 6
*b*c)/(21*a**2*(a + b*x**4)**(3/4)*(a*d - b*c)**2) + b**(3/2)*x**
3*(a/(b*x**4) + 1)**(3/4)*(47*a**2*d**2 - 38*a*b*c*d + 12*b**2*c*
*2)*elliptic_f(atan(sqrt(a)/(sqrt(b)*x**2))/2, 2)/(21*a**(5/2)*(a
 + b*x**4)**(3/4)*(a*d - b*c)**3)
```

---

**Mathematica [C]** time = 1.56098, size = 407, normalized size = 1.14

$$\frac{x\left(\frac{25ac\left(21a^2d^2-26abcd+12b^2c^2\right)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(c+dx^4)\left(5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}+\frac{5b\left(-16a^2d+ab\left(9c-13dx^4\right)+6b^2cx^4\right)}{a+bx^4}+\cdots\right)}{105a^2\,(a+bx^4)^{3/4}\,(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^4)^(11/4)*(c + d*x^4)),x]
```

```
[Out] (x*((5*b*(-16*a^2*d + 6*b^2*c*x^4 + a*b*(9*c - 13*d*x^4)))/(a + b
*x^4) + (25*a*c*(12*b^2*c^2 - 26*a*b*c*d + 21*a^2*d^2)*AppellF1[1
/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/((c + d*x^4)*(5*a*c
*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - x^4*(4*
a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*
c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))) + (18
*a*b*c*d*(-6*b*c + 13*a*d)*x^4*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^
4)/a), -((d*x^4)/c)])/((c + d*x^4)*(-9*a*c*AppellF1[5/4, 3/4, 1,
9/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[9/4, 3/4,
2, 13/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[9/4, 7/4, 1
, 13/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(105*a^2*(b*c - a*d)^2*(
a + b*x^4)^(3/4))
```

---

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4 + c} \left(bx^4 + a\right)^{-\frac{11}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^4+a)^(11/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(11/4)/(d*x^4+c),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{11}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**4+a)**(11/4)/(d*x**4+c),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{11}{4}}(dx^4 + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)), x)
```

## 3.106 $\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2} dx$

**Optimal**. Leaf size=280

$$-\frac{b^{7/4}(8bc-11ad)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}-\frac{b^{7/4}(8bc-11ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}+\frac{(bc-ad)^{7/4}(3ad+8bc)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3}+\frac{(bc-ad)^{7/4}(3ad+8bc)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3}+\frac{bx\left(a+bx^4\right)^{3/4}(2bc-ad)}{4cd^2}-\frac{x\left(a+bx^4\right)^{7/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

```
[Out] (b*(2*b*c - a*d)*x*(a + b*x^4)^(3/4))/(4*c*d^2) - ((b*c - a*d)*x*
(a + b*x^4)^(7/4))/(4*c*d*(c + d*x^4)) - (b^(7/4)*(8*b*c - 11*a*d
)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^3) + ((b*c - a*d)^(
7/4)*(8*b*c + 3*a*d)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b
*x^4)^(1/4))])/(8*c^(7/4)*d^3) - (b^(7/4)*(8*b*c - 11*a*d)*ArcTan
h[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^3) + ((b*c - a*d)^(7/4)*(8
*b*c + 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^
(1/4))])/(8*c^(7/4)*d^3)
```

---

**Rubi [A]** time = 0.893411, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$

$$-\frac{b^{7/4}(8bc-11ad)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}-\frac{b^{7/4}(8bc-11ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}+\frac{(bc-ad)^{7/4}(3ad+8bc)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3}+\frac{(bc-ad)^{7/4}(3ad+8bc)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3}+\frac{bx\left(a+bx^4\right)^{3/4}(2bc-ad)}{4cd^2}-\frac{x\left(a+bx^4\right)^{7/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^(11/4)/(c + d*x^4)^2,x]
```

```
[Out] (b*(2*b*c - a*d)*x*(a + b*x^4)^(3/4))/(4*c*d^2) - ((b*c - a*d)*x*
(a + b*x^4)^(7/4))/(4*c*d*(c + d*x^4)) - (b^(7/4)*(8*b*c - 11*a*d
)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^3) + ((b*c - a*d)^(
7/4)*(8*b*c + 3*a*d)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b
*x^4)^(1/4))])/(8*c^(7/4)*d^3) - (b^(7/4)*(8*b*c - 11*a*d)*ArcTan
h[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^3) + ((b*c - a*d)^(7/4)*(8
*b*c + 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^
```

```
(1/4))])/(8*c^(7/4)*d^3)
```

---

**Rubi in Sympy [A]** time = 101.332, size = 255, normalized size = 0.91

$$\frac{b^{\frac{7}{4}}\left(11ad-8bc\right)\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}+\frac{b^{\frac{7}{4}}\left(11ad-8bc\right)\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}$$
$$-\frac{bx\left(a+bx^4\right)^{\frac{3}{4}}\left(ad-2bc\right)}{4cd^2}+\frac{x\left(a+bx^4\right)^{\frac{7}{4}}\left(ad-bc\right)}{4cd\left(c+dx^4\right)}$$
$$+\frac{\left(-ad+bc\right)^{\frac{7}{4}}\left(3ad+8bc\right)\operatorname{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{\frac{7}{4}}d^3}+\frac{\left(-ad+bc\right)^{\frac{7}{4}}\left(3ad+8bc\right)\operatorname{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{\frac{7}{4}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**(11/4)/(d*x**4+c)**2,x)
```

```
[Out] b**(7/4)*(11*a*d - 8*b*c)*atan(b**(1/4)*x/(a + b*x**4)**(1/4))/(8
*d**3) + b**(7/4)*(11*a*d - 8*b*c)*atanh(b**(1/4)*x/(a + b*x**4)*
*(1/4))/(8*d**3) - b*x*(a + b*x**4)**(3/4)*(a*d - 2*b*c)/(4*c*d**
2) + x*(a + b*x**4)**(7/4)*(a*d - b*c)/(4*c*d*(c + d*x**4)) + (-a
*d + b*c)**(7/4)*(3*a*d + 8*b*c)*atan(x*(-a*d + b*c)**(1/4)/(c**(
1/4)*(a + b*x**4)**(1/4)))/(8*c**(7/4)*d**3) + (-a*d + b*c)**(7/4
)*(3*a*d + 8*b*c)*atanh(x*(-a*d + b*c)**(1/4)/(c**(1/4)*(a + b*x*
*4)**(1/4)))/(8*c**(7/4)*d**3)
```

---

**Mathematica [C]** time = 4.04845, size = 735, normalized size = 2.62

$$\frac{5\left(3a^3d^3x^4\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+3a^3cd^2\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)-a\left(c+dx^4\right)\left(3a^2d^2+2abcd-2b^2c^2\right)\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+2a\left(c+dx^4\right)\left(3a^2d^2+2abc\ldots\right.\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^4)^(11/4)/(c + d*x^4)^2,x]
```

```
[Out] ((-36*a*b^2*c^(11/4)*(-8*b*c + 11*a*d)*x^5*AppellF1[5/4, 1/4, 1,
9/4, -((b*x^4)/a), -((d*x^4)/c)])/((a + b*x^4)^(1/4)*(-9*a*c*Appe
llF1[5/4, 1/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*A
ppellF1[9/4, 1/4, 2, 13/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*Appe
llF1[9/4, 5/4, 1, 13/4, -((b*x^4)/a), -((d*x^4)/c)]))) + (5*(8*b^
2*c^(11/4)*(b*c - a*d)^(1/4)*x*(a + b*x^4)^(3/4) - 8*a*b*c^(7/4)*
d*(b*c - a*d)^(1/4)*x*(a + b*x^4)^(3/4) + 4*a^2*c^(3/4)*d^2*(b*c
```

```
- a*d)^(1/4)*x*(a + b*x^4)^(3/4) + 4*b^2*c^(7/4)*d*(b*c - a*d)^(1
/4)*x^5*(a + b*x^4)^(3/4) + 2*a*(-2*b^2*c^2 + 2*a*b*c*d + 3*a^2*d
^2)*(c + d*x^4)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)
^(1/4))] - a*(-2*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*(c + d*x^4)*Log
[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] - 2*a*b^2*c^3
*Log[c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + 2*a^2*b
*c^2*d*Log[c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + 3
*a^3*c*d^2*Log[c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]
 - 2*a*b^2*c^2*d*x^4*Log[c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b + a*x
^4)^(1/4)] + 2*a^2*b*c*d^2*x^4*Log[c^(1/4) + ((b*c - a*d)^(1/4)*x
)/(b + a*x^4)^(1/4)] + 3*a^3*d^3*x^4*Log[c^(1/4) + ((b*c - a*d)^(
1/4)*x)/(b + a*x^4)^(1/4)]))/(b*c - a*d)^(1/4))/(80*c^(7/4)*d^2*(
c + d*x^4))
```

---

**Maple [F]** time = 0.096, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d x^{4}+c\right)^{2}} \left(b x^{4}+a\right)^{\frac{11}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^(11/4)/(d*x^4+c)^2,x)
```

```
[Out] int((b*x^4+a)^(11/4)/(d*x^4+c)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{4}+a\right)^{\frac{11}{4}}}{\left(d x^{4}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(11/4)/(d*x^4 + c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(11/4)/(d*x^4 + c)^2, x)
```

---

**Fricas [A]** time = 19.9538, size = 3868, normalized size = 13.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(11/4)/(d*x^4 + c)^2,x, algorithm="fricas")
```

[Out] -1/16*(4*(c*d^3*x^4 + c^2*d^2)*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(1/4)*arctan(c^5*d^9*x*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(3/4)/(x*sqrt(((4096*b^11*c^14*d^6 - 22528*a*b^10*c^13*d^7 + 46464*a^2*b^9*c^12*d^8 - 37664*a^3*b^8*c^11*d^9 - 5071*a^4*b^7*c^10*d^10 + 25641*a^5*b^6*c^9*d^11 - 7931*a^6*b^5*c^8*d^12 - 6259*a^7*b^4*c^7*d^13 + 2739*a^8*b^3*c^6*d^14 + 891*a^9*b^2*c^5*d^15 - 297*a^10*b*c^4*d^16 - 81*a^11*c^3*d^17)*x^2*sqrt((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12)) + (262144*b^16*c^16 - 2031616*a*b^15*c^15*d + 6451200*a^2*b^14*c^14*d^2 - 10168320*a^3*b^13*c^13*d^3 + 6467520*a^4*b^12*c^12*d^4 + 3123216*a^5*b^11*c^11*d^5 - 7258119*a^6*b^10*c^10*d^6 + 2307030*a^7*b^9*c^9*d^7 + 2428965*a^8*b^8*c^8*d^8 - 1607320*a^9*b^7*c^7*d^9 - 387134*a^10*b^6*c^6*d^10 + 436356*a^11*b^5*c^5*d^11 + 40770*a^12*b^4*c^4*d^12 - 63720*a^13*b^3*c^3*d^13 - 6075*a^14*b^2*c^2*d^14 + 4374*a^15*b*c*d^15 + 729*a^16*d^16)*sqrt(b*x^4 + a))/x^2) - (512*b^8*c^8 - 1984*a*b^7*c^7*d + 2456*a^2*b^6*c^6*d^2 - 413*a^3*b^5*c^5*d^3 - 1175*a^4*b^4*c^4*d^4 + 478*a^5*b^3*c^3*d^5 + 234*a^6*b^2*c^2*d^6 - 81*a^7*b*c*d^7 - 27*a^8*d^8)*(b*x^4 + a)^(1/4))) - 4*(c*d^3*x^4 + c^2*d^2)*((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12)^(1/4)*arctan(d^9*x*((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12)^(3/4)/(x*sqrt(((4096*b^11*c^4*d^6 - 22528*a*b^10*c^3*d^7 + 46464*a^2*b^9*c^2*d^8 - 42592*a^3*b^8*c*d^9 + 14641*a^4*b^7*d^10)*x^2*sqrt((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12) + (262144*b^16*c^6 - 2162688*a*b^15*c^5*d + 7434240*a^2*b^14*c^4*d^2 - 13629440*a^3*b^13*c^3*d^3 + 14055360*a^4*b^12*c^2*d^4 - 7730448*a^5*b^11*c*d^5 + 1771561*a^6*b^10*d^6)*sqrt(b*x^4 + a))/x^2) - (512*b^8*c^3 - 2112*a*b^7*c^2*d + 2904*a^2*b^6*c*d^2 - 1331*a^3*b^5*d^3)*(b*x^4 + a)^(1/4))) - (c*d^3*x^4 + c^2*d^2)*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(1/4)*log(-(c^5*d^9*x*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(3/4) + (512*b^8*c^8 - 1984*a*b^7*c^7*d + 2456*a^2*b^6*c^6*d^2 - 413*a^3*b^5*c^5*d^3 - 1175*a^4*b^4*c^4*d^4 + 478*a^5*b^3*c^3*d^5 + 234*a^6*b^2*c^2*d^6 - 81*a^7*b*c*d^7 - 27*a^8*d^8)*(b*x^4 + a)^(1/4))/x) + (c*d^3*x^4 + c^2*d^2)*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b

```
^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^
11)/(c^7*d^12))^(1/4)*log((c^5*d^9*x*((4096*b^11*c^11 - 22528*a*b
^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071
*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 -
 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^
9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(3/4) - (512*b^
8*c^8 - 1984*a*b^7*c^7*d + 2456*a^2*b^6*c^6*d^2 - 413*a^3*b^5*c^5
*d^3 - 1175*a^4*b^4*c^4*d^4 + 478*a^5*b^3*c^3*d^5 + 234*a^6*b^2*c
^2*d^6 - 81*a^7*b*c*d^7 - 27*a^8*d^8)*(b*x^4 + a)^(1/4))/x) + (c*
d^3*x^4 + c^2*d^2)*((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 46464*a
^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12)^(
1/4)*log(-(d^9*x*((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2
*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12)^(3/
4) + (512*b^8*c^3 - 2112*a*b^7*c^2*d + 2904*a^2*b^6*c*d^2 - 1331*
a^3*b^5*d^3)*(b*x^4 + a)^(1/4))/x) - (c*d^3*x^4 + c^2*d^2)*((4096
*b^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^
3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12)^(1/4)*log((d^9*x*((4096*b^
11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b
^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12)^(3/4) - (512*b^8*c^3 - 2112*a
*b^7*c^2*d + 2904*a^2*b^6*c*d^2 - 1331*a^3*b^5*d^3)*(b*x^4 + a)^(
1/4))/x) - 4*(b^2*c*d*x^5 + (2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)*
(b*x^4 + a)^(3/4))/(c*d^3*x^4 + c^2*d^2)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**(11/4)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{11}{4}}}{(dx^4 + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(11/4)/(d*x^4 + c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(11/4)/(d*x^4 + c)^2, x)
```

## 3.107 $\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx$

**Optimal**. Leaf size=230

$$\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}+\frac{b^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}-\frac{(bc-ad)^{3/4}(3ad+4bc)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}-\frac{(bc-ad)^{3/4}(3ad+4bc)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

```
[Out] -((b*c - a*d)*x*(a + b*x^4)^(3/4))/(4*c*d*(c + d*x^4)) + (b^(7/4)
*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d^2) - ((b*c - a*d)^(3
/4)*(4*b*c + 3*a*d)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*
x^4)^(1/4))])/(8*c^(7/4)*d^2) + (b^(7/4)*ArcTanh[(b^(1/4)*x)/(a +
 b*x^4)^(1/4)])/(2*d^2) - ((b*c - a*d)^(3/4)*(4*b*c + 3*a*d)*ArcT
anh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4
)*d^2)
```

---

**Rubi [A]** time = 0.438932, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}+\frac{b^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}-\frac{(bc-ad)^{3/4}(3ad+4bc)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}-\frac{(bc-ad)^{3/4}(3ad+4bc)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^(7/4)/(c + d*x^4)^2,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^4)^(3/4))/(4*c*d*(c + d*x^4)) + (b^(7/4)
*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d^2) - ((b*c - a*d)^(3
/4)*(4*b*c + 3*a*d)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*
x^4)^(1/4))])/(8*c^(7/4)*d^2) + (b^(7/4)*ArcTanh[(b^(1/4)*x)/(a +
 b*x^4)^(1/4)])/(2*d^2) - ((b*c - a*d)^(3/4)*(4*b*c + 3*a*d)*ArcT
anh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4
)*d^2)
```

---

**Rubi in Sympy [A]** time = 63.5151, size = 206, normalized size = 0.9

$$\frac{b^{\frac{7}{4}}\operatorname{atan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}+\frac{b^{\frac{7}{4}}\operatorname{atanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}+\frac{x\left(a+bx^4\right)^{\frac{3}{4}}(ad-bc)}{4cd\left(c+dx^4\right)}$$
$$-\frac{(-ad+bc)^{\frac{3}{4}}(3ad+4bc)\operatorname{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{\frac{7}{4}}d^2}-\frac{(-ad+bc)^{\frac{3}{4}}(3ad+4bc)\operatorname{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{\frac{7}{4}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**(7/4)/(d*x**4+c)**2,x)
```

```
[Out] b**(7/4)*atan(b**(1/4)*x/(a + b*x**4)**(1/4))/(2*d**2) + b**(7/4)
*atanh(b**(1/4)*x/(a + b*x**4)**(1/4))/(2*d**2) + x*(a + b*x**4)*
*(3/4)*(a*d - b*c)/(4*c*d*(c + d*x**4)) - (-a*d + b*c)**(3/4)*(3*
a*d + 4*b*c)*atan(x*(-a*d + b*c)**(1/4)/(c**(1/4)*(a + b*x**4)**(
1/4)))/(8*c**(7/4)*d**2) - (-a*d + b*c)**(3/4)*(3*a*d + 4*b*c)*at
anh(x*(-a*d + b*c)**(1/4)/(c**(1/4)*(a + b*x**4)**(1/4)))/(8*c**(
7/4)*d**2)
```

---

**Mathematica [C]** time = 0.81934, size = 462, normalized size = 2.01

$$\frac{3a^2\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)\right)}{16c^{7/4}\sqrt[4]{bc-ad}}$$
$$-\frac{9ab^2cx^5F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5d\sqrt[4]{a+bx^4}\left(c+dx^4\right)\left(x^4\left(4adF_1\left(\frac{9}{4};\frac{1}{4},2;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{9}{4};\frac{5}{4},1;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-9acF_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx}{c}\right.\right.}$$
$$+\frac{ab\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)\right)}{16c^{3/4}d\sqrt[4]{bc-ad}}$$
$$-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^4)^(7/4)/(c + d*x^4)^2,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^4)^(3/4))/(4*c*d*(c + d*x^4)) - (9*a*b^2
*c*x^5*AppellF1[5/4, 1/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])/(5
*d*(a + b*x^4)^(1/4)*(c + d*x^4)*(-9*a*c*AppellF1[5/4, 1/4, 1, 9/
4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[9/4, 1/4, 2,
 13/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[9/4, 5/4, 1, 13
/4, -((b*x^4)/a), -((d*x^4)/c)]))) + (3*a^2*(2*ArcTan[((b*c - a*d
```

```
)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))] - Log[c^(1/4) - ((b*c - a
*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4) + ((b*c - a*d)^(1/4
)*x)/(b + a*x^4)^(1/4)]))/(16*c^(7/4)*(b*c - a*d)^(1/4)) + (a*b*(
2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))] - Log
[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4)
 + ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(16*c^(3/4)*d*(b*c
- a*d)^(1/4))
```

---

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{(dx^4+c)^2}\left(bx^4+a\right)^{\frac{7}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^(7/4)/(d*x^4+c)^2,x)

[Out] int((b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{7}{4}}}{(dx^4+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2,x, algorithm="maxima")

[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2, x)
```

---

**Fricas [A]** time = 1.42465, size = 1972, normalized size = 8.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2,x, algorithm="fricas")

[Out] -1/16*(4*(b*x^4 + a)^(3/4)*(b*c - a*d)*x + 4*(c*d^2*x^4 + c^2*d)*
((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a
^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d
```

```
^7)/(c^7*d^8))^(1/4)*arctan(c^5*d^6*x*((256*b^7*c^7 - 672*a^2*b^5
*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^
2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(3/4)/(x*sqr
t(((256*b^7*c^10*d^4 - 672*a^2*b^5*c^8*d^6 - 112*a^3*b^4*c^7*d^7
+ 609*a^4*b^3*c^6*d^8 + 189*a^5*b^2*c^5*d^9 - 189*a^6*b*c^4*d^10
- 81*a^7*c^3*d^11)*x^2*sqrt((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 -
112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 -
 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8)) + (4096*b^10*c^10 + 204
8*a*b^9*c^9*d - 14592*a^2*b^8*c^8*d^2 - 9472*a^3*b^7*c^7*d^3 + 18
928*a^4*b^6*c^6*d^4 + 15624*a^5*b^5*c^5*d^5 - 9639*a^6*b^4*c^4*d^
6 - 11124*a^7*b^3*c^3*d^7 + 486*a^8*b^2*c^2*d^8 + 2916*a^9*b*c*d^
9 + 729*a^10*d^10)*sqrt(b*x^4 + a))/x^2) + (64*b^5*c^5 + 16*a*b^4
*c^4*d - 116*a^2*b^3*c^3*d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c*d^
4 + 27*a^5*d^5)*(b*x^4 + a)^(1/4))) - 16*(c*d^2*x^4 + c^2*d)*(b^7
/d^8)^(1/4)*arctan(d^6*x*(b^7/d^8)^(3/4)/((b*x^4 + a)^(1/4)*b^5 +
 x*sqrt((b^7*d^4*x^2*sqrt(b^7/d^8) + sqrt(b*x^4 + a)*b^10)/x^2)))
 + (c*d^2*x^4 + c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*
a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189
*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4)*log((c^5*d^6*x*((256*
b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3
*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c
^7*d^8))^(3/4) + (64*b^5*c^5 + 16*a*b^4*c^4*d - 116*a^2*b^3*c^3*d
^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*(b*x^4 + a
)^(1/4))/x) - (c*d^2*x^4 + c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5
*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^
2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4)*log(-(c^5*
d^6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 +
 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81
*a^7*d^7)/(c^7*d^8))^(3/4) - (64*b^5*c^5 + 16*a*b^4*c^4*d - 116*a
^2*b^3*c^3*d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5
)*(b*x^4 + a)^(1/4))/x) - 4*(c*d^2*x^4 + c^2*d)*(b^7/d^8)^(1/4)*l
og((d^6*x*(b^7/d^8)^(3/4) + (b*x^4 + a)^(1/4)*b^5)/x) + 4*(c*d^2*
x^4 + c^2*d)*(b^7/d^8)^(1/4)*log(-(d^6*x*(b^7/d^8)^(3/4) - (b*x^4
 + a)^(1/4)*b^5)/x))/(c*d^2*x^4 + c^2*d)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**(7/4)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4 + a\right)^{\frac{7}{4}}}{\left(dx^4 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2,x, algorithm="giac")

[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2, x)
```

## 3.108 $\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx$

**Optimal**. Leaf size=135

$$\frac{3a\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{3a\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}$$

```
[Out] (x*(a + b*x^4)^(3/4))/(4*c*(c + d*x^4)) + (3*a*ArcTan[((b*c - a*d
)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(
1/4)) + (3*a*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(
1/4))])/(8*c^(7/4)*(b*c - a*d)^(1/4))
```

---

**Rubi [A]** time = 0.203699, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{3a\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{3a\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^(3/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*(a + b*x^4)^(3/4))/(4*c*(c + d*x^4)) + (3*a*ArcTan[((b*c - a*d
)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(
1/4)) + (3*a*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(
1/4))])/(8*c^(7/4)*(b*c - a*d)^(1/4))
```

---

**Rubi in Sympy [A]** time = 31.2364, size = 119, normalized size = 0.88

$$\frac{3a\,\mathrm{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{\frac{7}{4}}\sqrt[4]{-ad+bc}}+\frac{3a\,\mathrm{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{\frac{7}{4}}\sqrt[4]{-ad+bc}}+\frac{x\left(a+bx^4\right)^{\frac{3}{4}}}{4c\left(c+dx^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**(3/4)/(d*x**4+c)**2,x)
```

```
[Out] 3*a*atan(x*(-a*d + b*c)**(1/4)/(c**(1/4)*(a + b*x**4)**(1/4)))/(8
*c**(7/4)*(-a*d + b*c)**(1/4)) + 3*a*atanh(x*(-a*d + b*c)**(1/4)/
(c**(1/4)*(a + b*x**4)**(1/4)))/(8*c**(7/4)*(-a*d + b*c)**(1/4))
```

```
+ x*(a + b*x**4)**(3/4)/(4*c*(c + d*x**4))
```

---

**Mathematica [A]** time = 0.350861, size = 152, normalized size = 1.13

$$\frac{3a\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)\right)}{16c^{7/4}\sqrt[4]{bc-ad}}+\frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^4)^(3/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*(a + b*x^4)^(3/4))/(4*c*(c + d*x^4)) + (3*a*(2*ArcTan[((b*c -
a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))] - Log[c^(1/4) - ((b*c
- a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4) + ((b*c - a*d)^(
1/4)*x)/(b + a*x^4)^(1/4)]))/(16*c^(7/4)*(b*c - a*d)^(1/4))
```

---

**Maple [F]** time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^4+c\right)^2}\left(bx^4+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
```

```
[Out] int((b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**(3/4)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4 + a\right)^{\frac{3}{4}}}{\left(dx^4 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2, x)
```

## 3.109 $\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)^2}\,dx$

**Optimal**. Leaf size=162

$$\frac{(4bc-3ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}+\frac{(4bc-3ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}-\frac{dx\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)(bc-ad)}$$

```
[Out] -(d*x*(a + b*x^4)^(3/4))/(4*c*(b*c - a*d)*(c + d*x^4)) + ((4*b*c
- 3*a*d)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))
])/(8*c^(7/4)*(b*c - a*d)^(5/4)) + ((4*b*c - 3*a*d)*ArcTanh[((b*c
 - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c -
a*d)^(5/4))
```

---

**Rubi [A]** time = 0.300709, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{(4bc-3ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}+\frac{(4bc-3ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}-\frac{dx\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x]
```

```
[Out] -(d*x*(a + b*x^4)^(3/4))/(4*c*(b*c - a*d)*(c + d*x^4)) + ((4*b*c
- 3*a*d)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))
])/(8*c^(7/4)*(b*c - a*d)^(5/4)) + ((4*b*c - 3*a*d)*ArcTanh[((b*c
 - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c -
a*d)^(5/4))
```

---

**Rubi in Sympy [A]** time = 36.6484, size = 141, normalized size = 0.87

$$\frac{dx\left(a+bx^4\right)^{\frac{3}{4}}}{4c\left(c+dx^4\right)(ad-bc)}-\frac{(3ad-4bc)\,\mathrm{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{\frac{7}{4}}\left(-ad+bc\right)^{\frac{5}{4}}}-\frac{(3ad-4bc)\,\mathrm{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{\frac{7}{4}}\left(-ad+bc\right)^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**4+a)**(1/4)/(d*x**4+c)**2,x)
```

```
[Out] d*x*(a + b*x**4)**(3/4)/(4*c*(c + d*x**4)*(a*d - b*c)) - (3*a*d -
 4*b*c)*atan(x*(-a*d + b*c)**(1/4)/(c**(1/4)*(a + b*x**4)**(1/4))
```

```
)/(8*c**(7/4)*(-a*d + b*c)**(5/4)) - (3*a*d - 4*b*c)*atanh(x*(-a*
d + b*c)**(1/4)/(c**(1/4)*(a + b*x**4)**(1/4)))/(8*c**(7/4)*(-a*d
 + b*c)**(5/4))
```

---

**Mathematica [A]** time = 0.285242, size = 171, normalized size = 1.06

$$\frac{(4bc-3ad)\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)\right)}{16c^{7/4}(bc-ad)^{5/4}} - \frac{dx\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x]
```

```
[Out] -(d*x*(a + b*x^4)^(3/4))/(4*c*(b*c - a*d)*(c + d*x^4)) + ((4*b*c
- 3*a*d)*(2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/
4))] - Log[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + L
og[c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(16*c^(7/
4)*(b*c - a*d)^(5/4))
```

---

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^4+c\right)^2}\frac{1}{\sqrt[4]{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

```
[Out] int(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(1/4)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{1}{4}}(dx^4 + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2), x)
```

## 3.110 $\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx$

**Optimal**. Leaf size=205

$$-\frac{d(8bc-3ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}-\frac{d(8bc-3ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}+\frac{bx(ad+4bc)}{4ac\sqrt[4]{a+bx^4}(bc-ad)^2}-\frac{dx}{4c\sqrt[4]{a+bx^4}\,(c+dx^4)\,(bc-ad)}$$

```
[Out] (b*(4*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(a + b*x^4)^(1/4)) - (d*
x)/(4*c*(b*c - a*d)*(a + b*x^4)^(1/4)*(c + d*x^4)) - (d*(8*b*c -
3*a*d)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])
/(8*c^(7/4)*(b*c - a*d)^(9/4)) - (d*(8*b*c - 3*a*d)*ArcTanh[((b*c
 - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c -
a*d)^(9/4))
```

---

**Rubi [A]** time = 0.495085, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{d(8bc-3ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}-\frac{d(8bc-3ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}+\frac{bx(ad+4bc)}{4ac\sqrt[4]{a+bx^4}(bc-ad)^2}-\frac{dx}{4c\sqrt[4]{a+bx^4}\,(c+dx^4)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x]
```

```
[Out] (b*(4*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(a + b*x^4)^(1/4)) - (d*
x)/(4*c*(b*c - a*d)*(a + b*x^4)^(1/4)*(c + d*x^4)) - (d*(8*b*c -
3*a*d)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])
/(8*c^(7/4)*(b*c - a*d)^(9/4)) - (d*(8*b*c - 3*a*d)*ArcTanh[((b*c
 - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c -
a*d)^(9/4))
```

---

**Rubi in Sympy [A]** time = 77.0004, size = 180, normalized size = 0.88

$$\frac{dx}{4c\sqrt[4]{a+bx^4}\,(c+dx^4)\,(ad-bc)}+\frac{d\,(3ad-8bc)\operatorname{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{\frac{7}{4}}\,(-ad+bc)^{\frac{9}{4}}}+\frac{d\,(3ad-8bc)\operatorname{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{\frac{7}{4}}\,(-ad+bc)^{\frac{9}{4}}}+\frac{bx\,(ad+4bc)}{4ac\sqrt[4]{a+bx^4}\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c)**2,x)
```

```
[Out] d*x/(4*c*(a + b*x**4)**(1/4)*(c + d*x**4)*(a*d - b*c)) + d*(3*a*d
 - 8*b*c)*atan(x*(-a*d + b*c)**(1/4)/(c**(1/4)*(a + b*x**4)**(1/4
)))/(8*c**(7/4)*(-a*d + b*c)**(9/4)) + d*(3*a*d - 8*b*c)*atanh(x*
(-a*d + b*c)**(1/4)/(c**(1/4)*(a + b*x**4)**(1/4)))/(8*c**(7/4)*(
-a*d + b*c)**(9/4)) + b*x*(a*d + 4*b*c)/(4*a*c*(a + b*x**4)**(1/4
)*(a*d - b*c)**2)
```

---

**Mathematica [A]** time = 0.449289, size = 204, normalized size = 1.

$$\frac{x\left(a^2d^2+abd^2x^4+4b^2c\left(c+dx^4\right)\right)}{4ac\sqrt[4]{a+bx^4}\,(c+dx^4)\,(bc-ad)^2}+\frac{d(3ad-8bc)\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)\right)}{16c^{7/4}(bc-ad)^{9/4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x]
```

```
[Out] (x*(a^2*d^2 + a*b*d^2*x^4 + 4*b^2*c*(c + d*x^4)))/(4*a*c*(b*c - a
*d)^2*(a + b*x^4)^(1/4)*(c + d*x^4)) + (d*(-8*b*c + 3*a*d)*(2*Arc
Tan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))] - Log[c^(1
/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4) + ((
b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(16*c^(7/4)*(b*c - a*d)^
(9/4))
```

---

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int\frac{1}{\left(dx^4+c\right)^2}\left(bx^4+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

```
[Out] int(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)^{\frac{5}{4}}(dx^4+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{5}{4}}(dx^4 + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2), x)
```

## 3.111 $\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx$

**Optimal**. Leaf size=266

$$\frac{bx\left(-5a^2d^2-56abcd+16b^2c^2\right)}{20a^2c\sqrt[4]{a+bx^4}(bc-ad)^3}+\frac{3d^2(4bc-ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}+\frac{3d^2(4bc-ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}-\frac{dx}{4c\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)(bc-ad)}+\frac{bx(5ad+4bc)}{20ac\left(a+bx^4\right)^{5/4}(bc-ad)^2}$$

```
[Out] (b*(4*b*c + 5*a*d)*x)/(20*a*c*(b*c - a*d)^2*(a + b*x^4)^(5/4)) +
(b*(16*b^2*c^2 - 56*a*b*c*d - 5*a^2*d^2)*x)/(20*a^2*c*(b*c - a*d)
^3*(a + b*x^4)^(1/4)) - (d*x)/(4*c*(b*c - a*d)*(a + b*x^4)^(5/4)*
(c + d*x^4)) + (3*d^2*(4*b*c - a*d)*ArcTan[((b*c - a*d)^(1/4)*x)/
(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(13/4)) + (3
*d^2*(4*b*c - a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*
x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(13/4))
```

---

**Rubi [A]** time = 0.884767, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333


$$\frac{bx\left(-5a^2d^2-56abcd+16b^2c^2\right)}{20a^2c\sqrt[4]{a+bx^4}(bc-ad)^3}+\frac{3d^2(4bc-ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}+\frac{3d^2(4bc-ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}-\frac{dx}{4c\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)(bc-ad)}+\frac{bx(5ad+4bc)}{20ac\left(a+bx^4\right)^{5/4}(bc-ad)^2}$$


Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x]
```


```
[Out] (b*(4*b*c + 5*a*d)*x)/(20*a*c*(b*c - a*d)^2*(a + b*x^4)^(5/4)) +
(b*(16*b^2*c^2 - 56*a*b*c*d - 5*a^2*d^2)*x)/(20*a^2*c*(b*c - a*d)
^3*(a + b*x^4)^(1/4)) - (d*x)/(4*c*(b*c - a*d)*(a + b*x^4)^(5/4)*
(c + d*x^4)) + (3*d^2*(4*b*c - a*d)*ArcTan[((b*c - a*d)^(1/4)*x)/
(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(13/4)) + (3
*d^2*(4*b*c - a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*
x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(13/4))
```

**Rubi in Sympy [A]** time = 144.937, size = 240, normalized size = 0.9

$$\frac{dx}{4c\left(a+bx^4\right)^{\frac{5}{4}}\left(c+dx^4\right)\left(ad-bc\right)}-\frac{3d^2\left(ad-4bc\right)\operatorname{atan}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{\frac{7}{4}}\left(-ad+bc\right)^{\frac{13}{4}}}-\frac{3d^2\left(ad-4bc\right)\operatorname{atanh}\left(\frac{x\sqrt[4]{-ad+bc}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{\frac{7}{4}}\left(-ad+bc\right)^{\frac{13}{4}}}+\frac{bx\left(5ad+4bc\right)}{20ac\left(a+bx^4\right)^{\frac{5}{4}}\left(ad-bc\right)^2}+\frac{bx\left(5a^2d^2+56abcd-16b^2c^2\right)}{20a^2c\sqrt[4]{a+bx^4}\left(ad-bc\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**4+a)**(9/4)/(d*x**4+c)**2,x)
```

```
[Out] d*x/(4*c*(a + b*x**4)**(5/4)*(c + d*x**4)*(a*d - b*c)) - 3*d**2*(
a*d - 4*b*c)*atan(x*(-a*d + b*c)**(1/4)/(c**(1/4)*(a + b*x**4)**(
1/4)))/(8*c**(7/4)*(-a*d + b*c)**(13/4)) - 3*d**2*(a*d - 4*b*c)*a
tanh(x*(-a*d + b*c)**(1/4)/(c**(1/4)*(a + b*x**4)**(1/4)))/(8*c**
(7/4)*(-a*d + b*c)**(13/4)) + b*x*(5*a*d + 4*b*c)/(20*a*c*(a + b*
x**4)**(5/4)*(a*d - b*c)**2) + b*x*(5*a**2*d**2 + 56*a*b*c*d - 16
*b**2*c**2)/(20*a**2*c*(a + b*x**4)**(1/4)*(a*d - b*c)**3)
```

**Mathematica [A]** time = 0.822363, size = 242, normalized size = 0.91

$$\frac{1}{20}x\left(a+bx^4\right)^{3/4}\left(\frac{8b^2(7ad-2bc)}{a^2\left(a+bx^4\right)(ad-bc)^3}+\frac{4b^2}{a\left(a+bx^4\right)^2(bc-ad)^2}-\frac{5d^3}{c\left(c+dx^4\right)(bc-ad)^3}\right)+\frac{3d^2(4bc-ad)\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)\right)}{16c^{7/4}(bc-ad)^{13/4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x]
```

```
[Out] (x*(a + b*x^4)^(3/4)*((4*b^2)/(a*(b*c - a*d)^2*(a + b*x^4)^2) + (
8*b^2*(-2*b*c + 7*a*d))/(a^2*(-(b*c) + a*d)^3*(a + b*x^4)) - (5*d
^3)/(c*(b*c - a*d)^3*(c + d*x^4))))/20 + (3*d^2*(4*b*c - a*d)*(2*
ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))] - Log[c
^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4) +
 ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(16*c^(7/4)*(b*c - a*
d)^(13/4))
```

**Maple [F]** time = 0.064, size = 0, normalized size = 0.

$$\int \frac{1}{(dx^4+c)^2}\left(bx^4+a\right)^{-\frac{9}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

```
[Out] int(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)^{\frac{9}{4}}(dx^4+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**4+a)**(9/4)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)^{\frac{9}{4}}(dx^4+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2), x)
```

## 3.112 $\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx$

**Optimal**. Leaf size=353

$$-\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3bc-ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4cd^2\left(a+bx^4\right)^{3/4}}-\frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)(ad+2bc)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^2}-\frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)(ad+2bc)\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^2}+\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{4cd^2}-\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

```
[Out] (b*(3*b*c - a*d)*x*(a + b*x^4)^(1/4))/(4*c*d^2) - ((b*c - a*d)*x*
(a + b*x^4)^(5/4))/(4*c*d*(c + d*x^4)) - (Sqrt[a]*b^(3/2)*(3*b*c
- a*d)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/S
qrt[a]]/2, 2])/(4*c*d^2*(a + b*x^4)^(3/4)) - (3*(b*c - a*d)*(2*b*
c + a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*
c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)
], -1])/(8*b^(1/4)*c^2*d^2) - (3*(b*c - a*d)*(2*b*c + a*d)*Sqrt[a
/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]
*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)
*c^2*d^2)
```

---

**Rubi [A]** time = 0.987495, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476

$$-\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3bc-ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4cd^2\left(a+bx^4\right)^{3/4}}-\frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)(ad+2bc)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^2}-\frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)(ad+2bc)\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^2}+\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{4cd^2}-\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^(9/4)/(c + d*x^4)^2,x]
```

```
[Out] (b*(3*b*c - a*d)*x*(a + b*x^4)^(1/4))/(4*c*d^2) - ((b*c - a*d)*x*
(a + b*x^4)^(5/4))/(4*c*d*(c + d*x^4)) - (Sqrt[a]*b^(3/2)*(3*b*c
- a*d)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/S
qrt[a]]/2, 2])/(4*c*d^2*(a + b*x^4)^(3/4)) - (3*(b*c - a*d)*(2*b*
c + a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*
c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)
], -1])/(8*b^(1/4)*c^2*d^2) - (3*(b*c - a*d)*(2*b*c + a*d)*Sqrt[a
/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]
*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)
*c^2*d^2)
```

---

**Rubi in Sympy [A]** time = 112.913, size = 316, normalized size = 0.9

$$\frac{\sqrt{a}b^{\frac{3}{2}}x^{3}\left(ad-3bc\right)\left(\frac{a}{bx^{4}}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x^{2}}\right)}{2}\middle|2\right)}{4cd^{2}\left(a+bx^{4}\right)^{\frac{3}{4}}}-\frac{bx\sqrt[4]{a+bx^{4}}\left(ad-3bc\right)}{4cd^{2}}+\frac{x\left(a+bx^{4}\right)^{\frac{5}{4}}\left(ad-bc\right)}{4cd\left(c+dx^{4}\right)}+\frac{3\sqrt{\frac{a}{a+bx^{4}}}\sqrt{a+bx^{4}}\left(ad-bc\right)\left(ad+2bc\right)\Pi\left(-\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^{4}}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^{2}d^{2}}+\frac{3\sqrt{\frac{a}{a+bx^{4}}}\sqrt{a+bx^{4}}\left(ad-bc\right)\left(ad+2bc\right)\Pi\left(\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^{4}}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**(9/4)/(d*x**4+c)**2,x)
```

```
[Out] sqrt(a)*b**(3/2)*x**3*(a*d - 3*b*c)*(a/(b*x**4) + 1)**(3/4)*ellip
tic_f(atan(sqrt(a)/(sqrt(b)*x**2))/2, 2)/(4*c*d**2*(a + b*x**4)**
(3/4)) - b*x*(a + b*x**4)**(1/4)*(a*d - 3*b*c)/(4*c*d**2) + x*(a
+ b*x**4)**(5/4)*(a*d - b*c)/(4*c*d*(c + d*x**4)) + 3*sqrt(a/(a +
 b*x**4))*sqrt(a + b*x**4)*(a*d - b*c)*(a*d + 2*b*c)*elliptic_pi(
-sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)
**(1/4)), -1)/(8*b**(1/4)*c**2*d**2) + 3*sqrt(a/(a + b*x**4))*sqr
t(a + b*x**4)*(a*d - b*c)*(a*d + 2*b*c)*elliptic_pi(sqrt(-a*d + b
*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/
(8*b**(1/4)*c**2*d**2)
```

---

**Mathematica [C]** time = 1.29411, size = 506, normalized size = 1.43

$$\frac{x\left(\frac{5x^{4}\left(a+bx^{4}\right)\left(a^{2}d^{2}-2abcd+b^{2}c\left(3c+2dx^{4}\right)\right)\left(4adF_{1}\left(\frac{9}{4};\frac{3}{4},2;\frac{13}{4};-\frac{bx^{4}}{a},-\frac{dx^{4}}{c}\right)+3bcF_{1}\left(\frac{9}{4};\frac{7}{4},1;\frac{13}{4};-\frac{bx^{4}}{a},-\frac{dx^{4}}{c}\right)\right)-9ac\left(5a^{3}d^{2}+a^{2}bd\left(7dx^{4}-10c\right)+3ab^{2}c\left(5c+2d\right.\right.}{c\left(x^{4}\left(4adF_{1}\left(\frac{9}{4};\frac{3}{4},2;\frac{13}{4};-\frac{bx^{4}}{a},-\frac{dx^{4}}{c}\right)+3bcF_{1}\left(\frac{9}{4};\frac{7}{4},1;\frac{13}{4};-\frac{bx^{4}}{a},-\frac{dx^{4}}{c}\right)\right)-9acF_{1}\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^{4}}{a},-\frac{dx^{4}}{c}\right)\right)}\right.}{20d^{2}\left(a+bx^{4}\right)^{3/}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^4)^(9/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*((-25*a^2*(-3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*AppellF1[1/4, 3
/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(-5*a*c*AppellF1[1/4, 3/
4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4,
 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7
/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])) + (-9*a*c*(5*a^3*d^2 +
3*a*b^2*c*(5*c + 2*d*x^4) + a^2*b*d*(-10*c + 7*d*x^4) + b^3*c*x^4
*(9*c + 10*d*x^4))*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*
x^4)/c)] + 5*x^4*(a + b*x^4)*(-2*a*b*c*d + a^2*d^2 + b^2*c*(3*c +
 2*d*x^4))*(4*a*d*AppellF1[9/4, 3/4, 2, 13/4, -((b*x^4)/a), -((d*
x^4)/c)] + 3*b*c*AppellF1[9/4, 7/4, 1, 13/4, -((b*x^4)/a), -((d*x
^4)/c)]))/(c*(-9*a*c*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((
d*x^4)/c)] + x^4*(4*a*d*AppellF1[9/4, 3/4, 2, 13/4, -((b*x^4)/a),
 -((d*x^4)/c)] + 3*b*c*AppellF1[9/4, 7/4, 1, 13/4, -((b*x^4)/a),
-((d*x^4)/c)])))))/(20*d^2*(a + b*x^4)^(3/4)*(c + d*x^4))
```

---

**Maple [F]** time = 0.094, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d x^{4}+c\right)^{2}} \left(b x^{4}+a\right)^{\frac{9}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

```
[Out] int((b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{4}+a\right)^{\frac{9}{4}}}{\left(d x^{4}+c\right)^{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**(9/4)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{9}{4}}}{(dx^4 + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2, x)
```

# 3.113 $\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx$

**Optimal**. Leaf size=298

$$\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4cd\left(a+bx^4\right)^{3/4}}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(3ad+2bc)\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(3ad+2bc)\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d}-\frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

```
[Out] -((b*c - a*d)*x*(a + b*x^4)^(1/4))/(4*c*d*(c + d*x^4)) + (Sqrt[a]
*b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)
/Sqrt[a]]/2, 2])/(4*c*d*(a + b*x^4)^(3/4)) + ((2*b*c + 3*a*d)*Sqr
t[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sq
rt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b
^(1/4)*c^2*d) + ((2*b*c + 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x
^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)
*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d)
```

---

**Rubi [A]** time = 0.703559, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4cd\left(a+bx^4\right)^{3/4}}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(3ad+2bc)\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(3ad+2bc)\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d}-\frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^(5/4)/(c + d*x^4)^2,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^4)^(1/4))/(4*c*d*(c + d*x^4)) + (Sqrt[a]
*b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)
/Sqrt[a]]/2, 2])/(4*c*d*(a + b*x^4)^(3/4)) + ((2*b*c + 3*a*d)*Sqr
t[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sq
rt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b
```

```
^(1/4)*c^2*d) + ((2*b*c + 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x
^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)
*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d)
```

---

**Rubi in Sympy [A]** time = 83.37, size = 260, normalized size = 0.87

$$\frac{\sqrt{a}b^{\frac{3}{2}}x^3\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}\middle|2\right)}{4cd\left(a+bx^4\right)^{\frac{3}{4}}}+\frac{x\sqrt[4]{a+bx^4}\left(ad-bc\right)}{4cd\left(c+dx^4\right)}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(3ad+2bc\right)\left(-\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(3ad+2bc\right)\left(\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**(5/4)/(d*x**4+c)**2,x)
```

```
[Out] sqrt(a)*b**(3/2)*x**3*(a/(b*x**4) + 1)**(3/4)*elliptic_f(atan(sqr
t(a)/(sqrt(b)*x**2))/2, 2)/(4*c*d*(a + b*x**4)**(3/4)) + x*(a + b
*x**4)**(1/4)*(a*d - b*c)/(4*c*d*(c + d*x**4)) + sqrt(a/(a + b*x*
*4))*sqrt(a + b*x**4)*(3*a*d + 2*b*c)*elliptic_pi(-sqrt(-a*d + b*
c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(
8*b**(1/4)*c**2*d) + sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*(3*a*d
 + 2*b*c)*elliptic_pi(sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asin(b*
*(1/4)*x/(a + b*x**4)**(1/4)), -1)/(8*b**(1/4)*c**2*d)
```

---

**Mathematica [C]** time = 0.646114, size = 440, normalized size = 1.48

$$\frac{x\left(\frac{9ac\left(5a^2d+ab\left(7dx^4-5c\right)-3b^2cx^4\right)F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+5x^4\left(a+bx^4\right)\left(bc-ad\right)\left(4adF_1\left(\frac{9}{4};\frac{3}{4},2;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{9}{4};\frac{7}{4},1;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}{c\left(9acF_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-x^4\left(4adF_1\left(\frac{9}{4};\frac{3}{4},2;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{9}{4};\frac{7}{4},1;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}-\frac{\cdots}{x^4\cdots}\right)}{20d\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^4)^(5/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*((-25*a^2*(b*c + 3*a*d)*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a
), -((d*x^4)/c)])/(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a)
, -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)
/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a
```

```
), -((d*x^4)/c)])) + (9*a*c*(5*a^2*d - 3*b^2*c*x^4 + a*b*(-5*c +
7*d*x^4))*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]
+ 5*(b*c - a*d)*x^4*(a + b*x^4)*(4*a*d*AppellF1[9/4, 3/4, 2, 13/4
, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[9/4, 7/4, 1, 13/4,
 -((b*x^4)/a), -((d*x^4)/c)]))/(c*(9*a*c*AppellF1[5/4, 3/4, 1, 9/
4, -((b*x^4)/a), -((d*x^4)/c)] - x^4*(4*a*d*AppellF1[9/4, 3/4, 2,
 13/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[9/4, 7/4, 1,
13/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(20*d*(a + b*x^4)^(3/4)*(c
 + d*x^4))
```

---

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d x^{4}+c\right)^{2}} \left(b x^{4}+a\right)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

```
[Out] int((b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{4}+a\right)^{\frac{5}{4}}}{\left(d x^{4}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(5/4)/(d*x^4 + c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/(d*x^4 + c)^2, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(5/4)/(d*x^4 + c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**(5/4)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(5/4)/(d*x^4 + c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/(d*x^4 + c)^2, x)
```

## 3.114 $\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx$

**Optimal**. Leaf size=308

$$-\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4c\left(a+bx^4\right)^{3/4}(bc-ad)}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-3ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-3ad)\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)}+\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

```
[Out] (x*(a + b*x^4)^(1/4))/(4*c*(c + d*x^4)) - (Sqrt[a]*b^(3/2)*(1 + a
/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2]
)/(4*c*(b*c - a*d)*(a + b*x^4)^(3/4)) + ((2*b*c - 3*a*d)*Sqrt[a/(
a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]
*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4
)*c^2*(b*c - a*d)) + ((2*b*c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a
+ b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^
(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d))
```

---

**Rubi [A]** time = 0.636605, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$-\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4c\left(a+bx^4\right)^{3/4}(bc-ad)}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-3ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-3ad)\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)}+\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^(1/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*(a + b*x^4)^(1/4))/(4*c*(c + d*x^4)) - (Sqrt[a]*b^(3/2)*(1 + a
/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2]
)/(4*c*(b*c - a*d)*(a + b*x^4)^(3/4)) + ((2*b*c - 3*a*d)*Sqrt[a/(
a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]
```

```
*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4
)*c^2*(b*c - a*d)) + ((2*b*c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a
+ b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^
(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d))
```

---

**Rubi in Sympy [A]** time = 85.7406, size = 267, normalized size = 0.87

$$\frac{\sqrt{a}b^{\frac{3}{2}}x^{3}\left(\frac{a}{bx^{4}}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x^{2}}\right)}{2}\middle|2\right)}{4c\left(a+bx^{4}\right)^{\frac{3}{4}}\left(ad-bc\right)}+\frac{x\sqrt[4]{a+bx^{4}}}{4c\left(c+dx^{4}\right)}+\frac{\sqrt{\frac{a}{a+bx^{4}}}\sqrt{a+bx^{4}}\left(3ad-2bc\right)\Pi\left(-\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^{4}}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^{2}\left(ad-bc\right)}+\frac{\sqrt{\frac{a}{a+bx^{4}}}\sqrt{a+bx^{4}}\left(3ad-2bc\right)\Pi\left(\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^{4}}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^{2}\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**(1/4)/(d*x**4+c)**2,x)
```

```
[Out] sqrt(a)*b**(3/2)*x**3*(a/(b*x**4) + 1)**(3/4)*elliptic_f(atan(sqr
t(a)/(sqrt(b)*x**2))/2, 2)/(4*c*(a + b*x**4)**(3/4)*(a*d - b*c))
+ x*(a + b*x**4)**(1/4)/(4*c*(c + d*x**4)) + sqrt(a/(a + b*x**4))
*sqrt(a + b*x**4)*(3*a*d - 2*b*c)*elliptic_pi(-sqrt(-a*d + b*c)/(
sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(8*b*
*(1/4)*c**2*(a*d - b*c)) + sqrt(a/(a + b*x**4))*sqrt(a + b*x**4)*
(3*a*d - 2*b*c)*elliptic_pi(sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), a
sin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(8*b**(1/4)*c**2*(a*d -
b*c))
```

---

**Mathematica [C]** time = 0.287786, size = 322, normalized size = 1.05

$$\frac{x\left(-\frac{75a^{2}F_{1}\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^{4}}{a},-\frac{dx^{4}}{c}\right)}{x^{4}\left(4adF_{1}\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^{4}}{a},-\frac{dx^{4}}{c}\right)+3bcF_{1}\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^{4}}{a},-\frac{dx^{4}}{c}\right)\right)-5acF_{1}\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^{4}}{a},-\frac{dx^{4}}{c}\right)}-\frac{18abx^{4}F_{1}\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4}\right.}{x^{4}\left(4adF_{1}\left(\frac{9}{4};\frac{3}{4},2;\frac{13}{4};-\frac{bx^{4}}{a},-\frac{dx^{4}}{c}\right)+3bcF_{1}\left(\frac{9}{4};\frac{7}{4},1;\frac{13}{4}\right.\right.}\right.}{20\left(a+bx^{4}\right)^{3/4}\left(c+dx^{4}\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^4)^(1/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*((5*(a + b*x^4))/c - (75*a^2*AppellF1[1/4, 3/4, 1, 5/4, -((b*x
^4)/a), -((d*x^4)/c)])/(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^
```

```
4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b
*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x
^4)/a), -((d*x^4)/c)])) - (18*a*b*x^4*AppellF1[5/4, 3/4, 1, 9/4,
-((b*x^4)/a), -((d*x^4)/c)])/(-9*a*c*AppellF1[5/4, 3/4, 1, 9/4, -
((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[9/4, 3/4, 2, 13/
4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[9/4, 7/4, 1, 13/4
, -((b*x^4)/a), -((d*x^4)/c)]))))/(20*(a + b*x^4)^(3/4)*(c + d*x^
4))
```

---

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d x^{4}+c\right)^{2}} \sqrt[4]{b x^{4}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

```
[Out] int((b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{4}+a\right)^{\frac{1}{4}}}{\left(d x^{4}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(1/4)/(d*x^4 + c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c)^2, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(1/4)/(d*x^4 + c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**(1/4)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4 + a\right)^{\frac{1}{4}}}{\left(dx^4 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^(1/4)/(d*x^4 + c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c)^2, x)
```

## 3.115 $\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx$

**Optimal**. Leaf size=330

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(4bc-ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4\sqrt{a}c\left(a+bx^4\right)^{3/4}(bc-ad)^2}-\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2}-\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2}-\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

```
[Out] -(d*x*(a + b*x^4)^(1/4))/(4*c*(b*c - a*d)*(c + d*x^4)) - (b^(3/2)
*(4*b*c - a*d)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b
]*x^2)/Sqrt[a]]/2, 2])/(4*Sqrt[a]*c*(b*c - a*d)^2*(a + b*x^4)^(3/
4)) - (3*d*(2*b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*Elli
pticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(
a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)^2) - (3*d*(2*b
*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c
 - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)],
 -1])/(8*b^(1/4)*c^2*(b*c - a*d)^2)
```

---

**Rubi [A]** time = 0.752525, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(4bc-ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4\sqrt{a}c\left(a+bx^4\right)^{3/4}(bc-ad)^2}-\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2}-\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2}-\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^4)^(3/4)*(c + d*x^4)^2),x]
```

```
[Out] -(d*x*(a + b*x^4)^(1/4))/(4*c*(b*c - a*d)*(c + d*x^4)) - (b^(3/2)
*(4*b*c - a*d)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b
]*x^2)/Sqrt[a]]/2, 2])/(4*Sqrt[a]*c*(b*c - a*d)^2*(a + b*x^4)^(3/
4)) - (3*d*(2*b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*Elli
```

```
pticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(
a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)^2) - (3*d*(2*b
*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c
 - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)],
 -1])/(8*b^(1/4)*c^2*(b*c - a*d)^2)
```

---

**Rubi in Sympy [A]** time = 96.9919, size = 292, normalized size = 0.88

$$\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)\left(ad-bc\right)}+\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(ad-2bc\right)\Pi\left(-\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\left(ad-bc\right)^2}+\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(ad-2bc\right)\Pi\left(\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\left(ad-bc\right)^2}+\frac{b^{\frac{3}{2}}x^3\left(ad-4bc\right)\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}\middle|2\right)}{4\sqrt{a}c\left(a+bx^4\right)^{\frac{3}{4}}\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**4+a)**(3/4)/(d*x**4+c)**2,x)
```

```
[Out] d*x*(a + b*x**4)**(1/4)/(4*c*(c + d*x**4)*(a*d - b*c)) + 3*d*sqrt
(a/(a + b*x**4))*sqrt(a + b*x**4)*(a*d - 2*b*c)*elliptic_pi(-sqrt
(-a*d + b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/
4)), -1)/(8*b**(1/4)*c**2*(a*d - b*c)**2) + 3*d*sqrt(a/(a + b*x**
4))*sqrt(a + b*x**4)*(a*d - 2*b*c)*elliptic_pi(sqrt(-a*d + b*c)/(
sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -1)/(8*b*
*(1/4)*c**2*(a*d - b*c)**2) + b**(3/2)*x**3*(a*d - 4*b*c)*(a/(b*x
**4) + 1)**(3/4)*elliptic_f(atan(sqrt(a)/(sqrt(b)*x**2))/2, 2)/(4
*sqrt(a)*c*(a + b*x**4)**(3/4)*(a*d - b*c)**2)
```

---

**Mathematica [C]** time = 0.890998, size = 341, normalized size = 1.03

$$\frac{x\left(\frac{18abdx^4F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{x^4\left(4adF_1\left(\frac{9}{4};\frac{3}{4},2;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{9}{4};\frac{7}{4},1;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-9acF_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}+\frac{25a(3ad-4bc)F_1\left(\frac{1}{4};\frac{3}{4},1;\right.}{x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\right.\right.}}{20\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)\left(bc-ad\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^4)^(3/4)*(c + d*x^4)^2),x]
```

```
[Out] (x*((-5*d*(a + b*x^4))/c + (25*a*(-4*b*c + 3*a*d)*AppellF1[1/4, 3
/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(-5*a*c*AppellF1[1/4, 3/
```

```
4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4,
 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7
/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])) + (18*a*b*d*x^4*AppellF
1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])/(-9*a*c*AppellF1
[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*Appel
lF1[9/4, 3/4, 2, 13/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*Appell
F1[9/4, 7/4, 1, 13/4, -((b*x^4)/a), -((d*x^4)/c)]))))/(20*(b*c -
a*d)*(a + b*x^4)^(3/4)*(c + d*x^4))
```

---

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{(dx^4+c)^2}\left(bx^4+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x)

[Out] int(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)^{\frac{3}{4}}(dx^4+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2),x, algorithm="maxima")

[Out] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2),x, algorithm="fricas")

[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**4+a)**(3/4)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)^{\frac{3}{4}}(dx^4+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2), x)
```

## 3.116 $\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx$

**Optimal**. Leaf size=390

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(3a^2d^2-32abcd+8b^2c^2\right)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}c\left(a+bx^4\right)^{3/4}(bc-ad)^3}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(10bc-3ad)\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(10bc-3ad)\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3}+\frac{bx(3ad+4bc)}{12ac\left(a+bx^4\right)^{3/4}(bc-ad)^2}-\frac{dx}{4c\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)(bc-ad)}$$

```
[Out] (b*(4*b*c + 3*a*d)*x)/(12*a*c*(b*c - a*d)^2*(a + b*x^4)^(3/4)) -
(d*x)/(4*c*(b*c - a*d)*(a + b*x^4)^(3/4)*(c + d*x^4)) - (b^(3/2)*
(8*b^2*c^2 - 32*a*b*c*d + 3*a^2*d^2)*(1 + a/(b*x^4))^(3/4)*x^3*El
lipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(12*a^(3/2)*c*(b*c -
 a*d)^3*(a + b*x^4)^(3/4)) + (d^2*(10*b*c - 3*a*d)*Sqrt[a/(a + b*
x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[
c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*
(b*c - a*d)^3) + (d^2*(10*b*c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a
 + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b
^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)^3)
```

---

**Rubi [A]** time = 1.16421, antiderivative size = 390, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(3a^2d^2-32abcd+8b^2c^2\right)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}c\left(a+bx^4\right)^{3/4}(bc-ad)^3}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(10bc-3ad)\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(10bc-3ad)\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3}+\frac{bx(3ad+4bc)}{12ac\left(a+bx^4\right)^{3/4}(bc-ad)^2}-\frac{dx}{4c\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^4)^(7/4)*(c + d*x^4)^2),x]
```

```
[Out] (b*(4*b*c + 3*a*d)*x)/(12*a*c*(b*c - a*d)^2*(a + b*x^4)^(3/4)) -
(d*x)/(4*c*(b*c - a*d)*(a + b*x^4)^(3/4)*(c + d*x^4)) - (b^(3/2)*
(8*b^2*c^2 - 32*a*b*c*d + 3*a^2*d^2)*(1 + a/(b*x^4))^(3/4)*x^3*El
lipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(12*a^(3/2)*c*(b*c -
 a*d)^3*(a + b*x^4)^(3/4)) + (d^2*(10*b*c - 3*a*d)*Sqrt[a/(a + b*
x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[
c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*
(b*c - a*d)^3) + (d^2*(10*b*c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a
 + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b
^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)^3)
```

---

**Rubi in Sympy [A]** time = 157.932, size = 350, normalized size = 0.9

$$\frac{dx}{4c\left(a+bx^4\right)^{\frac{3}{4}}\left(c+dx^4\right)\left(ad-bc\right)} + \frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(3ad-10bc\right)\Pi\left(-\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\left(ad-bc\right)^3} + \frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(3ad-10bc\right)\Pi\left(\frac{\sqrt{-ad+bc}}{\sqrt{b}\sqrt{c}};\operatorname{asin}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\left(ad-bc\right)^3} + \frac{bx\left(3ad+4bc\right)}{12ac\left(a+bx^4\right)^{\frac{3}{4}}\left(ad-bc\right)^2} + \frac{b^{\frac{3}{2}}x^3\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\left(3a^2d^2-32abcd+8b^2c^2\right)F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}\middle|2\right)}{12a^{\frac{3}{2}}c\left(a+bx^4\right)^{\frac{3}{4}}\left(ad-bc\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**4+a)**(7/4)/(d*x**4+c)**2,x)
```

```
[Out] d*x/(4*c*(a + b*x**4)**(3/4)*(c + d*x**4)*(a*d - b*c)) + d**2*sqr
t(a/(a + b*x**4))*sqrt(a + b*x**4)*(3*a*d - 10*b*c)*elliptic_pi(-
sqrt(-a*d + b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)*
*(1/4)), -1)/(8*b**(1/4)*c**2*(a*d - b*c)**3) + d**2*sqrt(a/(a +
b*x**4))*sqrt(a + b*x**4)*(3*a*d - 10*b*c)*elliptic_pi(sqrt(-a*d
+ b*c)/(sqrt(b)*sqrt(c)), asin(b**(1/4)*x/(a + b*x**4)**(1/4)), -
1)/(8*b**(1/4)*c**2*(a*d - b*c)**3) + b*x*(3*a*d + 4*b*c)/(12*a*c
*(a + b*x**4)**(3/4)*(a*d - b*c)**2) + b**(3/2)*x**3*(a/(b*x**4)
+ 1)**(3/4)*(3*a**2*d**2 - 32*a*b*c*d + 8*b**2*c**2)*elliptic_f(a
tan(sqrt(a)/(sqrt(b)*x**2))/2, 2)/(12*a**(3/2)*c*(a + b*x**4)**(3
/4)*(a*d - b*c)**3)
```

---

**Mathematica [C]** time = 1.03826, size = 485, normalized size = 1.24

$$\frac{x\left(\frac{9ac(15a^2d^2+21abd^2x^4+4b^2c(5c+7dx^4))F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-5x^4(3a^2d^2+3abd^2x^4+4b^2c(c+dx^4))\left(4adF_1\left(\frac{9}{4};\frac{3}{4},2;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{9}{4};\frac{7}{4},1;\frac{13}{4}\right.\right.}{ac\left(9acF_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-x^4\left(4adF_1\left(\frac{9}{4};\frac{3}{4},2;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{9}{4};\frac{7}{4},1;\frac{13}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}}{60(a+bx^4)^{3/4}(c+dx^4)(bc-ad}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^4)^(7/4)*(c + d*x^4)^2),x]
```

```
[Out] (x*((-25*(8*b^2*c^2 - 24*a*b*c*d + 9*a^2*d^2)*AppellF1[1/4, 3/4,
1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(-5*a*c*AppellF1[1/4, 3/4, 1
, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4
, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4,
1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])) + (9*a*c*(15*a^2*d^2 + 21*a
*b*d^2*x^4 + 4*b^2*c*(5*c + 7*d*x^4))*AppellF1[5/4, 3/4, 1, 9/4,
-((b*x^4)/a), -((d*x^4)/c)] - 5*x^4*(3*a^2*d^2 + 3*a*b*d^2*x^4 +
4*b^2*c*(c + d*x^4))*(4*a*d*AppellF1[9/4, 3/4, 2, 13/4, -((b*x^4)
/a), -((d*x^4)/c)] + 3*b*c*AppellF1[9/4, 7/4, 1, 13/4, -((b*x^4)/
a), -((d*x^4)/c)]))/(a*c*(9*a*c*AppellF1[5/4, 3/4, 1, 9/4, -((b*x
^4)/a), -((d*x^4)/c)] - x^4*(4*a*d*AppellF1[9/4, 3/4, 2, 13/4, -(
(b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[9/4, 7/4, 1, 13/4, -((
b*x^4)/a), -((d*x^4)/c)])))))/(60*(b*c - a*d)^2*(a + b*x^4)^(3/4)
*(c + d*x^4))
```

---

**Maple [F]** time = 0.06, size = 0, normalized size = 0.

$$\int \frac{1}{(dx^4+c)^2}\left(bx^4+a\right)^{-\frac{7}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

```
[Out] int(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)^{\frac{7}{4}}(dx^4+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(7/4)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4 + a)^{\frac{7}{4}}(dx^4 + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2), x)
```

## 3.117 $\int \frac{1}{\sqrt[4]{1+x^4}(2+x^4)}\,dx$

**Optimal**. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\ 2^{3/4}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\ 2^{3/4}}$$

```
[Out] ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))]/(2*2^(3/4)) + ArcTanh[x/(2^(1
/4)*(1 + x^4)^(1/4))]/(2*2^(3/4))
```

---

**Rubi [A]** time = 0.0568344, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\ 2^{3/4}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x^4)^(1/4)*(2 + x^4)),x]
```

```
[Out] ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))]/(2*2^(3/4)) + ArcTanh[x/(2^(1
/4)*(1 + x^4)^(1/4))]/(2*2^(3/4))
```

---

**Rubi in Sympy [A]** time = 4.79372, size = 49, normalized size = 0.92

$$\frac{\sqrt[4]{2}\,\mathrm{atan}\left(\frac{2^{\frac{3}{4}}x}{2\sqrt[4]{x^4+1}}\right)}{4}+\frac{\sqrt[4]{2}\,\mathrm{atanh}\left(\frac{2^{\frac{3}{4}}x}{2\sqrt[4]{x^4+1}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+1)**(1/4)/(x**4+2),x)
```

```
[Out] 2**(1/4)*atan(2**(3/4)*x/(2*(x**4 + 1)**(1/4)))/4 + 2**(1/4)*atan
h(2**(3/4)*x/(2*(x**4 + 1)**(1/4)))/4
```

---

**Mathematica [A]** time = 0.08512, size = 70, normalized size = 1.32

$$\frac{-\log\left(2-\frac{2^{3/4}x}{\sqrt[4]{x^4+1}}\right)+\log\left(\frac{2^{3/4}x}{\sqrt[4]{x^4+1}}+2\right)+2\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 + x^4)^(1/4)*(2 + x^4)),x]
```

```
[Out] (2*ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))] - Log[2 - (2^(3/4)*x)/(1 +
 x^4)^(1/4)] + Log[2 + (2^(3/4)*x)/(1 + x^4)^(1/4)])/(4*2^(3/4))
```

---

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^4+2}\frac{1}{\sqrt[4]{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+1)^(1/4)/(x^4+2),x)
```

```
[Out] int(1/(x^4+1)^(1/4)/(x^4+2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+2)(x^4+1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x)
```

---

**Fricas [A]** time = 4.31309, size = 273, normalized size = 5.15

$$\frac{1}{64} \cdot 8^{\frac{3}{4}}\left(4 \arctan\left(-\frac{4\sqrt{x^4+1}x^2-\sqrt{2}\left(3x^4+2\right)}{2\cdot 8^{\frac{1}{4}}(x^4+1)^{\frac{1}{4}}x^3-8^{\frac{3}{4}}(x^4+1)^{\frac{3}{4}}x-\sqrt{2}(x^4+2)}\right)+\log\left(\frac{2\left(2\cdot 8^{\frac{1}{4}}\left(x^4+1\right)^{\frac{1}{4}}x^3+8^{\frac{3}{4}}\left(x^4+1\right)^{\frac{3}{4}}x+4\sqrt{x^4+1}x^2\right.}{x^4+2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)),x, algorithm="fricas")
```

```
[Out] 1/64*8^(3/4)*(4*arctan(-(4*sqrt(x^4 + 1)*x^2 - sqrt(2)*(3*x^4 + 2
))/(2*8^(1/4)*(x^4 + 1)^(1/4)*x^3 - 8^(3/4)*(x^4 + 1)^(3/4)*x - s
qrt(2)*(x^4 + 2))) + log(2*(2*8^(1/4)*(x^4 + 1)^(1/4)*x^3 + 8^(3/
4)*(x^4 + 1)^(3/4)*x + 4*sqrt(x^4 + 1)*x^2 + sqrt(2)*(3*x^4 + 2))
/(x^4 + 2)) - log(2*(2*8^(1/4)*(x^4 + 1)^(1/4)*x^3 + 8^(3/4)*(x^4
 + 1)^(3/4)*x - 4*sqrt(x^4 + 1)*x^2 - sqrt(2)*(3*x^4 + 2))/(x^4 +
 2)))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{x^4+1}\,(x^4+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+1)**(1/4)/(x**4+2),x)
```

```
[Out] Integral(1/((x**4 + 1)**(1/4)*(x**4 + 2)), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^4+2)(x^4+1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x)
```

## 3.118 $\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx$

**Optimal**. Leaf size=57

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

```
[Out] ArcTan[(a^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*a^(5/4)) + ArcTanh[(a^(1
/4)*x)/(a + b*x^4)^(1/4)]/(2*a^(5/4))
```

---

**Rubi [A]** time = 0.0694466, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a - (a - b)*x^4)*(a + b*x^4)^(1/4)),x]
```

```
[Out] ArcTan[(a^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*a^(5/4)) + ArcTanh[(a^(1
/4)*x)/(a + b*x^4)^(1/4)]/(2*a^(5/4))
```

---

**Rubi in Sympy [A]** time = 9.03974, size = 49, normalized size = 0.86

$$\frac{\operatorname{atan}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{\frac{5}{4}}}+\frac{\operatorname{atanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a-(a-b)*x**4)/(b*x**4+a)**(1/4),x)
```

```
[Out] atan(a**(1/4)*x/(a + b*x**4)**(1/4))/(2*a**(5/4)) + atanh(a**(1/4
)*x/(a + b*x**4)**(1/4))/(2*a**(5/4))
```

---

**Mathematica [A]** time = 0.109993, size = 76, normalized size = 1.33

$$\frac{-\log\left(1-\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}+1\right)+2\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{ax^4+b}}\right)}{4a^{5/4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a - (a - b)*x^4)*(a + b*x^4)^(1/4)),x]
```

```
[Out] (2*ArcTan[(a^(1/4)*x)/(b + a*x^4)^(1/4)] - Log[1 - (a^(1/4)*x)/(b
 + a*x^4)^(1/4)] + Log[1 + (a^(1/4)*x)/(b + a*x^4)^(1/4)])/(4*a^(
5/4))
```

---

**Maple [F]** time = 0.076, size = 0, normalized size = 0.

$$\int \frac{1}{a-(a-b)\,x^{4}}\frac{1}{\sqrt[4]{b x^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x)
```

```
[Out] int(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left((a-b)x^{4}-a\right)\left(bx^{4}+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(((a - b)*x^4 - a)*(b*x^4 + a)^(1/4)),x, algorithm="maxima")
```

```
[Out] -integrate(1/(((a - b)*x^4 - a)*(b*x^4 + a)^(1/4)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(((a - b)*x^4 - a)*(b*x^4 + a)^(1/4)),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{ax^4\sqrt[4]{a+bx^4}-a\sqrt[4]{a+bx^4}-bx^4\sqrt[4]{a+bx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a-(a-b)*x**4)/(b*x**4+a)**(1/4),x)
```

```
[Out] -Integral(1/(a*x**4*(a + b*x**4)**(1/4) - a*(a + b*x**4)**(1/4) -
 b*x**4*(a + b*x**4)**(1/4)), x)
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{((a-b)x^4-a)(bx^4+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(((a - b)*x^4 - a)*(b*x^4 + a)^(1/4)),x, algorithm="giac")
```

```
[Out] integrate(-1/(((a - b)*x^4 - a)*(b*x^4 + a)^(1/4)), x)
```

## 3.119 $\int (a + bx^4)^p (c + dx^4)^q \, dx$

**Optimal**. Leaf size=79

$$x \left(a + bx^4\right)^p \left(\frac{bx^4}{a} + 1\right)^{-p} \left(c + dx^4\right)^q \left(\frac{dx^4}{c} + 1\right)^{-q} F_1\left(\frac{1}{4}; -p, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)$$

```
[Out] (x*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^
4)/a), -((d*x^4)/c)])/((1 + (b*x^4)/a)^p*(1 + (d*x^4)/c)^q)
```

---

**Rubi [A]** time = 0.12334, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$x \left(a + bx^4\right)^p \left(\frac{bx^4}{a} + 1\right)^{-p} \left(c + dx^4\right)^q \left(\frac{dx^4}{c} + 1\right)^{-q} F_1\left(\frac{1}{4}; -p, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^p*(c + d*x^4)^q,x]
```

```
[Out] (x*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^
4)/a), -((d*x^4)/c)])/((1 + (b*x^4)/a)^p*(1 + (d*x^4)/c)^q)
```

---

**Rubi in Sympy [A]** time = 21.9447, size = 61, normalized size = 0.77

$$x \left(1 + \frac{bx^4}{a}\right)^{-p} \left(1 + \frac{dx^4}{c}\right)^{-q} \left(a + bx^4\right)^p \left(c + dx^4\right)^q \operatorname{appellf}_1\left(\frac{1}{4}, -p, -q, \frac{5}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**p*(d*x**4+c)**q,x)
```

```
[Out] x*(1 + b*x**4/a)**(-p)*(1 + d*x**4/c)**(-q)*(a + b*x**4)**p*(c +
d*x**4)**q*appellf1(1/4, -p, -q, 5/4, -b*x**4/a, -d*x**4/c)
```

---

**Mathematica [B]** time = 0.358687, size = 172, normalized size = 2.18

$$\frac{5acx \left(a + bx^4\right)^p \left(c + dx^4\right)^q F_1\left(\frac{1}{4}; -p, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{4x^4 \left(bcpF_1\left(\frac{5}{4}; 1 - p, -q; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + adqF_1\left(\frac{5}{4}; -p, 1 - q; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right) + 5acF_1\left(\frac{1}{4}; -p, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^4)^p*(c + d*x^4)^q,x]
```

```
[Out] (5*a*c*x*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[1/4, -p, -q, 5/4, -
((b*x^4)/a), -((d*x^4)/c)])/(5*a*c*AppellF1[1/4, -p, -q, 5/4, -((
b*x^4)/a), -((d*x^4)/c)] + 4*x^4*(b*c*p*AppellF1[5/4, 1 - p, -q,
9/4, -((b*x^4)/a), -((d*x^4)/c)] + a*d*q*AppellF1[5/4, -p, 1 - q,
 9/4, -((b*x^4)/a), -((d*x^4)/c)]))
```

---

**Maple [F]** time = 0.115, size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)^p \left(dx^4+c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^p*(d*x^4+c)^q,x)
```

```
[Out] int((b*x^4+a)^p*(d*x^4+c)^q,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)^p \left(dx^4+c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^p*(d*x^4 + c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^4+a\right)^p \left(dx^4+c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^p*(d*x^4 + c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**p*(d*x**4+c)**q,x)
```

```
[Out] Timed out
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)^p (dx^4 + c)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^p*(d*x^4 + c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

# 3.120 $\int \left(a+b x^4\right)^2 \left(c+d x^4\right)^q \, dx$

**Optimal**. Leaf size=176

$$\frac{x \left(c+d x^4\right)^q \left(\frac{d x^4}{c}+1\right)^{-q} \left(a^2 d^2 \left(16 q^2+56 q+45\right)-2 a b c d (4 q+9)+5 b^2 c^2\right) \, _2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{d x^4}{c}\right)}{d^2 (4 q+5) (4 q+9)}-\frac{b x \left(c+d x^4\right)^{q+1} (5 b c-a d (4 q+13))}{d^2 (4 q+5) (4 q+9)}+\frac{b x \left(a+b x^4\right) \left(c+d x^4\right)^{q+1}}{d (4 q+9)}$$

```
[Out] -((b*(5*b*c - a*d*(13 + 4*q))*x*(c + d*x^4)^(1 + q))/(d^2*(5 + 4*
q)*(9 + 4*q))) + (b*x*(a + b*x^4)*(c + d*x^4)^(1 + q))/(d*(9 + 4*
q)) + ((5*b^2*c^2 - 2*a*b*c*d*(9 + 4*q) + a^2*d^2*(45 + 56*q + 16
*q^2))*x*(c + d*x^4)^q*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/
c)])/(d^2*(5 + 4*q)*(9 + 4*q)*(1 + (d*x^4)/c)^q)
```

---

**Rubi [A]** time = 0.316393, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{x \left(c+d x^4\right)^q \left(\frac{d x^4}{c}+1\right)^{-q} \left(a^2 d^2 \left(16 q^2+56 q+45\right)-2 a b c d (4 q+9)+5 b^2 c^2\right) \, _2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{d x^4}{c}\right)}{d^2 (4 q+5) (4 q+9)}-\frac{b x \left(c+d x^4\right)^{q+1} (5 b c-a d (4 q+13))}{d^2 (4 q+5) (4 q+9)}+\frac{b x \left(a+b x^4\right) \left(c+d x^4\right)^{q+1}}{d (4 q+9)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)^2*(c + d*x^4)^q,x]
```


```
[Out] -((b*(5*b*c - a*d*(13 + 4*q))*x*(c + d*x^4)^(1 + q))/(d^2*(5 + 4*
q)*(9 + 4*q))) + (b*x*(a + b*x^4)*(c + d*x^4)^(1 + q))/(d*(9 + 4*
q)) + ((5*b^2*c^2 - 2*a*b*c*d*(9 + 4*q) + a^2*d^2*(45 + 56*q + 16
*q^2))*x*(c + d*x^4)^q*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/
c)])/(d^2*(5 + 4*q)*(9 + 4*q)*(1 + (d*x^4)/c)^q)
```


---

**Rubi in Sympy [A]** time = 32.8228, size = 153, normalized size = 0.87

$$\frac{b x \left(a+b x^4\right) \left(c+d x^4\right)^{q+1}}{d \left(4 q+9\right)}-\frac{b x \left(c+d x^4\right)^{q+1} \left(-a d \left(4 q+13\right)+5 b c\right)}{d^2 \left(4 q+5\right) \left(4 q+9\right)}+\frac{x \left(1+\frac{d x^4}{c}\right)^{-q} \left(c+d x^4\right)^q \left(-a d \left(4 q+5\right) \left(-a d \left(4 q+9\right)+b c\right)+b c \left(-a d \left(4 q+13\right)+5 b c\right)\right) {}_2F_1\left(\begin{matrix} -q, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| -\frac{d x^4}{c}\right)}{d^2 \left(4 q+5\right) \left(4 q+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)**2*(d*x**4+c)**q,x)
```

```
[Out] b*x*(a + b*x**4)*(c + d*x**4)**(q + 1)/(d*(4*q + 9)) - b*x*(c + d
*x**4)**(q + 1)*(-a*d*(4*q + 13) + 5*b*c)/(d**2*(4*q + 5)*(4*q +
9)) + x*(1 + d*x**4/c)**(-q)*(c + d*x**4)**q*(-a*d*(4*q + 5)*(-a*
d*(4*q + 9) + b*c) + b*c*(-a*d*(4*q + 13) + 5*b*c))*hyper((-q, 1/
4), (5/4,), -d*x**4/c)/(d**2*(4*q + 5)*(4*q + 9))
```

---

**Mathematica [A]**   time = 0.0788624, size = 106, normalized size = 0.6

$$\frac{1}{45}x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}\left(45a^2\,{}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)+bx^4\left(18a\,{}_2F_1\left(\frac{5}{4},-q;\frac{9}{4};-\frac{dx^4}{c}\right)+5bx^4\,{}_2F_1\left(\frac{9}{4},-q;\frac{13}{4};-\frac{dx^4}{c}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^4)^2*(c + d*x^4)^q,x]
```

```
[Out] (x*(c + d*x^4)^q*(45*a^2*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4
)/c)] + b*x^4*(18*a*Hypergeometric2F1[5/4, -q, 9/4, -((d*x^4)/c)]
 + 5*b*x^4*Hypergeometric2F1[9/4, -q, 13/4, -((d*x^4)/c)])))/(45*
(1 + (d*x^4)/c)^q)
```

---

**Maple [F]**   time = 0.057, size = 0, normalized size = 0.

$$\int\left(bx^4+a\right)^2\left(dx^4+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)^2*(d*x^4+c)^q,x)
```

```
[Out] int((b*x^4+a)^2*(d*x^4+c)^q,x)
```

---

**Maxima [F]**   time = 0., size = 0, normalized size = 0.

$$\int\left(bx^4+a\right)^2\left(dx^4+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(d*x^4 + c)^q,x, algorithm="maxima")

[Out] integrate((b*x^4 + a)^2*(d*x^4 + c)^q, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^8 + 2\,abx^4 + a^2\right)\left(dx^4 + c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(d*x^4 + c)^q,x, algorithm="fricas")

[Out] integral((b^2*x^8 + 2*a*b*x^4 + a^2)*(d*x^4 + c)^q, x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)**2*(d*x**4+c)**q,x)

[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4 + a\right)^2 \left(dx^4 + c\right)^q \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)^2*(d*x^4 + c)^q,x, algorithm="giac")

[Out] integrate((b*x^4 + a)^2*(d*x^4 + c)^q, x)
```

# 3.121 $\int \left(a+bx^4\right) \left(c+dx^4\right)^q \, dx$

**Optimal**. Leaf size=93

$$\frac{bx\left(c+dx^4\right)^{q+1}}{d(4q+5)}-\frac{x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}(bc-ad(4q+5))\,{}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)}{d(4q+5)}$$

```
[Out] (b*x*(c + d*x^4)^(1 + q))/(d*(5 + 4*q)) - ((b*c - a*d*(5 + 4*q))*
x*(c + d*x^4)^q*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/c)])/(d
*(5 + 4*q)*(1 + (d*x^4)/c)^q)
```

---

**Rubi [A]** time = 0.111122, antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}\left(a-\frac{bc}{4dq+5d}\right)\,{}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)+\frac{bx\left(c+dx^4\right)^{q+1}}{d(4q+5)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^4)*(c + d*x^4)^q,x]
```

```
[Out] (b*x*(c + d*x^4)^(1 + q))/(d*(5 + 4*q)) + ((a - (b*c)/(5*d + 4*d*
q))*x*(c + d*x^4)^q*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/c)]
)/(1 + (d*x^4)/c)^q
```

---

**Rubi in Sympy [A]** time = 11.3714, size = 73, normalized size = 0.78

$$\frac{bx\left(c+dx^4\right)^{q+1}}{d\left(4q+5\right)}-\frac{x\left(1+\frac{dx^4}{c}\right)^{-q}\left(c+dx^4\right)^q\left(-ad\left(4q+5\right)+bc\right){}_2F_1\left(\begin{matrix}-q,\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|-\frac{dx^4}{c}\right)}{d\left(4q+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**4+a)*(d*x**4+c)**q,x)
```

```
[Out] b*x*(c + d*x**4)**(q + 1)/(d*(4*q + 5)) - x*(1 + d*x**4/c)**(-q)*
(c + d*x**4)**q*(-a*d*(4*q + 5) + b*c)*hyper((-q, 1/4), (5/4,), -
d*x**4/c)/(d*(4*q + 5))
```

---

**Mathematica [A]** time = 0.0347268, size = 75, normalized size = 0.81

$$\frac{1}{5}x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}\left(5a\,{}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)+bx^4\,{}_2F_1\left(\frac{5}{4},-q;\frac{9}{4};-\frac{dx^4}{c}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^4)*(c + d*x^4)^q,x]
```

```
[Out] (x*(c + d*x^4)^q*(5*a*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/c
)] + b*x^4*Hypergeometric2F1[5/4, -q, 9/4, -((d*x^4)/c)]))/(5*(1
+ (d*x^4)/c)^q)
```

---

**Maple [F]** time = 0.049, size = 0, normalized size = 0.

$$\int\left(bx^4+a\right)\left(dx^4+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^4+a)*(d*x^4+c)^q,x)
```

```
[Out] int((b*x^4+a)*(d*x^4+c)^q,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\left(bx^4+a\right)\left(dx^4+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)*(d*x^4 + c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)*(d*x^4 + c)^q, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^4+a\right)\left(dx^4+c\right)^q,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)*(d*x^4 + c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)*(d*x^4 + c)^q, x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**4+a)*(d*x**4+c)**q,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^4 + a)(dx^4 + c)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^4 + a)*(d*x^4 + c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)*(d*x^4 + c)^q, x)
```

## 3.122 $\int \frac{(c+dx^4)^q}{a+bx^4}\,dx$

**Optimal**. Leaf size=57

$$\frac{x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}F_1\left(\frac{1}{4};1,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a}$$

```
[Out] (x*(c + d*x^4)^q*AppellF1[1/4, 1, -q, 5/4, -((b*x^4)/a), -((d*x^4
)/c)])/(a*(1 + (d*x^4)/c)^q)
```

---

**Rubi [A]** time = 0.0838656, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}F_1\left(\frac{1}{4};1,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^4)^q/(a + b*x^4),x]
```

```
[Out] (x*(c + d*x^4)^q*AppellF1[1/4, 1, -q, 5/4, -((b*x^4)/a), -((d*x^4
)/c)])/(a*(1 + (d*x^4)/c)^q)
```

---

**Rubi in Sympy [A]** time = 21.9034, size = 42, normalized size = 0.74

$$\frac{x\left(1+\frac{dx^4}{c}\right)^{-q}\left(c+dx^4\right)^q\operatorname{appellf}_1\left(\frac{1}{4},1,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**4+c)**q/(b*x**4+a),x)
```

```
[Out] x*(1 + d*x**4/c)**(-q)*(c + d*x**4)**q*appellf1(1/4, 1, -q, 5/4,
-b*x**4/a, -d*x**4/c)/a
```

---

**Mathematica [B]** time = 0.280766, size = 162, normalized size = 2.84

$$\frac{5acx\left(c+dx^4\right)^qF_1\left(\frac{1}{4};-q,1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{(a+bx^4)\left(4x^4\left(adqF_1\left(\frac{5}{4};1-q,1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-bcF_1\left(\frac{5}{4};-q,2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)+5acF_1\left(\frac{1}{4};-q,1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x^4)^q/(a + b*x^4),x]
```

```
[Out] (5*a*c*x*(c + d*x^4)^q*AppellF1[1/4, -q, 1, 5/4, -((d*x^4)/c), -(
(b*x^4)/a)])/((a + b*x^4)*(5*a*c*AppellF1[1/4, -q, 1, 5/4, -((d*x
^4)/c), -((b*x^4)/a)] + 4*x^4*(a*d*q*AppellF1[5/4, 1 - q, 1, 9/4,
 -((d*x^4)/c), -((b*x^4)/a)] - b*c*AppellF1[5/4, -q, 2, 9/4, -((d
*x^4)/c), -((b*x^4)/a)])))
```

---

**Maple [F]** time = 0.072, size = 0, normalized size = 0.

$$\int \frac{\left(d x^{4}+c\right)^{q}}{b x^{4}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^4+c)^q/(b*x^4+a),x)
```

```
[Out] int((d*x^4+c)^q/(b*x^4+a),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d x^{4}+c\right)^{q}}{b x^{4}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^q/(b*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^4 + c)^q/(b*x^4 + a), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d x^{4}+c\right)^{q}}{b x^{4}+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^q/(b*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral((d*x^4 + c)^q/(b*x^4 + a), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**4+c)**q/(b*x**4+a),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^4 + c\right)^q}{bx^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^q/(b*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate((d*x^4 + c)^q/(b*x^4 + a), x)
```

## 3.123 $\int \frac{(c+dx^4)^q}{(a+bx^4)^2} \, dx$

**Optimal**. Leaf size=57

$$\frac{x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}F_1\left(\frac{1}{4};2,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2}$$

```
[Out] (x*(c + d*x^4)^q*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4
)/c)])/(a^2*(1 + (d*x^4)/c)^q)
```

---

**Rubi [A]** time = 0.0831709, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}F_1\left(\frac{1}{4};2,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^4)^q/(a + b*x^4)^2,x]
```

```
[Out] (x*(c + d*x^4)^q*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4
)/c)])/(a^2*(1 + (d*x^4)/c)^q)
```

---

**Rubi in Sympy [A]** time = 20.2223, size = 44, normalized size = 0.77

$$\frac{x\left(1+\frac{dx^4}{c}\right)^{-q}\left(c+dx^4\right)^q \operatorname{appellf}_1\left(\frac{1}{4},2,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**4+c)**q/(b*x**4+a)**2,x)
```

```
[Out] x*(1 + d*x**4/c)**(-q)*(c + d*x**4)**q*appellf1(1/4, 2, -q, 5/4,
-b*x**4/a, -d*x**4/c)/a**2
```

---

**Mathematica [B]** time = 0.307696, size = 162, normalized size = 2.84

$$\frac{5acx\left(c+dx^4\right)^q F_1\left(\frac{1}{4};2,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\left(a+bx^4\right)^2\left(4x^4\left(adqF_1\left(\frac{5}{4};2,1-q;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-2bcF_1\left(\frac{5}{4};3,-q;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};2,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x^4)^q/(a + b*x^4)^2,x]
```

```
[Out] (5*a*c*x*(c + d*x^4)^q*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -(
(d*x^4)/c)])/((a + b*x^4)^2*(5*a*c*AppellF1[1/4, 2, -q, 5/4, -((b
*x^4)/a), -((d*x^4)/c)] + 4*x^4*(a*d*q*AppellF1[5/4, 2, 1 - q, 9/
4, -((b*x^4)/a), -((d*x^4)/c)] - 2*b*c*AppellF1[5/4, 3, -q, 9/4,
-((b*x^4)/a), -((d*x^4)/c)])))
```

---

**Maple [F]** time = 0.064, size = 0, normalized size = 0.

$$\int \frac{\left(dx^4+c\right)^q}{\left(bx^4+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^4+c)^q/(b*x^4+a)^2,x)
```

```
[Out] int((d*x^4+c)^q/(b*x^4+a)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^4+c\right)^q}{\left(bx^4+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^q/(b*x^4 + a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^4 + c)^q/(b*x^4 + a)^2, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(dx^4+c\right)^q}{b^2x^8+2\,abx^4+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^q/(b*x^4 + a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^4 + c)^q/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**4+c)**q/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^4 + c\right)^q}{\left(bx^4 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^4 + c)^q/(b*x^4 + a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^4 + c)^q/(b*x^4 + a)^2, x)
```

## 3.124 $\int \frac{1}{\sqrt[5]{a+bx^5}(c+dx^5)}\,dx$

**Optimal**. Leaf size=545

$$-\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}-\frac{2\sqrt{\frac{2}{5+\sqrt{5}}}x\sqrt[5]{bc-ad}}{\sqrt[5]{c}\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}x\sqrt[5]{bc-ad}}{\sqrt[5]{c}\sqrt[5]{a+bx^5}}+\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}+\frac{\left(1-\sqrt{5}\right)\log\left(\frac{2c^{2/5}\left(a+bx^5\right)^{2/5}-\sqrt{5}\sqrt[5]{c}x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}+\sqrt[5]{c}x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}+2x^2(bc-ad)^{2/5}}{\left(a+bx^5\right)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}+\frac{\left(1+\sqrt{5}\right)\log\left(\frac{2c^{2/5}\left(a+bx^5\right)^{2/5}+\sqrt{5}\sqrt[5]{c}x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}+\sqrt[5]{c}x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}+2x^2(bc-ad)^{2/5}}{\left(a+bx^5\right)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}$$

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] - (2*Sqrt[
2/(5 + Sqrt[5])]*(b*c - a*d)^(1/5)*x)/(c^(1/5)*(a + b*x^5)^(1/5))
])/(5*c^(4/5)*(b*c - a*d)^(1/5)) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[
Sqrt[(5 + 2*Sqrt[5])/5] + (Sqrt[(2*(5 + Sqrt[5]))/5]*(b*c - a*d)^
(1/5)*x)/(c^(1/5)*(a + b*x^5)^(1/5))])/(5*c^(4/5)*(b*c - a*d)^(1/
5)) - Log[c^(1/5) - ((b*c - a*d)^(1/5)*x)/(a + b*x^5)^(1/5)]/(5*c
^(4/5)*(b*c - a*d)^(1/5)) + ((1 - Sqrt[5])*Log[(2*(b*c - a*d)^(2/
5)*x^2 + c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) - Sqrt[5]*
c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) + 2*c^(2/5)*(a + b*
x^5)^(2/5))/(a + b*x^5)^(2/5)])/(20*c^(4/5)*(b*c - a*d)^(1/5)) +
((1 + Sqrt[5])*Log[(2*(b*c - a*d)^(2/5)*x^2 + c^(1/5)*(b*c - a*d)
^(1/5)*x*(a + b*x^5)^(1/5) + Sqrt[5]*c^(1/5)*(b*c - a*d)^(1/5)*x*
(a + b*x^5)^(1/5) + 2*c^(2/5)*(a + b*x^5)^(2/5))/(a + b*x^5)^(2/5
)])/(20*c^(4/5)*(b*c - a*d)^(1/5))
```

---

**Rubi [A]** time = 2.3109, antiderivative size = 545, normalized size of antiderivative = 1., number of

steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}-\frac{2\sqrt{\frac{2}{5+\sqrt{5}}}x\sqrt[5]{bc-ad}}{\sqrt[5]{c}\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}$$
$$+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}x\sqrt[5]{bc-ad}}{\sqrt[5]{c}\sqrt[5]{a+bx^5}}+\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}$$
$$+\frac{\left(1-\sqrt{5}\right)\log\left(\frac{2c^{2/5}(a+bx^5)^{2/5}-\sqrt{5}\sqrt[5]{c}x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}+\sqrt[5]{c}x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}+2x^2(bc-ad)^{2/5}}{(a+bx^5)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}$$
$$+\frac{\left(1+\sqrt{5}\right)\log\left(\frac{2c^{2/5}(a+bx^5)^{2/5}+\sqrt{5}\sqrt[5]{c}x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}+\sqrt[5]{c}x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}+2x^2(bc-ad)^{2/5}}{(a+bx^5)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^5)^(1/5)*(c + d*x^5)),x]
```

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] - (2*Sqrt[
2/(5 + Sqrt[5])]*(b*c - a*d)^(1/5)*x)/(c^(1/5)*(a + b*x^5)^(1/5))
])/(5*c^(4/5)*(b*c - a*d)^(1/5)) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[
Sqrt[(5 + 2*Sqrt[5])/5] + (Sqrt[(2*(5 + Sqrt[5]))/5]*(b*c - a*d)^
(1/5)*x)/(c^(1/5)*(a + b*x^5)^(1/5))]])/(5*c^(4/5)*(b*c - a*d)^(1/
5)) - Log[c^(1/5) - ((b*c - a*d)^(1/5)*x)/(a + b*x^5)^(1/5)]/(5*c
^(4/5)*(b*c - a*d)^(1/5)) + ((1 - Sqrt[5])*Log[(2*(b*c - a*d)^(2/
5)*x^2 + c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) - Sqrt[5]*
c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) + 2*c^(2/5)*(a + b*
x^5)^(2/5))/(a + b*x^5)^(2/5)])/(20*c^(4/5)*(b*c - a*d)^(1/5)) +
((1 + Sqrt[5])*Log[(2*(b*c - a*d)^(2/5)*x^2 + c^(1/5)*(b*c - a*d)
^(1/5)*x*(a + b*x^5)^(1/5) + Sqrt[5]*c^(1/5)*(b*c - a*d)^(1/5)*x*
(a + b*x^5)^(1/5) + 2*c^(2/5)*(a + b*x^5)^(2/5))/(a + b*x^5)^(2/5
)])/(20*c^(4/5)*(b*c - a*d)^(1/5))
```

---

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**5+a)**(1/5)/(d*x**5+c),x)
```

```
[Out] Timed out
```

**Mathematica [A]** time = 1.6263, size = 379, normalized size = 0.7

$$\frac{-\left(\sqrt{5}-1\right)\log\left(-\frac{\left(\sqrt{5}-1\right)\sqrt[5]{c}x\sqrt[5]{bc-ad}}{2\sqrt[5]{ax^5+b}}+\frac{x^2(bc-ad)^{2/5}}{(ax^5+b)^{2/5}}+c^{2/5}\right)+\left(1+\sqrt{5}\right)\log\left(\frac{\left(1+\sqrt{5}\right)\sqrt[5]{c}x\sqrt[5]{bc-ad}}{2\sqrt[5]{ax^5+b}}+\frac{x^2(bc-ad)^{2/5}}{(ax^5+b)^{2/5}}+c^{2/5}\right)-}{2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^5)^(1/5)*(c + d*x^5)),x]
```

```
[Out] (2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(2*Sqrt[2/(5 + Sqrt[5])]*(-((-1 +
 Sqrt[5])*c^(1/5))/4 + ((b*c - a*d)^(1/5)*x)/(b + a*x^5)^(1/5)))/
c^(1/5)] + 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(Sqrt[2 + 2/Sqrt[5]]*(((
1 + Sqrt[5])*c^(1/5))/4 + ((b*c - a*d)^(1/5)*x)/(b + a*x^5)^(1/5)
))/c^(1/5)] - 4*Log[c^(1/5) - ((b*c - a*d)^(1/5)*x)/(b + a*x^5)^(
1/5)] - (-1 + Sqrt[5])*Log[c^(2/5) + ((b*c - a*d)^(2/5)*x^2)/(b +
 a*x^5)^(2/5) - ((-1 + Sqrt[5])*c^(1/5)*(b*c - a*d)^(1/5)*x)/(2*(
b + a*x^5)^(1/5))] + (1 + Sqrt[5])*Log[c^(2/5) + ((b*c - a*d)^(2/
5)*x^2)/(b + a*x^5)^(2/5) + ((1 + Sqrt[5])*c^(1/5)*(b*c - a*d)^(1
/5)*x)/(2*(b + a*x^5)^(1/5))])/(20*c^(4/5)*(b*c - a*d)^(1/5))
```

**Maple [F]** time = 0.055, size = 0, normalized size = 0.

$$\int \frac{1}{dx^5+c}\frac{1}{\sqrt[5]{bx^5+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^5+a)^(1/5)/(d*x^5+c),x)
```

```
[Out] int(1/(b*x^5+a)^(1/5)/(d*x^5+c),x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^5+a)^{\frac{1}{5}}(dx^5+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**5+a)**(1/5)/(d*x**5+c),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^5+a)^{\frac{1}{5}}(dx^5+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)), x)
```

## 3.125 $\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx$

**Optimal**. Leaf size=143

$$-\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+15abcd+57b^2c^2\right)+\frac{bd(2ad+33bc)}{x}\right)}{15b^2}+\frac{c^2(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3-\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2$$

```
[Out] (-7*d*Sqrt[a + b/x]*(c + d/x)^2)/5 - (d*Sqrt[a + b/x]*(2*(57*b^2*
c^2 + 15*a*b*c*d - 2*a^2*d^2) + (b*d*(33*b*c + 2*a*d))/x))/(15*b^
2) + Sqrt[a + b/x]*(c + d/x)^3*x + (c^2*(b*c + 6*a*d)*ArcTanh[Sqr
t[a + b/x]/Sqrt[a]])/Sqrt[a]
```

---

**Rubi [A]** time = 0.417175, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$-\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+15abcd+57b^2c^2\right)+\frac{bd(2ad+33bc)}{x}\right)}{15b^2}+\frac{c^2(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3-\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b/x]*(c + d/x)^3,x]
```

```
[Out] (-7*d*Sqrt[a + b/x]*(c + d/x)^2)/5 - (d*Sqrt[a + b/x]*(2*(57*b^2*
c^2 + 15*a*b*c*d - 2*a^2*d^2) + (b*d*(33*b*c + 2*a*d))/x))/(15*b^
2) + Sqrt[a + b/x]*(c + d/x)^3*x + (c^2*(b*c + 6*a*d)*ArcTanh[Sqr
t[a + b/x]/Sqrt[a]])/Sqrt[a]
```

---

**Rubi in Sympy [A]** time = 50.01, size = 131, normalized size = 0.92

$$-\frac{7d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{5}+x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3+\frac{8d\sqrt{a+\frac{b}{x}}\left(\frac{a^2d^2}{2}-\frac{15abcd}{4}-\frac{57b^2c^2}{4}-\frac{bd(2ad+33bc)}{8x}\right)}{15b^2}+\frac{c^2\left(6ad+bc\right)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d/x)**3*(a+b/x)**(1/2),x)
```

```
[Out] -7*d*sqrt(a + b/x)*(c + d/x)**2/5 + x*sqrt(a + b/x)*(c + d/x)**3
+ 8*d*sqrt(a + b/x)*(a**2*d**2/2 - 15*a*b*c*d/4 - 57*b**2*c**2/4
- b*d*(2*a*d + 33*b*c)/(8*x))/(15*b**2) + c**2*(6*a*d + b*c)*atan
h(sqrt(a + b/x)/sqrt(a))/sqrt(a)
```

---

**Mathematica [A]** time = 0.20535, size = 131, normalized size = 0.92

$$\sqrt{\frac{ax+b}{x}}\left(-\frac{2d\left(-2a^2d^2+15abcd+45b^2c^2\right)}{15b^2}-\frac{2d^2(ad+15bc)}{15bx}+c^3x-\frac{2d^3}{5x^2}\right)+\frac{c^2(6ad+bc)\log\left(2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+2ax+b\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b/x]*(c + d/x)^3,x]
```

```
[Out] Sqrt[(b + a*x)/x]*((-2*d*(45*b^2*c^2 + 15*a*b*c*d - 2*a^2*d^2))/(
15*b^2) - (2*d^3)/(5*x^2) - (2*d^2*(15*b*c + a*d))/(15*b*x) + c^3
*x) + (c^2*(b*c + 6*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*x*Sqrt[(b + a*
x)/x]])/(2*Sqrt[a])
```

---

**Maple [A]** time = 0.02, size = 248, normalized size = 1.7

$$\frac{1}{30x^3b^2}\sqrt{\frac{ax+b}{x}}\left(90dc^2a\ln\left(1/2\,\frac{2\sqrt{ax^2+bx}\sqrt{a}+2ax+b}{\sqrt{a}}\right)b^2x^4+180dc^2a^{3/2}\sqrt{ax^2+bx}bx^4+30c^3\sqrt{ax^2+bx}\sqrt{a}b^2x^4+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d/x)^3*(a+b/x)^(1/2),x)
```

```
[Out] 1/30*((a*x+b)/x)^(1/2)/x^3*(90*d*c^2*a*ln(1/2*(2*(a*x^2+b*x)^(1/2
)*a^(1/2)+2*a*x+b)/a^(1/2))*b^2*x^4+180*d*c^2*a^(3/2)*(a*x^2+b*x)
^(1/2)*b*x^4+30*c^3*(a*x^2+b*x)^(1/2)*a^(1/2)*b^2*x^4+15*c^3*b^3*
ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^4-180*d*c
^2*(a*x^2+b*x)^(3/2)*a^(1/2)*b*x^2+8*a^(3/2)*(a*x^2+b*x)^(3/2)*x*
d^3-60*d^2*c*(a*x^2+b*x)^(3/2)*a^(1/2)*b*x-12*a^(1/2)*(a*x^2+b*x)
^(3/2)*b*d^3)/(x*(a*x+b))^(1/2)/a^(1/2)/b^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)*(c + d/x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.260376, size = 1, normalized size = 0.01

$$\left[\frac{15\left(b^3c^3+6\,ab^2c^2d\right)x^2\log\left(2\,ax\sqrt{\frac{ax+b}{x}}+(2\,ax+b)\sqrt{a}\right)+2\left(15\,b^2c^3x^3-6\,b^2d^3-2\left(45\,b^2c^2d+15\,abcd^2-2\,a^2d^3\right)x^2-2\left(\right.\right.}{30\,\sqrt{a}b^2x^2}\right.$$

$$-\frac{15\left(b^3c^3+6\,ab^2c^2d\right)x^2\arctan\left(\frac{a}{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}\right)-\left(15\,b^2c^3x^3-6\,b^2d^3-2\left(45\,b^2c^2d+15\,abcd^2-2\,a^2d^3\right)x^2-2\left(15\,b^2cd^2+ab\right.\right.}{15\,\sqrt{-a}b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)*(c + d/x)^3,x, algorithm="fricas")
```

```
[Out] [1/30*(15*(b^3*c^3 + 6*a*b^2*c^2*d)*x^2*log(2*a*x*sqrt((a*x + b)/
x) + (2*a*x + b)*sqrt(a)) + 2*(15*b^2*c^3*x^3 - 6*b^2*d^3 - 2*(45
*b^2*c^2*d + 15*a*b*c*d^2 - 2*a^2*d^3)*x^2 - 2*(15*b^2*c*d^2 + a*
b*d^3)*x)*sqrt(a)*sqrt((a*x + b)/x))/(sqrt(a)*b^2*x^2), -1/15*(15
*(b^3*c^3 + 6*a*b^2*c^2*d)*x^2*arctan(a/(sqrt(-a)*sqrt((a*x + b)/
x))) - (15*b^2*c^3*x^3 - 6*b^2*d^3 - 2*(45*b^2*c^2*d + 15*a*b*c*d
^2 - 2*a^2*d^3)*x^2 - 2*(15*b^2*c*d^2 + a*b*d^3)*x)*sqrt(-a)*sqrt
((a*x + b)/x))/(sqrt(-a)*b^2*x^2)]
```

---

**Sympy [A]** time = 15.4931, size = 491, normalized size = 3.43

$$\frac{4a^{\frac{11}{2}}b^{\frac{3}{2}}d^3x^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\frac{2a^{\frac{9}{2}}b^{\frac{5}{2}}d^3x^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{8a^{\frac{7}{2}}b^{\frac{7}{2}}d^3x\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{6a^{\frac{5}{2}}b^{\frac{9}{2}}d^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}$$
$$+6\sqrt{a}c^2d\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)-\frac{4a^6bd^3x^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{4a^5b^2d^3x^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{6ac^2d\sqrt{x}}{\sqrt{b}\sqrt{\frac{ax}{b}+1}}$$
$$+\sqrt{b}c^3\sqrt{x}\sqrt{\frac{ax}{b}+1}-\frac{6\sqrt{b}c^2d}{\sqrt{x}\sqrt{\frac{ax}{b}+1}}+3cd^2\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+\frac{bc^3\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d/x)**3*(a+b/x)**(1/2),x)
```

```
[Out] 4*a**(11/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*
x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(5/2)*d**3*
x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**
4*x**(5/2)) - 8*a**(7/2)*b**(7/2)*d**3*x*sqrt(a*x/b + 1)/(15*a**(
7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6*a**(5/2)*b**(
9/2)*d**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2
)*b**4*x**(5/2)) + 6*sqrt(a)*c**2*d*asinh(sqrt(a)*sqrt(x)/sqrt(b)
) - 4*a**6*b*d**3*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/
2)*b**4*x**(5/2)) - 4*a**5*b**2*d**3*x**(5/2)/(15*a**(7/2)*b**3*x
**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6*a*c**2*d*sqrt(x)/(sqrt(b
)*sqrt(a*x/b + 1)) + sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1) - 6*sqr
t(b)*c**2*d/(sqrt(x)*sqrt(a*x/b + 1)) + 3*c*d**2*Piecewise((-sqrt
(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b*c**3*asi
nh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)*(c + d/x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

# 3.126 $\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx$

**Optimal**. Leaf size=99

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{c\sqrt{a+\frac{b}{x}}(4ad+bc)}{a}+\frac{c(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b}$$

```
[Out] -((c*(b*c + 4*a*d)*Sqrt[a + b/x])/a) - (2*d^2*(a + b/x)^(3/2))/(3
*b) + (c^2*(a + b/x)^(3/2)*x)/a + (c*(b*c + 4*a*d)*ArcTanh[Sqrt[a
 + b/x]/Sqrt[a]])/Sqrt[a]
```

---

**Rubi [A]** time = 0.218174, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{c\sqrt{a+\frac{b}{x}}(4ad+bc)}{a}+\frac{c(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b/x]*(c + d/x)^2,x]
```

```
[Out] -((c*(b*c + 4*a*d)*Sqrt[a + b/x])/a) - (2*d^2*(a + b/x)^(3/2))/(3
*b) + (c^2*(a + b/x)^(3/2)*x)/a + (c*(b*c + 4*a*d)*ArcTanh[Sqrt[a
 + b/x]/Sqrt[a]])/Sqrt[a]
```

---

**Rubi in Sympy [A]** time = 20.3938, size = 82, normalized size = 0.83

$$-\frac{2d^2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b}+\frac{c^2x\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{a}-\frac{c\sqrt{a+\frac{b}{x}}\,(4ad+bc)}{a}+\frac{c\,(4ad+bc)\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d/x)**2*(a+b/x)**(1/2),x)
```

```
[Out] -2*d**2*(a + b/x)**(3/2)/(3*b) + c**2*x*(a + b/x)**(3/2)/a - c*sq
rt(a + b/x)*(4*a*d + b*c)/a + c*(4*a*d + b*c)*atanh(sqrt(a + b/x)
/sqrt(a))/sqrt(a)
```

**Mathematica [A]** time = 0.133863, size = 85, normalized size = 0.86

$$\sqrt{a+\frac{b}{x}}\left(-\frac{2d^2(ax+b)}{3bx}+c^2x-4cd\right)+\frac{c(4ad+bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b/x]*(c + d/x)^2,x]
```

```
[Out] Sqrt[a + b/x]*(-4*c*d + c^2*x - (2*d^2*(b + a*x))/(3*b*x)) + (c*(
b*c + 4*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x])/(2*Sqrt[
a])
```

**Maple [B]** time = 0.017, size = 191, normalized size = 1.9

$$\frac{1}{6\,bx^2}\sqrt{\frac{ax+b}{x}}\left(12\,cda\ln\left(1/2\,\frac{2\,\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)x^3b+24\,cda^{3/2}\sqrt{ax^2+bx}x^3+6\,c^2\sqrt{ax^2+bx}\sqrt{a}x^3b+3\,c^2b^2\ln\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d/x)^2*(a+b/x)^(1/2),x)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)/x^2*(12*c*d*a*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a
^(1/2)+2*a*x+b)/a^(1/2))*x^3*b+24*c*d*a^(3/2)*(a*x^2+b*x)^(1/2)*x
^3+6*c^2*(a*x^2+b*x)^(1/2)*a^(1/2)*x^3*b+3*c^2*b^2*ln(1/2*(2*(a*x
^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^3-24*c*d*(a*x^2+b*x)^(3
/2)*a^(1/2)*x-4*d^2*(a*x^2+b*x)^(3/2)*a^(1/2))/(x*(a*x+b))^(1/2)/
a^(1/2)/b
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)*(c + d/x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 0.25669, size = 1, normalized size = 0.01

$$\left[\frac{3\left(b^2c^2+4\,abcd\right)x\log\left(2\,ax\sqrt{\frac{ax+b}{x}}+(2\,ax+b)\sqrt{a}\right)+2\left(3\,bc^2x^2-2\,bd^2-2\left(6\,bcd+ad^2\right)x\right)\sqrt{a}\sqrt{\frac{ax+b}{x}}}{6\sqrt{a}bx},\right.$$
$$\left.-\frac{3\left(b^2c^2+4\,abcd\right)x\arctan\left(\frac{a}{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}\right)-\left(3\,bc^2x^2-2\,bd^2-2\left(6\,bcd+ad^2\right)x\right)\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{3\sqrt{-a}bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)*(c + d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*c^2 + 4*a*b*c*d)*x*log(2*a*x*sqrt((a*x + b)/x) + (2*
a*x + b)*sqrt(a)) + 2*(3*b*c^2*x^2 - 2*b*d^2 - 2*(6*b*c*d + a*d^2
)*x)*sqrt(a)*sqrt((a*x + b)/x))/(sqrt(a)*b*x), -1/3*(3*(b^2*c^2 +
 4*a*b*c*d)*x*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x))) - (3*b*c^2*x
^2 - 2*b*d^2 - 2*(6*b*c*d + a*d^2)*x)*sqrt(-a)*sqrt((a*x + b)/x))
/(sqrt(-a)*b*x)]
```

---

**Sympy [A]** time = 12.0423, size = 156, normalized size = 1.58

$$4\sqrt{a}cd\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)-\frac{4acd\sqrt{x}}{\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\sqrt{b}c^2\sqrt{x}\sqrt{\frac{ax}{b}+1}$$
$$-\frac{4\sqrt{b}cd}{\sqrt{x}\sqrt{\frac{ax}{b}+1}}+d^2\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+\frac{bc^2\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d/x)**2*(a+b/x)**(1/2),x)
```

```
[Out] 4*sqrt(a)*c*d*asinh(sqrt(a)*sqrt(x)/sqrt(b)) - 4*a*c*d*sqrt(x)/(s
qrt(b)*sqrt(a*x/b + 1)) + sqrt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1) -
4*sqrt(b)*c*d/(sqrt(x)*sqrt(a*x/b + 1)) + d**2*Piecewise((-sqrt(a
)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b*c**2*asinh
(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)*(c + d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.127 $\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx$

**Optimal**. Leaf size=74

$$-\frac{\sqrt{a+\frac{b}{x}}(2ad+bc)}{a}+\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{cx\left(a+\frac{b}{x}\right)^{3/2}}{a}$$

```
[Out] -(((b*c + 2*a*d)*Sqrt[a + b/x])/a) + (c*(a + b/x)^(3/2)*x)/a + ((
b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

---

**Rubi [A]** time = 0.146402, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$-\frac{\sqrt{a+\frac{b}{x}}(2ad+bc)}{a}+\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{cx\left(a+\frac{b}{x}\right)^{3/2}}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b/x]*(c + d/x),x]
```

```
[Out] -(((b*c + 2*a*d)*Sqrt[a + b/x])/a) + (c*(a + b/x)^(3/2)*x)/a + ((
b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

---

**Rubi in Sympy [A]** time = 12.818, size = 63, normalized size = 0.85

$$\frac{cx\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{a}-\frac{2\sqrt{a+\frac{b}{x}}\left(ad+\frac{bc}{2}\right)}{a}+\frac{2\left(ad+\frac{bc}{2}\right)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d/x)*(a+b/x)**(1/2),x)
```

```
[Out] c*x*(a + b/x)**(3/2)/a - 2*sqrt(a + b/x)*(a*d + b*c/2)/a + 2*(a*d
 + b*c/2)*atanh(sqrt(a + b/x)/sqrt(a))/sqrt(a)
```

---

**Mathematica [A]** time = 0.122252, size = 63, normalized size = 0.85

$$\sqrt{a+\frac{b}{x}}(cx-2d)+\frac{(2ad+bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b/x]*(c + d/x),x]
```

```
[Out] Sqrt[a + b/x]*(-2*d + c*x) + ((b*c + 2*a*d)*Log[b + 2*a*x + 2*Sqr
t[a]*Sqrt[a + b/x]*x])/(2*Sqrt[a])
```

---

**Maple [B]** time = 0.016, size = 163, normalized size = 2.2

$$\frac{1}{2\,bx}\sqrt{\frac{ax+b}{x}}\left(2\,da\ln\left(1/2\,\frac{2\,\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)x^2b+4\,da^{3/2}\sqrt{ax^2+bx}x^2+2\,c\sqrt{ax^2+bx}\sqrt{a}x^2b+cb^2\ln\left(\frac{1}{2}\left(2\,\sqrt{a}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d/x)*(a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)/x*(2*d*a*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2
)+2*a*x+b)/a^(1/2))*x^2*b+4*d*a^(3/2)*(a*x^2+b*x)^(1/2)*x^2+2*c*(
a*x^2+b*x)^(1/2)*a^(1/2)*x^2*b+c*b^2*ln(1/2*(2*(a*x^2+b*x)^(1/2)*
a^(1/2)+2*a*x+b)/a^(1/2))*x^2-4*d*(a*x^2+b*x)^(3/2)*a^(1/2))/(x*(
a*x+b))^(1/2)/a^(1/2)/b
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)*(c + d/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.259379, size = 1, normalized size = 0.01

$$\left[\frac{2\,(cx-2\,d)\sqrt{a}\sqrt{\frac{ax+b}{x}}+(bc+2\,ad)\log\left(2\,ax\sqrt{\frac{ax+b}{x}}+(2\,ax+b)\sqrt{a}\right)}{2\,\sqrt{a}},\ \frac{(cx-2\,d)\sqrt{-a}\sqrt{\frac{ax+b}{x}}-(bc+2\,ad)\arctan\left(\frac{a}{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}\right)}{\sqrt{-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)*(c + d/x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(c*x - 2*d)*sqrt(a)*sqrt((a*x + b)/x) + (b*c + 2*a*d)*log
(2*a*x*sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(a)))/sqrt(a), ((c*x -
 2*d)*sqrt(-a)*sqrt((a*x + b)/x) - (b*c + 2*a*d)*arctan(a/(sqrt(-
a)*sqrt((a*x + b)/x))))/sqrt(-a)]
```

---

**Sympy [A]** time = 21.0534, size = 121, normalized size = 1.64

$$2\sqrt{a}d\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)-\frac{2ad\sqrt{x}}{\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1}-\frac{2\sqrt{b}d}{\sqrt{x}\sqrt{\frac{ax}{b}+1}}+\frac{bc\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d/x)*(a+b/x)**(1/2),x)
```

```
[Out] 2*sqrt(a)*d*asinh(sqrt(a)*sqrt(x)/sqrt(b)) - 2*a*d*sqrt(x)/(sqrt(
b)*sqrt(a*x/b + 1)) + sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1) - 2*sqrt(
b)*d/(sqrt(x)*sqrt(a*x/b + 1)) + b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b
))/sqrt(a)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)*(c + d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.128 $\int \sqrt{a+\frac{b}{x}}\,dx$

**Optimal**. Leaf size=39

$$x\sqrt{a+\frac{b}{x}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

---

**Rubi [A]** time = 0.0574037, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$x\sqrt{a+\frac{b}{x}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b/x],x]

[Out] Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

---

**Rubi in Sympy [A]** time = 5.90255, size = 31, normalized size = 0.79

$$x\sqrt{a+\frac{b}{x}}+\frac{b\,\text{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**(1/2),x)

[Out] x*sqrt(a + b/x) + b*atanh(sqrt(a + b/x)/sqrt(a))/sqrt(a)
```

---

**Mathematica [A]** time = 0.0350874, size = 50, normalized size = 1.28

$$x\sqrt{a+\frac{b}{x}}+\frac{b\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b/x],x]
```

```
[Out] Sqrt[a + b/x]*x + (b*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x])/
(2*Sqrt[a])
```

---

**Maple [B]** time = 0.006, size = 74, normalized size = 1.9

$$\frac{x}{2}\sqrt{\frac{ax+b}{x}}\left(2\sqrt{ax^2+bx}\sqrt{a}+b\ln\left(\frac{1}{2}\left(2\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b\right)\frac{1}{\sqrt{a}}\right)\right)\frac{1}{\sqrt{x\,(ax+b)}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+b*ln(1/2*(2*
(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)))/(x*(a*x+b))^(1/2)/a^
(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.257149, size = 1, normalized size = 0.03

$$\left[\frac{2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\log\left(2\,ax\sqrt{\frac{ax+b}{x}}+(2\,ax+b)\sqrt{a}\right)}{2\sqrt{a}},\ \frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}-b\arctan\left(\frac{a}{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}\right)}{\sqrt{-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(a)*x*sqrt((a*x + b)/x) + b*log(2*a*x*sqrt((a*x + b)/
x) + (2*a*x + b)*sqrt(a)))/sqrt(a), (sqrt(-a)*x*sqrt((a*x + b)/x)
 - b*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x))))/sqrt(-a)]
```

---

**Sympy [A]** time = 6.49426, size = 42, normalized size = 1.08

$$\sqrt{b}\sqrt{x}\sqrt{\frac{ax}{b}+1}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**(1/2),x)
```

```
[Out] sqrt(b)*sqrt(x)*sqrt(a*x/b + 1) + b*asinh(sqrt(a)*sqrt(x)/sqrt(b)
)/sqrt(a)
```

---

**GIAC/XCAS [A]** time = 0.232805, size = 86, normalized size = 2.21

$$-\frac{b\ln\left(\left|-2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}-b\right|\right)\operatorname{sign}(x)}{2\sqrt{a}}+\frac{b\ln(|b|)\operatorname{sign}(x)}{2\sqrt{a}}+\sqrt{ax^2+bx}\operatorname{sign}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x),x, algorithm="giac")
```

```
[Out] -1/2*b*ln(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))*si
gn(x)/sqrt(a) + 1/2*b*ln(abs(b))*sign(x)/sqrt(a) + sqrt(a*x^2 + b
*x)*sign(x)
```

## 3.129 $\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx$

**Optimal**. Leaf size=104

$$\frac{2\sqrt{d}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^2}+\frac{x\sqrt{a+\frac{b}{x}}}{c}$$

```
[Out] (Sqrt[a + b/x]*x)/c + (2*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*
Sqrt[a + b/x])/Sqrt[b*c - a*d]])/c^2 + ((b*c - 2*a*d)*ArcTanh[Sqr
t[a + b/x]/Sqrt[a]])/(Sqrt[a]*c^2)
```

---

**Rubi [A]** time = 0.354658, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{2\sqrt{d}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^2}+\frac{x\sqrt{a+\frac{b}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b/x]/(c + d/x),x]
```

```
[Out] (Sqrt[a + b/x]*x)/c + (2*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*
Sqrt[a + b/x])/Sqrt[b*c - a*d]])/c^2 + ((b*c - 2*a*d)*ArcTanh[Sqr
t[a + b/x]/Sqrt[a]])/(Sqrt[a]*c^2)
```

---

**Rubi in Sympy [A]** time = 41.2538, size = 90, normalized size = 0.87

$$\frac{x\sqrt{a+\frac{b}{x}}}{c}+\frac{2\sqrt{d}\sqrt{ad-bc}\,\mathrm{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{c^2}-\frac{2\left(ad-\frac{bc}{2}\right)\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**(1/2)/(c+d/x),x)
```

```
[Out] x*sqrt(a + b/x)/c + 2*sqrt(d)*sqrt(a*d - b*c)*atanh(sqrt(d)*sqrt(
a + b/x)/sqrt(a*d - b*c))/c**2 - 2*(a*d - b*c/2)*atanh(sqrt(a + b
/x)/sqrt(a))/(sqrt(a)*c**2)
```

**Mathematica [A]** time = 0.358711, size = 153, normalized size = 1.47

$$\frac{2\sqrt{d}\sqrt{ad-bc}\log(cx+d)+\frac{(bc-2ad)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{\sqrt{a}}-2\sqrt{d}\sqrt{ad-bc}\log\left(2\sqrt{d}x\sqrt{a+\frac{b}{x}}\sqrt{ad-bc}-2adx+bcx-bd\right)+}{2c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b/x]/(c + d/x),x]
```

```
[Out] (2*c*Sqrt[a + b/x]*x + 2*Sqrt[d]*Sqrt[-(b*c) + a*d]*Log[d + c*x]
+ ((b*c - 2*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x])/Sqrt
[a] - 2*Sqrt[d]*Sqrt[-(b*c) + a*d]*Log[-(b*d) + b*c*x - 2*a*d*x +
 2*Sqrt[d]*Sqrt[-(b*c) + a*d]*Sqrt[a + b/x]*x])/(2*c^2)
```

**Maple [B]** time = 0.03, size = 286, normalized size = 2.8

$$\frac{x}{2c^3}\sqrt{\frac{ax+b}{x}}\left(2\sqrt{x(ax+b)}c^2\sqrt{a}\sqrt{\frac{(ad-bc)d}{c^2}}-2\ln\left(1/2\,\frac{2\sqrt{x(ax+b)}\sqrt{a}+2ax+b}{\sqrt{a}}\right)\sqrt{\frac{(ad-bc)d}{c^2}}acd+\ln\left(\frac{1}{2}\left(2\sqrt{x(a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^(1/2)/(c+d/x),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(2*(x*(a*x+b))^(1/2)*c^2*a^(1/2)*((a*d-b*
c)*d/c^2)^(1/2)-2*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^
(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a*c*d+ln(1/2*(2*(x*(a*x+b))^(1/2)*
a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*b*c^2-2*ln((2*(
x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*
x+d))*a^(3/2)*d^2+2*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/
2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(1/2)*b*c*d)/(x*(a*x+b))^(1/2)
/c^3/a^(1/2)/((a*d-b*c)*d/c^2)^(1/2)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)/(c + d/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.288149, size = 1, normalized size = 0.01

$$\left[\frac{2\sqrt{a}cx\sqrt{\frac{ax+b}{x}} - (bc - 2ad)\log\left(-2ax\sqrt{\frac{ax+b}{x}} + (2ax+b)\sqrt{a}\right) + 2\sqrt{-bcd+ad^2}\sqrt{a}\log\left(\frac{bd-(bc-2ad)x+2\sqrt{-bcd+ad^2}x\sqrt{\frac{ax+b}{x}}}{cx+d}\right)}{2\sqrt{a}c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)/(c + d/x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(a)*c*x*sqrt((a*x + b)/x) - (b*c - 2*a*d)*log(-2*a*x*
sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(a)) + 2*sqrt(-b*c*d + a*d^2)
*sqrt(a)*log((b*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sq
rt((a*x + b)/x))/(c*x + d)))/(sqrt(a)*c^2), 1/2*(2*sqrt(a)*c*x*sq
rt((a*x + b)/x) - 4*sqrt(b*c*d - a*d^2)*sqrt(a)*arctan(sqrt(b*c*d
 - a*d^2)/(d*sqrt((a*x + b)/x))) - (b*c - 2*a*d)*log(-2*a*x*sqrt(
(a*x + b)/x) + (2*a*x + b)*sqrt(a)))/(sqrt(a)*c^2), (sqrt(-a)*c*x
*sqrt((a*x + b)/x) - (b*c - 2*a*d)*arctan(a/(sqrt(-a)*sqrt((a*x +
 b)/x))) + sqrt(-b*c*d + a*d^2)*sqrt(-a)*log((b*d - (b*c - 2*a*d)
*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x + d)))/(sqr
t(-a)*c^2), (sqrt(-a)*c*x*sqrt((a*x + b)/x) - (b*c - 2*a*d)*arcta
n(a/(sqrt(-a)*sqrt((a*x + b)/x))) - 2*sqrt(b*c*d - a*d^2)*sqrt(-a
)*arctan(sqrt(b*c*d - a*d^2)/(d*sqrt((a*x + b)/x))))/(sqrt(-a)*c^
2)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{a + \frac{b}{x}}}{cx + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**(1/2)/(c+d/x),x)
```

```
[Out] Integral(x*sqrt(a + b/x)/(c*x + d), x)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)/(c + d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

# 3.130 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2} dx$

**Optimal**. Leaf size=147

$$\frac{\sqrt{d}(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^3}+\frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)}+\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

```
[Out] (2*d*Sqrt[a + b/x])/(c^2*(c + d/x)) + (Sqrt[a + b/x]*x)/(c*(c + d
/x)) + (Sqrt[d]*(3*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sq
rt[b*c - a*d]])/(c^3*Sqrt[b*c - a*d]) + ((b*c - 4*a*d)*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/(Sqrt[a]*c^3)
```

---

**Rubi [A]** time = 0.606933, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{\sqrt{d}(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^3}+\frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)}+\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b/x]/(c + d/x)^2,x]
```

```
[Out] (2*d*Sqrt[a + b/x])/(c^2*(c + d/x)) + (Sqrt[a + b/x]*x)/(c*(c + d
/x)) + (Sqrt[d]*(3*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sq
rt[b*c - a*d]])/(c^3*Sqrt[b*c - a*d]) + ((b*c - 4*a*d)*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/(Sqrt[a]*c^3)
```

---

**Rubi in Sympy [A]** time = 67.7229, size = 122, normalized size = 0.83

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}+\frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)}+\frac{\sqrt{d}\,(4ad-3bc)\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{c^3\sqrt{ad-bc}}-\frac{(4ad-bc)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**(1/2)/(c+d/x)**2,x)
```

```
[Out] x*sqrt(a + b/x)/(c*(c + d/x)) + 2*d*sqrt(a + b/x)/(c**2*(c + d/x)
) + sqrt(d)*(4*a*d - 3*b*c)*atanh(sqrt(d)*sqrt(a + b/x)/sqrt(a*d
- b*c))/(c**3*sqrt(a*d - b*c)) - (4*a*d - b*c)*atanh(sqrt(a + b/x
)/sqrt(a))/(sqrt(a)*c**3)
```

---

**Mathematica [C]** time = 0.599008, size = 197, normalized size = 1.34

$$\frac{\frac{i\sqrt{d}(3bc-4ad)\log\left(-\frac{2ic^4\left(-2i\sqrt{d}x\sqrt{a+\frac{b}{x}}\sqrt{bc-ad}-2adx+bcx-bd\right)}{d^{3/2}(cx+d)(3bc-4ad)\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}+\frac{2cx\sqrt{a+\frac{b}{x}}(cx+2d)}{cx+d}+\frac{(bc-4ad)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{\sqrt{a}}}{2c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b/x]/(c + d/x)^2,x]
```

```
[Out] ((2*c*Sqrt[a + b/x]*x*(2*d + c*x))/(d + c*x) + ((b*c - 4*a*d)*Log
[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x])/Sqrt[a] + (I*Sqrt[d]*(3*
b*c - 4*a*d)*Log[((-2*I)*c^4*(-(b*d) + b*c*x - 2*a*d*x - (2*I)*Sq
rt[d]*Sqrt[b*c - a*d]*Sqrt[a + b/x]*x))/(d^(3/2)*(3*b*c - 4*a*d)*
Sqrt[b*c - a*d]*(d + c*x))])/Sqrt[b*c - a*d])/(2*c^3)
```

---

**Maple [B]** time = 0.028, size = 939, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^(1/2)/(c+d/x)^2,x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(-4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2
*a*x+b)/a^(1/2))*a^2*((a*d-b*c)*d/c^2)^(1/2)*x*c^2*d^2-2*(x*(a*x+
b))^(1/2)*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*x^2*c^4-4*ln(1/2*(2*(x*
(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*((a*d-b*c)*d/c^2)^(1
/2)*c*d^3+5*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))
*a*((a*d-b*c)*d/c^2)^(1/2)*x*b*c^3*d+2*c^4*(x*(a*x+b))^(3/2)*a^(1
/2)*((a*d-b*c)*d/c^2)^(1/2)+2*(x*(a*x+b))^(1/2)*a^(3/2)*((a*d-b*c
)*d/c^2)^(1/2)*x*c^3*d-4*(x*(a*x+b))^(1/2)*a^(1/2)*((a*d-b*c)*d/c
^2)^(1/2)*x*b*c^4+5*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/
a^(1/2))*a*((a*d-b*c)*d/c^2)^(1/2)*b*c^2*d^2-ln(1/2*(2*(x*(a*x+b)
)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x*b^2*c
^4-4*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*
c*x-b*d)/(c*x+d))*a^(5/2)*x*c*d^3+7*ln((2*(x*(a*x+b))^(1/2)*((a*d
-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(3/2)*x*b*c^2*
d^2-3*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b
*c*x-b*d)/(c*x+d))*a^(1/2)*x*b^2*c^3*d+4*(x*(a*x+b))^(1/2)*a^(3/2
)*((a*d-b*c)*d/c^2)^(1/2)*c^2*d^2-4*(x*(a*x+b))^(1/2)*a^(1/2)*((a
```

```
*d-b*c)*d/c^2)^(1/2)*b*c^3*d-ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+
2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*b^2*c^3*d-4*ln((2*(x*(a
*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d)
)*a^(5/2)*d^4+7*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c
-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(3/2)*b*c*d^3-3*ln((2*(x*(a*x+b))^
(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(1/
2)*b^2*c^2*d^2)/(x*(a*x+b))^(1/2)/c^4/a^(1/2)/((a*d-b*c)*d/c^2)^(
1/2)/(a*d-b*c)/(c*x+d)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)/(c + d/x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.300437, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)/(c + d/x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*sqrt(-
d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a
*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) - 2*(c^2*x^2 + 2*c
*d*x)*sqrt(a)*sqrt((a*x + b)/x) + (b*c*d - 4*a*d^2 + (b*c^2 - 4*a
*c*d)*x)*log(-2*a*x*sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(a)))/((c
^4*x + c^3*d)*sqrt(a)), -1/2*((3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a
*c*d)*x)*sqrt(-a)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt
(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x
+ d)) - 2*(c^2*x^2 + 2*c*d*x)*sqrt(-a)*sqrt((a*x + b)/x) + 2*(b*c
*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*arctan(a/(sqrt(-a)*sqrt((a*x
+ b)/x))))/((c^4*x + c^3*d)*sqrt(-a)), 1/2*(2*(3*b*c*d - 4*a*d^2
+ (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*sqrt(d/(b*c - a*d))*arctan(-(b*c
 - a*d)*sqrt(d/(b*c - a*d))/(d*sqrt((a*x + b)/x))) + 2*(c^2*x^2 +
 2*c*d*x)*sqrt(a)*sqrt((a*x + b)/x) - (b*c*d - 4*a*d^2 + (b*c^2 -
 4*a*c*d)*x)*log(-2*a*x*sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(a)))
/((c^4*x + c^3*d)*sqrt(a)), ((3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*
c*d)*x)*sqrt(-a)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d/(
b*c - a*d))/(d*sqrt((a*x + b)/x))) + (c^2*x^2 + 2*c*d*x)*sqrt(-a)
*sqrt((a*x + b)/x) - (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*arct
an(a/(sqrt(-a)*sqrt((a*x + b)/x))))/((c^4*x + c^3*d)*sqrt(-a))]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{a+\frac{b}{x}}}{(cx+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**(1/2)/(c+d/x)**2,x)
```

```
[Out] Integral(x**2*sqrt(a + b/x)/(c*x + d)**2, x)
```

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)/(c + d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.131 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3} \, dx$

**Optimal**. Leaf size=213

$$\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{3/2}}+\frac{(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^4}+\frac{d\sqrt{a+\frac{b}{x}}(11bc-12ad)}{4c^3\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2}+\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}$$

```
[Out] (3*d*Sqrt[a + b/x])/(2*c^2*(c + d/x)^2) + (d*(11*b*c - 12*a*d)*Sq
rt[a + b/x])/(4*c^3*(b*c - a*d)*(c + d/x)) + (Sqrt[a + b/x]*x)/(c
*(c + d/x)^2) + (Sqrt[d]*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*A
rcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(4*c^4*(b*c - a*d
)^(3/2)) + ((b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(Sqrt[a
]*c^4)
```

---

**Rubi [A]** time = 0.968426, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{3/2}}+\frac{(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^4}+\frac{d\sqrt{a+\frac{b}{x}}(11bc-12ad)}{4c^3\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2}+\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b/x]/(c + d/x)^3,x]
```

```
[Out] (3*d*Sqrt[a + b/x])/(2*c^2*(c + d/x)^2) + (d*(11*b*c - 12*a*d)*Sq
rt[a + b/x])/(4*c^3*(b*c - a*d)*(c + d/x)) + (Sqrt[a + b/x]*x)/(c
*(c + d/x)^2) + (Sqrt[d]*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*A
rcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(4*c^4*(b*c - a*d
)^(3/2)) + ((b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(Sqrt[a
]*c^4)
```

---

**Rubi in Sympy [A]** time = 111.143, size = 182, normalized size = 0.85

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}+\frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2}+\frac{d\sqrt{a+\frac{b}{x}}\,(12ad-11bc)}{4c^3\left(c+\frac{d}{x}\right)(ad-bc)}$$
$$+\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{4c^4\,(ad-bc)^{\frac{3}{2}}}-\frac{(6ad-bc)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**(1/2)/(c+d/x)**3,x)
```

```
[Out] x*sqrt(a + b/x)/(c*(c + d/x)**2) + 3*d*sqrt(a + b/x)/(2*c**2*(c +
 d/x)**2) + d*sqrt(a + b/x)*(12*a*d - 11*b*c)/(4*c**3*(c + d/x)*(
a*d - b*c)) + sqrt(d)*(24*a**2*d**2 - 40*a*b*c*d + 15*b**2*c**2)*
atanh(sqrt(d)*sqrt(a + b/x)/sqrt(a*d - b*c))/(4*c**4*(a*d - b*c)*
*(3/2)) - (6*a*d - b*c)*atanh(sqrt(a + b/x)/sqrt(a))/(sqrt(a)*c**
4)
```

---

**Mathematica [C]** time = 0.78896, size = 275, normalized size = 1.29

$$\frac{\frac{i\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\log\left(-\frac{8ic^5\sqrt{bc-ad}\left(-2i\sqrt{d}x\sqrt{a+\frac{b}{x}}\sqrt{bc-ad}-2adx+bcx-bd\right)}{d^{3/2}(cx+d)\left(24a^2d^2-40abcd+15b^2c^2\right)}\right)}{(bc-ad)^{3/2}}+\frac{2cx\sqrt{a+\frac{b}{x}}\left(bc\left(4c^2x^2+17cdx+11d^2\right)-2ad\left(2c^2x^2+9cdx+6d^2\right)\right)}{(cx+d)^2(bc-ad)}+4(}{8c^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b/x]/(c + d/x)^3,x]
```

```
[Out] ((2*c*Sqrt[a + b/x]*x*(-2*a*d*(6*d^2 + 9*c*d*x + 2*c^2*x^2) + b*c
*(11*d^2 + 17*c*d*x + 4*c^2*x^2)))/((b*c - a*d)*(d + c*x)^2) + (4
*(b*c - 6*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x])/Sqrt[a
] + (I*Sqrt[d]*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*Log[((-8*I)
*c^5*Sqrt[b*c - a*d]*(-(b*d) + b*c*x - 2*a*d*x - (2*I)*Sqrt[d]*Sq
rt[b*c - a*d]*Sqrt[a + b/x]*x))/(d^(3/2)*(15*b^2*c^2 - 40*a*b*c*d
 + 24*a^2*d^2)*(d + c*x))])/(b*c - a*d)^(3/2))/(8*c^4)
```

---

**Maple [B]** time = 0.028, size = 1963, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^(1/2)/(c+d/x)^3,x)
```

```
[Out] 1/8*((a*x+b)/x)^(1/2)*x*(104*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+
2*a*x+b)/a^(1/2))*a^2*((a*d-b*c)*d/c^2)^(1/2)*x*b*c^3*d^3-24*ln((
2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/
(c*x+d))*a^(7/2)*d^6+128*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2
)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(5/2)*x*b*c^2*d^4-110*ln(
(2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)
/(c*x+d))*a^(3/2)*x*b^2*c^3*d^3-46*(x*(a*x+b))^(1/2)*a^(3/2)*((a*
d-b*c)*d/c^2)^(1/2)*b*c^3*d^3-12*(x*(a*x+b))^(1/2)*a^(5/2)*((a*d-
b*c)*d/c^2)^(1/2)*x^3*c^5*d+14*(x*(a*x+b))^(1/2)*a^(3/2)*((a*d-b*
c)*d/c^2)^(1/2)*x^3*b*c^6+12*(x*(a*x+b))^(3/2)*a^(3/2)*((a*d-b*c)
*d/c^2)^(1/2)*x*c^5*d+64*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2
)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(5/2)*x^2*b*c^3*d^3-55*ln
((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d
)/(c*x+d))*a^(3/2)*x^2*b^2*c^4*d^2+36*(x*(a*x+b))^(1/2)*a^(5/2)*(
(a*d-b*c)*d/c^2)^(1/2)*x*c^3*d^3-48*ln(1/2*(2*(x*(a*x+b))^(1/2)*a
^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*x*c^2*d^4-14
*(x*(a*x+b))^(3/2)*a^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b*c^6+22*(x*
(a*x+b))^(1/2)*a^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^2*c^6+52*ln(
1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*((a*d-b*c)
*d/c^2)^(1/2)*b*c^2*d^4+15*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c
^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(1/2)*x^2*b^3*c^5*d-10*
(x*(a*x+b))^(3/2)*a^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*b*c^5*d-32*ln(1
/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*((a*d-b*c)*d/
c^2)^(1/2)*b^2*c^3*d^3+8*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*
x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x*b^3*c^5*d+30*ln((2*(x*(a*
x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))
*a^(1/2)*x*b^3*c^4*d^2+22*(x*(a*x+b))^(1/2)*a^(1/2)*((a*d-b*c)*d/
c^2)^(1/2)*b^2*c^4*d^2-24*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^
2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(7/2)*x^2*c^2*d^4+8*(x*(
a*x+b))^(3/2)*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*c^4*d^2-48*ln((2*(x
*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x
+d))*a^(7/2)*x*c*d^5+24*(x*(a*x+b))^(1/2)*a^(5/2)*((a*d-b*c)*d/c^
2)^(1/2)*c^2*d^4+64*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/
2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(5/2)*b*c*d^5-55*ln((2*(x*(a*x
+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*
a^(3/2)*b^2*c^2*d^4-24*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+
b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*c*d^5+4*ln(1/2*(2*(x*(a*x
+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^2*
b^3*c^6+4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(
(a*d-b*c)*d/c^2)^(1/2)*b^3*c^4*d^2+15*ln((2*(x*(a*x+b))^(1/2)*((a
*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(1/2)*b^3*c^
3*d^3+52*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^
2*((a*d-b*c)*d/c^2)^(1/2)*x^2*b*c^4*d^2-24*ln(1/2*(2*(x*(a*x+b))^
(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*x^2*c
^3*d^3-32*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a
*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^2*c^5*d-64*ln(1/2*(2*(x*(a*x+b))^(
1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*((a*d-b*c)*d/c^2)^(1/2)*x*b^2*c^
4*d^2+44*(x*(a*x+b))^(1/2)*a^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b^2*
c^5*d-78*(x*(a*x+b))^(1/2)*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b*c^
4*d^2-18*(x*(a*x+b))^(1/2)*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*x^2*b*
c^5*d)/(x*(a*x+b))^(1/2)/c^5/(a*d-b*c)^2/(c*x+d)^2/((a*d-b*c)*d/c
^2)^(1/2)/a^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)/(c + d/x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.402124, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)/(c + d/x)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((15*b^2*c^2*d^2 - 40*a*b*c*d^3 + 24*a^2*d^4 + (15*b^2*c^4
- 40*a*b*c^3*d + 24*a^2*c^2*d^2)*x^2 + 2*(15*b^2*c^3*d - 40*a*b*c
^2*d^2 + 24*a^2*c*d^3)*x)*sqrt(a)*sqrt(-d/(b*c - a*d))*log(-(2*(b
*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c -
 2*a*d)*x)/(c*x + d)) - 2*(4*(b*c^4 - a*c^3*d)*x^3 + (17*b*c^3*d
- 18*a*c^2*d^2)*x^2 + (11*b*c^2*d^2 - 12*a*c*d^3)*x)*sqrt(a)*sqrt
((a*x + b)/x) + 4*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c
^4 - 7*a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a*b*c^2*
d^2 + 6*a^2*c*d^3)*x)*log(-2*a*x*sqrt((a*x + b)/x) + (2*a*x + b)*
sqrt(a)))/((b*c^5*d^2 - a*c^4*d^3 + (b*c^7 - a*c^6*d)*x^2 + 2*(b*
c^6*d - a*c^5*d^2)*x)*sqrt(a)), 1/4*((15*b^2*c^2*d^2 - 40*a*b*c*d
^3 + 24*a^2*d^4 + (15*b^2*c^4 - 40*a*b*c^3*d + 24*a^2*c^2*d^2)*x^
2 + 2*(15*b^2*c^3*d - 40*a*b*c^2*d^2 + 24*a^2*c*d^3)*x)*sqrt(a)*s
qrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d/(b*c - a*d))/(d*sqr
t((a*x + b)/x))) + (4*(b*c^4 - a*c^3*d)*x^3 + (17*b*c^3*d - 18*a*
c^2*d^2)*x^2 + (11*b*c^2*d^2 - 12*a*c*d^3)*x)*sqrt(a)*sqrt((a*x +
 b)/x) - 2*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*
a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6
*a^2*c*d^3)*x)*log(-2*a*x*sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(a)
))/((b*c^5*d^2 - a*c^4*d^3 + (b*c^7 - a*c^6*d)*x^2 + 2*(b*c^6*d -
 a*c^5*d^2)*x)*sqrt(a)), -1/8*((15*b^2*c^2*d^2 - 40*a*b*c*d^3 + 2
4*a^2*d^4 + (15*b^2*c^4 - 40*a*b*c^3*d + 24*a^2*c^2*d^2)*x^2 + 2*
(15*b^2*c^3*d - 40*a*b*c^2*d^2 + 24*a^2*c*d^3)*x)*sqrt(-a)*sqrt(-
d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a
*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*(b*c^4 - a*
c^3*d)*x^3 + (17*b*c^3*d - 18*a*c^2*d^2)*x^2 + (11*b*c^2*d^2 - 12
*a*c*d^3)*x)*sqrt(-a)*sqrt((a*x + b)/x) + 8*(b^2*c^2*d^2 - 7*a*b*
c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 +
 2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*arctan(a/(sqrt(-a
)*sqrt((a*x + b)/x))))/((b*c^5*d^2 - a*c^4*d^3 + (b*c^7 - a*c^6*d
```

```
)*x^2 + 2*(b*c^6*d - a*c^5*d^2)*x)*sqrt(-a)), 1/4*((15*b^2*c^2*d^
2 - 40*a*b*c*d^3 + 24*a^2*d^4 + (15*b^2*c^4 - 40*a*b*c^3*d + 24*a
^2*c^2*d^2)*x^2 + 2*(15*b^2*c^3*d - 40*a*b*c^2*d^2 + 24*a^2*c*d^3
)*x)*sqrt(-a)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d/(b*c
 - a*d))/(d*sqrt((a*x + b)/x))) + (4*(b*c^4 - a*c^3*d)*x^3 + (17*
b*c^3*d - 18*a*c^2*d^2)*x^2 + (11*b*c^2*d^2 - 12*a*c*d^3)*x)*sqrt
(-a)*sqrt((a*x + b)/x) - 4*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*d^4
 + (b^2*c^4 - 7*a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7
*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*arctan(a/(sqrt(-a)*sqrt((a*x + b)/
x))))/((b*c^5*d^2 - a*c^4*d^3 + (b*c^7 - a*c^6*d)*x^2 + 2*(b*c^6*
d - a*c^5*d^2)*x)*sqrt(-a))]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**(1/2)/(c+d/x)**3,x)

[Out] Timed out
```

---

**GIAC/XCAS [A]** time = 0.561121, size = 4, normalized size = 0.02

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)/(c + d/x)^3,x, algorithm="giac")

[Out] sage0*x
```

## 3.132 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3 \, dx$

**Optimal**. Leaf size=164

$$-\frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{3bd(2ad+19bc)}{x}+2(13bc-ad)(2ad+5bc)\right)}{35b^2}-3c^2\sqrt{a+\frac{b}{x}}(2ad+bc)+3\sqrt{a}c^2(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+x\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3-\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2$$

```
[Out] -3*c^2*(b*c + 2*a*d)*Sqrt[a + b/x] - (9*d*(a + b/x)^(3/2)*(c + d/
x)^2)/7 - (d*(a + b/x)^(3/2)*(2*(13*b*c - a*d)*(5*b*c + 2*a*d) +
(3*b*d*(19*b*c + 2*a*d))/x))/(35*b^2) + (a + b/x)^(3/2)*(c + d/x)
^3*x + 3*Sqrt[a]*c^2*(b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

---

**Rubi [A]** time = 0.44844, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{3bd(2ad+19bc)}{x}+2(13bc-ad)(2ad+5bc)\right)}{35b^2}-3c^2\sqrt{a+\frac{b}{x}}(2ad+bc)+3\sqrt{a}c^2(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+x\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3-\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^(3/2)*(c + d/x)^3,x]
```

```
[Out] -3*c^2*(b*c + 2*a*d)*Sqrt[a + b/x] - (9*d*(a + b/x)^(3/2)*(c + d/
x)^2)/7 - (d*(a + b/x)^(3/2)*(2*(13*b*c - a*d)*(5*b*c + 2*a*d) +
(3*b*d*(19*b*c + 2*a*d))/x))/(35*b^2) + (a + b/x)^(3/2)*(c + d/x)
^3*x + 3*Sqrt[a]*c^2*(b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

---

**Rubi in Sympy [A]** time = 48.4643, size = 151, normalized size = 0.92

$$3\sqrt{a}c^2\left(2ad+bc\right)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-3c^2\sqrt{a+\frac{b}{x}}\left(2ad+bc\right)-\frac{9d\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(c+\frac{d}{x}\right)^2}{7}+x\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(c+\frac{d}{x}\right)^3+\frac{8d\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(-\frac{9bd(2ad+19bc)}{8x}+\left(\frac{3ad}{4}-\frac{39bc}{4}\right)\left(2ad+5bc\right)\right)}{105b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**(3/2)*(c+d/x)**3,x)
```

```
[Out] 3*sqrt(a)*c**2*(2*a*d + b*c)*atanh(sqrt(a + b/x)/sqrt(a)) - 3*c**
2*sqrt(a + b/x)*(2*a*d + b*c) - 9*d*(a + b/x)**(3/2)*(c + d/x)**2
/7 + x*(a + b/x)**(3/2)*(c + d/x)**3 + 8*d*(a + b/x)**(3/2)*(-9*b
*d*(2*a*d + 19*b*c)/(8*x) + (3*a*d/4 - 39*b*c/4)*(2*a*d + 5*b*c))
/(105*b**2)
```

---

**Mathematica [A]** time = 0.259178, size = 169, normalized size = 1.03

$$\frac{\sqrt{a+\frac{b}{x}}\left(4a^3d^3x^3-2a^2bd^2x^2(21cx+d)+ab^2x\left(35c^3x^3-280c^2dx^2-84cd^2x-16d^3\right)-2b^3\left(35c^3x^3+35c^2dx^2+21cd^2x+\right.\right.}{35b^2x^3}$$

$$+\frac{3}{2}\sqrt{a}c^2(2ad+bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x)^(3/2)*(c + d/x)^3,x]
```

```
[Out] (Sqrt[a + b/x]*(4*a^3*d^3*x^3 - 2*a^2*b*d^2*x^2*(d + 21*c*x) + a*
b^2*x*(-16*d^3 - 84*c*d^2*x - 280*c^2*d*x^2 + 35*c^3*x^3) - 2*b^3
*(5*d^3 + 21*c*d^2*x + 35*c^2*d*x^2 + 35*c^3*x^3)))/(35*b^2*x^3)
+ (3*Sqrt[a]*c^2*(b*c + 2*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a +
 b/x]*x])/2
```

---

**Maple [B]** time = 0.02, size = 332, normalized size = 2.

$$\frac{1}{70\,x^4b^2}\sqrt{\frac{ax+b}{x}}\left(210\,c^2a^{3/2}\ln\left(1/2\,\frac{2\,\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)db^2x^5+105\,\sqrt{a}c^3b^3\ln\left(1/2\,\frac{2\,\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)x^5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^(3/2)*(c+d/x)^3,x)
```

```
[Out] 1/70*((a*x+b)/x)^(1/2)/x^4/b^2*(210*c^2*a^(3/2)*ln(1/2*(2*(a*x^2+
b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*d*b^2*x^5+105*a^(1/2)*c^3*b^
3*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^5+420*c
^2*a^2*(a*x^2+b*x)^(1/2)*d*b*x^5+210*a*c^3*(a*x^2+b*x)^(1/2)*b^2*
x^5-420*c^2*(a*x^2+b*x)^(3/2)*a*d*b*x^3-140*c^3*(a*x^2+b*x)^(3/2)
*b^2*x^3+8*(a*x^2+b*x)^(3/2)*x^2*a^2*d^3-84*(a*x^2+b*x)^(3/2)*x^2
*a*b*c*d^2-140*(a*x^2+b*x)^(3/2)*x^2*b^2*c^2*d-12*(a*x^2+b*x)^(3/
```

```
2)*x*a*b*d^3-84*(a*x^2+b*x)^(3/2)*x*b^2*c*d^2-20*(a*x^2+b*x)^(3/2
)*b^2*d^3)/(x*(a*x+b))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2)*(c + d/x)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.255313, size = 1, normalized size = 0.01

$$\left[\frac{105\left(b^3c^3+2\,ab^2c^2d\right)\sqrt{a}x^3\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(35\,ab^2c^3x^4-10\,b^3d^3-2\left(35\,b^3c^3+140\,ab^2c^2d+21\,a^2bcd^2\right.\right.}{70\,b^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2)*(c + d/x)^3,x, algorithm="fricas")

[Out] [1/70*(105*(b^3*c^3 + 2*a*b^2*c^2*d)*sqrt(a)*x^3*log(2*a*x + 2*sq
rt(a)*x*sqrt((a*x + b)/x) + b) + 2*(35*a*b^2*c^3*x^4 - 10*b^3*d^3
 - 2*(35*b^3*c^3 + 140*a*b^2*c^2*d + 21*a^2*b*c*d^2 - 2*a^3*d^3)*
x^3 - 2*(35*b^3*c^2*d + 42*a*b^2*c*d^2 + a^2*b*d^3)*x^2 - 2*(21*b
^3*c*d^2 + 8*a*b^2*d^3)*x)*sqrt((a*x + b)/x))/(b^2*x^3), 1/35*(10
5*(b^3*c^3 + 2*a*b^2*c^2*d)*sqrt(-a)*x^3*arctan(sqrt((a*x + b)/x)
/sqrt(-a)) + (35*a*b^2*c^3*x^4 - 10*b^3*d^3 - 2*(35*b^3*c^3 + 140
*a*b^2*c^2*d + 21*a^2*b*c*d^2 - 2*a^3*d^3)*x^3 - 2*(35*b^3*c^2*d
+ 42*a*b^2*c*d^2 + a^2*b*d^3)*x^2 - 2*(21*b^3*c*d^2 + 8*a*b^2*d^3
)*x)*sqrt((a*x + b)/x))/(b^2*x^3)]
```

---

**Sympy [A]** time = 41.5849, size = 1862, normalized size = 11.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**(3/2)*(c+d/x)**3,x)
```

```
[Out] -16*a**(19/2)*b**(11/2)*d**3*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*
b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9
*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(17/2)*b**(13/2)
*d**3*x**5*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a*
*(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2
)*b**10*x**(7/2)) - 30*a**(15/2)*b**(15/2)*d**3*x**4*sqrt(a*x/b +
 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2)
+ 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*
a**(13/2)*b**(17/2)*d**3*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7
*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**
(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 4*a**(13/2)*b**(3/2)*d**3*
x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**
4*x**(5/2)) - 100*a**(11/2)*b**(19/2)*d**3*x**2*sqrt(a*x/b + 1)/(
105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315
*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 12*a**(1
1/2)*b**(5/2)*c*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7
/2) + 15*a**(5/2)*b**4*x**(5/2)) + 2*a**(11/2)*b**(5/2)*d**3*x**2
*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x*
*(5/2)) - 96*a**(9/2)*b**(21/2)*d**3*x*sqrt(a*x/b + 1)/(105*a**(1
3/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)
*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 6*a**(9/2)*b**(7/
2)*c*d**2*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a*
*(5/2)*b**4*x**(5/2)) - 8*a**(9/2)*b**(7/2)*d**3*x*sqrt(a*x/b + 1
)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 30*a*
*(7/2)*b**(23/2)*d**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/
2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) +
105*a**(7/2)*b**10*x**(7/2)) - 24*a**(7/2)*b**(9/2)*c*d**2*x*sqrt
(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2
)) - 6*a**(7/2)*b**(9/2)*d**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x
**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 18*a**(5/2)*b**(11/2)*c*d*
*2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*
x**(5/2)) + 6*a**(3/2)*c**2*d*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 3*
sqrt(a)*b*c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 16*a**10*b**5*d**
3*x**(13/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x*
*(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2
)) + 48*a**9*b**6*d**3*x**(11/2)/(105*a**(13/2)*b**7*x**(13/2) +
315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a
**(7/2)*b**10*x**(7/2)) + 48*a**8*b**7*d**3*x**(9/2)/(105*a**(13/
2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b
**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 16*a**7*b**8*d**3*x
**(7/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11
/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) -
 4*a**7*b*d**3*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*
b**4*x**(5/2)) - 12*a**6*b**2*c*d**2*x**(7/2)/(15*a**(7/2)*b**3*x
**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a**6*b**2*d**3*x**(5/2)/
(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 12*a**5
*b**3*c*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b*
*4*x**(5/2)) - 6*a**2*c**2*d*sqrt(x)/(sqrt(b)*sqrt(a*x/b + 1)) +
a*sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1) - 2*a*sqrt(b)*c**3*sqrt(x)
/sqrt(a*x/b + 1) - 6*a*sqrt(b)*c**2*d/(sqrt(x)*sqrt(a*x/b + 1)) +
 3*a*c*d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2
)/(3*b), True)) - 2*b**(3/2)*c**3/(sqrt(x)*sqrt(a*x/b + 1)) + 3*b
*c**2*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3
*b), True))
```

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2)*(c + d/x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

# 3.133 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2 \, dx$

**Optimal**. Leaf size=126

$$\frac{c^2 x \left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{c \left(a+\frac{b}{x}\right)^{3/2} (4 a d+3 b c)}{3 a}-c \sqrt{a+\frac{b}{x}} (4 a d+3 b c)+\sqrt{a} c (4 a d+3 b c) \tanh ^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{2 d^2 \left(a+\frac{b}{x}\right)^{5/2}}{5 b}$$

```
[Out] -(c*(3*b*c + 4*a*d)*Sqrt[a + b/x]) - (c*(3*b*c + 4*a*d)*(a + b/x)
^(3/2))/(3*a) - (2*d^2*(a + b/x)^(5/2))/(5*b) + (c^2*(a + b/x)^(5
/2)*x)/a + Sqrt[a]*c*(3*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a
]]
```

---

**Rubi [A]** time = 0.249053, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{c^2 x \left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{c \left(a+\frac{b}{x}\right)^{3/2} (4 a d+3 b c)}{3 a}-c \sqrt{a+\frac{b}{x}} (4 a d+3 b c)+\sqrt{a} c (4 a d+3 b c) \tanh ^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{2 d^2 \left(a+\frac{b}{x}\right)^{5/2}}{5 b}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^(3/2)*(c + d/x)^2,x]
```

```
[Out] -(c*(3*b*c + 4*a*d)*Sqrt[a + b/x]) - (c*(3*b*c + 4*a*d)*(a + b/x)
^(3/2))/(3*a) - (2*d^2*(a + b/x)^(5/2))/(5*b) + (c^2*(a + b/x)^(5
/2)*x)/a + Sqrt[a]*c*(3*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a
]]
```

---

**Rubi in Sympy [A]** time = 22.4613, size = 107, normalized size = 0.85

$$\sqrt{a} c \left(4 a d+3 b c\right) \operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-c \sqrt{a+\frac{b}{x}} \left(4 a d+3 b c\right)-\frac{2 d^2 \left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{5 b}+\frac{c^2 x \left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{a}-\frac{c \left(a+\frac{b}{x}\right)^{\frac{3}{2}} \left(4 a d+3 b c\right)}{3 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**(3/2)*(c+d/x)**2,x)
```

```
[Out] sqrt(a)*c*(4*a*d + 3*b*c)*atanh(sqrt(a + b/x)/sqrt(a)) - c*sqrt(a
 + b/x)*(4*a*d + 3*b*c) - 2*d**2*(a + b/x)**(5/2)/(5*b) + c**2*x*
(a + b/x)**(5/2)/a - c*(a + b/x)**(3/2)*(4*a*d + 3*b*c)/(3*a)
```

---

**Mathematica [A]** time = 0.238831, size = 115, normalized size = 0.91

$$\frac{1}{15}\sqrt{a+\frac{b}{x}}\left(-\frac{6a^2d^2}{b}-\frac{4d(3ad+5bc)}{x}+15ac^2x-80acd-30bc^2-\frac{6bd^2}{x^2}\right)+\frac{1}{2}\sqrt{a}c(4ad+3bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x)^(3/2)*(c + d/x)^2,x]
```

```
[Out] (Sqrt[a + b/x]*(-30*b*c^2 - 80*a*c*d - (6*a^2*d^2)/b - (6*b*d^2)/
x^2 - (4*d*(5*b*c + 3*a*d))/x + 15*a*c^2*x))/15 + (Sqrt[a]*c*(3*b
*c + 4*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x])/2
```

---

**Maple [B]** time = 0.019, size = 244, normalized size = 1.9

$$\frac{1}{30\,bx^3}\sqrt{\frac{ax+b}{x}}\left(60\,ca^{3/2}\ln\left(1/2\,\frac{2\,\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)dbx^4+45\,\sqrt{a}c^2b^2\ln\left(1/2\,\frac{2\,\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)x^4+12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^(3/2)*(c+d/x)^2,x)
```

```
[Out] 1/30*((a*x+b)/x)^(1/2)/x^3/b*(60*c*a^(3/2)*ln(1/2*(2*(a*x^2+b*x)^
(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*d*b*x^4+45*a^(1/2)*c^2*b^2*ln(1/2
*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^4+120*c*a^2*(a*
x^2+b*x)^(1/2)*d*x^4+90*a*c^2*(a*x^2+b*x)^(1/2)*b*x^4-120*c*(a*x^
2+b*x)^(3/2)*a*d*x^2-60*(a*x^2+b*x)^(3/2)*c^2*b*x^2-12*(a*x^2+b*x
)^(3/2)*x*a*d^2-40*(a*x^2+b*x)^(3/2)*x*b*c*d-12*(a*x^2+b*x)^(3/2)
*b*d^2)/(x*(a*x+b))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2)*(c + d/x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.258953, size = 1, normalized size = 0.01

$$\left[\frac{15\left(3b^2c^2+4abcd\right)\sqrt{a}x^2\log\left(2ax+2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(15abc^2x^3-6b^2d^2-2\left(15b^2c^2+40abcd+3a^2d^2\right)x^2-4\left(5\right.\right.}{30bx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2)*(c + d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/30*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(
a)*x*sqrt((a*x + b)/x) + b) + 2*(15*a*b*c^2*x^3 - 6*b^2*d^2 - 2*(
15*b^2*c^2 + 40*a*b*c*d + 3*a^2*d^2)*x^2 - 4*(5*b^2*c*d + 3*a*b*d
^2)*x)*sqrt((a*x + b)/x))/(b*x^2), 1/15*(15*(3*b^2*c^2 + 4*a*b*c*
d)*sqrt(-a)*x^2*arctan(sqrt((a*x + b)/x)/sqrt(-a)) + (15*a*b*c^2*
x^3 - 6*b^2*d^2 - 2*(15*b^2*c^2 + 40*a*b*c*d + 3*a^2*d^2)*x^2 - 4
*(5*b^2*c*d + 3*a*b*d^2)*x)*sqrt((a*x + b)/x))/(b*x^2)]
```

---

**Sympy [A]** time = 26.8339, size = 576, normalized size = 4.57

$$\frac{4a^{\frac{11}{2}}b^{\frac{5}{2}}d^2x^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\frac{2a^{\frac{9}{2}}b^{\frac{7}{2}}d^2x^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{8a^{\frac{7}{2}}b^{\frac{9}{2}}d^2x\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}$$
$$-\frac{6a^{\frac{5}{2}}b^{\frac{11}{2}}d^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+4a^{\frac{3}{2}}cd\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)+3\sqrt{a}bc^2\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)$$
$$-\frac{4a^6b^2d^2x^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{4a^5b^3d^2x^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}$$
$$-\frac{4a^2cd\sqrt{x}}{\sqrt{b}\sqrt{\frac{ax}{b}+1}}+a\sqrt{b}c^2\sqrt{x}\sqrt{\frac{ax}{b}+1}-\frac{2a\sqrt{b}c^2\sqrt{x}}{\sqrt{\frac{ax}{b}+1}}-\frac{4a\sqrt{b}cd}{\sqrt{x}\sqrt{\frac{ax}{b}+1}}$$
$$+ad^2\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)-\frac{2b^{\frac{3}{2}}c^2}{\sqrt{x}\sqrt{\frac{ax}{b}+1}}+2bcd\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**(3/2)*(c+d/x)**2,x)
```

```
[Out] 4*a**(11/2)*b**(5/2)*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*
x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(7/2)*d**2*
x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**
4*x**(5/2)) - 8*a**(7/2)*b**(9/2)*d**2*x*sqrt(a*x/b + 1)/(15*a**(
7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6*a**(5/2)*b**(
11/2)*d**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/
2)*b**4*x**(5/2)) + 4*a**(3/2)*c*d*asinh(sqrt(a)*sqrt(x)/sqrt(b))
 + 3*sqrt(a)*b*c**2*asinh(sqrt(a)*sqrt(x)/sqrt(b)) - 4*a**6*b**2*
d**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5
/2)) - 4*a**5*b**3*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*
a**(5/2)*b**4*x**(5/2)) - 4*a**2*c*d*sqrt(x)/(sqrt(b)*sqrt(a*x/b
+ 1)) + a*sqrt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1) - 2*a*sqrt(b)*c**2
*sqrt(x)/sqrt(a*x/b + 1) - 4*a*sqrt(b)*c*d/(sqrt(x)*sqrt(a*x/b +
1)) + a*d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/
2)/(3*b), True)) - 2*b**(3/2)*c**2/(sqrt(x)*sqrt(a*x/b + 1)) + 2*
b*c*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b
), True))
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2)*(c + d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

# 3.134 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)\, dx$

**Optimal**. Leaf size=100

$$-\frac{\left(a+\frac{b}{x}\right)^{3/2}(2ad+3bc)}{3a}-\sqrt{a+\frac{b}{x}}(2ad+3bc)+\sqrt{a}(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{cx\left(a+\frac{b}{x}\right)^{5/2}}{a}$$

```
[Out] -((3*b*c + 2*a*d)*Sqrt[a + b/x]) - ((3*b*c + 2*a*d)*(a + b/x)^(3/
2))/(3*a) + (c*(a + b/x)^(5/2)*x)/a + Sqrt[a]*(3*b*c + 2*a*d)*Arc
Tanh[Sqrt[a + b/x]/Sqrt[a]]
```

---

**Rubi [A]** time = 0.181951, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$-\frac{\left(a+\frac{b}{x}\right)^{3/2}(2ad+3bc)}{3a}-\sqrt{a+\frac{b}{x}}(2ad+3bc)+\sqrt{a}(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{cx\left(a+\frac{b}{x}\right)^{5/2}}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^(3/2)*(c + d/x),x]
```

```
[Out] -((3*b*c + 2*a*d)*Sqrt[a + b/x]) - ((3*b*c + 2*a*d)*(a + b/x)^(3/
2))/(3*a) + (c*(a + b/x)^(5/2)*x)/a + Sqrt[a]*(3*b*c + 2*a*d)*Arc
Tanh[Sqrt[a + b/x]/Sqrt[a]]
```

---

**Rubi in Sympy [A]** time = 14.7568, size = 87, normalized size = 0.87

$$2\sqrt{a}\left(ad+\frac{3bc}{2}\right)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\sqrt{a+\frac{b}{x}}\,(2ad+3bc)+\frac{cx\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{a}-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(ad+\frac{3bc}{2}\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**(3/2)*(c+d/x),x)
```

```
[Out] 2*sqrt(a)*(a*d + 3*b*c/2)*atanh(sqrt(a + b/x)/sqrt(a)) - sqrt(a +
 b/x)*(2*a*d + 3*b*c) + c*x*(a + b/x)**(5/2)/a - 2*(a + b/x)**(3/
2)*(a*d + 3*b*c/2)/(3*a)
```

**Mathematica [A]** time = 0.162674, size = 84, normalized size = 0.84

$$\frac{\sqrt{a+\frac{b}{x}}(ax(3cx-8d)-2b(3cx+d))}{3x}+\frac{1}{2}\sqrt{a}(2ad+3bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x)^(3/2)*(c + d/x),x]
```

```
[Out] (Sqrt[a + b/x]*(a*x*(-8*d + 3*c*x) - 2*b*(d + 3*c*x)))/(3*x) + (S
qrt[a]*(3*b*c + 2*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x]
)/2
```

**Maple [B]** time = 0.016, size = 194, normalized size = 1.9

$$\frac{1}{6bx^2}\sqrt{\frac{ax+b}{x}}\left(6a^{3/2}\ln\left(1/2\frac{2\sqrt{ax^2+bx}\sqrt{a}+2ax+b}{\sqrt{a}}\right)dx^3b+9\sqrt{a}\ln\left(1/2\frac{2\sqrt{ax^2+bx}\sqrt{a}+2ax+b}{\sqrt{a}}\right)b^2cx^3+12a^2\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^(3/2)*(c+d/x),x)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)/x^2*(6*a^(3/2)*ln(1/2*(2*(a*x^2+b*x)^(1/2)*
a^(1/2)+2*a*x+b)/a^(1/2))*d*x^3*b+9*a^(1/2)*ln(1/2*(2*(a*x^2+b*x)
^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^2*c*x^3+12*a^2*(a*x^2+b*x)^(1/
2)*d*x^3+18*a*(a*x^2+b*x)^(1/2)*c*x^3*b-12*(a*x^2+b*x)^(3/2)*a*d*
x-12*(a*x^2+b*x)^(3/2)*c*x*b-4*d*(a*x^2+b*x)^(3/2)*b)/(x*(a*x+b))
^(1/2)/b
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2)*(c + d/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 0.25885, size = 1, normalized size = 0.01

$$\left[\frac{3\,(3\,bc+2\,ad)\sqrt{a}x\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,\left(3\,acx^2-2\,bd-2\,(3\,bc+4\,ad)x\right)\sqrt{\frac{ax+b}{x}}}{6\,x},\ \frac{3\,(3\,bc+2\,ad)\sqrt{-a}x\arctan}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2)*(c + d/x),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(3*b*c + 2*a*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a
*x + b)/x) + b) + 2*(3*a*c*x^2 - 2*b*d - 2*(3*b*c + 4*a*d)*x)*sqr
t((a*x + b)/x))/x, 1/3*(3*(3*b*c + 2*a*d)*sqrt(-a)*x*arctan(sqrt(
(a*x + b)/x)/sqrt(-a)) + (3*a*c*x^2 - 2*b*d - 2*(3*b*c + 4*a*d)*x
)*sqrt((a*x + b)/x))/x]
```

---

**Sympy [A]** time = 32.5682, size = 204, normalized size = 2.04

$$2a^{\frac{3}{2}}d\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)+3\sqrt{a}bc\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)-\frac{2a^2d\sqrt{x}}{\sqrt{b}\sqrt{\frac{ax}{b}+1}}+a\sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1}$$
$$-\frac{2a\sqrt{b}c\sqrt{x}}{\sqrt{\frac{ax}{b}+1}}-\frac{2a\sqrt{b}d}{\sqrt{x}\sqrt{\frac{ax}{b}+1}}-\frac{2b^{\frac{3}{2}}c}{\sqrt{x}\sqrt{\frac{ax}{b}+1}}+bd\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**(3/2)*(c+d/x),x)
```

```
[Out] 2*a**(3/2)*d*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 3*sqrt(a)*b*c*asinh
(sqrt(a)*sqrt(x)/sqrt(b)) - 2*a**2*d*sqrt(x)/(sqrt(b)*sqrt(a*x/b
+ 1)) + a*sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1) - 2*a*sqrt(b)*c*sqrt(
x)/sqrt(a*x/b + 1) - 2*a*sqrt(b)*d/(sqrt(x)*sqrt(a*x/b + 1)) - 2*
b**(3/2)*c/(sqrt(x)*sqrt(a*x/b + 1)) + b*d*Piecewise((-sqrt(a)/x,
 Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2)*(c + d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

# 3.135 $\int \left(a+\frac{b}{x}\right)^{3/2} dx$

**Optimal**. Leaf size=54

$$x\left(a+\frac{b}{x}\right)^{3/2}-3b\sqrt{a+\frac{b}{x}}+3\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[Out] -3*b*Sqrt[a + b/x] + (a + b/x)^(3/2)*x + 3*Sqrt[a]*b*ArcTanh[Sqrt
[a + b/x]/Sqrt[a]]
```

---

**Rubi [A]** time = 0.0767684, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454

$$x\left(a+\frac{b}{x}\right)^{3/2}-3b\sqrt{a+\frac{b}{x}}+3\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^(3/2),x]
```

```
[Out] -3*b*Sqrt[a + b/x] + (a + b/x)^(3/2)*x + 3*Sqrt[a]*b*ArcTanh[Sqrt
[a + b/x]/Sqrt[a]]
```

---

**Rubi in Sympy [A]** time = 7.71145, size = 44, normalized size = 0.81

$$3\sqrt{a}b\,\text{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-3b\sqrt{a+\frac{b}{x}}+x\left(a+\frac{b}{x}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**(3/2),x)
```

```
[Out] 3*sqrt(a)*b*atanh(sqrt(a + b/x)/sqrt(a)) - 3*b*sqrt(a + b/x) + x*
(a + b/x)**(3/2)
```

---

**Mathematica [A]** time = 0.0458004, size = 56, normalized size = 1.04

$$\sqrt{a+\frac{b}{x}}(ax-2b)+\frac{3}{2}\sqrt{a}b\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x)^(3/2),x]
```

```
[Out] Sqrt[a + b/x]*(-2*b + a*x) + (3*Sqrt[a]*b*Log[b + 2*a*x + 2*Sqrt[
a]*Sqrt[a + b/x]*x])/2
```

---

**Maple [B]** time = 0.006, size = 94, normalized size = 1.7

$$\frac{1}{2x}\sqrt{\frac{ax+b}{x}}\left(3\sqrt{a}b\ln\left(1/2\,\frac{2\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)x^2+6\,a\sqrt{ax^2+bx}x^2-4\,\left(ax^2+bx\right)^{3/2}\right)\frac{1}{\sqrt{x\,(ax+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^(3/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)/x*(3*a^(1/2)*b*ln(1/2*(2*(a*x^2+b*x)^(1/2)*
a^(1/2)+2*a*x+b)/a^(1/2))*x^2+6*a*(a*x^2+b*x)^(1/2)*x^2-4*(a*x^2+
b*x)^(3/2))/(x*(a*x+b))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.263802, size = 1, normalized size = 0.02

$$\left[\frac{3}{2}\sqrt{a}b\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+(ax-2\,b)\sqrt{\frac{ax+b}{x}},\,3\,\sqrt{-a}b\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)+(ax-2\,b)\sqrt{\frac{ax+b}{x}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [3/2*sqrt(a)*b*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (
a*x - 2*b)*sqrt((a*x + b)/x), 3*sqrt(-a)*b*arctan(sqrt((a*x + b)/
x)/sqrt(-a)) + (a*x - 2*b)*sqrt((a*x + b)/x)]
```

---

**Sympy [A]** time = 9.18015, size = 92, normalized size = 1.7

$$3\sqrt{a}b\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)+\frac{a^2x^{\frac{3}{2}}}{\sqrt{b}\sqrt{\frac{ax}{b}+1}}-\frac{a\sqrt{b}\sqrt{x}}{\sqrt{\frac{ax}{b}+1}}-\frac{2b^{\frac{3}{2}}}{\sqrt{x}\sqrt{\frac{ax}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**(3/2),x)
```

```
[Out] 3*sqrt(a)*b*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + a**2*x**(3/2)/(sqrt(
b)*sqrt(a*x/b + 1)) - a*sqrt(b)*sqrt(x)/sqrt(a*x/b + 1) - 2*b**(3
/2)/(sqrt(x)*sqrt(a*x/b + 1))
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.136 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx$

**Optimal**. Leaf size=106

$$-\frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}}+\frac{\sqrt{a}(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{ax\sqrt{a+\frac{b}{x}}}{c}$$

```
[Out] (a*Sqrt[a + b/x]*x)/c - (2*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[d]*Sqrt
[a + b/x])/Sqrt[b*c - a*d]])/(c^2*Sqrt[d]) + (Sqrt[a]*(3*b*c - 2*
a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^2
```

---

**Rubi [A]** time = 0.384434, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$-\frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}}+\frac{\sqrt{a}(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{ax\sqrt{a+\frac{b}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^(3/2)/(c + d/x),x]
```

```
[Out] (a*Sqrt[a + b/x]*x)/c - (2*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[d]*Sqrt
[a + b/x])/Sqrt[b*c - a*d]])/(c^2*Sqrt[d]) + (Sqrt[a]*(3*b*c - 2*
a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^2
```

---

**Rubi in Sympy [A]** time = 37.9565, size = 92, normalized size = 0.87

$$-\frac{\sqrt{a}\,(2ad-3bc)\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{ax\sqrt{a+\frac{b}{x}}}{c}+\frac{2\,(ad-bc)^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{c^2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**(3/2)/(c+d/x),x)
```

```
[Out] -sqrt(a)*(2*a*d - 3*b*c)*atanh(sqrt(a + b/x)/sqrt(a))/c**2 + a*x*
sqrt(a + b/x)/c + 2*(a*d - b*c)**(3/2)*atanh(sqrt(d)*sqrt(a + b/x
```

```
)/sqrt(a*d - b*c))/(c**2*sqrt(d))
```

---

**Mathematica [A]** time = 0.49446, size = 156, normalized size = 1.47

$$\frac{\frac{2(ad-bc)^{3/2}\log(cx+d)}{\sqrt{d}} - \frac{2(ad-bc)^{3/2}\log\left(2\sqrt{d}x\sqrt{a+\frac{b}{x}}\sqrt{ad-bc}-2adx+bcx-bd\right)}{\sqrt{d}} - \sqrt{a}(2ad-3bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)+2acx\sqrt{a+\frac{b}{x}}}{2c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x)^(3/2)/(c + d/x),x]
```

```
[Out] (2*a*c*Sqrt[a + b/x]*x + (2*(-(b*c) + a*d)^(3/2)*Log[d + c*x])/Sq
rt[d] - Sqrt[a]*(-3*b*c + 2*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a
 + b/x]*x] - (2*(-(b*c) + a*d)^(3/2)*Log[-(b*d) + b*c*x - 2*a*d*x
 + 2*Sqrt[d]*Sqrt[-(b*c) + a*d]*Sqrt[a + b/x]*x])/Sqrt[d])/(2*c^2
)
```

---

**Maple [B]** time = 0.02, size = 528, normalized size = 5.

$$\frac{x}{2\,dc^3}\sqrt{\frac{ax+b}{x}}\left(-2\,d^2\ln\left(1/2\,\frac{2\,\sqrt{x\,(ax+b)}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)a^2c\sqrt{\frac{(ad-bc)\,d}{c^2}}+3\,\ln\left(1/2\,\frac{2\,\sqrt{x\,(ax+b)}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)abdc^2\sqrt{\frac{(ad-bc)\,d}{c^2}} \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^(3/2)/(c+d/x),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(-2*d^2*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/
2)+2*a*x+b)/a^(1/2))*a^2*c*((a*d-b*c)*d/c^2)^(1/2)+3*ln(1/2*(2*(x
*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b*d*c^2*((a*d-b*c)*d/
c^2)^(1/2)+2*(x*(a*x+b))^(1/2)*a^(3/2)*d*c^2*((a*d-b*c)*d/c^2)^(1
/2)-2*(x*(a*x+b))^(1/2)*b*a^(1/2)*c^3*((a*d-b*c)*d/c^2)^(1/2)-2*d
^3*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*
x-b*d)/(c*x+d))*a^(5/2)+4*d^2*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*
d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(3/2)*b*c-2*ln((2*(x
*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x
+d))*b^2*d*a^(1/2)*c^2+2*b*(a*x^2+b*x)^(1/2)*a^(1/2)*c^3*((a*d-b*
c)*d/c^2)^(1/2)+b^2*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/
a^(1/2))*c^3*((a*d-b*c)*d/c^2)^(1/2)-ln(1/2*(2*(x*(a*x+b))^(1/2)*
a^(1/2)+2*a*x+b)/a^(1/2))*b^2*c^3*((a*d-b*c)*d/c^2)^(1/2))/(x*(a*
x+b))^(1/2)/d/a^(1/2)/c^3/((a*d-b*c)*d/c^2)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2)/(c + d/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.296247, size = 1, normalized size = 0.01

$$\left[\frac{2\,acx\sqrt{\frac{ax+b}{x}} - (3\,bc - 2\,ad)\sqrt{a}\log\left(2\,ax - 2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) - 2\,(bc - ad)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{2\,dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+bd-(bc-2\,ad)x}{cx+d}\right)}{2\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2)/(c + d/x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*x*sqrt((a*x + b)/x) - (3*b*c - 2*a*d)*sqrt(a)*log(2*a
*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(b*c - a*d)*sqrt(-(b*
c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b
*d - (b*c - 2*a*d)*x)/(c*x + d)))/c^2, (a*c*x*sqrt((a*x + b)/x) +
 (3*b*c - 2*a*d)*sqrt(-a)*arctan(sqrt((a*x + b)/x)/sqrt(-a)) - (b
*c - a*d)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sq
rt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)))/c^2, 1/2*(2*
a*c*x*sqrt((a*x + b)/x) - 4*(b*c - a*d)*sqrt((b*c - a*d)/d)*arcta
n(sqrt((a*x + b)/x)/sqrt((b*c - a*d)/d)) - (3*b*c - 2*a*d)*sqrt(a
)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b))/c^2, (a*c*x*sqr
t((a*x + b)/x) + (3*b*c - 2*a*d)*sqrt(-a)*arctan(sqrt((a*x + b)/x
)/sqrt(-a)) - 2*(b*c - a*d)*sqrt((b*c - a*d)/d)*arctan(sqrt((a*x
+ b)/x)/sqrt((b*c - a*d)/d)))/c^2]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{cx + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**(3/2)/(c+d/x),x)
```

```
[Out] Integral(x*(a + b/x)**(3/2)/(c*x + d), x)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2)/(c + d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.137 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx$

**Optimal**. Leaf size=156

$$-\frac{(bc-4ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{d}}+\frac{\sqrt{a}(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}-\frac{\sqrt{a+\frac{b}{x}}(bc-2ad)}{c^2\left(c+\frac{d}{x}\right)}+\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

```
[Out] -(((b*c - 2*a*d)*Sqrt[a + b/x])/(c^2*(c + d/x))) + (a*Sqrt[a + b/
x]*x)/(c*(c + d/x)) - ((b*c - 4*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt
[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3*Sqrt[d]) + (Sqrt[a]*(3*
b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^3
```

---

**Rubi [A]** time = 0.670228, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{(bc-4ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{d}}+\frac{\sqrt{a}(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}-\frac{\sqrt{a+\frac{b}{x}}(bc-2ad)}{c^2\left(c+\frac{d}{x}\right)}+\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^(3/2)/(c + d/x)^2,x]
```

```
[Out] -(((b*c - 2*a*d)*Sqrt[a + b/x])/(c^2*(c + d/x))) + (a*Sqrt[a + b/
x]*x)/(c*(c + d/x)) - ((b*c - 4*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt
[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3*Sqrt[d]) + (Sqrt[a]*(3*
b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^3
```

---

**Rubi in Sympy [A]** time = 68.3839, size = 129, normalized size = 0.83

$$-\frac{\sqrt{a}\,(4ad-3bc)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}+\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}\,(2ad-bc)}{c^2\left(c+\frac{d}{x}\right)}+\frac{\sqrt{ad-bc}\,(4ad-bc)\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{c^3\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**(3/2)/(c+d/x)**2,x)
```

```
[Out] -sqrt(a)*(4*a*d - 3*b*c)*atanh(sqrt(a + b/x)/sqrt(a))/c**3 + a*x*
sqrt(a + b/x)/(c*(c + d/x)) + sqrt(a + b/x)*(2*a*d - b*c)/(c**2*(
c + d/x)) + sqrt(a*d - b*c)*(4*a*d - b*c)*atanh(sqrt(d)*sqrt(a +
b/x)/sqrt(a*d - b*c))/(c**3*sqrt(d))
```

---

**Mathematica [C]** time = 0.478662, size = 231, normalized size = 1.48

$$-\frac{\frac{i(4a^2d^2-5abcd+b^2c^2)\log\left(\frac{2c^4\left(2\sqrt{d}x\sqrt{a+\frac{b}{x}}\sqrt{bc-ad}-2iadx-ib(d-cx)\right)}{\sqrt{d}(cx+d)\sqrt{bc-ad}\left(4a^2d^2-5abcd+b^2c^2\right)}\right)}{\sqrt{d}\sqrt{bc-ad}}-\frac{2cx\sqrt{a+\frac{b}{x}}(acx+2ad-bc)}{cx+d}+\sqrt{a}(4ad-3bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+\right.}{2c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x)^(3/2)/(c + d/x)^2,x]
```

```
[Out] -((-2*c*Sqrt[a + b/x]*x*(-(b*c) + 2*a*d + a*c*x))/(d + c*x) + Sqr
t[a]*(-3*b*c + 4*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x]
+ (I*(b^2*c^2 - 5*a*b*c*d + 4*a^2*d^2)*Log[(2*c^4*((-2*I)*a*d*x +
 2*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[a + b/x]*x - I*b*(d - c*x)))/(Sqr
t[d]*Sqrt[b*c - a*d]*(b^2*c^2 - 5*a*b*c*d + 4*a^2*d^2)*(d + c*x))
])/(Sqrt[d]*Sqrt[b*c - a*d]))/(2*c^3)
```

---

**Maple [B]** time = 0.02, size = 1202, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^(3/2)/(c+d/x)^2,x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(-4*a^(5/2)*ln(1/2*(2*(x*(a*x+b))^(1/2)*a
^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*c*d^4+2*c^4*(x*(
a*x+b))^(3/2)*a*((a*d-b*c)*d/c^2)^(1/2)*d+2*(x*(a*x+b))^(1/2)*((a
*d-b*c)*d/c^2)^(1/2)*x*b^2*c^5-4*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*
c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*a^3*c*d^4+ln((2*(
x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*
x+d))*x*b^3*c^4*d+4*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*a^2
*c^2*d^3+2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*b^2*c^4*d+9*
ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b
*d)/(c*x+d))*a^2*b*c*d^4-6*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c
^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a*b^2*c^2*d^3-4*a^(5/2)*l
n(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d
/c^2)^(1/2)*x*c^2*d^3-2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)
*x^2*a^2*c^4*d+2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*x^2*a*
b*c^5+7*a^(3/2)*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1
```

```
/2))*((a*d-b*c)*d/c^2)^(1/2)*b*c^2*d^3+2*(x*(a*x+b))^(1/2)*((a*d-
b*c)*d/c^2)^(1/2)*x*a^2*c^3*d^2-3*a^(1/2)*ln(1/2*(2*(x*(a*x+b))^(
1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*b^2*c^3*d^
2+9*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c
*x-b*d)/(c*x+d))*x*a^2*b*c^2*d^3-6*ln((2*(x*(a*x+b))^(1/2)*((a*d-
b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*a*b^2*c^3*d^2-6
*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*a*b*c^3*d^2-4*ln((2*(x
*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x
+d))*a^3*d^5+7*a^(3/2)*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+
b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x*b*c^3*d^2-3*a^(1/2)*ln(1/2*
(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^
(1/2)*x*b^2*c^4*d-4*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*x*a
*b*c^4*d-2*c^5*(x*(a*x+b))^(3/2)*b*((a*d-b*c)*d/c^2)^(1/2)+ln((2*
(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c
*x+d))*b^3*c^3*d^2)/(x*(a*x+b))^(1/2)/c^4/((a*d-b*c)*d/c^2)^(1/2)
/(a*d-b*c)/(c*x+d)/d
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2)/(c + d/x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.292952, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2)/(c + d/x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*
a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (b*c*d - 4*a*d^2 + (b*
c^2 - 4*a*c*d)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*
d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(
a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqrt((a*x + b)/x))/(c^4*x + c^3*
d), -1/2*(2*(b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt((b*c - a
*d)/d)*arctan(sqrt((a*x + b)/x)/sqrt((b*c - a*d)/d)) + (3*b*c*d -
 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x
*sqrt((a*x + b)/x) + b) - 2*(a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqr
t((a*x + b)/x))/(c^4*x + c^3*d), 1/2*(2*(3*b*c*d - 4*a*d^2 + (3*b
*c^2 - 4*a*c*d)*x)*sqrt(-a)*arctan(sqrt((a*x + b)/x)/sqrt(-a)) -
(b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(-(b*c - a*d)/d)*log(
```

```
(2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*
d)*x)/(c*x + d)) + 2*(a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqrt((a*x
+ b)/x))/(c^4*x + c^3*d), ((3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*
d)*x)*sqrt(-a)*arctan(sqrt((a*x + b)/x)/sqrt(-a)) - (b*c*d - 4*a*
d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt((b*c - a*d)/d)*arctan(sqrt((a*x +
 b)/x)/sqrt((b*c - a*d)/d)) + (a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*s
qrt((a*x + b)/x))/(c^4*x + c^3*d)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{(cx + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**(3/2)/(c+d/x)**2,x)
```

```
[Out] Integral(x**2*(a + b/x)**(3/2)/(c*x + d)**2, x)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2)/(c + d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.138 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx$

**Optimal**. Leaf size=209

$$-\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4\sqrt{d}\sqrt{bc-ad}}+\frac{3\sqrt{a}(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}-\frac{3\sqrt{a+\frac{b}{x}}(bc-4ad)}{4c^3\left(c+\frac{d}{x}\right)}-\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{2c^2\left(c+\frac{d}{x}\right)^2}+\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}$$

```
[Out] -((b*c - 3*a*d)*Sqrt[a + b/x])/(2*c^2*(c + d/x)^2) - (3*(b*c - 4*
a*d)*Sqrt[a + b/x])/(4*c^3*(c + d/x)) + (a*Sqrt[a + b/x]*x)/(c*(c
 + d/x)^2) - (3*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]
*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(4*c^4*Sqrt[d]*Sqrt[b*c - a*d])
 + (3*Sqrt[a]*(b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^4
```

---

**Rubi [A]** time = 0.983858, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4\sqrt{d}\sqrt{bc-ad}}+\frac{3\sqrt{a}(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}-\frac{3\sqrt{a+\frac{b}{x}}(bc-4ad)}{4c^3\left(c+\frac{d}{x}\right)}-\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{2c^2\left(c+\frac{d}{x}\right)^2}+\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^(3/2)/(c + d/x)^3,x]
```

```
[Out] -((b*c - 3*a*d)*Sqrt[a + b/x])/(2*c^2*(c + d/x)^2) - (3*(b*c - 4*
a*d)*Sqrt[a + b/x])/(4*c^3*(c + d/x)) + (a*Sqrt[a + b/x]*x)/(c*(c
 + d/x)^2) - (3*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]
*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(4*c^4*Sqrt[d]*Sqrt[b*c - a*d])
 + (3*Sqrt[a]*(b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^4
```

---

**Rubi in Sympy [A]** time = 106.166, size = 182, normalized size = 0.87

$$-\frac{6\sqrt{a}\left(ad-\frac{bc}{2}\right)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}+\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}+\frac{\sqrt{a+\frac{b}{x}}\,(3ad-bc)}{2c^2\left(c+\frac{d}{x}\right)^2}+\frac{3\sqrt{a+\frac{b}{x}}\,(4ad-bc)}{4c^3\left(c+\frac{d}{x}\right)}+\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{4c^4\sqrt{d}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**(3/2)/(c+d/x)**3,x)
```

```
[Out] -6*sqrt(a)*(a*d - b*c/2)*atanh(sqrt(a + b/x)/sqrt(a))/c**4 + a*x*
sqrt(a + b/x)/(c*(c + d/x)**2) + sqrt(a + b/x)*(3*a*d - b*c)/(2*c
**2*(c + d/x)**2) + 3*sqrt(a + b/x)*(4*a*d - b*c)/(4*c**3*(c + d/
x)) + 3*(8*a**2*d**2 - 8*a*b*c*d + b**2*c**2)*atanh(sqrt(d)*sqrt(
a + b/x)/sqrt(a*d - b*c))/(4*c**4*sqrt(d)*sqrt(a*d - b*c))
```

---

**Mathematica [C]** time = 0.512074, size = 256, normalized size = 1.22

$$\frac{\frac{3i\left(8a^2d^2-8abcd+b^2c^2\right)\log\left(\frac{8c^5\left(2\sqrt{d}x\sqrt{a+\frac{b}{x}}\sqrt{bc-ad}+2iadx+ib(d-cx)\right)}{3\sqrt{d}(cx+d)\sqrt{bc-ad}\left(8a^2d^2-8abcd+b^2c^2\right)}\right)}{\sqrt{d}\sqrt{bc-ad}}+\frac{2cx\sqrt{a+\frac{b}{x}}\left(2a\left(2c^2x^2+9cdx+6d^2\right)-bc(5cx+3d)\right)}{(cx+d)^2}-12\sqrt{a}(2ad-bc)\log\left(2\sqrt{a}\right.}{8c^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x)^(3/2)/(c + d/x)^3,x]
```

```
[Out] ((2*c*Sqrt[a + b/x]*x*(-(b*c*(3*d + 5*c*x)) + 2*a*(6*d^2 + 9*c*d*
x + 2*c^2*x^2)))/(d + c*x)^2 - 12*Sqrt[a]*(-(b*c) + 2*a*d)*Log[b
+ 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x] + ((3*I)*(b^2*c^2 - 8*a*b*c*
d + 8*a^2*d^2)*Log[(8*c^5*((2*I)*a*d*x + 2*Sqrt[d]*Sqrt[b*c - a*d
]*Sqrt[a + b/x]*x + I*b*(d - c*x)))/(3*Sqrt[d]*Sqrt[b*c - a*d]*(b
^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*(d + c*x))])/(Sqrt[d]*Sqrt[b*c -
a*d]))/(8*c^4)
```

---

**Maple [B]** time = 0.021, size = 2373, normalized size = 11.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^(3/2)/(c+d/x)^3,x)
```

```
[Out] 1/8*((a*x+b)/x)^(1/2)*x*(-24*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d
/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^4*d^7+60*a^(5/2)*ln(1
/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^
2)^(1/2)*x^2*b*c^4*d^3+120*a^(5/2)*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^
(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x*b*c^3*d^4-48*a^
(3/2)*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d
-b*c)*d/c^2)^(1/2)*x^2*b^2*c^5*d^2+18*(x*(a*x+b))^(1/2)*((a*d-b*c
)*d/c^2)^(1/2)*x^3*a^2*b*c^6*d-96*a^(3/2)*ln(1/2*(2*(x*(a*x+b))^(
1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x*b^2*c^4*
d^3+12*a^(1/2)*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/
2))*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^3*c^6*d-18*(x*(a*x+b))^(3/2)*((
a*d-b*c)*d/c^2)^(1/2)*x*a*b*c^6*d-18*(x*(a*x+b))^(1/2)*((a*d-b*c)
*d/c^2)^(1/2)*x^2*a^2*b*c^5*d^2+24*(x*(a*x+b))^(1/2)*((a*d-b*c)*d
/c^2)^(1/2)*x^2*a*b^2*c^6*d+24*a^(1/2)*ln(1/2*(2*(x*(a*x+b))^(1/2
)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x*b^3*c^5*d^2
-90*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*x*a^2*b*c^4*d^3+66*
(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*x*a*b^2*c^5*d^2-48*a^(3
/2)*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b
*c)*d/c^2)^(1/2)*b^2*c^3*d^4-3*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)
*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^4*c^4*d^3-24*a^(7/2
)*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c
)*d/c^2)^(1/2)*c*d^6+6*(x*(a*x+b))^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*
x*b^2*c^7-6*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^3*c^7
-24*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c
*x-b*d)/(c*x+d))*x^2*a^4*c^2*d^5-3*ln((2*(x*(a*x+b))^(1/2)*((a*d-
b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*b^4*c^6*d+8*(
x*(a*x+b))^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*a^2*c^4*d^3+2*(x*(a*x+b)
)^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*b^2*c^6*d-48*ln((2*(x*(a*x+b))^(1
/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*a^4*c
*d^6-6*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+
b*c*x-b*d)/(c*x+d))*x*b^4*c^5*d^2+24*(x*(a*x+b))^(1/2)*((a*d-b*c)
*d/c^2)^(1/2)*a^3*c^2*d^5-6*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(
1/2)*b^3*c^5*d^2+72*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/
2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^3*b*c*d^6-75*ln((2*(x*(a*x+b))
^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^2*
b^2*c^2*d^5+30*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-
2*a*d*x+b*c*x-b*d)/(c*x+d))*a*b^3*c^3*d^4+12*(x*(a*x+b))^(3/2)*((
a*d-b*c)*d/c^2)^(1/2)*x*a^2*c^5*d^2+36*(x*(a*x+b))^(1/2)*((a*d-b*
c)*d/c^2)^(1/2)*x*a^3*c^3*d^4-12*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c
^2)^(1/2)*x*b^3*c^6*d+12*a^(1/2)*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1
/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*b^3*c^4*d^3+144*ln(
(2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)
/(c*x+d))*x*a^3*b*c^2*d^5-150*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*
d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*a^2*b^2*c^3*d^4+60*l
n((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*
d)/(c*x+d))*x*a*b^3*c^4*d^3-54*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2
)^(1/2)*a^2*b*c^3*d^4+36*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2
)*a*b^2*c^4*d^3-6*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*x^3*a
*b^2*c^7+60*a^(5/2)*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/
a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*b*c^2*d^5-48*a^(7/2)*ln(1/2*(2*(
x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2
```

```
)*x*c^2*d^5-12*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*x^3*a^3*
c^5*d^2-24*a^(7/2)*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a
^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^2*c^3*d^4+72*ln((2*(x*(a*x+b))^
(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*a
^3*b*c^3*d^4-75*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c
-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*a^2*b^2*c^4*d^3+30*ln((2*(x*(a*x
+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*
x^2*a*b^3*c^5*d^2-10*(x*(a*x+b))^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*a*
b*c^5*d^2)/(x*(a*x+b))^(1/2)/c^5/(a*d-b*c)^2/((a*d-b*c)*d/c^2)^(1
/2)/(c*x+d)^2/d
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2)/(c + d/x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.328166, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2)/(c + d/x)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(12*(b*c*d^2 - 2*a*d^3 + (b*c^3 - 2*a*c^2*d)*x^2 + 2*(b*c^2
*d - 2*a*c*d^2)*x)*sqrt(-b*c*d + a*d^2)*sqrt(a)*log(2*a*x - 2*sqr
t(a)*x*sqrt((a*x + b)/x) + b) - 2*(4*a*c^3*x^3 - (5*b*c^3 - 18*a*
c^2*d)*x^2 - 3*(b*c^2*d - 4*a*c*d^2)*x)*sqrt(-b*c*d + a*d^2)*sqrt
((a*x + b)/x) - 3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c
^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 8*a*b*c^2*
d^2 + 8*a^2*c*d^3)*x)*log(-(2*(b*c*d - a*d^2)*x*sqrt((a*x + b)/x)
 - sqrt(-b*c*d + a*d^2)*(b*d - (b*c - 2*a*d)*x))/(c*x + d)))/((c^
6*x^2 + 2*c^5*d*x + c^4*d^2)*sqrt(-b*c*d + a*d^2)), 1/8*(24*(b*c*
d^2 - 2*a*d^3 + (b*c^3 - 2*a*c^2*d)*x^2 + 2*(b*c^2*d - 2*a*c*d^2)
*x)*sqrt(-b*c*d + a*d^2)*sqrt(-a)*arctan(sqrt((a*x + b)/x)/sqrt(-
a)) + 2*(4*a*c^3*x^3 - (5*b*c^3 - 18*a*c^2*d)*x^2 - 3*(b*c^2*d -
4*a*c*d^2)*x)*sqrt(-b*c*d + a*d^2)*sqrt((a*x + b)/x) + 3*(b^2*c^2
*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c
^2*d^2)*x^2 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*log(
-(2*(b*c*d - a*d^2)*x*sqrt((a*x + b)/x) - sqrt(-b*c*d + a*d^2)*(b
*d - (b*c - 2*a*d)*x))/(c*x + d)))/((c^6*x^2 + 2*c^5*d*x + c^4*d^
2)*sqrt(-b*c*d + a*d^2)), -1/4*(6*(b*c*d^2 - 2*a*d^3 + (b*c^3 - 2
```

```
*a*c^2*d)*x^2 + 2*(b*c^2*d - 2*a*c*d^2)*x)*sqrt(b*c*d - a*d^2)*sq
rt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - (4*a*c^3*x
^3 - (5*b*c^3 - 18*a*c^2*d)*x^2 - 3*(b*c^2*d - 4*a*c*d^2)*x)*sqrt
(b*c*d - a*d^2)*sqrt((a*x + b)/x) + 3*(b^2*c^2*d^2 - 8*a*b*c*d^3
+ 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^
2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*arctan(-(b*c - a*d)/(sq
rt(b*c*d - a*d^2)*sqrt((a*x + b)/x))))/((c^6*x^2 + 2*c^5*d*x + c^
4*d^2)*sqrt(b*c*d - a*d^2)), 1/4*(12*(b*c*d^2 - 2*a*d^3 + (b*c^3
- 2*a*c^2*d)*x^2 + 2*(b*c^2*d - 2*a*c*d^2)*x)*sqrt(b*c*d - a*d^2)
*sqrt(-a)*arctan(sqrt((a*x + b)/x)/sqrt(-a)) + (4*a*c^3*x^3 - (5*
b*c^3 - 18*a*c^2*d)*x^2 - 3*(b*c^2*d - 4*a*c*d^2)*x)*sqrt(b*c*d -
 a*d^2)*sqrt((a*x + b)/x) - 3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*
d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d
- 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*arctan(-(b*c - a*d)/(sqrt(b*c*d
 - a*d^2)*sqrt((a*x + b)/x))))/((c^6*x^2 + 2*c^5*d*x + c^4*d^2)*s
qrt(b*c*d - a*d^2))]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{(cx + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**(3/2)/(c+d/x)**3,x)
```

```
[Out] Integral(x**3*(a + b/x)**(3/2)/(c*x + d)**3, x)
```

---

**GIAC/XCAS [A]** time = 0.550936, size = 4, normalized size = 0.02

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(3/2)/(c + d/x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

# 3.139 $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 \, dx$

**Optimal**. Leaf size=198

$$a^{3/2}c^2(6ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(-10a^2d^2+135abcd+469b^2c^2\right)+\frac{5bd(10ad+89bc)}{x}\right)}{315b^2}$$
$$-\frac{1}{3}c^2\left(a+\frac{b}{x}\right)^{3/2}(6ad+5bc)-ac^2\sqrt{a+\frac{b}{x}}(6ad+5bc)+x\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3-\frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2$$

```
[Out] -(a*c^2*(5*b*c + 6*a*d)*Sqrt[a + b/x]) - (c^2*(5*b*c + 6*a*d)*(a
+ b/x)^(3/2))/3 - (11*d*(a + b/x)^(5/2)*(c + d/x)^2)/9 - (d*(a +
b/x)^(5/2)*(2*(469*b^2*c^2 + 135*a*b*c*d - 10*a^2*d^2) + (5*b*d*(
89*b*c + 10*a*d))/x))/(315*b^2) + (a + b/x)^(5/2)*(c + d/x)^3*x +
 a^(3/2)*c^2*(5*b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

---

**Rubi [A]** time = 0.499922, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$a^{3/2}c^2(6ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(-10a^2d^2+135abcd+469b^2c^2\right)+\frac{5bd(10ad+89bc)}{x}\right)}{315b^2}$$
$$-\frac{1}{3}c^2\left(a+\frac{b}{x}\right)^{3/2}(6ad+5bc)-ac^2\sqrt{a+\frac{b}{x}}(6ad+5bc)+x\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3-\frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^(5/2)*(c + d/x)^3,x]
```

```
[Out] -(a*c^2*(5*b*c + 6*a*d)*Sqrt[a + b/x]) - (c^2*(5*b*c + 6*a*d)*(a
+ b/x)^(3/2))/3 - (11*d*(a + b/x)^(5/2)*(c + d/x)^2)/9 - (d*(a +
b/x)^(5/2)*(2*(469*b^2*c^2 + 135*a*b*c*d - 10*a^2*d^2) + (5*b*d*(
89*b*c + 10*a*d))/x))/(315*b^2) + (a + b/x)^(5/2)*(c + d/x)^3*x +
 a^(3/2)*c^2*(5*b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

---

**Rubi in Sympy [A]** time = 54.8412, size = 184, normalized size = 0.93

$$a^{\frac{3}{2}}c^2(6ad+5bc)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-ac^2\sqrt{a+\frac{b}{x}}(6ad+5bc)-\frac{c^2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}(6ad+5bc)}{3}-\frac{11d\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(c+\frac{d}{x}\right)^2}{9}+x\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(c+\frac{d}{x}\right)^3+\frac{8d\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(\frac{5a^2d^2}{2}-\frac{135abcd}{4}-\frac{469b^2c^2}{4}-\frac{5bd(10ad+89bc)}{8x}\right)}{315b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**(5/2)*(c+d/x)**3,x)
```

```
[Out] a**(3/2)*c**2*(6*a*d + 5*b*c)*atanh(sqrt(a + b/x)/sqrt(a)) - a*c*
*2*sqrt(a + b/x)*(6*a*d + 5*b*c) - c**2*(a + b/x)**(3/2)*(6*a*d +
 5*b*c)/3 - 11*d*(a + b/x)**(5/2)*(c + d/x)**2/9 + x*(a + b/x)**(
5/2)*(c + d/x)**3 + 8*d*(a + b/x)**(5/2)*(5*a**2*d**2/2 - 135*a*b
*c*d/4 - 469*b**2*c**2/4 - 5*b*d*(10*a*d + 89*b*c)/(8*x))/(315*b*
*2)
```

---

**Mathematica [A]** time = 0.354917, size = 212, normalized size = 1.07

$$\frac{1}{2}a^{3/2}c^2(6ad+5bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)+\frac{\sqrt{a+\frac{b}{x}}\left(20a^4d^3x^4-10a^3bd^2x^3(27cx+d)-3a^2b^2x^2\left(-105c^3x^3+966c^2dx^2+270cd^2x+50d^3\right)-2ab^3x\left(735c^3x^3+693c^2d\right.\right.}{315b^2x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x)^(5/2)*(c + d/x)^3,x]
```

```
[Out] (Sqrt[a + b/x]*(20*a^4*d^3*x^4 - 10*a^3*b*d^2*x^3*(d + 27*c*x) -
3*a^2*b^2*x^2*(50*d^3 + 270*c*d^2*x + 966*c^2*d*x^2 - 105*c^3*x^3
) - 2*b^4*(35*d^3 + 135*c*d^2*x + 189*c^2*d*x^2 + 105*c^3*x^3) -
2*a*b^3*x*(95*d^3 + 405*c*d^2*x + 693*c^2*d*x^2 + 735*c^3*x^3)))/
(315*b^2*x^4) + (a^(3/2)*c^2*(5*b*c + 6*a*d)*Log[b + 2*a*x + 2*Sq
rt[a]*Sqrt[a + b/x]*x])/2
```

---

**Maple [B]** time = 0.022, size = 434, normalized size = 2.2

$$\frac{1}{630\,x^5b^2}\sqrt{\frac{ax+b}{x}}\left(1890\,a^{5/2}c^2\ln\left(1/2\,\frac{2\,\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)dx^6b^2+1575\,a^{3/2}c^3b^3\ln\left(1/2\,\frac{2\,\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^(5/2)*(c+d/x)^3,x)
```

```
[Out] 1/630*((a*x+b)/x)^(1/2)/x^5/b^2*(1890*a^(5/2)*c^2*ln(1/2*(2*(a*x^
2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*d*x^6*b^2+1575*a^(3/2)*c^3
*b^3*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^6+37
80*a^3*c^2*(a*x^2+b*x)^(1/2)*d*x^6*b+3150*a^2*c^3*(a*x^2+b*x)^(1/
2)*x^6*b^2-3780*a^2*c^2*(a*x^2+b*x)^(3/2)*d*x^4*b-2520*a*c^3*(a*x
^2+b*x)^(3/2)*x^4*b^2+40*(a*x^2+b*x)^(3/2)*x^3*a^3*d^3-540*(a*x^2
+b*x)^(3/2)*x^3*a^2*b*c*d^2-2016*(a*x^2+b*x)^(3/2)*x^3*a*b^2*c^2*
d-420*(a*x^2+b*x)^(3/2)*x^3*b^3*c^3-60*(a*x^2+b*x)^(3/2)*x^2*a^2*
b*d^3-1080*(a*x^2+b*x)^(3/2)*x^2*a*b^2*c*d^2-756*(a*x^2+b*x)^(3/2
)*x^2*b^3*c^2*d-240*(a*x^2+b*x)^(3/2)*x*a*b^2*d^3-540*(a*x^2+b*x)
^(3/2)*x*b^3*c*d^2-140*(a*x^2+b*x)^(3/2)*b^3*d^3)/(x*(a*x+b))^(1/
2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2)*(c + d/x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.261345, size = 1, normalized size = 0.01

$$\left[\frac{315\left(5\,ab^3c^3+6\,a^2b^2c^2d\right)\sqrt{a}x^4\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(315\,a^2b^2c^3x^5-70\,b^4d^3-2\left(735\,ab^3c^3+1449\,a^2b^2c^2d+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2)*(c + d/x)^3,x, algorithm="fricas")
```

```
[Out] [1/630*(315*(5*a*b^3*c^3 + 6*a^2*b^2*c^2*d)*sqrt(a)*x^4*log(2*a*x
 + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(315*a^2*b^2*c^3*x^5 -
70*b^4*d^3 - 2*(735*a*b^3*c^3 + 1449*a^2*b^2*c^2*d + 135*a^3*b*c*
d^2 - 10*a^4*d^3)*x^4 - 2*(105*b^4*c^3 + 693*a*b^3*c^2*d + 405*a^
2*b^2*c*d^2 + 5*a^3*b*d^3)*x^3 - 6*(63*b^4*c^2*d + 135*a*b^3*c*d^
2 + 25*a^2*b^2*d^3)*x^2 - 10*(27*b^4*c*d^2 + 19*a*b^3*d^3)*x)*sqr
t((a*x + b)/x))/(b^2*x^4), 1/315*(315*(5*a*b^3*c^3 + 6*a^2*b^2*c^
2*d)*sqrt(-a)*x^4*arctan(sqrt((a*x + b)/x)/sqrt(-a)) + (315*a^2*b
^2*c^3*x^5 - 70*b^4*d^3 - 2*(735*a*b^3*c^3 + 1449*a^2*b^2*c^2*d +
 135*a^3*b*c*d^2 - 10*a^4*d^3)*x^4 - 2*(105*b^4*c^3 + 693*a*b^3*c
^2*d + 405*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^3 - 6*(63*b^4*c^2*d + 1
35*a*b^3*c*d^2 + 25*a^2*b^2*d^3)*x^2 - 10*(27*b^4*c*d^2 + 19*a*b^
3*d^3)*x)*sqrt((a*x + b)/x))/(b^2*x^4)]
```

---

**Sympy [A]** time = 82.497, size = 5557, normalized size = 28.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**(5/2)*(c+d/x)**3,x)
```

```
[Out] 32*a**(29/2)*b**(27/2)*d**3*x**10*sqrt(a*x/b + 1)/(315*a**(21/2)*
b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)
*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2
)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)
*b**21*x**(9/2)) + 176*a**(27/2)*b**(29/2)*d**3*x**9*sqrt(a*x/b +
 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/
2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15
/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(1
1/2) + 315*a**(9/2)*b**21*x**(9/2)) + 396*a**(25/2)*b**(31/2)*d**
3*x**8*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(
19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**
(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a*
*(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 462*a**(
23/2)*b**(33/2)*d**3*x**7*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x*
*(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x
**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*
x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x
**(9/2)) + 210*a**(21/2)*b**(35/2)*d**3*x**6*sqrt(a*x/b + 1)/(315
*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 472
5*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 47
25*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 3
15*a**(9/2)*b**21*x**(9/2)) - 32*a**(21/2)*b**(11/2)*d**3*x**6*sq
rt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*
x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7
/2)) - 378*a**(19/2)*b**(37/2)*d**3*x**5*sqrt(a*x/b + 1)/(315*a**
(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a*
*(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a
**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a
**(9/2)*b**21*x**(9/2)) - 48*a**(19/2)*b**(13/2)*c*d**2*x**6*sqrt
(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x*
```

```
*(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2
)) - 80*a**(19/2)*b**(13/2)*d**3*x**5*sqrt(a*x/b + 1)/(105*a**(13
/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*
b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 1134*a**(17/2)*b**
(39/2)*d**3*x**4*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) +
 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2)
+ 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2)
 + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2))
- 120*a**(17/2)*b**(15/2)*c*d**2*x**5*sqrt(a*x/b + 1)/(105*a**(13
/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*
b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 60*a**(17/2)*b**(1
5/2)*d**3*x**4*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 31
5*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**
(7/2)*b**10*x**(7/2)) - 1494*a**(15/2)*b**(41/2)*d**3*x**3*sqrt(a
*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x
**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*
x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20
*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 90*a**(15/2)*b**(17/2
)*c*d**2*x**4*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315
*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(
7/2)*b**10*x**(7/2)) - 80*a**(15/2)*b**(17/2)*d**3*x**3*sqrt(a*x/
b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/
2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) +
4*a**(15/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*
x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 1098*a**(13/2)*b**(43/2)*
d**3*x**2*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a
**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*
a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890
*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 120*a
**(13/2)*b**(19/2)*c*d**2*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**
7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x*
*(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 200*a**(13/2)*b**(19/2)*d
**3*x**2*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(
11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*
b**10*x**(7/2)) + 24*a**(13/2)*b**(5/2)*c*d**2*x**3*sqrt(a*x/b +
1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + 2*a*
*(13/2)*b**(5/2)*d**3*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(
7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 430*a**(11/2)*b**(45/2)*d**3*
x*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)
*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2
)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/
2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 300*a**(11/2)
*b**(21/2)*c*d**2*x**2*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13
/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) +
 105*a**(7/2)*b**10*x**(7/2)) - 192*a**(11/2)*b**(21/2)*d**3*x*sq
rt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*
x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7
/2)) + 12*a**(11/2)*b**(7/2)*c**2*d*x**3*sqrt(a*x/b + 1)/(15*a**(
7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + 12*a**(11/2)*b*
*(7/2)*c*d**2*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 1
5*a**(5/2)*b**4*x**(5/2)) - 8*a**(11/2)*b**(7/2)*d**3*x*sqrt(a*x/
b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) -
70*a**(9/2)*b**(47/2)*d**3*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x
**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*
x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19
*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*
```

x**(9/2)) - 288*a**(9/2)*b**(23/2)*c*d**2*x*sqrt(a*x/b + 1)/(105*
a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 60*a**(9/2)*
b**(23/2)*d**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 31
5*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**
(7/2)*b**10*x**(7/2)) + 6*a**(9/2)*b**(9/2)*c**2*d*x**2*sqrt(a*x/
b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) -
48*a**(9/2)*b**(9/2)*c*d**2*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x
**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6*a**(9/2)*b**(9/2)*d**3*s
qrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(
5/2)) - 90*a**(7/2)*b**(25/2)*c*d**2*sqrt(a*x/b + 1)/(105*a**(13/
2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b
**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 24*a**(7/2)*b**(11/
2)*c**2*d*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5
/2)*b**4*x**(5/2)) - 36*a**(7/2)*b**(11/2)*c*d**2*sqrt(a*x/b + 1)
/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 18*a**
(5/2)*b**(13/2)*c**2*d*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2)
 + 15*a**(5/2)*b**4*x**(5/2)) + 6*a**(5/2)*c**2*d*asinh(sqrt(a)*s
qrt(x)/sqrt(b)) + 5*a**(3/2)*b*c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b)
) - 32*a**15*b**13*d**3*x**(21/2)/(315*a**(21/2)*b**15*x**(21/2)
+ 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2)
 + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2
) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2))
 - 192*a**14*b**14*d**3*x**(19/2)/(315*a**(21/2)*b**15*x**(21/2)
+ 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2)
 + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2
) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2))
 - 480*a**13*b**15*d**3*x**(17/2)/(315*a**(21/2)*b**15*x**(21/2)
+ 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2)
 + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2
) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2))
 - 640*a**12*b**16*d**3*x**(15/2)/(315*a**(21/2)*b**15*x**(21/2)
+ 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2)
 + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2
) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2))
 - 480*a**11*b**17*d**3*x**(13/2)/(315*a**(21/2)*b**15*x**(21/2)
+ 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2)
 + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2
) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2))
 + 32*a**11*b**5*d**3*x**(13/2)/(105*a**(13/2)*b**7*x**(13/2) + 3
15*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a*
*(7/2)*b**10*x**(7/2)) - 192*a**10*b**18*d**3*x**(11/2)/(315*a**(
21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**
(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a*
*(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a*
*(9/2)*b**21*x**(9/2)) + 48*a**10*b**6*c*d**2*x**(13/2)/(105*a**(
13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2
)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 96*a**10*b**6*d*
*3*x**(11/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x
**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/
2)) - 32*a**9*b**19*d**3*x**(9/2)/(315*a**(21/2)*b**15*x**(21/2)
+ 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2)
 + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2
) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2))
 + 144*a**9*b**7*c*d**2*x**(11/2)/(105*a**(13/2)*b**7*x**(13/2) +
 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*

```
a**(7/2)*b**10*x**(7/2)) + 96*a**9*b**7*d**3*x**(9/2)/(105*a**(13
/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*
b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 144*a**8*b**8*c*d*
*2*x**(9/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x*
*(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2
)) + 32*a**8*b**8*d**3*x**(7/2)/(105*a**(13/2)*b**7*x**(13/2) + 3
15*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a*
*(7/2)*b**10*x**(7/2)) - 4*a**8*b*d**3*x**(7/2)/(15*a**(7/2)*b**3
*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + 48*a**7*b**9*c*d**2*x**(
7/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2)
 + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 24
*a**7*b**2*c*d**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/
2)*b**4*x**(5/2)) - 4*a**7*b**2*d**3*x**(5/2)/(15*a**(7/2)*b**3*x
**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 12*a**6*b**3*c**2*d*x**(7/
2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 24*a
**6*b**3*c*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)
*b**4*x**(5/2)) - 12*a**5*b**4*c**2*d*x**(5/2)/(15*a**(7/2)*b**3*
x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6*a**3*c**2*d*sqrt(x)/(sq
rt(b)*sqrt(a*x/b + 1)) + a**2*sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1
) - 4*a**2*sqrt(b)*c**3*sqrt(x)/sqrt(a*x/b + 1) - 6*a**2*sqrt(b)*
c**2*d/(sqrt(x)*sqrt(a*x/b + 1)) + 3*a**2*c*d**2*Piecewise((-sqrt
(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) - 4*a*b**(3/
2)*c**3/(sqrt(x)*sqrt(a*x/b + 1)) + 6*a*b*c**2*d*Piecewise((-sqrt
(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b**2*c**3*
Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), Tru
e))
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2)*(c + d/x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.140 $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^2 \, dx$

**Optimal**. Leaf size=152

$$a^{3/2}c(4ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c\left(a+\frac{b}{x}\right)^{5/2}(4ad+5bc)}{5a}$$
$$-\frac{1}{3}c\left(a+\frac{b}{x}\right)^{3/2}(4ad+5bc)-ac\sqrt{a+\frac{b}{x}}(4ad+5bc)-\frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}$$

```
[Out] -(a*c*(5*b*c + 4*a*d)*Sqrt[a + b/x]) - (c*(5*b*c + 4*a*d)*(a + b/
x)^(3/2))/3 - (c*(5*b*c + 4*a*d)*(a + b/x)^(5/2))/(5*a) - (2*d^2*
(a + b/x)^(7/2))/(7*b) + (c^2*(a + b/x)^(7/2)*x)/a + a^(3/2)*c*(5
*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

---

**Rubi [A]** time = 0.299007, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$a^{3/2}c(4ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c\left(a+\frac{b}{x}\right)^{5/2}(4ad+5bc)}{5a}$$
$$-\frac{1}{3}c\left(a+\frac{b}{x}\right)^{3/2}(4ad+5bc)-ac\sqrt{a+\frac{b}{x}}(4ad+5bc)-\frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^(5/2)*(c + d/x)^2,x]
```


```
[Out] -(a*c*(5*b*c + 4*a*d)*Sqrt[a + b/x]) - (c*(5*b*c + 4*a*d)*(a + b/
x)^(3/2))/3 - (c*(5*b*c + 4*a*d)*(a + b/x)^(5/2))/(5*a) - (2*d^2*
(a + b/x)^(7/2))/(7*b) + (c^2*(a + b/x)^(7/2)*x)/a + a^(3/2)*c*(5
*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```


---

**Rubi in Sympy [A]** time = 26.7544, size = 131, normalized size = 0.86

$$a^{\frac{3}{2}}c\,(4ad+5bc)\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-ac\sqrt{a+\frac{b}{x}}\,(4ad+5bc)-\frac{c\left(a+\frac{b}{x}\right)^{\frac{3}{2}}(4ad+5bc)}{3}$$
$$-\frac{2d^2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{7b}+\frac{c^2x\left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{a}-\frac{c\left(a+\frac{b}{x}\right)^{\frac{5}{2}}(4ad+5bc)}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**(5/2)*(c+d/x)**2,x)
```

```
[Out] a**(3/2)*c*(4*a*d + 5*b*c)*atanh(sqrt(a + b/x)/sqrt(a)) - a*c*sqr
t(a + b/x)*(4*a*d + 5*b*c) - c*(a + b/x)**(3/2)*(4*a*d + 5*b*c)/3
 - 2*d**2*(a + b/x)**(7/2)/(7*b) + c**2*x*(a + b/x)**(7/2)/a - c*
(a + b/x)**(5/2)*(4*a*d + 5*b*c)/(5*a)
```

---

**Mathematica [A]** time = 0.249973, size = 156, normalized size = 1.03

$$\frac{1}{2}a^{3/2}c(4ad+5bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)$$
$$+\frac{\sqrt{a+\frac{b}{x}}\left(-30a^3d^2x^3+a^2bx^2\left(105c^2x^2-644cdx-90d^2\right)-2ab^2x\left(245c^2x^2+154cdx+45d^2\right)-2b^3\left(35c^2x^2+42cdx+15c\right.\right.}{105bx^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x)^(5/2)*(c + d/x)^2,x]
```

```
[Out] (Sqrt[a + b/x]*(-30*a^3*d^2*x^3 - 2*b^3*(15*d^2 + 42*c*d*x + 35*c
^2*x^2) + a^2*b*x^2*(-90*d^2 - 644*c*d*x + 105*c^2*x^2) - 2*a*b^2
*x*(45*d^2 + 154*c*d*x + 245*c^2*x^2)))/(105*b*x^3) + (a^(3/2)*c*
(5*b*c + 4*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x])/2
```

---

**Maple [B]** time = 0.02, size = 318, normalized size = 2.1

$$\frac{1}{210\,bx^4}\sqrt{\frac{ax+b}{x}}\left(420\,a^{5/2}c\ln\left(1/2\,\frac{2\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)dbx^5+525\,a^{3/2}c^2b^2\ln\left(1/2\,\frac{2\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)x^5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^(5/2)*(c+d/x)^2,x)
```

```
[Out] 1/210*((a*x+b)/x)^(1/2)/x^4/b*(420*a^(5/2)*c*ln(1/2*(2*(a*x^2+b*x
)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*d*b*x^5+525*a^(3/2)*c^2*b^2*ln(
1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^5+840*a^3*c*
(a*x^2+b*x)^(1/2)*d*x^5+1050*a^2*c^2*(a*x^2+b*x)^(1/2)*b*x^5-840*
a^2*c*(a*x^2+b*x)^(3/2)*d*x^3-840*a*c^2*(a*x^2+b*x)^(3/2)*b*x^3-6
0*(a*x^2+b*x)^(3/2)*x^2*a^2*d^2-448*(a*x^2+b*x)^(3/2)*x^2*a*b*c*d
-140*(a*x^2+b*x)^(3/2)*x^2*b^2*c^2-120*(a*x^2+b*x)^(3/2)*x*a*b*d^
2-168*(a*x^2+b*x)^(3/2)*x*b^2*c*d-60*(a*x^2+b*x)^(3/2)*b^2*d^2)/(
x*(a*x+b))^(1/2)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2)*(c + d/x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 0.252331, size = 1, normalized size = 0.01

$$\left[\frac{105\left(5\,ab^2c^2+4\,a^2bcd\right)\sqrt{a}x^3\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(105\,a^2bc^2x^4-30\,b^3d^2-2\left(245\,ab^2c^2+322\,a^2bcd+15\,a^3c\right.\right.}{210\,bx^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2)*(c + d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/210*(105*(5*a*b^2*c^2 + 4*a^2*b*c*d)*sqrt(a)*x^3*log(2*a*x + 2
*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(105*a^2*b*c^2*x^4 - 30*b^3
*d^2 - 2*(245*a*b^2*c^2 + 322*a^2*b*c*d + 15*a^3*d^2)*x^3 - 2*(35
*b^3*c^2 + 154*a*b^2*c*d + 45*a^2*b*d^2)*x^2 - 6*(14*b^3*c*d + 15
*a*b^2*d^2)*x)*sqrt((a*x + b)/x))/(b*x^3), 1/105*(105*(5*a*b^2*c^
2 + 4*a^2*b*c*d)*sqrt(-a)*x^3*arctan(sqrt((a*x + b)/x)/sqrt(-a))
+ (105*a^2*b*c^2*x^4 - 30*b^3*d^2 - 2*(245*a*b^2*c^2 + 322*a^2*b*
c*d + 15*a^3*d^2)*x^3 - 2*(35*b^3*c^2 + 154*a*b^2*c*d + 45*a^2*b*
d^2)*x^2 - 6*(14*b^3*c*d + 15*a*b^2*d^2)*x)*sqrt((a*x + b)/x))/(b
*x^3)]
```

**Sympy [A]** time = 48.817, size = 1884, normalized size = 12.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**(5/2)*(c+d/x)**2,x)
```

```
[Out] -16*a**(19/2)*b**(13/2)*d**2*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*
b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9
```

```
*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(17/2)*b**(15/2)
*d**2*x**5*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a*
*(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2
)*b**10*x**(7/2)) - 30*a**(15/2)*b**(17/2)*d**2*x**4*sqrt(a*x/b +
 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2)
+ 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*
a**(13/2)*b**(19/2)*d**2*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7
*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**
(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 8*a**(13/2)*b**(5/2)*d**2*
x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**
4*x**(5/2)) - 100*a**(11/2)*b**(21/2)*d**2*x**2*sqrt(a*x/b + 1)/(
105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315
*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 8*a**(11
/2)*b**(7/2)*c*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2)
+ 15*a**(5/2)*b**4*x**(5/2)) + 4*a**(11/2)*b**(7/2)*d**2*x**2*sqr
t(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/
2)) - 96*a**(9/2)*b**(23/2)*d**2*x*sqrt(a*x/b + 1)/(105*a**(13/2)
*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**
9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 4*a**(9/2)*b**(9/2)*c
*d*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*
b**4*x**(5/2)) - 16*a**(9/2)*b**(9/2)*d**2*x*sqrt(a*x/b + 1)/(15*
a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 30*a**(7/2)
*b**(25/2)*d**2*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 3
15*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a*
*(7/2)*b**10*x**(7/2)) - 16*a**(7/2)*b**(11/2)*c*d*x*sqrt(a*x/b +
 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 12*
a**(7/2)*b**(11/2)*d**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2
) + 15*a**(5/2)*b**4*x**(5/2)) - 12*a**(5/2)*b**(13/2)*c*d*sqrt(a
*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2))
 + 4*a**(5/2)*c*d*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 5*a**(3/2)*b*c
**2*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 16*a**10*b**6*d**2*x**(13/2)
/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 3
15*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**
9*b**7*d**2*x**(11/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/
2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**
10*x**(7/2)) + 48*a**8*b**8*d**2*x**(9/2)/(105*a**(13/2)*b**7*x**
(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2
) + 105*a**(7/2)*b**10*x**(7/2)) + 16*a**7*b**9*d**2*x**(7/2)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a
**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 8*a**7*b**
2*d**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**
(5/2)) - 8*a**6*b**3*c*d*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15
*a**(5/2)*b**4*x**(5/2)) - 8*a**6*b**3*d**2*x**(5/2)/(15*a**(7/2)
*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**5*b**4*c*d*x**
(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 4
*a**3*c*d*sqrt(x)/(sqrt(b)*sqrt(a*x/b + 1)) + a**2*sqrt(b)*c**2*s
qrt(x)*sqrt(a*x/b + 1) - 4*a**2*sqrt(b)*c**2*sqrt(x)/sqrt(a*x/b +
 1) - 4*a**2*sqrt(b)*c*d/(sqrt(x)*sqrt(a*x/b + 1)) + a**2*d**2*Pi
ecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)
) - 4*a*b**(3/2)*c**2/(sqrt(x)*sqrt(a*x/b + 1)) + 4*a*b*c*d*Piece
wise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) +
 b**2*c**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)
/(3*b), True))
```

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2)*(c + d/x)^2,x, algorithm="giac")

[Out] Exception raised: TypeError
```

## 3.141 $\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)\,dx$

**Optimal**. Leaf size=125

$$a^{3/2}(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{\left(a+\frac{b}{x}\right)^{5/2}(2ad+5bc)}{5a}$$
$$-\frac{1}{3}\left(a+\frac{b}{x}\right)^{3/2}(2ad+5bc)-a\sqrt{a+\frac{b}{x}}(2ad+5bc)+\frac{cx\left(a+\frac{b}{x}\right)^{7/2}}{a}$$

```
[Out] -(a*(5*b*c + 2*a*d)*Sqrt[a + b/x]) - ((5*b*c + 2*a*d)*(a + b/x)^(
3/2))/3 - ((5*b*c + 2*a*d)*(a + b/x)^(5/2))/(5*a) + (c*(a + b/x)^
(7/2)*x)/a + a^(3/2)*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a
]]
```

---

**Rubi [A]** time = 0.208579, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$a^{3/2}(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{\left(a+\frac{b}{x}\right)^{5/2}(2ad+5bc)}{5a}$$
$$-\frac{1}{3}\left(a+\frac{b}{x}\right)^{3/2}(2ad+5bc)-a\sqrt{a+\frac{b}{x}}(2ad+5bc)+\frac{cx\left(a+\frac{b}{x}\right)^{7/2}}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^(5/2)*(c + d/x),x]
```

```
[Out] -(a*(5*b*c + 2*a*d)*Sqrt[a + b/x]) - ((5*b*c + 2*a*d)*(a + b/x)^(
3/2))/3 - ((5*b*c + 2*a*d)*(a + b/x)^(5/2))/(5*a) + (c*(a + b/x)^
(7/2)*x)/a + a^(3/2)*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a
]]
```

---

**Rubi in Sympy [A]** time = 17.6048, size = 110, normalized size = 0.88

$$2a^{\frac{3}{2}}\left(ad+\frac{5bc}{2}\right)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-a\sqrt{a+\frac{b}{x}}(2ad+5bc)$$
$$-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(\frac{2ad}{3}+\frac{5bc}{3}\right)+\frac{cx\left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{a}-\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(ad+\frac{5bc}{2}\right)}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**(5/2)*(c+d/x),x)
```

```
[Out] 2*a**(3/2)*(a*d + 5*b*c/2)*atanh(sqrt(a + b/x)/sqrt(a)) - a*sqrt(
a + b/x)*(2*a*d + 5*b*c) - (a + b/x)**(3/2)*(2*a*d/3 + 5*b*c/3) +
 c*x*(a + b/x)**(7/2)/a - 2*(a + b/x)**(5/2)*(a*d + 5*b*c/2)/(5*a
)
```

---

**Mathematica [A]** time = 0.252027, size = 105, normalized size = 0.84

$$\frac{1}{2}a^{3/2}(2ad+5bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)+\frac{\sqrt{a+\frac{b}{x}}\left(a^2x^2(15cx-46d)-2abx(35cx+11d)-2b^2(5cx+3d)\right)}{15x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x)^(5/2)*(c + d/x),x]
```

```
[Out] (Sqrt[a + b/x]*(-2*b^2*(3*d + 5*c*x) + a^2*x^2*(-46*d + 15*c*x) -
 2*a*b*x*(11*d + 35*c*x)))/(15*x^2) + (a^(3/2)*(5*b*c + 2*a*d)*Lo
g[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x])/2
```

---

**Maple [B]** time = 0.016, size = 240, normalized size = 1.9

$$\frac{1}{30\,bx^3}\sqrt{\frac{ax+b}{x}}\left(30\,a^{5/2}\ln\left(1/2\,\frac{2\,\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)dbx^4+75\,a^{3/2}\ln\left(1/2\,\frac{2\,\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)b^2cx^4+60\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^(5/2)*(c+d/x),x)
```

```
[Out] 1/30*((a*x+b)/x)^(1/2)/x^3/b*(30*a^(5/2)*ln(1/2*(2*(a*x^2+b*x)^(1
/2)*a^(1/2)+2*a*x+b)/a^(1/2))*d*b*x^4+75*a^(3/2)*ln(1/2*(2*(a*x^2
+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^2*c*x^4+60*a^3*(a*x^2+b*x
)^(1/2)*d*x^4+150*a^2*(a*x^2+b*x)^(1/2)*c*b*x^4-60*(a*x^2+b*x)^(3
/2)*a^2*d*x^2-120*a*(a*x^2+b*x)^(3/2)*c*b*x^2-32*(a*x^2+b*x)^(3/2
)*x*a*b*d-20*(a*x^2+b*x)^(3/2)*x*b^2*c-12*(a*x^2+b*x)^(3/2)*b^2*d
)/(x*(a*x+b))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2)*(c + d/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.262862, size = 1, normalized size = 0.01

$$\left[\frac{15\left(5\,abc+2\,a^{2}d\right)\sqrt{a}x^{2}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(15\,a^{2}cx^{3}-6\,b^{2}d-2\left(35\,abc+23\,a^{2}d\right)x^{2}-2\left(5\,b^{2}c+11\,abd\right)x\right.}{30\,x^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2)*(c + d/x),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(5*a*b*c + 2*a^2*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x
*sqrt((a*x + b)/x) + b) + 2*(15*a^2*c*x^3 - 6*b^2*d - 2*(35*a*b*c
 + 23*a^2*d)*x^2 - 2*(5*b^2*c + 11*a*b*d)*x)*sqrt((a*x + b)/x))/x
^2, 1/15*(15*(5*a*b*c + 2*a^2*d)*sqrt(-a)*x^2*arctan(sqrt((a*x +
b)/x)/sqrt(-a)) + (15*a^2*c*x^3 - 6*b^2*d - 2*(35*a*b*c + 23*a^2*
d)*x^2 - 2*(5*b^2*c + 11*a*b*d)*x)*sqrt((a*x + b)/x))/x^2]
```

---

**Sympy [A]** time = 47.6987, size = 561, normalized size = 4.49

$$\frac{4a^{\frac{11}{2}}b^{\frac{7}{2}}dx^{3}\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^{3}x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^{4}x^{\frac{5}{2}}}+\frac{2a^{\frac{9}{2}}b^{\frac{9}{2}}dx^{2}\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^{3}x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^{4}x^{\frac{5}{2}}}-\frac{8a^{\frac{7}{2}}b^{\frac{11}{2}}dx\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^{3}x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^{4}x^{\frac{5}{2}}}$$
$$-\frac{6a^{\frac{5}{2}}b^{\frac{13}{2}}d\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^{3}x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^{4}x^{\frac{5}{2}}}+2a^{\frac{5}{2}}d\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)+5a^{\frac{3}{2}}bc\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)-\frac{4a^{6}b^{3}dx^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^{3}x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^{4}x^{\frac{5}{2}}}$$
$$-\frac{4a^{5}b^{4}dx^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^{3}x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^{4}x^{\frac{5}{2}}}-\frac{2a^{3}d\sqrt{x}}{\sqrt{b}\sqrt{\frac{ax}{b}+1}}+a^{2}\sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1}-\frac{4a^{2}\sqrt{b}c\sqrt{x}}{\sqrt{\frac{ax}{b}+1}}-\frac{2a^{2}\sqrt{b}d}{\sqrt{x}\sqrt{\frac{ax}{b}+1}}$$
$$-\frac{4ab^{\frac{3}{2}}c}{\sqrt{x}\sqrt{\frac{ax}{b}+1}}+2abd\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for}\: b=0\\-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+b^{2}c\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for}\: b=0\\-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**(5/2)*(c+d/x),x)
```

```
[Out] 4*a**(11/2)*b**(7/2)*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**
(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(9/2)*d*x**2*s
qrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(
5/2)) - 8*a**(7/2)*b**(11/2)*d*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**
3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6*a**(5/2)*b**(13/2)*d*
sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**
(5/2)) + 2*a**(5/2)*d*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 5*a**(3/2)
*b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b)) - 4*a**6*b**3*d*x**(7/2)/(15*
a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a**5*b**4
*d*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2
)) - 2*a**3*d*sqrt(x)/(sqrt(b)*sqrt(a*x/b + 1)) + a**2*sqrt(b)*c*
sqrt(x)*sqrt(a*x/b + 1) - 4*a**2*sqrt(b)*c*sqrt(x)/sqrt(a*x/b + 1
) - 2*a**2*sqrt(b)*d/(sqrt(x)*sqrt(a*x/b + 1)) - 4*a*b**(3/2)*c/(
sqrt(x)*sqrt(a*x/b + 1)) + 2*a*b*d*Piecewise((-sqrt(a)/x, Eq(b, 0
)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b**2*c*Piecewise((-sqrt(
a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2)*(c + d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.142 $\int \left(a+\frac{b}{x}\right)^{5/2} dx$

**Optimal**. Leaf size=71

$$5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2}-5ab\sqrt{a+\frac{b}{x}}$$

```
[Out] -5*a*b*Sqrt[a + b/x] - (5*b*(a + b/x)^(3/2))/3 + (a + b/x)^(5/2)*
x + 5*a^(3/2)*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

---

**Rubi [A]** time = 0.101272, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$

$$5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2}-5ab\sqrt{a+\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^(5/2),x]
```

```
[Out] -5*a*b*Sqrt[a + b/x] - (5*b*(a + b/x)^(3/2))/3 + (a + b/x)^(5/2)*
x + 5*a^(3/2)*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

---

**Rubi in Sympy [A]** time = 9.5882, size = 60, normalized size = 0.85

$$5a^{\frac{3}{2}}b\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-5ab\sqrt{a+\frac{b}{x}}-\frac{5b\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3}+x\left(a+\frac{b}{x}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**(5/2),x)
```

```
[Out] 5*a**(3/2)*b*atanh(sqrt(a + b/x)/sqrt(a)) - 5*a*b*sqrt(a + b/x) -
 5*b*(a + b/x)**(3/2)/3 + x*(a + b/x)**(5/2)
```

---

**Mathematica [A]** time = 0.102443, size = 71, normalized size = 1.

$$\frac{5}{2}a^{3/2}b\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)+\sqrt{a+\frac{b}{x}}\left(a^2x-\frac{14ab}{3}-\frac{2b^2}{3x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x)^(5/2),x]
```

```
[Out] Sqrt[a + b/x]*((-14*a*b)/3 - (2*b^2)/(3*x) + a^2*x) + (5*a^(3/2)*
b*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x])/2
```

---

**Maple [A]** time = 0.005, size = 112, normalized size = 1.6

$$\frac{1}{6x^2}\sqrt{\frac{ax+b}{x}}\left(15a^{3/2}\ln\left(1/2\,\frac{2\sqrt{ax^2+bx}\sqrt{a}+2ax+b}{\sqrt{a}}\right)x^3b+30a^2\sqrt{ax^2+bx}x^3-24a\left(ax^2+bx\right)^{3/2}x-4\left(ax^2+bx\right)^{3/}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^(5/2),x)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)/x^2*(15*a^(3/2)*ln(1/2*(2*(a*x^2+b*x)^(1/2)
*a^(1/2)+2*a*x+b)/a^(1/2))*x^3*b+30*a^2*(a*x^2+b*x)^(1/2)*x^3-24*
a*(a*x^2+b*x)^(3/2)*x-4*(a*x^2+b*x)^(3/2)*b)/(x*(a*x+b))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.256117, size = 1, normalized size = 0.01

$$\left[\frac{15\,a^{\frac{3}{2}}bx\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(3\,a^2x^2-14\,abx-2\,b^2\right)\sqrt{\frac{ax+b}{x}}}{6\,x},\frac{15\,\sqrt{-a}abx\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)+\left(3\,a^2x^2-14\,ab\right.}{3\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*a^(3/2)*b*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) +
b) + 2*(3*a^2*x^2 - 14*a*b*x - 2*b^2)*sqrt((a*x + b)/x))/x, 1/3*(
15*sqrt(-a)*a*b*x*arctan(sqrt((a*x + b)/x)/sqrt(-a)) + (3*a^2*x^2
 - 14*a*b*x - 2*b^2)*sqrt((a*x + b)/x))/x]
```

---

**Sympy [A]** time = 13.3606, size = 99, normalized size = 1.39

$$a^{\frac{5}{2}}x\sqrt{1+\frac{b}{ax}}-\frac{14a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax}}}{3}-\frac{5a^{\frac{3}{2}}b\log\left(\frac{b}{ax}\right)}{2}+5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{b}{ax}}+1\right)-\frac{2\sqrt{a}b^{2}\sqrt{1+\frac{b}{ax}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**(5/2),x)
```

```
[Out] a**(5/2)*x*sqrt(1 + b/(a*x)) - 14*a**(3/2)*b*sqrt(1 + b/(a*x))/3
- 5*a**(3/2)*b*log(b/(a*x))/2 + 5*a**(3/2)*b*log(sqrt(1 + b/(a*x)
) + 1) - 2*sqrt(a)*b**2*sqrt(1 + b/(a*x))/(3*x)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.143 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx$

**Optimal**. Leaf size=134

$$\frac{a^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{2(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}}-\frac{b\sqrt{a+\frac{b}{x}}(ad+2bc)}{cd}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}$$

```
[Out] -((b*(2*b*c + a*d)*Sqrt[a + b/x])/(c*d)) + (a*(a + b/x)^(3/2)*x)/
c + (2*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c
- a*d]])/(c^2*d^(3/2)) + (a^(3/2)*(5*b*c - 2*a*d)*ArcTanh[Sqrt[a
+ b/x]/Sqrt[a]])/c^2
```

---

**Rubi [A]** time = 0.639039, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{a^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{2(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}}-\frac{b\sqrt{a+\frac{b}{x}}(ad+2bc)}{cd}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^(5/2)/(c + d/x),x]
```

```
[Out] -((b*(2*b*c + a*d)*Sqrt[a + b/x])/(c*d)) + (a*(a + b/x)^(3/2)*x)/
c + (2*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c
- a*d]])/(c^2*d^(3/2)) + (a^(3/2)*(5*b*c - 2*a*d)*ArcTanh[Sqrt[a
+ b/x]/Sqrt[a]])/c^2
```

---

**Rubi in Sympy [A]** time = 62.0592, size = 114, normalized size = 0.85

$$-\frac{a^{\frac{3}{2}}\left(2ad-5bc\right)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{ax\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{c}-\frac{b\sqrt{a+\frac{b}{x}}\left(ad+2bc\right)}{cd}+\frac{2\left(ad-bc\right)^{\frac{5}{2}}\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{c^2d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**(5/2)/(c+d/x),x)
```

```
[Out] -a**(3/2)*(2*a*d - 5*b*c)*atanh(sqrt(a + b/x)/sqrt(a))/c**2 + a*x
*(a + b/x)**(3/2)/c - b*sqrt(a + b/x)*(a*d + 2*b*c)/(c*d) + 2*(a*
```

```
d - b*c)**(5/2)*atanh(sqrt(d)*sqrt(a + b/x)/sqrt(a*d - b*c))/(c**
2*d**(3/2))
```

---

**Mathematica [A]** time = 0.228124, size = 172, normalized size = 1.28

$$-\frac{a^{3/2}(2ad-5bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{2c^2}+\sqrt{a+\frac{b}{x}}\left(\frac{a^2x}{c}-\frac{2b^2}{d}\right)+\frac{(ad-bc)^{5/2}\log(cx+d)}{c^2d^{3/2}}-\frac{(ad-bc)^{5/2}\log\left(2\sqrt{d}x\sqrt{a+\frac{b}{x}}\sqrt{ad-bc}-2adx+bcx-bd\right)}{c^2d^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x)^(5/2)/(c + d/x),x]
```

```
[Out] Sqrt[a + b/x]*((-2*b^2)/d + (a^2*x)/c) + ((-(b*c) + a*d)^(5/2)*Lo
g[d + c*x])/(c^2*d^(3/2)) - (a^(3/2)*(-5*b*c + 2*a*d)*Log[b + 2*a
*x + 2*Sqrt[a]*Sqrt[a + b/x]*x])/(2*c^2) - ((-(b*c) + a*d)^(5/2)*
Log[-(b*d) + b*c*x - 2*a*d*x + 2*Sqrt[d]*Sqrt[-(b*c) + a*d]*Sqrt[
a + b/x]*x])/(c^2*d^(3/2))
```

---

**Maple [B]** time = 0.023, size = 859, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^(5/2)/(c+d/x),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)/x*(-2*d^3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/
2)+2*a*x+b)/a^(1/2))*a^3*x^2*c*((a*d-b*c)*d/c^2)^(1/2)+5*ln(1/2*(
2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b*x^2*d^2*c^2*(
(a*d-b*c)*d/c^2)^(1/2)+4*b^2*a*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2
)+2*a*x+b)/a^(1/2))*x^2*d*c^3*((a*d-b*c)*d/c^2)^(1/2)-4*ln(1/2*(2
*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^2*x^2*d*c^3*((a*
d-b*c)*d/c^2)^(1/2)+2*(x*(a*x+b))^(1/2)*a^(5/2)*x^2*d^2*c^2*((a*d
-b*c)*d/c^2)^(1/2)-4*(x*(a*x+b))^(1/2)*a^(3/2)*b*x^2*d*c^3*((a*d-
b*c)*d/c^2)^(1/2)+2*c^4*(x*(a*x+b))^(1/2)*b^2*x^2*a^(1/2)*((a*d-b
*c)*d/c^2)^(1/2)-2*d^4*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^
(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(7/2)*x^2+6*d^3*ln((2*(x*(a
*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d)
)*a^(5/2)*b*x^2*c-6*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/
2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(3/2)*b^2*x^2*d^2*c^2+2*ln((2*
(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c
*x+d))*b^3*x^2*d*c^3*a^(1/2)+8*b*a^(3/2)*(a*x^2+b*x)^(1/2)*x^2*d*
```

```
c^3*((a*d-b*c)*d/c^2)^(1/2)-2*b^2*(a*x^2+b*x)^(1/2)*c^4*x^2*a^(1/
2)*((a*d-b*c)*d/c^2)^(1/2)-b^3*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2
)+2*a*x+b)/a^(1/2))*c^4*x^2*((a*d-b*c)*d/c^2)^(1/2)+c^4*ln(1/2*(2
*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^3*x^2*((a*d-b*c)*d
/c^2)^(1/2)-4*b*(a*x^2+b*x)^(3/2)*d*c^3*a^(1/2)*((a*d-b*c)*d/c^2)
^(1/2))/(x*(a*x+b))^(1/2)/d^2/c^3/a^(1/2)/((a*d-b*c)*d/c^2)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2)/(c + d/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.435569, size = 1, normalized size = 0.01

$$\left[-\frac{\left(5\,abcd-2\,a^2d^2\right)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{2\,dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+bd-(bc-2}{cx+d}\right.}{2\,c^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2)/(c + d/x),x, algorithm="fricas")
```

```
[Out] [-1/2*((5*a*b*c*d - 2*a^2*d^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sq
rt((a*x + b)/x) + b) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b
*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) +
b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a^2*c*d*x - 2*b^2*c^2)*sqr
t((a*x + b)/x))/(c^2*d), ((5*a*b*c*d - 2*a^2*d^2)*sqrt(-a)*arctan
(sqrt((a*x + b)/x)/sqrt(-a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sq
rt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)
/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) + (a^2*c*d*x - 2*b^2*c^2)
*sqrt((a*x + b)/x))/(c^2*d), 1/2*(4*(b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*sqrt((b*c - a*d)/d)*arctan(sqrt((a*x + b)/x)/sqrt((b*c - a*d)/
d)) - (5*a*b*c*d - 2*a^2*d^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqr
t((a*x + b)/x) + b) + 2*(a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x)
)/(c^2*d), ((5*a*b*c*d - 2*a^2*d^2)*sqrt(-a)*arctan(sqrt((a*x + b
)/x)/sqrt(-a)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*
d)/d)*arctan(sqrt((a*x + b)/x)/sqrt((b*c - a*d)/d)) + (a^2*c*d*x
- 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d)]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{cx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**(5/2)/(c+d/x),x)
```

```
[Out] Integral(x*(a + b/x)**(5/2)/(c*x + d), x)
```

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2)/(c + d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.144 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2} dx$

**Optimal**. Leaf size=166

$$\frac{a^{3/2}(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}-\frac{(bc-ad)^{3/2}(4ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3d^{3/2}}+\frac{\sqrt{a+\frac{b}{x}}(bc-2ad)(bc-ad)}{c^2d\left(c+\frac{d}{x}\right)}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}$$

```
[Out] ((b*c - 2*a*d)*(b*c - a*d)*Sqrt[a + b/x])/(c^2*d*(c + d/x)) + (a*
(a + b/x)^(3/2)*x)/(c*(c + d/x)) - ((b*c - a*d)^(3/2)*(b*c + 4*a*
d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3*d^(3/2))
 + (a^(3/2)*(5*b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^3
```

---

**Rubi [A]** time = 0.668661, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{a^{3/2}(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}-\frac{(bc-ad)^{3/2}(4ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3d^{3/2}}+\frac{\sqrt{a+\frac{b}{x}}(bc-2ad)(bc-ad)}{c^2d\left(c+\frac{d}{x}\right)}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^(5/2)/(c + d/x)^2,x]
```

```
[Out] ((b*c - 2*a*d)*(b*c - a*d)*Sqrt[a + b/x])/(c^2*d*(c + d/x)) + (a*
(a + b/x)^(3/2)*x)/(c*(c + d/x)) - ((b*c - a*d)^(3/2)*(b*c + 4*a*
d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3*d^(3/2))
 + (a^(3/2)*(5*b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^3
```

---

**Rubi in Sympy [A]** time = 63.2027, size = 138, normalized size = 0.83

$$-\frac{a^{\frac{3}{2}}\left(4ad-5bc\right)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^{3}}+\frac{ax\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{c\left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}\left(ad-bc\right)\left(2ad-bc\right)}{c^{2}d\left(c+\frac{d}{x}\right)}+\frac{\left(ad-bc\right)^{\frac{3}{2}}\left(4ad+bc\right)\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{c^{3}d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**(5/2)/(c+d/x)**2,x)
```

```
[Out] -a**(3/2)*(4*a*d - 5*b*c)*atanh(sqrt(a + b/x)/sqrt(a))/c**3 + a*x
*(a + b/x)**(3/2)/(c*(c + d/x)) + sqrt(a + b/x)*(a*d - b*c)*(2*a*
d - b*c)/(c**2*d*(c + d/x)) + (a*d - b*c)**(3/2)*(4*a*d + b*c)*at
anh(sqrt(d)*sqrt(a + b/x)/sqrt(a*d - b*c))/(c**3*d**(3/2))
```

---

**Mathematica [C]** time = 0.604074, size = 219, normalized size = 1.32

$$-\frac{a^{3/2}(4ad-5bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)-\frac{2cx\sqrt{a+\frac{b}{x}}\left(a^{2}d(cx+2d)-2abcd+b^{2}c^{2}\right)}{d(cx+d)}+\frac{i(bc-ad)^{3/2}(4ad+bc)\log\left(\frac{2c^{4}\left(2dx\sqrt{a+\frac{b}{x}}\sqrt{bc-ad}-2ia\cdots\right.}{(cx+d)(bc-ad)^{5/\cdots}}\right.}{d^{3/2}}}{2c^{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x)^(5/2)/(c + d/x)^2,x]
```

```
[Out] -((-2*c*Sqrt[a + b/x]*x*(b^2*c^2 - 2*a*b*c*d + a^2*d*(2*d + c*x))
)/(d*(d + c*x)) + a^(3/2)*(-5*b*c + 4*a*d)*Log[b + 2*a*x + 2*Sqrt
[a]*Sqrt[a + b/x]*x] + (I*(b*c - a*d)^(3/2)*(b*c + 4*a*d)*Log[(2*
c^4*((-2*I)*a*d^(3/2)*x + 2*d*Sqrt[b*c - a*d]*Sqrt[a + b/x]*x - I
*b*Sqrt[d]*(d - c*x)))/((b*c - a*d)^(5/2)*(b*c + 4*a*d)*(d + c*x)
)])/d^(3/2))/(2*c^3)
```

---

**Maple [B]** time = 0.024, size = 2014, normalized size = 12.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^(5/2)/(c+d/x)^2,x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(-4*a^(9/2)*ln((2*(x*(a*x+b))^(1/2)*((a*d
-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*d^6+2*c^4*(x*(a*
x+b))^(3/2)*a^(5/2)*d^2*((a*d-b*c)*d/c^2)^(1/2)-4*a^(9/2)*ln((2*(
x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*
x+d))*x*c*d^5+4*(x*(a*x+b))^(1/2)*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)
*c^2*d^4+11*a^(7/2)*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/
2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c*d^5-9*a^(5/2)*ln((2*(x*(a*x+
b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b
^2*c^2*d^4+a^(3/2)*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2
)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^3*c^3*d^3-4*a^4*ln(1/2*(2*(x*(a
*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*c*
d^5+2*c^6*(x*(a*x+b))^(3/2)*b^2*a^(1/2)*((a*d-b*c)*d/c^2)^(1/2)+l
n(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d
/c^2)^(1/2)*x*b^4*c^6-ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b
)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x*b^4*c^6+a^(1/2)*ln((2*(x*(a*
x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))
*b^4*c^4*d^2+ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)
)*((a*d-b*c)*d/c^2)^(1/2)*b^4*c^5*d-ln(1/2*(2*(a*x^2+b*x)^(1/2)*a
^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*b^4*c^5*d-5*a^2*
ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*
d/c^2)^(1/2)*x*b^2*c^4*d^2-5*a^2*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1
/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*b^2*c^3*d^3+a^(1/2)
*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-
b*d)/(c*x+d))*x*b^4*c^5*d-2*a^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*(a*x^
2+b*x)^(1/2)*x*b^3*c^6-2*a^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*(a*x^2+b
*x)^(1/2)*b^3*c^5*d-ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/
a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a*b^3*c^4*d^2+ln(1/2*(2*(a*x^2+b
*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a*b^3
*c^4*d^2+11*a^(7/2)*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/
2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b*c^2*d^4-9*a^(5/2)*ln((2*(x*(
a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d
))*x*b^2*c^3*d^3-2*(x*(a*x+b))^(1/2)*a^(7/2)*((a*d-b*c)*d/c^2)^(1
/2)*x^2*c^4*d^2-2*(x*(a*x+b))^(1/2)*a^(3/2)*((a*d-b*c)*d/c^2)^(1/
2)*x^2*b^2*c^6-4*c^5*(x*(a*x+b))^(3/2)*a^(3/2)*b*d*((a*d-b*c)*d/c
^2)^(1/2)+2*(x*(a*x+b))^(1/2)*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*x*c
^3*d^3-8*(x*(a*x+b))^(1/2)*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*b*c^3*
d^3+4*(x*(a*x+b))^(1/2)*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*b^2*c^4*d
^2+2*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*(a*x^2+b*x)^(1/2)*b^2*c^4*d^
2-4*a^4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a
*d-b*c)*d/c^2)^(1/2)*x*c^2*d^4+9*a^3*ln(1/2*(2*(x*(a*x+b))^(1/2)*
a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*b*c^2*d^4-ln(1/
2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2
)^(1/2)*x*a*b^3*c^5*d+ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b
)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x*a*b^3*c^5*d-4*(x*(a*x+b))^(1
/2)*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b*c^4*d^2+4*(x*(a*x+b))^(1/
2)*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*x^2*b*c^5*d+2*a^(3/2)*((a*d-b*
c)*d/c^2)^(1/2)*(a*x^2+b*x)^(1/2)*x*b^2*c^5*d+2*(x*(a*x+b))^(1/2)
*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b^2*c^5*d+9*a^3*ln(1/2*(2*(x*(
a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x
*b*c^3*d^3+a^(3/2)*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2
)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b^3*c^4*d^2)/(x*(a*x+b))^(1/2)/
c^4/d^2/a^(1/2)/(a*d-b*c)/((a*d-b*c)*d/c^2)^(1/2)/(c*x+d)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2)/(c + d/x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 0.373037, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2)/(c + d/x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*
sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (b^2*c^2
*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d
^2)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt(
(a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a^2*c^2*d*x
^2 + (b^2*c^3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/
(c^4*d*x + c^3*d^2), 1/2*(2*(5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2
*d - 4*a^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt((a*x + b)/x)/sqrt(-a))
- (b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d -
 4*a^2*c*d^2)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d
)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) + 2*(a
^2*c^2*d*x^2 + (b^2*c^3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x
 + b)/x))/(c^4*d*x + c^3*d^2), -1/2*(2*(b^2*c^2*d + 3*a*b*c*d^2 -
 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt((b*c -
 a*d)/d)*arctan(sqrt((a*x + b)/x)/sqrt((b*c - a*d)/d)) + (5*a*b*c
*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*log(2*a
*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*c^2*d*x^2 + (b^2
*c^3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(c^4*d*x
+ c^3*d^2), ((5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^
2)*x)*sqrt(-a)*arctan(sqrt((a*x + b)/x)/sqrt(-a)) - (b^2*c^2*d +
3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x
)*sqrt((b*c - a*d)/d)*arctan(sqrt((a*x + b)/x)/sqrt((b*c - a*d)/d
)) + (a^2*c^2*d*x^2 + (b^2*c^3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x)*sq
rt((a*x + b)/x))/(c^4*d*x + c^3*d^2)]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{(cx+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**(5/2)/(c+d/x)**2,x)
```

```
[Out] Integral(x**2*(a + b/x)**(5/2)/(c*x + d)**2, x)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2)/(c + d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.145 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx$

**Optimal**. Leaf size=237

$$\frac{a^{3/2}(5bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}-\frac{\sqrt{bc-ad}\left(-24a^2d^2+8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4d^{3/2}}-\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abcd+b^2c^2\right)}{4c^3d\left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)(bc-ad)}{2c^2d\left(c+\frac{d}{x}\right)^2}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}$$

```
[Out] ((b*c - 3*a*d)*(b*c - a*d)*Sqrt[a + b/x])/(2*c^2*d*(c + d/x)^2) -
 ((b^2*c^2 + 7*a*b*c*d - 12*a^2*d^2)*Sqrt[a + b/x])/(4*c^3*d*(c +
 d/x)) + (a*(a + b/x)^(3/2)*x)/(c*(c + d/x)^2) - (Sqrt[b*c - a*d]
*(b^2*c^2 + 8*a*b*c*d - 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x]
)/Sqrt[b*c - a*d]])/(4*c^4*d^(3/2)) + (a^(3/2)*(5*b*c - 6*a*d)*Ar
cTanh[Sqrt[a + b/x]/Sqrt[a]])/c^4
```

---

**Rubi [A]** time = 1.0225, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381

$$\frac{a^{3/2}(5bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}-\frac{\sqrt{bc-ad}\left(-24a^2d^2+8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4d^{3/2}}-\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abcd+b^2c^2\right)}{4c^3d\left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)(bc-ad)}{2c^2d\left(c+\frac{d}{x}\right)^2}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^(5/2)/(c + d/x)^3,x]
```

```
[Out] ((b*c - 3*a*d)*(b*c - a*d)*Sqrt[a + b/x])/(2*c^2*d*(c + d/x)^2) -
 ((b^2*c^2 + 7*a*b*c*d - 12*a^2*d^2)*Sqrt[a + b/x])/(4*c^3*d*(c +
 d/x)) + (a*(a + b/x)^(3/2)*x)/(c*(c + d/x)^2) - (Sqrt[b*c - a*d]
*(b^2*c^2 + 8*a*b*c*d - 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x]
)/Sqrt[b*c - a*d]])/(4*c^4*d^(3/2)) + (a^(3/2)*(5*b*c - 6*a*d)*Ar
cTanh[Sqrt[a + b/x]/Sqrt[a]])/c^4
```

---

**Rubi in Sympy [A]** time = 106.304, size = 204, normalized size = 0.86

$$-\frac{a^{\frac{3}{2}}(6ad-5bc)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}+\frac{ax\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{c\left(c+\frac{d}{x}\right)^2}+\frac{\sqrt{a+\frac{b}{x}}(ad-bc)(3ad-bc)}{2c^2d\left(c+\frac{d}{x}\right)^2}+\frac{\sqrt{a+\frac{b}{x}}\left(12a^2d^2-7abcd-b^2c^2\right)}{4c^3d\left(c+\frac{d}{x}\right)}+\frac{\sqrt{ad-bc}\left(24a^2d^2-8abcd-b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{4c^4d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**(5/2)/(c+d/x)**3,x)
```

```
[Out] -a**(3/2)*(6*a*d - 5*b*c)*atanh(sqrt(a + b/x)/sqrt(a))/c**4 + a*x
*(a + b/x)**(3/2)/(c*(c + d/x)**2) + sqrt(a + b/x)*(a*d - b*c)*(3
*a*d - b*c)/(2*c**2*d*(c + d/x)**2) + sqrt(a + b/x)*(12*a**2*d**2
 - 7*a*b*c*d - b**2*c**2)/(4*c**3*d*(c + d/x)) + sqrt(a*d - b*c)*
(24*a**2*d**2 - 8*a*b*c*d - b**2*c**2)*atanh(sqrt(d)*sqrt(a + b/x
)/sqrt(a*d - b*c))/(4*c**4*d**(3/2))
```

---

**Mathematica [C]** time = 0.651414, size = 304, normalized size = 1.28

$$\frac{-4a^{3/2}(6ad-5bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)+\frac{2cx\sqrt{a+\frac{b}{x}}\left(2a^2d\left(2c^2x^2+9cdx+6d^2\right)-abcd(11cx+7d)+b^2c^2(cx-d)\right)}{d(cx+d)^2}-\frac{i\left(24a^3d^3-32a^2bcd^2\right.}{}}{8c^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x)^(5/2)/(c + d/x)^3,x]
```

```
[Out] ((2*c*Sqrt[a + b/x]*x*(b^2*c^2*(-d + c*x) - a*b*c*d*(7*d + 11*c*x
) + 2*a^2*d*(6*d^2 + 9*c*d*x + 2*c^2*x^2)))/(d*(d + c*x)^2) - 4*a
^(3/2)*(-5*b*c + 6*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x
] - (I*(b^3*c^3 + 7*a*b^2*c^2*d - 32*a^2*b*c*d^2 + 24*a^3*d^3)*Lo
g[(8*c^5*((-2*I)*a*d^(3/2)*x + 2*d*Sqrt[b*c - a*d]*Sqrt[a + b/x]*
x - I*b*Sqrt[d]*(d - c*x)))/(Sqrt[b*c - a*d]*(b^3*c^3 + 7*a*b^2*c
^2*d - 32*a^2*b*c*d^2 + 24*a^3*d^3)*(d + c*x))])/(d^(3/2)*Sqrt[b*
c - a*d]))/(8*c^4)
```

---

**Maple [B]** time = 0.022, size = 2807, normalized size = 11.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^(5/2)/(c+d/x)^3,x)
```

```
[Out] 1/8*((a*x+b)/x)^(1/2)*x*(-22*(x*(a*x+b))^(3/2)*((a*d-b*c)*d/c^2)^
(1/2)*x*a^2*b*c^6*d^2+8*(x*(a*x+b))^(3/2)*((a*d-b*c)*d/c^2)^(1/2)
*x*a*b^2*c^7*d-ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-
2*a*d*x+b*c*x-b*d)/(c*x+d))*b^5*c^5*d^3+20*a^(3/2)*ln(1/2*(2*(x*(
a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x
^2*b^3*c^6*d^2+22*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*x^3*a
^3*b*c^6*d^2-128*a^(5/2)*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*
x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x*b^2*c^4*d^4-24*a^(9/2)*ln
(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/
c^2)^(1/2)*c*d^7+2*(x*(a*x+b))^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b^
3*c^8-2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^4*c^8-24*
ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b
*d)/(c*x+d))*x^2*a^5*c^2*d^6-ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d
/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*b^5*c^7*d+8*(x*(a*x
+b))^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*a^3*c^4*d^4+6*(x*(a*x+b))^(3/2
)*((a*d-b*c)*d/c^2)^(1/2)*b^3*c^7*d-48*ln((2*(x*(a*x+b))^(1/2)*((
a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*a^5*c*d^7-2
*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-
b*d)/(c*x+d))*x*b^5*c^6*d^2+24*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2
)^(1/2)*a^4*c^2*d^6-2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*b
^4*c^6*d^2+80*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2
*a*d*x+b*c*x-b*d)/(c*x+d))*a^4*b*c*d^7-95*ln((2*(x*(a*x+b))^(1/2)
*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^3*b^2*c^
2*d^6+45*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*
x+b*c*x-b*d)/(c*x+d))*a^2*b^3*c^3*d^5-5*ln((2*(x*(a*x+b))^(1/2)*(
(a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a*b^4*c^4*d^
4+68*a^(7/2)*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)
)*((a*d-b*c)*d/c^2)^(1/2)*x^2*b*c^4*d^4+136*a^(7/2)*ln(1/2*(2*(x*
(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*
x*b*c^3*d^5-64*a^(5/2)*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+
b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^2*c^5*d^3-102*(x*(a*x+b
))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*x*a^3*b*c^4*d^4+92*(x*(a*x+b))^(
1/2)*((a*d-b*c)*d/c^2)^(1/2)*x*a^2*b^2*c^5*d^3-22*(x*(a*x+b))^(1/
2)*((a*d-b*c)*d/c^2)^(1/2)*x*a*b^3*c^6*d^2-8*(x*(a*x+b))^(1/2)*((
a*d-b*c)*d/c^2)^(1/2)*x^3*a^2*b^2*c^7*d+40*a^(3/2)*ln(1/2*(2*(x*(
a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x
*b^3*c^5*d^3-190*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*
c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*a^3*b^2*c^3*d^5+90*ln((2*(x*(a*x+
b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x
*a^2*b^3*c^4*d^4-10*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/
2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*a*b^4*c^5*d^3-62*(x*(a*x+b))^(
1/2)*((a*d-b*c)*d/c^2)^(1/2)*a^3*b*c^3*d^5+50*(x*(a*x+b))^(1/2)*(
(a*d-b*c)*d/c^2)^(1/2)*a^2*b^2*c^4*d^4-10*(x*(a*x+b))^(1/2)*((a*d
-b*c)*d/c^2)^(1/2)*a*b^3*c^5*d^3-24*a^(9/2)*ln(1/2*(2*(x*(a*x+b))
^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^2*c^3*
d^5-24*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+
b*c*x-b*d)/(c*x+d))*a^5*d^8-48*a^(9/2)*ln(1/2*(2*(x*(a*x+b))^(1/2
)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x*c^2*d^6+68*
a^(7/2)*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a
*d-b*c)*d/c^2)^(1/2)*b*c^2*d^6-12*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/
c^2)^(1/2)*x^3*a^4*c^5*d^3-2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^
```

```
(1/2)*x^3*a*b^3*c^8-64*a^(5/2)*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2
)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*b^2*c^3*d^5+12*(x*(a*
x+b))^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*x*a^3*c^5*d^3+20*a^(3/2)*ln(1
/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^
2)^(1/2)*b^3*c^4*d^4+80*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)
^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*a^4*b*c^3*d^5-95*ln((2*(
x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*
x+d))*x^2*a^3*b^2*c^4*d^4+45*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d
/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*a^2*b^3*c^5*d^3-5*l
n((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*
d)/(c*x+d))*x^2*a*b^4*c^6*d^2-10*(x*(a*x+b))^(3/2)*((a*d-b*c)*d/c
^2)^(1/2)*a^2*b*c^5*d^3-4*(x*(a*x+b))^(3/2)*((a*d-b*c)*d/c^2)^(1/
2)*a*b^2*c^6*d^2+36*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*x*a
^4*c^3*d^5-4*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b^4*c^7*
d+160*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b
*c*x-b*d)/(c*x+d))*x*a^4*b*c^2*d^6-18*(x*(a*x+b))^(1/2)*((a*d-b*c
)*d/c^2)^(1/2)*x^2*a^3*b*c^5*d^3-14*(x*(a*x+b))^(1/2)*((a*d-b*c)*
d/c^2)^(1/2)*x^2*a*b^3*c^7*d+34*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^
2)^(1/2)*x^2*a^2*b^2*c^6*d^2)/(x*(a*x+b))^(1/2)/c^5/(a*d-b*c)^2/d
^2/((a*d-b*c)*d/c^2)^(1/2)/(c*x+d)^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2)/(c + d/x)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.342362, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2)/(c + d/x)^3,x, algorithm="fricas")

[Out] [-1/8*(4*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)
*x^2 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x)*sqrt(a)*log(2*a*x - 2*s
qrt(a)*x*sqrt((a*x + b)/x) + b) + (b^2*c^2*d^2 + 8*a*b*c*d^3 - 24
*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*
c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*sqrt(-(b*c - a*d)/d)*log
((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a
*d)*x)/(c*x + d)) - 2*(4*a^2*c^3*d*x^3 + (b^2*c^4 - 11*a*b*c^3*d
+ 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*a^2*c*d^3
```

```
)*x)*sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3), -1/4
*((b^2*c^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*
d - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c
*d^3)*x)*sqrt((b*c - a*d)/d)*arctan(sqrt((a*x + b)/x)/sqrt((b*c -
 a*d)/d)) + 2*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2
*d^2)*x^2 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x)*sqrt(a)*log(2*a*x
- 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - (4*a^2*c^3*d*x^3 + (b^2*c^
4 - 11*a*b*c^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d
^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x
 + c^4*d^3), 1/8*(8*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a
^2*c^2*d^2)*x^2 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x)*sqrt(-a)*arc
tan(sqrt((a*x + b)/x)/sqrt(-a)) - (b^2*c^2*d^2 + 8*a*b*c*d^3 - 24
*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*
c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*sqrt(-(b*c - a*d)/d)*log
((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a
*d)*x)/(c*x + d)) + 2*(4*a^2*c^3*d*x^3 + (b^2*c^4 - 11*a*b*c^3*d
+ 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*a^2*c*d^3
)*x)*sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3), 1/4*
(4*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 +
 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x)*sqrt(-a)*arctan(sqrt((a*x + b
)/x)/sqrt(-a)) - (b^2*c^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c
^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2
*d^2 - 24*a^2*c*d^3)*x)*sqrt((b*c - a*d)/d)*arctan(sqrt((a*x + b)
/x)/sqrt((b*c - a*d)/d)) + (4*a^2*c^3*d*x^3 + (b^2*c^4 - 11*a*b*c
^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*a^2*
c*d^3)*x)*sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(a + \frac{b}{x}\right)^{\frac{5}{2}}}{(cx + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**(5/2)/(c+d/x)**3,x)
```

```
[Out] Integral(x**3*(a + b/x)**(5/2)/(c*x + d)**3, x)
```

---

**GIAC/XCAS [A]** time = 0.549039, size = 4, normalized size = 0.02

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(5/2)/(c + d/x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

## 3.146 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx$

**Optimal**. Leaf size=126

$$-\frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+9abcd+3b^2c^2\right)+\frac{bd(2ad+3bc)}{x}\right)}{3ab^2}+\frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}$$

```
[Out] -(d*Sqrt[a + b/x]*(2*(3*b^2*c^2 + 9*a*b*c*d - 2*a^2*d^2) + (b*d*(
3*b*c + 2*a*d))/x))/(3*a*b^2) + (c*Sqrt[a + b/x]*(c + d/x)^2*x)/a
 - (c^2*(b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

---

**Rubi [A]** time = 0.283658, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$-\frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+9abcd+3b^2c^2\right)+\frac{bd(2ad+3bc)}{x}\right)}{3ab^2}+\frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d/x)^3/Sqrt[a + b/x],x]
```

```
[Out] -(d*Sqrt[a + b/x]*(2*(3*b^2*c^2 + 9*a*b*c*d - 2*a^2*d^2) + (b*d*(
3*b*c + 2*a*d))/x))/(3*a*b^2) + (c*Sqrt[a + b/x]*(c + d/x)^2*x)/a
 - (c^2*(b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

---

**Rubi in Sympy [A]** time = 25.5374, size = 114, normalized size = 0.9

$$\frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}+\frac{4d\sqrt{a+\frac{b}{x}}\left(a^2d^2-\frac{9abcd}{2}-\frac{3b^2c^2}{2}-\frac{bd(2ad+3bc)}{4x}\right)}{3ab^2}+\frac{c^2\left(6ad-bc\right)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d/x)**3/(a+b/x)**(1/2),x)
```

```
[Out] c*x*sqrt(a + b/x)*(c + d/x)**2/a + 4*d*sqrt(a + b/x)*(a**2*d**2 -
 9*a*b*c*d/2 - 3*b**2*c**2/2 - b*d*(2*a*d + 3*b*c)/(4*x))/(3*a*b*
```

```
*2) + c**2*(6*a*d - b*c)*atanh(sqrt(a + b/x)/sqrt(a))/a**(3/2)
```

---

**Mathematica [A]** time = 0.288788, size = 99, normalized size = 0.79

$$\frac{c^2(6ad-bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{2a^{3/2}}+\sqrt{a+\frac{b}{x}}\left(\frac{4ad^3}{3b^2}+\frac{c^3x}{a}-\frac{2d^2(9cx+d)}{3bx}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d/x)^3/Sqrt[a + b/x],x]
```

```
[Out] Sqrt[a + b/x]*((4*a*d^3)/(3*b^2) + (c^3*x)/a - (2*d^2*(d + 9*c*x)
)/(3*b*x)) + (c^2*(-(b*c) + 6*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt
[a + b/x]*x])/(2*a^(3/2))
```

---

**Maple [B]** time = 0.021, size = 540, normalized size = 4.3

$$\frac{1}{6\,b^3x^2}\sqrt{\frac{ax+b}{x}}\left(3\,a^4\ln\left(1/2\,\frac{2\,\sqrt{x\,(ax+b)}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)d^3bx^3-3\,d^3\ln\left(1/2\,\frac{2\,\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)a^4bx^3-9\,a^3\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d/x)^3/(a+b/x)^(1/2),x)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)/x^2*(3*a^4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1
/2)+2*a*x+b)/a^(1/2))*d^3*b*x^3-3*d^3*ln(1/2*(2*(a*x^2+b*x)^(1/2)
*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*b*x^3-9*a^3*ln(1/2*(2*(x*(a*x+b))^
(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*c*d^2*b^2*x^3+9*d^2*ln(1/2*(2*(a*
x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*c*b^2*x^3+9*ln(1/2*(
2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*c^2*d*b^3*x^3*a^2+9
*d*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*c^2*b^3*
x^3*a^2-6*d^3*(a*x^2+b*x)^(1/2)*a^(9/2)*x^3+18*d^2*(a*x^2+b*x)^(1
/2)*c*a^(7/2)*b*x^3+18*d*(a*x^2+b*x)^(1/2)*c^2*b^2*x^3*a^(5/2)-6*
a^(9/2)*(x*(a*x+b))^(1/2)*d^3*x^3+18*a^(7/2)*(x*(a*x+b))^(1/2)*c*
d^2*b*x^3-18*(x*(a*x+b))^(1/2)*c^2*d*b^2*x^3*a^(5/2)+6*(x*(a*x+b)
)^(1/2)*c^3*b^3*x^3*a^(3/2)-3*b^4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(
1/2)+2*a*x+b)/a^(1/2))*c^3*x^3*a+12*d^3*(a*x^2+b*x)^(3/2)*a^(7/2)
*x-36*d^2*(a*x^2+b*x)^(3/2)*c*b*x*a^(5/2)-4*d^3*(a*x^2+b*x)^(3/2)
*b*a^(5/2))/(x*(a*x+b))^(1/2)/b^3/a^(5/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)^3/sqrt(a + b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.253138, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(b^{3}c^{3}-6\,ab^{2}c^{2}d\right)x\log\left(2\,ax\sqrt{\frac{ax+b}{x}}+(2\,ax+b)\sqrt{a}\right)-2\left(3\,b^{2}c^{3}x^{2}-2\,abd^{3}-2\left(9\,abcd^{2}-2\,a^{2}d^{3}\right)x\right)\sqrt{a}\sqrt{\frac{ax+b}{x}}}{6\,a^{\frac{3}{2}}b^{2}x},\ \frac{3\left(b^{3}c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)^3/sqrt(a + b/x),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b^3*c^3 - 6*a*b^2*c^2*d)*x*log(2*a*x*sqrt((a*x + b)/x)
+ (2*a*x + b)*sqrt(a)) - 2*(3*b^2*c^3*x^2 - 2*a*b*d^3 - 2*(9*a*b*
c*d^2 - 2*a^2*d^3)*x)*sqrt(a)*sqrt((a*x + b)/x))/(a^(3/2)*b^2*x),
 1/3*(3*(b^3*c^3 - 6*a*b^2*c^2*d)*x*arctan(a/(sqrt(-a)*sqrt((a*x
+ b)/x))) + (3*b^2*c^3*x^2 - 2*a*b*d^3 - 2*(9*a*b*c*d^2 - 2*a^2*d
^3)*x)*sqrt(-a)*sqrt((a*x + b)/x))/(sqrt(-a)*a*b^2*x)]
```

---

**Sympy [A]** time = 15.5631, size = 377, normalized size = 2.99

$$\frac{4a^{\frac{7}{2}}b^{\frac{3}{2}}d^{3}x^{2}\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^{3}x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^{4}x^{\frac{3}{2}}}+\frac{2a^{\frac{5}{2}}b^{\frac{5}{2}}d^{3}x\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^{3}x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^{4}x^{\frac{3}{2}}}-\frac{2a^{\frac{3}{2}}b^{\frac{7}{2}}d^{3}\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^{3}x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^{4}x^{\frac{3}{2}}}-\frac{4a^{4}bd^{3}x^{\frac{5}{2}}}{3a^{\frac{5}{2}}b^{3}x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^{4}x^{\frac{3}{2}}}-\frac{4a^{3}b^{2}d^{3}x^{\frac{3}{2}}}{3a^{\frac{5}{2}}b^{3}x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^{4}x^{\frac{3}{2}}}+3cd^{2}\left(\begin{cases}-\frac{1}{\sqrt{ax}} & \text{for}\ b=0\\-\frac{2\sqrt{a+\frac{b}{x}}}{b} & \text{otherwise}\end{cases}\right)+\frac{\sqrt{b}c^{3}\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a}+\frac{6c^{2}d\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}-\frac{bc^{3}\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d/x)**3/(a+b/x)**(1/2),x)
```

```
[Out] 4*a**(7/2)*b**(3/2)*d**3*x**2*sqrt(a*x/b + 1)/(3*a**(5/2)*b**3*x*
*(5/2) + 3*a**(3/2)*b**4*x**(3/2)) + 2*a**(5/2)*b**(5/2)*d**3*x*s
qrt(a*x/b + 1)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/
2)) - 2*a**(3/2)*b**(7/2)*d**3*sqrt(a*x/b + 1)/(3*a**(5/2)*b**3*x
```

```
**(5/2) + 3*a**(3/2)*b**4*x**(3/2)) - 4*a**4*b*d**3*x**(5/2)/(3*a
**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2)) - 4*a**3*b**2*d
**3*x**(3/2)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2)
) + 3*c*d**2*Piecewise((-1/(sqrt(a)*x), Eq(b, 0)), (-2*sqrt(a + b
/x)/b, True)) + sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1)/a + 6*c**2*d
*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a) - b*c**3*asinh(sqrt(a)*sq
rt(x)/sqrt(b))/a**(3/2)
```

---

**GIAC/XCAS [A]** time = 0.256988, size = 207, normalized size = 1.64

$$-\frac{1}{3}\left(\frac{3\,c^{3}\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a}-\frac{3\left(bc^{3}-6\,ac^{2}d\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}ab}+\frac{2\left(9\,b^{7}cd^{2}\sqrt{\frac{ax+b}{x}}-3\,ab^{6}d^{3}\sqrt{\frac{ax+b}{x}}+\frac{(ax+b)b^{6}d^{3}\sqrt{\frac{ax+b}{x}}}{x}\right)}{b^{9}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)^3/sqrt(a + b/x),x, algorithm="giac")
```

```
[Out] -1/3*(3*c^3*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a) - 3*(b*c^3 -
6*a*c^2*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a*b) + 2*
(9*b^7*c*d^2*sqrt((a*x + b)/x) - 3*a*b^6*d^3*sqrt((a*x + b)/x) +
(a*x + b)*b^6*d^3*sqrt((a*x + b)/x)/x)/b^9)*b
```

## 3.147 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx$

**Optimal**. Leaf size=73

$$-\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}-\frac{2d^2\sqrt{a+\frac{b}{x}}}{b}$$

```
[Out] (-2*d^2*Sqrt[a + b/x])/b + (c^2*Sqrt[a + b/x]*x)/a - (c*(b*c - 4*
a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

---

**Rubi [A]** time = 0.180524, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$-\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}-\frac{2d^2\sqrt{a+\frac{b}{x}}}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d/x)^2/Sqrt[a + b/x],x]
```

```
[Out] (-2*d^2*Sqrt[a + b/x])/b + (c^2*Sqrt[a + b/x]*x)/a - (c*(b*c - 4*
a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

---

**Rubi in Sympy [A]** time = 16.5146, size = 60, normalized size = 0.82

$$-\frac{2d^2\sqrt{a+\frac{b}{x}}}{b}+\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}+\frac{c\,(4ad-bc)\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d/x)**2/(a+b/x)**(1/2),x)
```

```
[Out] -2*d**2*sqrt(a + b/x)/b + c**2*x*sqrt(a + b/x)/a + c*(4*a*d - b*c
)*atanh(sqrt(a + b/x)/sqrt(a))/a**(3/2)
```

---

**Mathematica [A]** time = 0.161446, size = 75, normalized size = 1.03

$$\frac{c(4ad-bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{2a^{3/2}}+\sqrt{a+\frac{b}{x}}\left(\frac{c^2x}{a}-\frac{2d^2}{b}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d/x)^2/Sqrt[a + b/x],x]
```

```
[Out] Sqrt[a + b/x]*((-2*d^2)/b + (c^2*x)/a) + (c*(-(b*c) + 4*a*d)*Log[
b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x])/(2*a^(3/2))
```

---

**Maple [B]** time = 0.02, size = 354, normalized size = 4.9

$$\frac{1}{2\,x b^2}\sqrt{\frac{ax+b}{x}}\left(d^2a^3\ln\left(\frac{1}{2}\left(2\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b\right)\frac{1}{\sqrt{a}}\right)x^2b-a^3\ln\left(\frac{1}{2}\left(2\sqrt{x\,(ax+b)}\sqrt{a}+2\,ax+b\right)\frac{1}{\sqrt{a}}\right)d^2x^2b+2\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d/x)^2/(a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)/x*(d^2*a^3*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1
/2)+2*a*x+b)/a^(1/2))*x^2*b-a^3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/
2)+2*a*x+b)/a^(1/2))*d^2*x^2*b+2*d^2*a^(7/2)*(a*x^2+b*x)^(1/2)*x^
2+4*d*(a*x^2+b*x)^(1/2)*c*x^2*b*a^(5/2)+2*a^(7/2)*(x*(a*x+b))^(1/
2)*d^2*x^2-4*(x*(a*x+b))^(1/2)*c*d*x^2*b*a^(5/2)+2*(x*(a*x+b))^(1
/2)*c^2*x^2*b^2*a^(3/2)+2*d*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2
*a*x+b)/a^(1/2))*c*x^2*b^2*a^2+2*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1
/2)+2*a*x+b)/a^(1/2))*c*d*x^2*b^2*a^2-4*d^2*(a*x^2+b*x)^(3/2)*a^(
5/2)-b^3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*c^
2*x^2*a)/(x*(a*x+b))^(1/2)/b^2/a^(5/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)^2/sqrt(a + b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 0.248367, size = 1, normalized size = 0.01

$$\left[\frac{2\left(bc^2x - 2\,ad^2\right)\sqrt{a}\sqrt{\frac{ax+b}{x}} - \left(b^2c^2 - 4\,abcd\right)\log\left(2\,ax\sqrt{\frac{ax+b}{x}} + (2\,ax + b)\sqrt{a}\right)}{2\,a^{\frac{3}{2}}b}, \frac{\left(bc^2x - 2\,ad^2\right)\sqrt{-a}\sqrt{\frac{ax+b}{x}} + \left(b^2c^2 - 4\,abc\right.}{\sqrt{-a}ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)^2/sqrt(a + b/x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b*c^2*x - 2*a*d^2)*sqrt(a)*sqrt((a*x + b)/x) - (b^2*c^2
- 4*a*b*c*d)*log(2*a*x*sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(a)))/
(a^(3/2)*b), ((b*c^2*x - 2*a*d^2)*sqrt(-a)*sqrt((a*x + b)/x) + (b
^2*c^2 - 4*a*b*c*d)*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x))))/(sqrt
(-a)*a*b)]
```

**Sympy [A]** time = 12.382, size = 105, normalized size = 1.44

$$d^2\left(\begin{cases} -\frac{1}{\sqrt{a}x} & \text{for}\ b = 0 \\ -\frac{2\sqrt{a+\frac{b}{x}}}{b} & \text{otherwise} \end{cases}\right) + \frac{\sqrt{b}c^2\sqrt{x}\sqrt{\frac{ax}{b} + 1}}{a} + \frac{4cd\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}} - \frac{bc^2\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d/x)**2/(a+b/x)**(1/2),x)
```

```
[Out] d**2*Piecewise((-1/(sqrt(a)*x), Eq(b, 0)), (-2*sqrt(a + b/x)/b, T
rue)) + sqrt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1)/a + 4*c*d*asinh(sqrt
(a)*sqrt(x)/sqrt(b))/sqrt(a) - b*c**2*asinh(sqrt(a)*sqrt(x)/sqrt(
b))/a**(3/2)
```

**GIAC/XCAS [A]** time = 0.255677, size = 131, normalized size = 1.79

$$-b\left(\frac{c^2\sqrt{\frac{ax+b}{x}}}{\left(a - \frac{ax+b}{x}\right)a} + \frac{2\,d^2\sqrt{\frac{ax+b}{x}}}{b^2} - \frac{\left(bc^2 - 4\,acd\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)^2/sqrt(a + b/x),x, algorithm="giac")
```

```
[Out] -b*(c^2*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a) + 2*d^2*sqrt((a*x
 + b)/x)/b^2 - (b*c^2 - 4*a*c*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a
))/(sqrt(-a)*a*b))
```

## 3.148 $\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx$

**Optimal**. Leaf size=51

$$\frac{cx\sqrt{a+\frac{b}{x}}}{a}-\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] (c*Sqrt[a + b/x]*x)/a - ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt
[a]])/a^(3/2)
```

---

**Rubi [A]** time = 0.109843, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{cx\sqrt{a+\frac{b}{x}}}{a}-\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d/x)/Sqrt[a + b/x],x]
```

```
[Out] (c*Sqrt[a + b/x]*x)/a - ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt
[a]])/a^(3/2)
```

---

**Rubi in Sympy [A]** time = 9.44179, size = 42, normalized size = 0.82

$$\frac{cx\sqrt{a+\frac{b}{x}}}{a}+\frac{2\left(ad-\frac{bc}{2}\right)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d/x)/(a+b/x)**(1/2),x)
```

```
[Out] c*x*sqrt(a + b/x)/a + 2*(a*d - b*c/2)*atanh(sqrt(a + b/x)/sqrt(a)
)/a**(3/2)
```

---

**Mathematica [A]** time = 0.0753147, size = 62, normalized size = 1.22

$$\frac{(2ad-bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{2a^{3/2}}+\frac{cx\sqrt{a+\frac{b}{x}}}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d/x)/Sqrt[a + b/x],x]
```

```
[Out] (c*Sqrt[a + b/x]*x)/a + ((-(b*c) + 2*a*d)*Log[b + 2*a*x + 2*Sqrt[
a]*Sqrt[a + b/x]*x])/(2*a^(3/2))
```

---

**Maple [B]** time = 0.017, size = 179, normalized size = 3.5

$$-\frac{x}{2\,b}\sqrt{\frac{ax+b}{x}}\left(2\sqrt{x\,(ax+b)}da^{5/2}-2\sqrt{x\,(ax+b)}cba^{3/2}-2\,d\sqrt{ax^2+bx}a^{5/2}-d\ln\left(\frac{1}{2}\left(2\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b\right)\frac{1}{\sqrt{a}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d/x)/(a+b/x)^(1/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(2*(x*(a*x+b))^(1/2)*d*a^(5/2)-2*(x*(a*x
+b))^(1/2)*c*b*a^(3/2)-2*d*(a*x^2+b*x)^(1/2)*a^(5/2)-d*ln(1/2*(2*
(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b*a^2-ln(1/2*(2*(x*(a
*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*d*b*a^2+b^2*ln(1/2*(2*(x*(
a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*c*a)/(x*(a*x+b))^(1/2)/b/
a^(5/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)/sqrt(a + b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.249634, size = 1, normalized size = 0.02

$$\left[\frac{2\,\sqrt{a}cx\sqrt{\frac{ax+b}{x}} - (bc-2\,ad)\log\left(2\,ax\sqrt{\frac{ax+b}{x}} + (2\,ax+b)\sqrt{a}\right)}{2\,a^{\frac{3}{2}}}, \frac{\sqrt{-a}cx\sqrt{\frac{ax+b}{x}} + (bc-2\,ad)\arctan\left(\frac{a}{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}\right)}{\sqrt{-a}a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)/sqrt(a + b/x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(a)*c*x*sqrt((a*x + b)/x) - (b*c - 2*a*d)*log(2*a*x*s
qrt((a*x + b)/x) + (2*a*x + b)*sqrt(a)))/a^(3/2), (sqrt(-a)*c*x*s
qrt((a*x + b)/x) + (b*c - 2*a*d)*arctan(a/(sqrt(-a)*sqrt((a*x + b
)/x))))/(sqrt(-a)*a)]
```

---

**Sympy [A]** time = 25.0676, size = 73, normalized size = 1.43

$$\frac{\sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a} + \frac{2d\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}} - \frac{bc\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d/x)/(a+b/x)**(1/2),x)
```

```
[Out] sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1)/a + 2*d*asinh(sqrt(a)*sqrt(x)/s
qrt(b))/sqrt(a) - b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/2)
```

---

**GIAC/XCAS [A]** time = 0.250741, size = 99, normalized size = 1.94

$$-b\left(\frac{c\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a} - \frac{(bc-2\,ad)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)/sqrt(a + b/x),x, algorithm="giac")
```

```
[Out] -b*(c*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a) - (b*c - 2*a*d)*arc
tan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a*b))
```

## 3.149 $\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx$

**Optimal**. Leaf size=43

$$\frac{x\sqrt{a+\frac{b}{x}}}{a}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] (Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

---

**Rubi [A]** time = 0.059137, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$\frac{x\sqrt{a+\frac{b}{x}}}{a}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a + b/x],x]

[Out] (Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

---

**Rubi in Sympy [A]** time = 5.35715, size = 32, normalized size = 0.74

$$\frac{x\sqrt{a+\frac{b}{x}}}{a}-\frac{b\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b/x)**(1/2),x)

[Out] x*sqrt(a + b/x)/a - b*atanh(sqrt(a + b/x)/sqrt(a))/a**(3/2)
```

---

**Mathematica [A]** time = 0.0467834, size = 53, normalized size = 1.23

$$\frac{x\sqrt{a+\frac{b}{x}}}{a}-\frac{b\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a + b/x],x]
```

```
[Out] (Sqrt[a + b/x]*x)/a - (b*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*
x])/(2*a^(3/2))
```

---

**Maple [A]** time = 0.005, size = 70, normalized size = 1.6

$$-\frac{x}{2}\sqrt{\frac{ax+b}{x}}\left(b\ln\left(\frac{1}{2}\left(2\sqrt{x(ax+b)}\sqrt{a}+2ax+b\right)\frac{1}{\sqrt{a}}\right)-2\sqrt{x(ax+b)}\sqrt{a}\right)\frac{1}{\sqrt{x(ax+b)}}a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b/x)^(1/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(b*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2
*a*x+b)/a^(1/2))-2*(x*(a*x+b))^(1/2)*a^(1/2))/(x*(a*x+b))^(1/2)/a
^(3/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(a + b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.247426, size = 1, normalized size = 0.02

$$\left[\frac{2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\log\left(-2ax\sqrt{\frac{ax+b}{x}}+(2ax+b)\sqrt{a}\right)}{2a^{\frac{3}{2}}},\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}+b\arctan\left(\frac{a}{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}\right)}{\sqrt{-a}a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(a + b/x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(a)*x*sqrt((a*x + b)/x) + b*log(-2*a*x*sqrt((a*x + b)
/x) + (2*a*x + b)*sqrt(a)))/a^(3/2), (sqrt(-a)*x*sqrt((a*x + b)/x
) + b*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x))))/(sqrt(-a)*a)]
```

---

**Sympy [A]** time = 7.83873, size = 44, normalized size = 1.02

$$\frac{\sqrt{b}\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a} - \frac{b\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b/x)**(1/2),x)
```

```
[Out] sqrt(b)*sqrt(x)*sqrt(a*x/b + 1)/a - b*asinh(sqrt(a)*sqrt(x)/sqrt(
b))/a**(3/2)
```

---

**GIAC/XCAS [A]** time = 0.237001, size = 96, normalized size = 2.23

$$-\frac{b\ln\left(|b|\right)\operatorname{sign}\left(x\right)}{2\,a^{\frac{3}{2}}} + \frac{b\ln\left(\left|-2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}-b\right|\right)}{2\,a^{\frac{3}{2}}\operatorname{sign}\left(x\right)} + \frac{\sqrt{ax^2+bx}}{a\operatorname{sign}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(a + b/x),x, algorithm="giac")
```

```
[Out] -1/2*b*ln(abs(b))*sign(x)/a^(3/2) + 1/2*b*ln(abs(-2*(sqrt(a)*x -
sqrt(a*x^2 + b*x))*sqrt(a) - b))/(a^(3/2)*sign(x)) + sqrt(a*x^2 +
 b*x)/(a*sign(x))
```

## 3.150 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx$

**Optimal**. Leaf size=108

$$-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2}-\frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

```
[Out] (Sqrt[a + b/x]*x)/(a*c) - (2*d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x
])/Sqrt[b*c - a*d]])/(c^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/(a^(3/2)*c^2)
```

---

**Rubi [A]** time = 0.316674, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2}-\frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a + b/x]*(c + d/x)),x]
```

```
[Out] (Sqrt[a + b/x]*x)/(a*c) - (2*d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x
])/Sqrt[b*c - a*d]])/(c^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/(a^(3/2)*c^2)
```

---

**Rubi in Sympy [A]** time = 40.7588, size = 92, normalized size = 0.85

$$\frac{2d^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{c^2\sqrt{ad-bc}}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}-\frac{2\left(ad+\frac{bc}{2}\right)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+d/x)/(a+b/x)**(1/2),x)
```

```
[Out] 2*d**(3/2)*atanh(sqrt(d)*sqrt(a + b/x)/sqrt(a*d - b*c))/(c**2*sqr
t(a*d - b*c)) + x*sqrt(a + b/x)/(a*c) - 2*(a*d + b*c/2)*atanh(sqr
t(a + b/x)/sqrt(a))/(a**(3/2)*c**2)
```

**Mathematica [A]** time = 0.467142, size = 157, normalized size = 1.45

$$\frac{-\frac{(2ad+bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{a^{3/2}}+\frac{2d^{3/2}\log(cx+d)}{\sqrt{ad-bc}}-\frac{2d^{3/2}\log\left(2\sqrt{d}x\sqrt{a+\frac{b}{x}}\sqrt{ad-bc}-2adx+bcx-bd\right)}{\sqrt{ad-bc}}+\frac{2cx\sqrt{a+\frac{b}{x}}}{a}}{2c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[a + b/x]*(c + d/x)),x]
```

```
[Out] ((2*c*Sqrt[a + b/x]*x)/a + (2*d^(3/2)*Log[d + c*x])/Sqrt[-(b*c) +
 a*d] - ((b*c + 2*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x]
)/a^(3/2) - (2*d^(3/2)*Log[-(b*d) + b*c*x - 2*a*d*x + 2*Sqrt[d]*S
qrt[-(b*c) + a*d]*Sqrt[a + b/x]*x])/Sqrt[-(b*c) + a*d])/(2*c^2)
```

**Maple [B]** time = 0.019, size = 389, normalized size = 3.6

$$\frac{x}{2\,c^3\,(ad-bc)}\sqrt{\frac{ax+b}{x}}\left(-2\,d^2\ln\left(1/2\,\frac{2\,\sqrt{x\,(ax+b)}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)a^3c\sqrt{\frac{(ad-bc)\,d}{c^2}}+2\,d\sqrt{x\,(ax+b)}c^2a^{5/2}\sqrt{\frac{(ad-bc)\,d}{c^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+d/x)/(a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(-2*d^2*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/
2)+2*a*x+b)/a^(1/2))*a^3*c*((a*d-b*c)*d/c^2)^(1/2)+2*d*(x*(a*x+b)
)^(1/2)*c^2*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)-2*b*(x*(a*x+b))^(1/2)
*c^3*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)-2*d^3*ln((2*(x*(a*x+b))^(1/2
)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(7/2)+2
*d^2*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*
c*x-b*d)/(c*x+d))*b*c*a^(5/2)+d*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/
2)+2*a*x+b)/a^(1/2))*b*c^2*a^2*((a*d-b*c)*d/c^2)^(1/2)+b^2*ln(1/2
*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*c^3*((a*d-b*c)*d/
c^2)^(1/2)*a)/(x*(a*x+b))^(1/2)/c^3/(a*d-b*c)/a^(5/2)/((a*d-b*c)*
d/c^2)^(1/2)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a + b/x)*(c + d/x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.277234, size = 1, normalized size = 0.01

$$\left[\frac{2\,a^{\frac{3}{2}}d\sqrt{-\frac{d}{bc-ad}}\log\left(-\frac{2\,(bc-ad)x\sqrt{-\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}-bd+(bc-2\,ad)x}{cx+d}\right)+2\,\sqrt{a}cx\sqrt{\frac{ax+b}{x}}+(bc+2\,ad)\log\left(-2\,ax\sqrt{\frac{ax+b}{x}}+(2\,ax+b)\sqrt{a}\right)}{2\,a^{\frac{3}{2}}c^2}\right.$$

$$-\frac{4\,a^{\frac{3}{2}}d\sqrt{\frac{d}{bc-ad}}\arctan\left(-\frac{(bc-ad)\sqrt{\frac{d}{bc-ad}}}{d\sqrt{\frac{ax+b}{x}}}\right)-2\,\sqrt{a}cx\sqrt{\frac{ax+b}{x}}-(bc+2\,ad)\log\left(-2\,ax\sqrt{\frac{ax+b}{x}}+(2\,ax+b)\sqrt{a}\right)}{2\,a^{\frac{3}{2}}c^2},$$

$$\left.-\frac{2\,\sqrt{-a}ad\sqrt{\frac{d}{bc-ad}}\arctan\left(-\frac{(bc-ad)\sqrt{\frac{d}{bc-ad}}}{d\sqrt{\frac{ax+b}{x}}}\right)-\sqrt{-a}cx\sqrt{\frac{ax+b}{x}}-(bc+2\,ad)\arctan\left(\frac{a}{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}\right)}{\sqrt{-a}ac^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a + b/x)*(c + d/x)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a^(3/2)*d*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt
(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x
+ d)) + 2*sqrt(a)*c*x*sqrt((a*x + b)/x) + (b*c + 2*a*d)*log(-2*a*
x*sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(a)))/(a^(3/2)*c^2), (sqrt(
-a)*a*d*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c -
 a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + sq
rt(-a)*c*x*sqrt((a*x + b)/x) + (b*c + 2*a*d)*arctan(a/(sqrt(-a)*s
qrt((a*x + b)/x))))/(sqrt(-a)*a*c^2), -1/2*(4*a^(3/2)*d*sqrt(d/(b
*c - a*d))*arctan(-(b*c - a*d)*sqrt(d/(b*c - a*d))/(d*sqrt((a*x +
 b)/x))) - 2*sqrt(a)*c*x*sqrt((a*x + b)/x) - (b*c + 2*a*d)*log(-2
*a*x*sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(a)))/(a^(3/2)*c^2), -(2
*sqrt(-a)*a*d*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d/(b*c
 - a*d))/(d*sqrt((a*x + b)/x))) - sqrt(-a)*c*x*sqrt((a*x + b)/x)
- (b*c + 2*a*d)*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x))))/(sqrt(-a)
*a*c^2)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+\frac{b}{x}}\,(cx+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+d/x)/(a+b/x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b/x)*(c*x + d)), x)
```

---

**GIAC/XCAS [A]** time = 0.252986, size = 174, normalized size = 1.61

$$-b\left(\frac{2\,d^2\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}bc^2}+\frac{\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)ac}-\frac{(bc+2\,ad)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}abc^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a + b/x)*(c + d/x)),x, algorithm="giac")
```

```
[Out] -b*(2*d^2*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))/(sqrt(b
*c*d - a*d^2)*b*c^2) + sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a*c)
- (b*c + 2*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a*b*
c^2))
```

## 3.151 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx$

**Optimal**. Leaf size=172

$$-\frac{(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^3}-\frac{d^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}}+\frac{d\sqrt{a+\frac{b}{x}}(bc-2ad)}{ac^2\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

```
[Out] (d*(b*c - 2*a*d)*Sqrt[a + b/x])/(a*c^2*(b*c - a*d)*(c + d/x)) + (
Sqrt[a + b/x]*x)/(a*c*(c + d/x)) - (d^(3/2)*(5*b*c - 4*a*d)*ArcTa
n[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(3/2
)) - ((b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(3/2)*c^3)
```

---

**Rubi [A]** time = 0.650209, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^3}-\frac{d^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}}+\frac{d\sqrt{a+\frac{b}{x}}(bc-2ad)}{ac^2\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a + b/x]*(c + d/x)^2),x]
```

```
[Out] (d*(b*c - 2*a*d)*Sqrt[a + b/x])/(a*c^2*(b*c - a*d)*(c + d/x)) + (
Sqrt[a + b/x]*x)/(a*c*(c + d/x)) - (d^(3/2)*(5*b*c - 4*a*d)*ArcTa
n[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(3/2
)) - ((b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(3/2)*c^3)
```

---

**Rubi in Sympy [A]** time = 68.0127, size = 141, normalized size = 0.82

$$-\frac{dx\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)(ad-bc)}+\frac{d^{\frac{3}{2}}(4ad-5bc)\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{c^3(ad-bc)^{\frac{3}{2}}}+\frac{x\sqrt{a+\frac{b}{x}}(2ad-bc)}{ac^2(ad-bc)}-\frac{(4ad+bc)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+d/x)**2/(a+b/x)**(1/2),x)
```

```
[Out] -d*x*sqrt(a + b/x)/(c*(c + d/x)*(a*d - b*c)) + d**(3/2)*(4*a*d -
5*b*c)*atanh(sqrt(d)*sqrt(a + b/x)/sqrt(a*d - b*c))/(c**3*(a*d -
b*c)**(3/2)) + x*sqrt(a + b/x)*(2*a*d - b*c)/(a*c**2*(a*d - b*c))
 - (4*a*d + b*c)*atanh(sqrt(a + b/x)/sqrt(a))/(a**(3/2)*c**3)
```

---

**Mathematica [C]** time = 0.84081, size = 224, normalized size = 1.3

$$-\frac{\frac{(4ad+bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{a^{3/2}}+\frac{id^{3/2}(5bc-4ad)\log\left(\frac{2c^4\sqrt{bc-ad}\left(2\sqrt{d}x\sqrt{a+\frac{b}{x}}\sqrt{bc-ad}-2iadx-ib(d-cx)\right)}{d^{5/2}(cx+d)(5bc-4ad)}\right)}{(bc-ad)^{3/2}}+\frac{2cx\sqrt{a+\frac{b}{x}}(bc(cx+d)-ad(cx+2d))}{a(cx+d)(ad-bc)}}{2c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[a + b/x]*(c + d/x)^2),x]
```

```
[Out] -((2*c*Sqrt[a + b/x]*x*(b*c*(d + c*x) - a*d*(2*d + c*x)))/(a*(-(b
*c) + a*d)*(d + c*x)) + ((b*c + 4*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*
Sqrt[a + b/x]*x])/a^(3/2) + (I*d^(3/2)*(5*b*c - 4*a*d)*Log[(2*c^4
*Sqrt[b*c - a*d]*((-2*I)*a*d*x + 2*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[a
 + b/x]*x - I*b*(d - c*x)))/(d^(5/2)*(5*b*c - 4*a*d)*(d + c*x))])
/(b*c - a*d)^(3/2))/(2*c^3)
```

---

**Maple [B]** time = 0.029, size = 1137, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+d/x)^2/(a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(-4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2
*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^4*x*c^2*d^3-2*(x*(a*x+
b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*a^(7/2)*x^2*c^4*d-4*ln(1/2*(2*(
x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2
)*a^4*c*d^4+7*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2
))*((a*d-b*c)*d/c^2)^(1/2)*a^3*x*b*c^3*d^2+2*d*c^4*(x*(a*x+b))^(3
/2)*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)+2*(x*(a*x+b))^(1/2)*((a*d-b*c
)*d/c^2)^(1/2)*a^(7/2)*x*c^3*d^2-6*(x*(a*x+b))^(1/2)*((a*d-b*c)*d
/c^2)^(1/2)*a^(5/2)*x*b*c^4*d+2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^
2)^(1/2)*a^(3/2)*x*b^2*c^5+7*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+
2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^3*b*c^2*d^3-2*ln(1/2*
(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^
(1/2)*a^2*x*b^2*c^4*d-4*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)
^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(9/2)*x*c*d^4+9*ln((2*(x*(
a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d
))*a^(7/2)*x*b*c^2*d^3-5*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2
```

```
)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(5/2)*x*b^2*c^3*d^2+4*(x*
(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*a^(7/2)*c^2*d^3-6*(x*(a*x+
b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*a^(5/2)*b*c^3*d^2+2*(x*(a*x+b))
^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*a^(3/2)*b^2*c^4*d-2*ln(1/2*(2*(x*(
a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a
^2*b^2*c^3*d^2-ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/
2))*((a*d-b*c)*d/c^2)^(1/2)*x*a*b^3*c^5-4*ln((2*(x*(a*x+b))^(1/2)
*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(9/2)*d^
5+9*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c
*x-b*d)/(c*x+d))*a^(7/2)*b*c*d^4-5*ln((2*(x*(a*x+b))^(1/2)*((a*d-
b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(5/2)*b^2*c^2*d
^3-ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*
c)*d/c^2)^(1/2)*a*b^3*c^4*d)/(x*(a*x+b))^(1/2)/a^(5/2)/(a*d-b*c)^
2/c^4/((a*d-b*c)*d/c^2)^(1/2)/(c*x+d)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a + b/x)*(c + d/x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.359771, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a + b/x)*(c + d/x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*s
qrt(a)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c -
a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(
(b*c^3 - a*c^2*d)*x^2 + (b*c^2*d - 2*a*c*d^2)*x)*sqrt(a)*sqrt((a*
x + b)/x) + (b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a
*b*c^2*d - 4*a^2*c*d^2)*x)*log(-2*a*x*sqrt((a*x + b)/x) + (2*a*x
+ b)*sqrt(a)))/((a*b*c^4*d - a^2*c^3*d^2 + (a*b*c^5 - a^2*c^4*d)*
x)*sqrt(a)), -1/2*(2*(5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*
a^2*c*d^2)*x)*sqrt(a)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*sqr
t(d/(b*c - a*d))/(d*sqrt((a*x + b)/x))) - 2*((b*c^3 - a*c^2*d)*x^
2 + (b*c^2*d - 2*a*c*d^2)*x)*sqrt(a)*sqrt((a*x + b)/x) - (b^2*c^2
*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d
^2)*x)*log(-2*a*x*sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(a)))/((a*b
*c^4*d - a^2*c^3*d^2 + (a*b*c^5 - a^2*c^4*d)*x)*sqrt(a)), 1/2*((5
```

```
*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(-a)*
sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*s
qrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((b*c^3
- a*c^2*d)*x^2 + (b*c^2*d - 2*a*c*d^2)*x)*sqrt(-a)*sqrt((a*x + b)
/x) + 2*(b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c
^2*d - 4*a^2*c*d^2)*x)*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x))))/((
a*b*c^4*d - a^2*c^3*d^2 + (a*b*c^5 - a^2*c^4*d)*x)*sqrt(-a)), -((
5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(-a)
*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d/(b*c - a*d))/(d*s
qrt((a*x + b)/x))) - ((b*c^3 - a*c^2*d)*x^2 + (b*c^2*d - 2*a*c*d^
2)*x)*sqrt(-a)*sqrt((a*x + b)/x) - (b^2*c^2*d + 3*a*b*c*d^2 - 4*a
^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)*arctan(a/(sqrt(
-a)*sqrt((a*x + b)/x))))/((a*b*c^4*d - a^2*c^3*d^2 + (a*b*c^5 - a
^2*c^4*d)*x)*sqrt(-a))]
```

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+d/x)**2/(a+b/x)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

---

**GIAC/XCAS [A]** time = 0.25707, size = 393, normalized size = 2.28

$$-b\left(\frac{\left(5\,bcd^2-4\,ad^3\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^2c^4-abc^3d\right)\sqrt{bcd-ad^2}}+\frac{b^2c^2\sqrt{\frac{ax+b}{x}}-2\,abcd\sqrt{\frac{ax+b}{x}}+2\,a^2d^2\sqrt{\frac{ax+b}{x}}+\frac{(ax+b)bcd\sqrt{\frac{ax+b}{x}}}{x}-\frac{2\,(ax+b)ad^2\sqrt{\frac{ax+b}{x}}}{x}}{\left(abc^3-a^2c^2d\right)\left(abc-a^2d-\frac{(ax+b)bc}{x}+\frac{2\,(ax+b)ad}{x}-\frac{(ax+b)^2d}{x^2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a + b/x)*(c + d/x)^2),x, algorithm="giac")
```

```
[Out] -b*((5*b*c*d^2 - 4*a*d^3)*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d -
 a*d^2))/((b^2*c^4 - a*b*c^3*d)*sqrt(b*c*d - a*d^2)) + (b^2*c^2*s
qrt((a*x + b)/x) - 2*a*b*c*d*sqrt((a*x + b)/x) + 2*a^2*d^2*sqrt((
a*x + b)/x) + (a*x + b)*b*c*d*sqrt((a*x + b)/x)/x - 2*(a*x + b)*a
*d^2*sqrt((a*x + b)/x)/x)/((a*b*c^3 - a^2*c^2*d)*(a*b*c - a^2*d -
 (a*x + b)*b*c/x + 2*(a*x + b)*a*d/x - (a*x + b)^2*d/x^2)) - (b*c
 + 4*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a*b*c^3))
```

## 3.152 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3} dx$

**Optimal**. Leaf size=250

$$-\frac{(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^4}-\frac{d^{3/2}\left(24a^2d^2-56abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{5/2}}+\frac{d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{4ac^3\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{2ac^2\left(c+\frac{d}{x}\right)^2(bc-ad)}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

```
[Out] (d*(2*b*c - 3*a*d)*Sqrt[a + b/x])/(2*a*c^2*(b*c - a*d)*(c + d/x)^
2) + (d*(b*c - 4*a*d)*(4*b*c - 3*a*d)*Sqrt[a + b/x])/(4*a*c^3*(b*
c - a*d)^2*(c + d/x)) + (Sqrt[a + b/x]*x)/(a*c*(c + d/x)^2) - (d^
(3/2)*(35*b^2*c^2 - 56*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt
[a + b/x])/Sqrt[b*c - a*d]])/(4*c^4*(b*c - a*d)^(5/2)) - ((b*c +
6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(3/2)*c^4)
```

---

**Rubi [A]** time = 1.08312, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^4}-\frac{d^{3/2}\left(24a^2d^2-56abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{5/2}}+\frac{d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{4ac^3\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{2ac^2\left(c+\frac{d}{x}\right)^2(bc-ad)}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a + b/x]*(c + d/x)^3),x]
```


```
[Out] (d*(2*b*c - 3*a*d)*Sqrt[a + b/x])/(2*a*c^2*(b*c - a*d)*(c + d/x)^
2) + (d*(b*c - 4*a*d)*(4*b*c - 3*a*d)*Sqrt[a + b/x])/(4*a*c^3*(b*
c - a*d)^2*(c + d/x)) + (Sqrt[a + b/x]*x)/(a*c*(c + d/x)^2) - (d^
(3/2)*(35*b^2*c^2 - 56*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt
[a + b/x])/Sqrt[b*c - a*d]])/(4*c^4*(b*c - a*d)^(5/2)) - ((b*c +
6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(3/2)*c^4)
```


---

**Rubi in Sympy [A]** time = 117.385, size = 218, normalized size = 0.87

$$-\frac{dx\sqrt{a+\frac{b}{x}}}{2c\left(c+\frac{d}{x}\right)^2(ad-bc)}-\frac{3dx\sqrt{a+\frac{b}{x}}\,(2ad-3bc)}{4c^2\left(c+\frac{d}{x}\right)(ad-bc)^2}+\frac{d^{\frac{3}{2}}\left(24a^2d^2-56abcd+35b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{4c^4(ad-bc)^{\frac{5}{2}}}+\frac{x\sqrt{a+\frac{b}{x}}\,(3ad-4bc)(4ad-bc)}{4ac^3(ad-bc)^2}-\frac{(6ad+bc)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c+d/x)**3/(a+b/x)**(1/2),x)
```

```
[Out] -d*x*sqrt(a + b/x)/(2*c*(c + d/x)**2*(a*d - b*c)) - 3*d*x*sqrt(a
+ b/x)*(2*a*d - 3*b*c)/(4*c**2*(c + d/x)*(a*d - b*c)**2) + d**(3/
2)*(24*a**2*d**2 - 56*a*b*c*d + 35*b**2*c**2)*atanh(sqrt(d)*sqrt(
a + b/x)/sqrt(a*d - b*c))/(4*c**4*(a*d - b*c)**(5/2)) + x*sqrt(a
+ b/x)*(3*a*d - 4*b*c)*(4*a*d - b*c)/(4*a*c**3*(a*d - b*c)**2) -
(6*a*d + b*c)*atanh(sqrt(a + b/x)/sqrt(a))/(a**(3/2)*c**4)
```

---

**Mathematica [C]** time = 0.950155, size = 301, normalized size = 1.2

$$\frac{-\frac{4(6ad+bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{a^{3/2}}+\frac{2cx\sqrt{a+\frac{b}{x}}\left(2a^2d^2\left(2c^2x^2+9cdx+6d^2\right)-abcd\left(8c^2x^2+29cdx+19d^2\right)+4b^2c^2(cx+d)^2\right)}{a(cx+d)^2(bc-ad)^2}-\frac{id^{3/2}\left(24a^2d^2-56abcd+35b^2\cdots\right.}{\cdots}}{8c^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[a + b/x]*(c + d/x)^3),x]
```

```
[Out] ((2*c*Sqrt[a + b/x]*x*(4*b^2*c^2*(d + c*x)^2 + 2*a^2*d^2*(6*d^2 +
 9*c*d*x + 2*c^2*x^2) - a*b*c*d*(19*d^2 + 29*c*d*x + 8*c^2*x^2)))
/(a*(b*c - a*d)^2*(d + c*x)^2) - (4*(b*c + 6*a*d)*Log[b + 2*a*x +
 2*Sqrt[a]*Sqrt[a + b/x]*x])/a^(3/2) - (I*d^(3/2)*(35*b^2*c^2 - 5
6*a*b*c*d + 24*a^2*d^2)*Log[(8*c^5*(b*c - a*d)^(3/2)*((-2*I)*a*d*
x + 2*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[a + b/x]*x - I*b*(d - c*x)))/(
d^(5/2)*(35*b^2*c^2 - 56*a*b*c*d + 24*a^2*d^2)*(d + c*x))])/(b*c
- a*d)^(5/2))/(8*c^4)
```

---

**Maple [B]** time = 0.025, size = 2269, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c+d/x)^3/(a+b/x)^(1/2),x)
```

```
[Out] 1/8*((a*x+b)/x)^(1/2)*x*(-12*(x*(a*x+b))^(1/2)*a^(9/2)*((a*d-b*c)
*d/c^2)^(1/2)*x^3*c^5*d^2+12*(x*(a*x+b))^(3/2)*a^(7/2)*((a*d-b*c)
*d/c^2)^(1/2)*x*c^5*d^2+68*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*
a*x+b)/a^(1/2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*b*c^2*d^5+80*ln((2*(x
*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x
+d))*a^(9/2)*x^2*b*c^3*d^4-91*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*
d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(7/2)*x^2*b^2*c^4*d^
3+35*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*
c*x-b*d)/(c*x+d))*a^(5/2)*x^2*b^3*c^5*d^2-18*(x*(a*x+b))^(3/2)*a^
(5/2)*((a*d-b*c)*d/c^2)^(1/2)*b*c^5*d^2+36*(x*(a*x+b))^(1/2)*a^(9
/2)*((a*d-b*c)*d/c^2)^(1/2)*x*c^3*d^4-60*ln(1/2*(2*(x*(a*x+b))^(1
/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*b^2*c^3
*d^4+160*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*
x+b*c*x-b*d)/(c*x+d))*a^(9/2)*x*b*c^2*d^5-182*ln((2*(x*(a*x+b))^(
1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(7/2
)*x*b^2*c^3*d^4+70*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2
)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(5/2)*x*b^3*c^4*d^3-62*(x*(a*x+
b))^(1/2)*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*b*c^3*d^4-8*(x*(a*x+b))
^(1/2)*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^3*c^7+4*ln(1/2*(2*(x
*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)
*x^2*a*b^4*c^7-8*(x*(a*x+b))^(1/2)*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2
)*b^3*c^5*d^2+4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1
/2))*((a*d-b*c)*d/c^2)^(1/2)*a*b^4*c^5*d^2-24*ln((2*(x*(a*x+b))^(
1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(11/
2)*d^7+12*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a
^2*((a*d-b*c)*d/c^2)^(1/2)*b^3*c^4*d^3-24*ln(1/2*(2*(x*(a*x+b))^(
1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5*((a*d-b*c)*d/c^2)^(1/2)*x^2*c^
3*d^4-48*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^
5*((a*d-b*c)*d/c^2)^(1/2)*x*c^2*d^5-18*(x*(a*x+b))^(1/2)*a^(7/2)*
((a*d-b*c)*d/c^2)^(1/2)*x^2*b*c^5*d^2+46*(x*(a*x+b))^(1/2)*a^(5/2
)*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^2*c^6*d+68*ln(1/2*(2*(x*(a*x+b))^
(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*x^2*b
*c^4*d^3+22*(x*(a*x+b))^(1/2)*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*x^3
*b*c^6*d+136*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)
)*a^4*((a*d-b*c)*d/c^2)^(1/2)*x*b*c^3*d^4-60*ln(1/2*(2*(x*(a*x+b)
)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*x^2
*b^2*c^5*d^2-22*(x*(a*x+b))^(3/2)*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)
*x*b*c^6*d+92*(x*(a*x+b))^(1/2)*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*x
*b^2*c^5*d^2+24*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1
/2))*a^2*((a*d-b*c)*d/c^2)^(1/2)*x*b^3*c^5*d^2-120*ln(1/2*(2*(x*(
a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/
2)*x*b^2*c^4*d^3+12*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/
a^(1/2))*a^2*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^3*c^6*d-102*(x*(a*x+b)
)^(1/2)*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b*c^4*d^3-16*(x*(a*x+b)
)^(1/2)*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b^3*c^6*d+8*ln(1/2*(2*(
x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2
)*x*a*b^4*c^6*d-24*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a
^(1/2))*a^5*((a*d-b*c)*d/c^2)^(1/2)*c*d^6-24*ln((2*(x*(a*x+b))^(1
/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(11/2
)*x^2*c^2*d^5+8*(x*(a*x+b))^(3/2)*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)
*c^4*d^3-48*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a
```

```
*d*x+b*c*x-b*d)/(c*x+d))*a^(11/2)*x*c*d^6+24*(x*(a*x+b))^(1/2)*a^
(9/2)*((a*d-b*c)*d/c^2)^(1/2)*c^2*d^5+80*ln((2*(x*(a*x+b))^(1/2)*
((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(9/2)*b*c
*d^6-91*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x
+b*c*x-b*d)/(c*x+d))*a^(7/2)*b^2*c^2*d^5+35*ln((2*(x*(a*x+b))^(1/
2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(5/2)*
b^3*c^3*d^4+46*(x*(a*x+b))^(1/2)*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*
b^2*c^4*d^3)/(x*(a*x+b))^(1/2)/c^5/(a*d-b*c)^3/(c*x+d)^2/((a*d-b*
c)*d/c^2)^(1/2)/a^(5/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a + b/x)*(c + d/x)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.766872, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a + b/x)*(c + d/x)^3),x, algorithm="fricas")
```

```
[Out] [1/8*((35*a*b^2*c^2*d^3 - 56*a^2*b*c*d^4 + 24*a^3*d^5 + (35*a*b^2
*c^4*d - 56*a^2*b*c^3*d^2 + 24*a^3*c^2*d^3)*x^2 + 2*(35*a*b^2*c^3
*d^2 - 56*a^2*b*c^2*d^3 + 24*a^3*c*d^4)*x)*sqrt(a)*sqrt(-d/(b*c -
 a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/
x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(4*(b^2*c^5 - 2*a*b*c^
4*d + a^2*c^3*d^2)*x^3 + (8*b^2*c^4*d - 29*a*b*c^3*d^2 + 18*a^2*c
^2*d^3)*x^2 + (4*b^2*c^3*d^2 - 19*a*b*c^2*d^3 + 12*a^2*c*d^4)*x)*
sqrt(a)*sqrt((a*x + b)/x) + 4*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11
*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^
3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*b^2*c^3*d^2 - 11*
a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*log(-2*a*x*sqrt((a*x + b)/x) + (2
*a*x + b)*sqrt(a)))/((a*b^2*c^6*d^2 - 2*a^2*b*c^5*d^3 + a^3*c^4*d
^4 + (a*b^2*c^8 - 2*a^2*b*c^7*d + a^3*c^6*d^2)*x^2 + 2*(a*b^2*c^7
*d - 2*a^2*b*c^6*d^2 + a^3*c^5*d^3)*x)*sqrt(a)), 1/8*((35*a*b^2*c
^2*d^3 - 56*a^2*b*c*d^4 + 24*a^3*d^5 + (35*a*b^2*c^4*d - 56*a^2*b
*c^3*d^2 + 24*a^3*c^2*d^3)*x^2 + 2*(35*a*b^2*c^3*d^2 - 56*a^2*b*c
^2*d^3 + 24*a^3*c*d^4)*x)*sqrt(-a)*sqrt(-d/(b*c - a*d))*log(-(2*(
b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c
- 2*a*d)*x)/(c*x + d)) + 2*(4*(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^
```

```
2)*x^3 + (8*b^2*c^4*d - 29*a*b*c^3*d^2 + 18*a^2*c^2*d^3)*x^2 + (4
*b^2*c^3*d^2 - 19*a*b*c^2*d^3 + 12*a^2*c*d^4)*x)*sqrt(-a)*sqrt((a
*x + b)/x) + 8*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 +
6*a^3*d^5 + (b^3*c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c
^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 +
 6*a^3*c*d^4)*x)*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x))))/((a*b^2*
c^6*d^2 - 2*a^2*b*c^5*d^3 + a^3*c^4*d^4 + (a*b^2*c^8 - 2*a^2*b*c^
7*d + a^3*c^6*d^2)*x^2 + 2*(a*b^2*c^7*d - 2*a^2*b*c^6*d^2 + a^3*c
^5*d^3)*x)*sqrt(-a)), -1/4*((35*a*b^2*c^2*d^3 - 56*a^2*b*c*d^4 +
24*a^3*d^5 + (35*a*b^2*c^4*d - 56*a^2*b*c^3*d^2 + 24*a^3*c^2*d^3)
*x^2 + 2*(35*a*b^2*c^3*d^2 - 56*a^2*b*c^2*d^3 + 24*a^3*c*d^4)*x)*
sqrt(a)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d/(b*c - a*d
))/(d*sqrt((a*x + b)/x))) - (4*(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d
^2)*x^3 + (8*b^2*c^4*d - 29*a*b*c^3*d^2 + 18*a^2*c^2*d^3)*x^2 + (
4*b^2*c^3*d^2 - 19*a*b*c^2*d^3 + 12*a^2*c*d^4)*x)*sqrt(a)*sqrt((a
*x + b)/x) - 2*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 +
6*a^3*d^5 + (b^3*c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c
^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 +
 6*a^3*c*d^4)*x)*log(-2*a*x*sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(
a)))/((a*b^2*c^6*d^2 - 2*a^2*b*c^5*d^3 + a^3*c^4*d^4 + (a*b^2*c^8
 - 2*a^2*b*c^7*d + a^3*c^6*d^2)*x^2 + 2*(a*b^2*c^7*d - 2*a^2*b*c^
6*d^2 + a^3*c^5*d^3)*x)*sqrt(a)), -1/4*((35*a*b^2*c^2*d^3 - 56*a^
2*b*c*d^4 + 24*a^3*d^5 + (35*a*b^2*c^4*d - 56*a^2*b*c^3*d^2 + 24*
a^3*c^2*d^3)*x^2 + 2*(35*a*b^2*c^3*d^2 - 56*a^2*b*c^2*d^3 + 24*a^
3*c*d^4)*x)*sqrt(-a)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt
(d/(b*c - a*d))/(d*sqrt((a*x + b)/x))) - (4*(b^2*c^5 - 2*a*b*c^4*
d + a^2*c^3*d^2)*x^3 + (8*b^2*c^4*d - 29*a*b*c^3*d^2 + 18*a^2*c^2
*d^3)*x^2 + (4*b^2*c^3*d^2 - 19*a*b*c^2*d^3 + 12*a^2*c*d^4)*x)*sq
rt(-a)*sqrt((a*x + b)/x) - 4*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*
a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3
*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*b^2*c^3*d^2 - 11*a
^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*arctan(a/(sqrt(-a)*sqrt((a*x + b)/
x))))/((a*b^2*c^6*d^2 - 2*a^2*b*c^5*d^3 + a^3*c^4*d^4 + (a*b^2*c^
8 - 2*a^2*b*c^7*d + a^3*c^6*d^2)*x^2 + 2*(a*b^2*c^7*d - 2*a^2*b*c
^6*d^2 + a^3*c^5*d^3)*x)*sqrt(-a))]
```

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c+d/x)**3/(a+b/x)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

---

**GIAC/XCAS [A]** time = 0.261172, size = 459, normalized size = 1.84

$$-\frac{1}{4}\,b\left(\frac{\left(35\,b^2c^2d^2-56\,abcd^3+24\,a^2d^4\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^3c^6-2\,ab^2c^5d+a^2bc^4d^2\right)\sqrt{bcd-ad^2}}+\frac{13\,b^2c^2d^2\sqrt{\frac{ax+b}{x}}-21\,abcd^3\sqrt{\frac{ax+b}{x}}+8\,a^2d^4\sqrt{\frac{ax+b}{x}}+\frac{11\,(ax+b)\cdots}{\cdots}}{\left(b^2c^5-2\,abc^4d+a^2c^3d^2\right)\left(bc-ad+\frac{(ax\cdots}{\cdots}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a + b/x)*(c + d/x)^3),x, algorithm="giac")
```

```
[Out] -1/4*b*((35*b^2*c^2*d^2 - 56*a*b*c*d^3 + 24*a^2*d^4)*arctan(d*sqr
t((a*x + b)/x)/sqrt(b*c*d - a*d^2))/((b^3*c^6 - 2*a*b^2*c^5*d + a
^2*b*c^4*d^2)*sqrt(b*c*d - a*d^2)) + (13*b^2*c^2*d^2*sqrt((a*x +
b)/x) - 21*a*b*c*d^3*sqrt((a*x + b)/x) + 8*a^2*d^4*sqrt((a*x + b)
/x) + 11*(a*x + b)*b*c*d^3*sqrt((a*x + b)/x)/x - 8*(a*x + b)*a*d^
4*sqrt((a*x + b)/x)/x)/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*(b*
c - a*d + (a*x + b)*d/x)^2) + 4*sqrt((a*x + b)/x)/((a - (a*x + b)
/x)*a*c^3) - 4*(b*c + 6*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(
sqrt(-a)*a*b*c^4))
```

## 3.153 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=132

$$-\frac{3c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{(bc-2ad)\left(2a^2d^2-2abcd+3b^2c^2\right)-\frac{abd^2(2ad+bc)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}$$

```
[Out] ((b*c - 2*a*d)*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2) - (a*b*d^2*(b*
c + 2*a*d))/x)/(a^2*b^2*Sqrt[a + b/x]) + (c*(c + d/x)^2*x)/(a*Sqr
t[a + b/x]) - (3*c^2*(b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
)/a^(5/2)
```

---

**Rubi [A]** time = 0.326522, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$-\frac{3c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{(bc-2ad)\left(2a^2d^2-2abcd+3b^2c^2\right)-\frac{abd^2(2ad+bc)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d/x)^3/(a + b/x)^(3/2),x]
```

```
[Out] ((b*c - 2*a*d)*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2) - (a*b*d^2*(b*
c + 2*a*d))/x)/(a^2*b^2*Sqrt[a + b/x]) + (c*(c + d/x)^2*x)/(a*Sqr
t[a + b/x]) - (3*c^2*(b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
)/a^(5/2)
```

---

**Rubi in Sympy [A]** time = 27.2067, size = 122, normalized size = 0.92

$$\frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}-\frac{4\left(\frac{abd^2(2ad+bc)}{4x}+\left(\frac{ad}{2}-\frac{bc}{4}\right)\left(2a^2d^2-2abcd+3b^2c^2\right)\right)}{a^2b^2\sqrt{a+\frac{b}{x}}}+\frac{3c^2\,(2ad-bc)\,\text{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d/x)**3/(a+b/x)**(3/2),x)
```

```
[Out] c*x*(c + d/x)**2/(a*sqrt(a + b/x)) - 4*(a*b*d**2*(2*a*d + b*c)/(4
*x) + (a*d/2 - b*c/4)*(2*a**2*d**2 - 2*a*b*c*d + 3*b**2*c**2))/(a
**2*b**2*sqrt(a + b/x)) + 3*c**2*(2*a*d - b*c)*atanh(sqrt(a + b/x
)/sqrt(a))/a**(5/2)
```

---

**Mathematica [A]** time = 0.179282, size = 123, normalized size = 0.93

$$\frac{3c^2(2ad-bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{2a^{5/2}}+\frac{\sqrt{a+\frac{b}{x}}\left(-4a^3d^3x-2a^2bd^2(d-3cx)+ab^2c^2x(cx-6d)+3b^3c^3x\right)}{a^2b^2(ax+b)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d/x)^3/(a + b/x)^(3/2),x]
```

```
[Out] (Sqrt[a + b/x]*(3*b^3*c^3*x - 4*a^3*d^3*x - 2*a^2*b*d^2*(d - 3*c*
x) + a*b^2*c^2*x*(-6*d + c*x)))/(a^2*b^2*(b + a*x)) + (3*c^2*(-(b
*c) + 2*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x])/(2*a^(5/
2))
```

---

**Maple [B]** time = 0.025, size = 976, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d/x)^3/(a+b/x)^(3/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)/x/a^(9/2)*(3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a
^(1/2)+2*a*x+b)/a^(1/2))*x^4*a^4*b^4*c^3-6*a^(5/2)*(x*(a*x+b))^(1
/2)*x^2*b^5*c^3+6*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^
(1/2))*x^3*a^3*b^5*c^3+3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*
x+b)/a^(1/2))*x^2*a^2*b^6*c^3-6*a^(9/2)*(x*(a*x+b))^(1/2)*x^4*b^3
*c^3+4*a^(7/2)*(x*(a*x+b))^(3/2)*c^3*x^2*b^3-12*a^(7/2)*(x*(a*x+b
))^(1/2)*x^3*b^4*c^3+8*a^(11/2)*(a*x^2+b*x)^(3/2)*x*b*d^3-3*ln(1/
2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*x^2*b^4*c*d^
2-6*a^(9/2)*(a*x^2+b*x)^(1/2)*x^2*b^3*c*d^2-6*a^(9/2)*(x*(a*x+b))
^(1/2)*x^2*b^3*c*d^2+12*a^(7/2)*(x*(a*x+b))^(1/2)*x^2*b^4*c^2*d-6
*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*x^2*b^
5*c^2*d-12*a^(11/2)*(x*(a*x+b))^(1/2)*x^3*b^2*c*d^2+24*a^(9/2)*(x
*(a*x+b))^(1/2)*x^3*b^3*c^2*d+3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/
2)+2*a*x+b)/a^(1/2))*a^4*x^2*b^4*c*d^2-12*ln(1/2*(2*(x*(a*x+b))^(
1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*x^3*b^4*c^2*d+4*a^(13/2)*(a*x^
2+b*x)^(3/2)*x^2*d^3-4*a^(13/2)*(x*(a*x+b))^(3/2)*d^3*x^2+4*a^(9/
```

```
2)*(a*x^2+b*x)^(3/2)*b^2*d^3-12*a^(9/2)*(x*(a*x+b))^(3/2)*c^2*d*x
^2*b^2+3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^
6*x^4*b^2*c*d^2-3*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^
(1/2))*a^6*x^4*b^2*c*d^2+12*a^(11/2)*(x*(a*x+b))^(3/2)*c*d^2*x^2*
b-12*a^(11/2)*(a*x^2+b*x)^(1/2)*x^3*b^2*c*d^2-6*a^(13/2)*(a*x^2+b
*x)^(1/2)*x^4*b*c*d^2-6*a^(13/2)*(x*(a*x+b))^(1/2)*x^4*b*c*d^2+12
*a^(11/2)*(x*(a*x+b))^(1/2)*x^4*b^2*c^2*d+6*ln(1/2*(2*(x*(a*x+b))
^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5*x^3*b^3*c*d^2-6*ln(1/2*(2*(x
*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5*x^4*b^3*c^2*d-6*ln(
1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5*x^3*b^3*c*
d^2)/(x*(a*x+b))^(1/2)/b^3/(a*x+b)^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)^3/(a + b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.256538, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(b^3c^3-2\,ab^2c^2d\right)x\sqrt{\frac{ax+b}{x}}\log\left(2\,ax\sqrt{\frac{ax+b}{x}}+(2\,ax+b)\sqrt{a}\right)-2\left(ab^2c^3x^2-2\,a^2bd^3+\left(3\,b^3c^3-6\,ab^2c^2d+6\,a^2bcd^2-4\right.\right.}{2\,a^{\frac{5}{2}}b^2x\sqrt{\frac{ax+b}{x}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)^3/(a + b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(b^3*c^3 - 2*a*b^2*c^2*d)*x*sqrt((a*x + b)/x)*log(2*a*x*
sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(a)) - 2*(a*b^2*c^3*x^2 - 2*a
^2*b*d^3 + (3*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 4*a^3*d^3
)*x)*sqrt(a))/(a^(5/2)*b^2*x*sqrt((a*x + b)/x)), (3*(b^3*c^3 - 2*
a*b^2*c^2*d)*x*sqrt((a*x + b)/x)*arctan(a/(sqrt(-a)*sqrt((a*x + b
)/x))) + (a*b^2*c^3*x^2 - 2*a^2*b*d^3 + (3*b^3*c^3 - 6*a*b^2*c^2*
d + 6*a^2*b*c*d^2 - 4*a^3*d^3)*x)*sqrt(-a))/(sqrt(-a)*a^2*b^2*x*s
qrt((a*x + b)/x))]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx+d)^3}{x^3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d/x)**3/(a+b/x)**(3/2),x)
```

```
[Out] Integral((c*x + d)**3/(x**3*(a + b/x)**(3/2)), x)
```

---

**GIAC/XCAS [A]** time = 0.252264, size = 297, normalized size = 2.25

$$-b\left(\frac{2\,d^3\sqrt{\frac{ax+b}{x}}}{b^3}-\frac{3\left(bc^3-2\,ac^2d\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2b}-\frac{2\,ab^3c^3-6\,a^2b^2c^2d+6\,a^3bcd^2-2\,a^4d^3-\frac{3\,(ax+b)b^3c^3}{x}+\frac{6\,(ax+b)ab^2c^2d}{x}}{\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^2b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)^3/(a + b/x)^(3/2),x, algorithm="giac")
```

```
[Out] -b*(2*d^3*sqrt((a*x + b)/x)/b^3 - 3*(b*c^3 - 2*a*c^2*d)*arctan(sq
rt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2*b) - (2*a*b^3*c^3 - 6*a^2
*b^2*c^2*d + 6*a^3*b*c*d^2 - 2*a^4*d^3 - 3*(a*x + b)*b^3*c^3/x +
6*(a*x + b)*a*b^2*c^2*d/x - 6*(a*x + b)*a^2*b*c*d^2/x + 2*(a*x +
b)*a^3*d^3/x)/((a*sqrt((a*x + b)/x) - (a*x + b)*sqrt((a*x + b)/x)
/x)*a^2*b^3))
```

## 3.154 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=94

$$-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2a^2d^2+bc(3bc-4ad)}{a^2b\sqrt{a+\frac{b}{x}}}+\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}$$

```
[Out] (2*a^2*d^2 + b*c*(3*b*c - 4*a*d))/(a^2*b*Sqrt[a + b/x]) + (c^2*x)
/(a*Sqrt[a + b/x]) - (c*(3*b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqr
t[a]])/a^(5/2)
```

---

**Rubi [A]** time = 0.247795, antiderivative size = 90, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{\frac{c(3bc-4ad)}{a^2}+\frac{2d^2}{b}}{\sqrt{a+\frac{b}{x}}}+\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d/x)^2/(a + b/x)^(3/2),x]
```

```
[Out] ((2*d^2)/b + (c*(3*b*c - 4*a*d))/a^2)/Sqrt[a + b/x] + (c^2*x)/(a*
Sqrt[a + b/x]) - (c*(3*b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]
])/a^(5/2)
```

---

**Rubi in Sympy [A]** time = 18.8213, size = 82, normalized size = 0.87

$$\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}+\frac{2\left(a^2d^2-\frac{bc(4ad-3bc)}{2}\right)}{a^2b\sqrt{a+\frac{b}{x}}}+\frac{c\,(4ad-3bc)\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d/x)**2/(a+b/x)**(3/2),x)
```

```
[Out] c**2*x/(a*sqrt(a + b/x)) + 2*(a**2*d**2 - b*c*(4*a*d - 3*b*c)/2)/
(a**2*b*sqrt(a + b/x)) + c*(4*a*d - 3*b*c)*atanh(sqrt(a + b/x)/sq
rt(a))/a**(5/2)
```

---

**Mathematica [A]** time = 0.187898, size = 100, normalized size = 1.06

$$\frac{c(4ad-3bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{2a^{5/2}}+\frac{x\sqrt{a+\frac{b}{x}}\left(2a^2d^2+abc(cx-4d)+3b^2c^2\right)}{a^2b(ax+b)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d/x)^2/(a + b/x)^(3/2),x]
```

```
[Out] (Sqrt[a + b/x]*x*(3*b^2*c^2 + 2*a^2*d^2 + a*b*c*(-4*d + c*x)))/(a
^2*b*(b + a*x)) + (c*(-3*b*c + 4*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*S
qrt[a + b/x]*x])/(2*a^(5/2))
```

---

**Maple [B]** time = 0.022, size = 796, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d/x)^2/(a+b/x)^(3/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x/a^(9/2)*(6*a^(9/2)*(x*(a*x+b))^(1/2)*x^2*
b^2*c^2+a^6*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))
*x^2*b*d^2-a^6*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/
2))*x^2*b*d^2+4*a^(11/2)*(a*x^2+b*x)^(1/2)*x*b*d^2+8*a^(9/2)*(x*(
a*x+b))^(3/2)*c*d*b+4*a^(11/2)*(x*(a*x+b))^(1/2)*x*b*d^2-3*ln(1/2
*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^2*a^4*b^3*c^2-6
*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x*a^3*b^4*
c^2+6*a^(5/2)*(x*(a*x+b))^(1/2)*b^4*c^2-3*ln(1/2*(2*(x*(a*x+b))^(
1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b^5*c^2+2*a^(13/2)*(a*x^2+b*x)
^(1/2)*x^2*d^2+2*a^(13/2)*(x*(a*x+b))^(1/2)*x^2*d^2-4*a^(7/2)*(x*
(a*x+b))^(3/2)*c^2*b^2+2*a^(9/2)*(a*x^2+b*x)^(1/2)*b^2*d^2+2*a^(9
/2)*(x*(a*x+b))^(1/2)*b^2*d^2+a^4*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(
1/2)+2*a*x+b)/a^(1/2))*b^3*d^2-a^4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^
(1/2)+2*a*x+b)/a^(1/2))*b^3*d^2-4*a^(11/2)*(x*(a*x+b))^(3/2)*d^2+
12*a^(7/2)*(x*(a*x+b))^(1/2)*x*b^3*c^2+2*a^5*ln(1/2*(2*(a*x^2+b*x
)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x*b^2*d^2-2*a^5*ln(1/2*(2*(x*(a
*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x*b^2*d^2-8*a^(7/2)*(x*(a*
x+b))^(1/2)*b^3*c*d+4*a^3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a
*x+b)/a^(1/2))*b^4*c*d-16*a^(9/2)*(x*(a*x+b))^(1/2)*x*b^2*c*d+4*a
^5*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^2*b^2*
c*d+8*a^4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x
```

```
*b^3*c*d-8*a^(11/2)*(x*(a*x+b))^(1/2)*x^2*b*c*d)/(x*(a*x+b))^(1/2
)/b^2/(a*x+b)^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)^2/(a + b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.243, size = 1, normalized size = 0.01

$$\left[-\frac{\left(3\,b^2c^2-4\,abcd\right)\sqrt{\frac{ax+b}{x}}\log\left(2\,ax\sqrt{\frac{ax+b}{x}}+(2\,ax+b)\sqrt{a}\right)-2\left(abc^2x+3\,b^2c^2-4\,abcd+2\,a^2d^2\right)\sqrt{a}}{2\,a^{\frac{5}{2}}b\sqrt{\frac{ax+b}{x}}},\frac{\left(3\,b^2c^2-4\,abcd\right)\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)^2/(a + b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((3*b^2*c^2 - 4*a*b*c*d)*sqrt((a*x + b)/x)*log(2*a*x*sqrt((
a*x + b)/x) + (2*a*x + b)*sqrt(a)) - 2*(a*b*c^2*x + 3*b^2*c^2 - 4
*a*b*c*d + 2*a^2*d^2)*sqrt(a))/(a^(5/2)*b*sqrt((a*x + b)/x)), ((3
*b^2*c^2 - 4*a*b*c*d)*sqrt((a*x + b)/x)*arctan(a/(sqrt(-a)*sqrt((
a*x + b)/x))) + (a*b*c^2*x + 3*b^2*c^2 - 4*a*b*c*d + 2*a^2*d^2)*s
qrt(-a))/(sqrt(-a)*a^2*b*sqrt((a*x + b)/x))]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{(cx+d)^2}{x^2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d/x)**2/(a+b/x)**(3/2),x)
```

```
[Out] Integral((c*x + d)**2/(x**2*(a + b/x)**(3/2)), x)
```

---

**GIAC/XCAS [A]** time = 0.251661, size = 217, normalized size = 2.31

$$b\left(\frac{\left(3\,bc^2 - 4\,acd\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2b} + \frac{2\,ab^2c^2 - 4\,a^2bcd + 2\,a^3d^2 - \frac{3\,(ax+b)b^2c^2}{x} + \frac{4\,(ax+b)abcd}{x} - \frac{2\,(ax+b)a^2d^2}{x}}{\left(a\sqrt{\frac{ax+b}{x}} - \frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^2b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)^2/(a + b/x)^(3/2),x, algorithm="giac")
```

```
[Out] b*((3*b*c^2 - 4*a*c*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-
a)*a^2*b) + (2*a*b^2*c^2 - 4*a^2*b*c*d + 2*a^3*d^2 - 3*(a*x + b)*
b^2*c^2/x + 4*(a*x + b)*a*b*c*d/x - 2*(a*x + b)*a^2*d^2/x)/((a*sq
rt((a*x + b)/x) - (a*x + b)*sqrt((a*x + b)/x)/x)*a^2*b^2))
```

## 3.155 $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=76

$$-\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}$$

```
[Out] (3*b*c - 2*a*d)/(a^2*Sqrt[a + b/x]) + (c*x)/(a*Sqrt[a + b/x]) - (
(3*b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

---

**Rubi [A]** time = 0.154825, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$-\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d/x)/(a + b/x)^(3/2),x]
```

```
[Out] (3*b*c - 2*a*d)/(a^2*Sqrt[a + b/x]) + (c*x)/(a*Sqrt[a + b/x]) - (
(3*b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

---

**Rubi in Sympy [A]** time = 12.5111, size = 66, normalized size = 0.87

$$\frac{cx}{a\sqrt{a+\frac{b}{x}}}-\frac{2\left(ad-\frac{3bc}{2}\right)}{a^2\sqrt{a+\frac{b}{x}}}+\frac{2\left(ad-\frac{3bc}{2}\right)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d/x)/(a+b/x)**(3/2),x)
```

```
[Out] c*x/(a*sqrt(a + b/x)) - 2*(a*d - 3*b*c/2)/(a**2*sqrt(a + b/x)) +
2*(a*d - 3*b*c/2)*atanh(sqrt(a + b/x)/sqrt(a))/a**(5/2)
```

**Mathematica [A]** time = 0.116594, size = 81, normalized size = 1.07

$$\frac{(2ad-3bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{2a^{5/2}}+\frac{x\sqrt{a+\frac{b}{x}}(acx-2ad+3bc)}{a^2(ax+b)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d/x)/(a + b/x)^(3/2),x]
```

```
[Out] (Sqrt[a + b/x]*x*(3*b*c - 2*a*d + a*c*x))/(a^2*(b + a*x)) + ((-3*
b*c + 2*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x])/(2*a^(5/
2))
```

**Maple [B]** time = 0.018, size = 394, normalized size = 5.2

$$-\frac{x}{2\,b\,(ax+b)^2}\sqrt{\frac{ax+b}{x}}\left(4\,a^{11/2}\sqrt{x\,(ax+b)}x^2d-6\,a^{9/2}\sqrt{x\,(ax+b)}x^2bc-4\,a^{9/2}\,(x\,(ax+b))^{3/2}\,d+8\,a^{9/2}\sqrt{x\,(ax+b)}xbd+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d/x)/(a+b/x)^(3/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(4*a^(11/2)*(x*(a*x+b))^(1/2)*x^2*d-6*a^
(9/2)*(x*(a*x+b))^(1/2)*x^2*b*c-4*a^(9/2)*(x*(a*x+b))^(3/2)*d+8*a
^(9/2)*(x*(a*x+b))^(1/2)*x*b*d+4*a^(7/2)*(x*(a*x+b))^(3/2)*c*b-12
*a^(7/2)*(x*(a*x+b))^(1/2)*x*b^2*c+4*a^(7/2)*(x*(a*x+b))^(1/2)*b^
2*d-6*a^(5/2)*(x*(a*x+b))^(1/2)*b^3*c-2*a^5*ln(1/2*(2*(x*(a*x+b))
^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^2*b*d+3*ln(1/2*(2*(x*(a*x+b))^
(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^2*a^4*b^2*c-4*a^4*ln(1/2*(2*(x*
(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x*b^2*d+6*ln(1/2*(2*(x*(
a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x*a^3*b^3*c-2*a^3*ln(1/2*
(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^3*d+3*ln(1/2*(2*
(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b^4*c)/a^(9/2)/(x
*(a*x+b))^(1/2)/b/(a*x+b)^2
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)/(a + b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.24653, size = 1, normalized size = 0.01

$$\left[-\frac{(3\,bc-2\,ad)\sqrt{\frac{ax+b}{x}}\log\left(2\,ax\sqrt{\frac{ax+b}{x}}+(2\,ax+b)\sqrt{a}\right)-2\,(acx+3\,bc-2\,ad)\sqrt{a}}{2\,a^{\frac{5}{2}}\sqrt{\frac{ax+b}{x}}},\ \frac{(3\,bc-2\,ad)\sqrt{\frac{ax+b}{x}}\arctan\left(\frac{a}{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}\right)}{\sqrt{-a}a^{2}\sqrt{\frac{ax+b}{x}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)/(a + b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((3*b*c - 2*a*d)*sqrt((a*x + b)/x)*log(2*a*x*sqrt((a*x + b)
/x) + (2*a*x + b)*sqrt(a)) - 2*(a*c*x + 3*b*c - 2*a*d)*sqrt(a))/(
a^(5/2)*sqrt((a*x + b)/x)), ((3*b*c - 2*a*d)*sqrt((a*x + b)/x)*ar
ctan(a/(sqrt(-a)*sqrt((a*x + b)/x))) + (a*c*x + 3*b*c - 2*a*d)*sq
rt(-a))/(sqrt(-a)*a^2*sqrt((a*x + b)/x))]
```

---

**Sympy [A]** time = 21.4323, size = 224, normalized size = 2.95

$$c\left(\frac{x^{\frac{3}{2}}}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{3\sqrt{b}\sqrt{x}}{a^{2}\sqrt{\frac{ax}{b}+1}}-\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}\right)+d\left(-\frac{2a^{3}x\sqrt{1+\frac{b}{ax}}}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}-\frac{a^{3}x\log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}+\frac{2a^{3}x\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}-\frac{a^{2}b\log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}+\frac{2a^{2}b\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d/x)/(a+b/x)**(3/2),x)
```

```
[Out] c*(x**(3/2)/(a*sqrt(b)*sqrt(a*x/b + 1)) + 3*sqrt(b)*sqrt(x)/(a**2
*sqrt(a*x/b + 1)) - 3*b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(5/2))
+ d*(-2*a**3*x*sqrt(1 + b/(a*x))/(a**(9/2)*x + a**(7/2)*b) - a**3
*x*log(b/(a*x))/(a**(9/2)*x + a**(7/2)*b) + 2*a**3*x*log(sqrt(1 +
 b/(a*x)) + 1)/(a**(9/2)*x + a**(7/2)*b) - a**2*b*log(b/(a*x))/(a
**(9/2)*x + a**(7/2)*b) + 2*a**2*b*log(sqrt(1 + b/(a*x)) + 1)/(a*
*(9/2)*x + a**(7/2)*b))
```

---

**GIAC/XCAS [A]** time = 0.254238, size = 165, normalized size = 2.17

$$b\left(\frac{(3\,bc-2\,ad)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^{2}b}+\frac{2\,abc-2\,a^{2}d-\frac{3\,(ax+b)bc}{x}+\frac{2\,(ax+b)ad}{x}}{\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^{2}b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)/(a + b/x)^(3/2),x, algorithm="giac")
```

```
[Out] b*((3*b*c - 2*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a
^2*b) + (2*a*b*c - 2*a^2*d - 3*(a*x + b)*b*c/x + 2*(a*x + b)*a*d/
x)/((a*sqrt((a*x + b)/x) - (a*x + b)*sqrt((a*x + b)/x)/x)*a^2*b))
```

## 3.156 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=61

$$-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{3x\sqrt{a+\frac{b}{x}}}{a^2}-\frac{2x}{a\sqrt{a+\frac{b}{x}}}$$

```
[Out] (-2*x)/(a*Sqrt[a + b/x]) + (3*Sqrt[a + b/x]*x)/a^2 - (3*b*ArcTanh
[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

---

**Rubi [A]** time = 0.0834458, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{3x\sqrt{a+\frac{b}{x}}}{a^2}-\frac{2x}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^(-3/2),x]
```

```
[Out] (-2*x)/(a*Sqrt[a + b/x]) + (3*Sqrt[a + b/x]*x)/a^2 - (3*b*ArcTanh
[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

---

**Rubi in Sympy [A]** time = 8.025, size = 51, normalized size = 0.84

$$-\frac{2x}{a\sqrt{a+\frac{b}{x}}}+\frac{3x\sqrt{a+\frac{b}{x}}}{a^2}-\frac{3b\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b/x)**(3/2),x)
```

```
[Out] -2*x/(a*sqrt(a + b/x)) + 3*x*sqrt(a + b/x)/a**2 - 3*b*atanh(sqrt(
a + b/x)/sqrt(a))/a**(5/2)
```

---

**Mathematica [A]** time = 0.100055, size = 67, normalized size = 1.1

$$\frac{x\sqrt{a+\frac{b}{x}}(ax+3b)}{a^2(ax+b)}-\frac{3b\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{2a^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x)^(-3/2),x]
```

```
[Out] (Sqrt[a + b/x]*x*(3*b + a*x))/(a^2*(b + a*x)) - (3*b*Log[b + 2*a*
x + 2*Sqrt[a]*Sqrt[a + b/x]*x])/(2*a^(5/2))
```

---

**Maple [B]** time = 0.005, size = 203, normalized size = 3.3

$$-\frac{x}{2\,(ax+b)^2}\sqrt{\frac{ax+b}{x}}\left(-6\,a^{9/2}\sqrt{x\,(ax+b)}x^2+4\,a^{7/2}\,(x\,(ax+b))^{3/2}-12\,a^{7/2}\sqrt{x\,(ax+b)}xb+3\,\ln\left(1/2\,\frac{2\,\sqrt{x\,(ax+b)}\sqrt{a}+}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b/x)^(3/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x/a^(9/2)*(-6*a^(9/2)*(x*(a*x+b))^(1/2)*x^
2+4*a^(7/2)*(x*(a*x+b))^(3/2)-12*a^(7/2)*(x*(a*x+b))^(1/2)*x*b+3*
ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^2*a^4*b-6
*a^(5/2)*(x*(a*x+b))^(1/2)*b^2+6*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1
/2)+2*a*x+b)/a^(1/2))*x*a^3*b^2+3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(
1/2)+2*a*x+b)/a^(1/2))*a^2*b^3)/(x*(a*x+b))^(1/2)/(a*x+b)^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(-3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.251474, size = 1, normalized size = 0.02

$$\left[\frac{3\,b\sqrt{\frac{ax+b}{x}}\log\left(-2\,ax\sqrt{\frac{ax+b}{x}}+(2\,ax+b)\sqrt{a}\right)+2\,(ax+3\,b)\sqrt{a}}{2\,a^{\frac{5}{2}}\sqrt{\frac{ax+b}{x}}},\ \frac{3\,b\sqrt{\frac{ax+b}{x}}\arctan\left(\frac{a}{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}\right)+(ax+3\,b)\sqrt{-a}}{\sqrt{-a}a^{2}\sqrt{\frac{ax+b}{x}}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(-3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*b*sqrt((a*x + b)/x)*log(-2*a*x*sqrt((a*x + b)/x) + (2*a*x
 + b)*sqrt(a)) + 2*(a*x + 3*b)*sqrt(a))/(a^(5/2)*sqrt((a*x + b)/x
)), (3*b*sqrt((a*x + b)/x)*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x)))
 + (a*x + 3*b)*sqrt(-a))/(sqrt(-a)*a^2*sqrt((a*x + b)/x))]
```

---

**Sympy [A]** time = 11.4526, size = 71, normalized size = 1.16

$$\frac{x^{\frac{3}{2}}}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{3\sqrt{b}\sqrt{x}}{a^{2}\sqrt{\frac{ax}{b}+1}}-\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b/x)**(3/2),x)
```

```
[Out] x**(3/2)/(a*sqrt(b)*sqrt(a*x/b + 1)) + 3*sqrt(b)*sqrt(x)/(a**2*sq
rt(a*x/b + 1)) - 3*b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(5/2)
```

---

**GIAC/XCAS [A]** time = 0.250133, size = 116, normalized size = 1.9

$$b\left(\frac{3\,\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^{2}}+\frac{2\,a-\frac{3\,(ax+b)}{x}}{\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(-3/2),x, algorithm="giac")
```

```
[Out] b*(3*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2) + (2*a - 3
*(a*x + b)/x)/((a*sqrt((a*x + b)/x) - (a*x + b)*sqrt((a*x + b)/x)
```

/x)*a^2))

## 3.157 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx$

**Optimal**. Leaf size=147

$$-\frac{(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2}+\frac{b(3bc-ad)}{a^2c\sqrt{a+\frac{b}{x}}(bc-ad)}+\frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{3/2}}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

```
[Out] (b*(3*b*c - a*d))/(a^2*c*(b*c - a*d)*Sqrt[a + b/x]) + x/(a*c*Sqrt
[a + b/x]) + (2*d^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c -
 a*d]])/(c^2*(b*c - a*d)^(3/2)) - ((3*b*c + 2*a*d)*ArcTanh[Sqrt[a
 + b/x]/Sqrt[a]])/(a^(5/2)*c^2)
```

---

**Rubi [A]** time = 0.618832, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2}+\frac{b(3bc-ad)}{a^2c\sqrt{a+\frac{b}{x}}(bc-ad)}+\frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{3/2}}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b/x)^(3/2)*(c + d/x)),x]
```

```
[Out] (b*(3*b*c - a*d))/(a^2*c*(b*c - a*d)*Sqrt[a + b/x]) + x/(a*c*Sqrt
[a + b/x]) + (2*d^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c -
 a*d]])/(c^2*(b*c - a*d)^(3/2)) - ((3*b*c + 2*a*d)*ArcTanh[Sqrt[a
 + b/x]/Sqrt[a]])/(a^(5/2)*c^2)
```

---

**Rubi in Sympy [A]** time = 67.6655, size = 122, normalized size = 0.83

$$\frac{2d^{\frac{5}{2}}\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{c^2\left(ad-bc\right)^{\frac{3}{2}}}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}+\frac{b\left(ad-3bc\right)}{a^2c\sqrt{a+\frac{b}{x}}\left(ad-bc\right)}-\frac{(2ad+3bc)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b/x)**(3/2)/(c+d/x),x)
```

```
[Out] 2*d**(5/2)*atanh(sqrt(d)*sqrt(a + b/x)/sqrt(a*d - b*c))/(c**2*(a*
d - b*c)**(3/2)) + x/(a*c*sqrt(a + b/x)) + b*(a*d - 3*b*c)/(a**2*
c*sqrt(a + b/x)*(a*d - b*c)) - (2*a*d + 3*b*c)*atanh(sqrt(a + b/x
)/sqrt(a))/(a**(5/2)*c**2)
```

---

**Mathematica [A]** time = 1.22763, size = 197, normalized size = 1.34

$$\frac{-\frac{(2ad+3bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{a^{5/2}}+\frac{2cx\sqrt{a+\frac{b}{x}}\left(a^2dx+ab(d-cx)-3b^2c\right)}{a^2(ax+b)(ad-bc)}+\frac{2d^{5/2}\log(cx+d)}{(ad-bc)^{3/2}}-\frac{2d^{5/2}\log\left(2\sqrt{d}x\sqrt{a+\frac{b}{x}}\sqrt{ad-bc}-2adx+bcx-bd\right)}{(ad-bc)^{3/2}}}{2c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b/x)^(3/2)*(c + d/x)),x]
```

```
[Out] ((2*c*Sqrt[a + b/x]*x*(-3*b^2*c + a^2*d*x + a*b*(d - c*x)))/(a^2*
(-(b*c) + a*d)*(b + a*x)) + (2*d^(5/2)*Log[d + c*x])/(-(b*c) + a*
d)^(3/2) - ((3*b*c + 2*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/
x]*x])/a^(5/2) - (2*d^(5/2)*Log[-(b*d) + b*c*x - 2*a*d*x + 2*Sqrt
[d]*Sqrt[-(b*c) + a*d]*Sqrt[a + b/x]*x])/(-(b*c) + a*d)^(3/2))/(2
*c^2)
```

---

**Maple [B]** time = 0.026, size = 1480, normalized size = 10.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b/x)^(3/2)/(c+d/x),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x/a^(9/2)*(-3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a
^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^2*b^5*c^4-4*(x
*(a*x+b))^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*a^(7/2)*b^2*c^4-4*ln((2*(
x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*
x+d))*a^(13/2)*x*b*d^4+2*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2
)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(9/2)*b^3*c*d^3-2*ln((2*(
x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*
x+d))*a^(15/2)*x^2*d^4-2*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2
)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(11/2)*b^2*d^4+4*(x*(a*x+
b))^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*a^(9/2)*b*c^3*d+12*(x*(a*x+b))^
(1/2)*((a*d-b*c)*d/c^2)^(1/2)*a^(7/2)*x*b^3*c^4-2*ln(1/2*(2*(x*(a
*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^
5*b^2*c*d^3+4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2
))*((a*d-b*c)*d/c^2)^(1/2)*a^3*b^4*c^3*d+4*ln((2*(x*(a*x+b))^(1/2
)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(11/2)*
x*b^2*c*d^3+2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*a^(9/2)*b
^2*c^2*d^2-8*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*a^(7/2)*b^
```

```
3*c^3*d+2*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d
*x+b*c*x-b*d)/(c*x+d))*a^(13/2)*x^2*b*c*d^3+6*(x*(a*x+b))^(1/2)*(
(a*d-b*c)*d/c^2)^(1/2)*a^(5/2)*b^4*c^4+4*(x*(a*x+b))^(1/2)*((a*d-
b*c)*d/c^2)^(1/2)*a^(11/2)*x*b*c^2*d^2+ln(1/2*(2*(x*(a*x+b))^(1/2
)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^6*x^2*b*c^2
*d^2-4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*
d-b*c)*d/c^2)^(1/2)*a^6*x*b*c*d^3-8*(x*(a*x+b))^(1/2)*((a*d-b*c)*
d/c^2)^(1/2)*a^(11/2)*x^2*b*c^3*d-16*(x*(a*x+b))^(1/2)*((a*d-b*c)
*d/c^2)^(1/2)*a^(9/2)*x*b^2*c^3*d+2*ln(1/2*(2*(x*(a*x+b))^(1/2)*a
^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^5*x*b^2*c^2*d^
2+4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b
*c)*d/c^2)^(1/2)*a^5*x^2*b^2*c^3*d+8*ln(1/2*(2*(x*(a*x+b))^(1/2)*
a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^4*x*b^3*c^3*d
+ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)
*d/c^2)^(1/2)*a^4*b^3*c^2*d^2-3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/
2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^2*a^4*b^3*c^4-6*ln
(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/
c^2)^(1/2)*x*a^3*b^4*c^4-2*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*
a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^7*x^2*c*d^3+2*(x*(a*x+b
))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*a^(13/2)*x^2*c^2*d^2+6*(x*(a*x+b
))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*a^(9/2)*x^2*b^2*c^4)/(x*(a*x+b))
^(1/2)/(a*d-b*c)^2/c^3/((a*d-b*c)*d/c^2)^(1/2)/(a*x+b)^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + b/x)^(3/2)*(c + d/x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.38584, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + b/x)^(3/2)*(c + d/x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a^(5/2)*d^2*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x)*log(-
(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (
b*c - 2*a*d)*x)/(c*x + d)) - (3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*sq
rt((a*x + b)/x)*log(-2*a*x*sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(a
)) - 2*(3*b^2*c^2 - a*b*c*d + (a*b*c^2 - a^2*c*d)*x)*sqrt(a))/((a
^2*b*c^3 - a^3*c^2*d)*sqrt(a)*sqrt((a*x + b)/x)), -(sqrt(-a)*a^2*
```

```
d^2*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x)*log(-(2*(b*c - a*d)*x*
sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(
c*x + d)) - (3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*sqrt((a*x + b)/x)*a
rctan(a/(sqrt(-a)*sqrt((a*x + b)/x))) - (3*b^2*c^2 - a*b*c*d + (a
*b*c^2 - a^2*c*d)*x)*sqrt(-a))/((a^2*b*c^3 - a^3*c^2*d)*sqrt(-a)*
sqrt((a*x + b)/x)), 1/2*(4*a^(5/2)*d^2*sqrt(d/(b*c - a*d))*sqrt((
a*x + b)/x)*arctan(-(b*c - a*d)*sqrt(d/(b*c - a*d))/(d*sqrt((a*x
+ b)/x))) + (3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*sqrt((a*x + b)/x)*l
og(-2*a*x*sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(a)) + 2*(3*b^2*c^2
 - a*b*c*d + (a*b*c^2 - a^2*c*d)*x)*sqrt(a))/((a^2*b*c^3 - a^3*c^
2*d)*sqrt(a)*sqrt((a*x + b)/x)), (2*sqrt(-a)*a^2*d^2*sqrt(d/(b*c
- a*d))*sqrt((a*x + b)/x)*arctan(-(b*c - a*d)*sqrt(d/(b*c - a*d))
/(d*sqrt((a*x + b)/x))) + (3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*sqrt(
(a*x + b)/x)*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x))) + (3*b^2*c^2
- a*b*c*d + (a*b*c^2 - a^2*c*d)*x)*sqrt(-a))/((a^2*b*c^3 - a^3*c^
2*d)*sqrt(-a)*sqrt((a*x + b)/x))]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a + \frac{b}{x}\right)^{\frac{3}{2}} (cx + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b/x)**(3/2)/(c+d/x),x)
```

```
[Out] Integral(x/((a + b/x)**(3/2)*(c*x + d)), x)
```

---

**GIAC/XCAS [A]** time = 0.254063, size = 261, normalized size = 1.78

$$\left(\frac{2\, d^3 \arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{(b^2c^3 - abc^2d)\sqrt{bcd - ad^2}} + \frac{2\, abc - \frac{3\,(ax+b)bc}{x} + \frac{(ax+b)ad}{x}}{(a^2bc^2 - a^3cd)\left(a\sqrt{\frac{ax+b}{x}} - \frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)} + \frac{(3\, bc + 2\, ad) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2bc^2}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + b/x)^(3/2)*(c + d/x)),x, algorithm="giac")
```

```
[Out] (2*d^3*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))/((b^2*c^3
- a*b*c^2*d)*sqrt(b*c*d - a*d^2)) + (2*a*b*c - 3*(a*x + b)*b*c/x
+ (a*x + b)*a*d/x)/((a^2*b*c^2 - a^3*c*d)*(a*sqrt((a*x + b)/x) -
(a*x + b)*sqrt((a*x + b)/x)/x)) + (3*b*c + 2*a*d)*arctan(sqrt((a*
x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2*b*c^2))*b
```

## 3.158 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx$

**Optimal**. Leaf size=224

$$-\frac{(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^3}+\frac{b\left(2a^2d^2-2abcd+3b^2c^2\right)}{a^2c^2\sqrt{a+\frac{b}{x}}(bc-ad)^2}+\frac{d^{5/2}(7bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{5/2}}+\frac{d(bc-2ad)}{ac^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

```
[Out] (b*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2))/(a^2*c^2*(b*c - a*d)^2*Sq
rt[a + b/x]) + (d*(b*c - 2*a*d))/(a*c^2*(b*c - a*d)*Sqrt[a + b/x]
*(c + d/x)) + x/(a*c*Sqrt[a + b/x]*(c + d/x)) + (d^(5/2)*(7*b*c -
 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3*(b*
c - a*d)^(5/2)) - ((3*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
)/(a^(5/2)*c^3)
```

---

**Rubi [A]** time = 0.96161, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$-\frac{(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^3}+\frac{b\left(2a^2d^2-2abcd+3b^2c^2\right)}{a^2c^2\sqrt{a+\frac{b}{x}}(bc-ad)^2}+\frac{d^{5/2}(7bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{5/2}}+\frac{d(bc-2ad)}{ac^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b/x)^(3/2)*(c + d/x)^2),x]
```


```
[Out] (b*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2))/(a^2*c^2*(b*c - a*d)^2*Sq
rt[a + b/x]) + (d*(b*c - 2*a*d))/(a*c^2*(b*c - a*d)*Sqrt[a + b/x]
*(c + d/x)) + x/(a*c*Sqrt[a + b/x]*(c + d/x)) + (d^(5/2)*(7*b*c -
 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3*(b*
c - a*d)^(5/2)) - ((3*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
)/(a^(5/2)*c^3)
```


---

**Rubi in Sympy [A]** time = 110.149, size = 194, normalized size = 0.87

$$-\frac{dx}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(ad-bc)}+\frac{d^{\frac{5}{2}}(4ad-7bc)\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{c^{3}(ad-bc)^{\frac{5}{2}}}+\frac{x(2ad-bc)}{ac^{2}\sqrt{a+\frac{b}{x}}(ad-bc)}+\frac{b\left(2a^{2}d^{2}-2abcd+3b^{2}c^{2}\right)}{a^{2}c^{2}\sqrt{a+\frac{b}{x}}(ad-bc)^{2}}-\frac{(4ad+3bc)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b/x)**(3/2)/(c+d/x)**2,x)
```

```
[Out] -d*x/(c*sqrt(a + b/x)*(c + d/x)*(a*d - b*c)) + d**(5/2)*(4*a*d -
7*b*c)*atanh(sqrt(d)*sqrt(a + b/x)/sqrt(a*d - b*c))/(c**3*(a*d -
b*c)**(5/2)) + x*(2*a*d - b*c)/(a*c**2*sqrt(a + b/x)*(a*d - b*c))
 + b*(2*a**2*d**2 - 2*a*b*c*d + 3*b**2*c**2)/(a**2*c**2*sqrt(a +
b/x)*(a*d - b*c)**2) - (4*a*d + 3*b*c)*atanh(sqrt(a + b/x)/sqrt(a
))/(a**(5/2)*c**3)
```

---

**Mathematica [C]** time = 1.10813, size = 290, normalized size = 1.29

$$\frac{-\frac{(4ad+3bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{a^{5/2}}+\frac{2cx\sqrt{a+\frac{b}{x}}\left(a^{3}d^{2}x(cx+2d)+a^{2}bd\left(-2c^{2}x^{2}-cdx+2d^{2}\right)+ab^{2}c\left(c^{2}x^{2}-cdx-2d^{2}\right)+3b^{3}c^{2}(cx+d)\right)}{a^{2}(ax+b)(cx+d)(bc-ad)^{2}}+\frac{id^{5/2}(7bc-4ad)\log}{}}{2c^{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b/x)^(3/2)*(c + d/x)^2),x]
```

```
[Out] ((2*c*Sqrt[a + b/x]*x*(3*b^3*c^2*(d + c*x) + a^3*d^2*x*(2*d + c*x
) + a^2*b*d*(2*d^2 - c*d*x - 2*c^2*x^2) + a*b^2*c*(-2*d^2 - c*d*x
 + c^2*x^2)))/(a^2*(b*c - a*d)^2*(b + a*x)*(d + c*x)) - ((3*b*c +
 4*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x])/a^(5/2) + (I*
d^(5/2)*(7*b*c - 4*a*d)*Log[((-2*I)*c^4*(b*c - a*d)^(3/2)*(-(b*d)
 + b*c*x - 2*a*d*x - (2*I)*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[a + b/x]*
x))/(d^(7/2)*(7*b*c - 4*a*d)*(d + c*x))])/(b*c - a*d)^(5/2))/(2*c
^3)
```

---

**Maple [B]** time = 0.026, size = 3121, normalized size = 13.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b/x)^(3/2)/(c+d/x)^2,x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x/a^(9/2)*(-6*(x*(a*x+b))^(1/2)*a^(5/2)*((a
*d-b*c)*d/c^2)^(1/2)*b^5*c^5*d+6*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1
/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^2*a^3*b^5*c^6+3*l
n(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d
/c^2)^(1/2)*x*a^2*b^6*c^6+3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2
*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^2*b^6*c^5*d-4*ln(1/2*(
2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^8*((a*d-b*c)*d/c^
2)^(1/2)*x^2*c*d^5+2*(x*(a*x+b))^(3/2)*a^(13/2)*((a*d-b*c)*d/c^2)
^(1/2)*x^2*c^4*d^2+2*(x*(a*x+b))^(1/2)*a^(15/2)*((a*d-b*c)*d/c^2)
^(1/2)*x^3*c^3*d^3-6*(x*(a*x+b))^(1/2)*a^(9/2)*((a*d-b*c)*d/c^2)^
(1/2)*x^3*b^3*c^6+11*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1
/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(15/2)*x^3*b*c^2*d^4-7*ln((2*
(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c
*x+d))*a^(13/2)*x^3*b^2*c^3*d^3+4*(x*(a*x+b))^(3/2)*a^(7/2)*((a*d
-b*c)*d/c^2)^(1/2)*x*b^3*c^6+4*(x*(a*x+b))^(1/2)*a^(15/2)*((a*d-b
*c)*d/c^2)^(1/2)*x^2*c^2*d^4+9*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2
)+2*a*x+b)/a^(1/2))*a^7*((a*d-b*c)*d/c^2)^(1/2)*x^3*b*c^3*d^3+14*
(x*(a*x+b))^(1/2)*a^(9/2)*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^3*c^5*d+3
*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5*((a*d-
b*c)*d/c^2)^(1/2)*x*b^3*c^3*d^3-11*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^
(1/2)+2*a*x+b)/a^(1/2))*a^5*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^3*c^4*d
^2-5*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5*((
a*d-b*c)*d/c^2)^(1/2)*x^3*b^3*c^5*d-12*(x*(a*x+b))^(1/2)*a^(13/2)
*((a*d-b*c)*d/c^2)^(1/2)*x^3*b*c^4*d^2+ln(1/2*(2*(x*(a*x+b))^(1/2
)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*x*b^5*c^5
*d-7*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*((
a*d-b*c)*d/c^2)^(1/2)*x^2*b^4*c^5*d-8*ln(1/2*(2*(x*(a*x+b))^(1/2)
*a^(1/2)+2*a*x+b)/a^(1/2))*a^7*((a*d-b*c)*d/c^2)^(1/2)*x*b*c*d^5+
14*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^6*((a*
d-b*c)*d/c^2)^(1/2)*x*b^2*c^2*d^4+15*ln(1/2*(2*(x*(a*x+b))^(1/2)*
a^(1/2)+2*a*x+b)/a^(1/2))*a^6*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^2*c^3
*d^3-3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^6*
((a*d-b*c)*d/c^2)^(1/2)*x^3*b^2*c^4*d^2+4*(x*(a*x+b))^(3/2)*a^(11
/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b*c^4*d^2+10*(x*(a*x+b))^(1/2)*a^(1
1/2)*((a*d-b*c)*d/c^2)^(1/2)*x^3*b^2*c^5*d-4*(x*(a*x+b))^(3/2)*a^
(9/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b^2*c^5*d-4*(x*(a*x+b))^(1/2)*a^(
13/2)*((a*d-b*c)*d/c^2)^(1/2)*x^2*b*c^3*d^3-8*(x*(a*x+b))^(1/2)*a
^(11/2)*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^2*c^4*d^2-5*ln(1/2*(2*(x*(a
*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2
)*b^5*c^4*d^2-4*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c
-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(17/2)*x^3*c*d^5-8*ln((2*(x*(a*x+b
))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^
(15/2)*x*b*d^6+11*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)
*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(11/2)*b^3*c*d^5-7*ln((2*(x*(a*x
+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*
a^(9/2)*b^4*c^2*d^4-12*(x*(a*x+b))^(1/2)*a^(7/2)*((a*d-b*c)*d/c^2
)^(1/2)*x^2*b^4*c^6-4*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b
)/a^(1/2))*a^6*((a*d-b*c)*d/c^2)^(1/2)*b^2*c*d^5+3*ln((2*(x*(a*x+
b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a
^(15/2)*x^2*b*c*d^5+15*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^
(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(13/2)*x^2*b^2*c^2*d^4-14*l
```

```
n((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*
d)/(c*x+d))*a^(11/2)*x^2*b^3*c^3*d^3-2*(x*(a*x+b))^(3/2)*a^(9/2)*
((a*d-b*c)*d/c^2)^(1/2)*b^2*c^4*d^2+4*(x*(a*x+b))^(3/2)*a^(7/2)*(
(a*d-b*c)*d/c^2)^(1/2)*b^3*c^5*d+9*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^
(1/2)+2*a*x+b)/a^(1/2))*a^5*((a*d-b*c)*d/c^2)^(1/2)*b^3*c^2*d^4-4
*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^8*((a*d-
b*c)*d/c^2)^(1/2)*x^3*c^2*d^4-2*(x*(a*x+b))^(1/2)*a^(15/2)*((a*d-
b*c)*d/c^2)^(1/2)*x^4*c^4*d^2+18*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*
c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(13/2)*x*b^2*c*d^
5-3*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c
*x-b*d)/(c*x+d))*a^(11/2)*x*b^3*c^2*d^4-7*ln((2*(x*(a*x+b))^(1/2)
*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(9/2)*x*
b^4*c^3*d^3+4*(x*(a*x+b))^(1/2)*a^(11/2)*((a*d-b*c)*d/c^2)^(1/2)*
b^2*c^2*d^4-8*(x*(a*x+b))^(1/2)*a^(9/2)*((a*d-b*c)*d/c^2)^(1/2)*b
^3*c^3*d^3+10*(x*(a*x+b))^(1/2)*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*b
^4*c^4*d^2-3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)
)*a^4*((a*d-b*c)*d/c^2)^(1/2)*b^4*c^3*d^3+8*(x*(a*x+b))^(1/2)*a^(
13/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b*c^2*d^4-14*(x*(a*x+b))^(1/2)*a^
(11/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b^2*c^3*d^3+12*(x*(a*x+b))^(1/2)
*a^(9/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b^3*c^4*d^2-2*(x*(a*x+b))^(1/2
)*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b^4*c^5*d-13*ln(1/2*(2*(x*(a*
x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*((a*d-b*c)*d/c^2)^(1/2)
*x*b^4*c^4*d^2+ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/
2))*a^7*((a*d-b*c)*d/c^2)^(1/2)*x^2*b*c^2*d^4-6*(x*(a*x+b))^(1/2)
*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b^5*c^6+3*ln(1/2*(2*(x*(a*x+b)
)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^3*a^4
*b^4*c^6-4*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*
d*x+b*c*x-b*d)/(c*x+d))*a^(17/2)*x^2*d^6-4*ln((2*(x*(a*x+b))^(1/2
)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(13/2)*
b^2*d^6)/(x*(a*x+b))^(1/2)/(a*d-b*c)^3/c^4/((a*d-b*c)*d/c^2)^(1/2
)/(a*x+b)^2/(c*x+d)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + b/x)^(3/2)*(c + d/x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.812128, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + b/x)^(3/2)*(c + d/x)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((7*a^2*b*c*d^3 - 4*a^3*d^4 + (7*a^2*b*c^2*d^2 - 4*a^3*c*d^
3)*x)*sqrt(a)*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x)*log(-(2*(b*c
 - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2
*a*d)*x)/(c*x + d)) - (3*b^3*c^3*d - 2*a*b^2*c^2*d^2 - 5*a^2*b*c*
d^3 + 4*a^3*d^4 + (3*b^3*c^4 - 2*a*b^2*c^3*d - 5*a^2*b*c^2*d^2 +
4*a^3*c*d^3)*x)*sqrt((a*x + b)/x)*log(-2*a*x*sqrt((a*x + b)/x) +
(2*a*x + b)*sqrt(a)) - 2*(3*b^3*c^3*d - 2*a*b^2*c^2*d^2 + 2*a^2*b
*c*d^3 + (a*b^2*c^4 - 2*a^2*b*c^3*d + a^3*c^2*d^2)*x^2 + (3*b^3*c
^4 - a*b^2*c^3*d - a^2*b*c^2*d^2 + 2*a^3*c*d^3)*x)*sqrt(a))/((a^2
*b^2*c^5*d - 2*a^3*b*c^4*d^2 + a^4*c^3*d^3 + (a^2*b^2*c^6 - 2*a^3
*b*c^5*d + a^4*c^4*d^2)*x)*sqrt(a)*sqrt((a*x + b)/x)), 1/2*(2*(7*
a^2*b*c*d^3 - 4*a^3*d^4 + (7*a^2*b*c^2*d^2 - 4*a^3*c*d^3)*x)*sqrt
(a)*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)*arctan(-(b*c - a*d)*sqr
t(d/(b*c - a*d))/(d*sqrt((a*x + b)/x))) + (3*b^3*c^3*d - 2*a*b^2*
c^2*d^2 - 5*a^2*b*c*d^3 + 4*a^3*d^4 + (3*b^3*c^4 - 2*a*b^2*c^3*d
- 5*a^2*b*c^2*d^2 + 4*a^3*c*d^3)*x)*sqrt((a*x + b)/x)*log(-2*a*x*
sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(a)) + 2*(3*b^3*c^3*d - 2*a*b
^2*c^2*d^2 + 2*a^2*b*c*d^3 + (a*b^2*c^4 - 2*a^2*b*c^3*d + a^3*c^2
*d^2)*x^2 + (3*b^3*c^4 - a*b^2*c^3*d - a^2*b*c^2*d^2 + 2*a^3*c*d^
3)*x)*sqrt(a))/((a^2*b^2*c^5*d - 2*a^3*b*c^4*d^2 + a^4*c^3*d^3 +
(a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^4*d^2)*x)*sqrt(a)*sqrt((a*x
+ b)/x)), -1/2*((7*a^2*b*c*d^3 - 4*a^3*d^4 + (7*a^2*b*c^2*d^2 - 4
*a^3*c*d^3)*x)*sqrt(-a)*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x)*lo
g(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d
+ (b*c - 2*a*d)*x)/(c*x + d)) - 2*(3*b^3*c^3*d - 2*a*b^2*c^2*d^2
- 5*a^2*b*c*d^3 + 4*a^3*d^4 + (3*b^3*c^4 - 2*a*b^2*c^3*d - 5*a^2*
b*c^2*d^2 + 4*a^3*c*d^3)*x)*sqrt((a*x + b)/x)*arctan(a/(sqrt(-a)*
sqrt((a*x + b)/x))) - 2*(3*b^3*c^3*d - 2*a*b^2*c^2*d^2 + 2*a^2*b*
c*d^3 + (a*b^2*c^4 - 2*a^2*b*c^3*d + a^3*c^2*d^2)*x^2 + (3*b^3*c^
4 - a*b^2*c^3*d - a^2*b*c^2*d^2 + 2*a^3*c*d^3)*x)*sqrt(-a))/((a^2
*b^2*c^5*d - 2*a^3*b*c^4*d^2 + a^4*c^3*d^3 + (a^2*b^2*c^6 - 2*a^3
*b*c^5*d + a^4*c^4*d^2)*x)*sqrt(-a)*sqrt((a*x + b)/x)), ((7*a^2*b
*c*d^3 - 4*a^3*d^4 + (7*a^2*b*c^2*d^2 - 4*a^3*c*d^3)*x)*sqrt(-a)*
sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)*arctan(-(b*c - a*d)*sqrt(d/
(b*c - a*d))/(d*sqrt((a*x + b)/x))) + (3*b^3*c^3*d - 2*a*b^2*c^2*
d^2 - 5*a^2*b*c*d^3 + 4*a^3*d^4 + (3*b^3*c^4 - 2*a*b^2*c^3*d - 5*
a^2*b*c^2*d^2 + 4*a^3*c*d^3)*x)*sqrt((a*x + b)/x)*arctan(a/(sqrt(
-a)*sqrt((a*x + b)/x))) + (3*b^3*c^3*d - 2*a*b^2*c^2*d^2 + 2*a^2*
b*c*d^3 + (a*b^2*c^4 - 2*a^2*b*c^3*d + a^3*c^2*d^2)*x^2 + (3*b^3*
c^4 - a*b^2*c^3*d - a^2*b*c^2*d^2 + 2*a^3*c*d^3)*x)*sqrt(-a))/((a
^2*b^2*c^5*d - 2*a^3*b*c^4*d^2 + a^4*c^3*d^3 + (a^2*b^2*c^6 - 2*a
^3*b*c^5*d + a^4*c^4*d^2)*x)*sqrt(-a)*sqrt((a*x + b)/x))]
```

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b/x)**(3/2)/(c+d/x)**2,x)
```

[Out] Exception raised: ValueError

---

**GIAC/XCAS [A]** time = 0.256391, size = 560, normalized size = 2.5

$$b\left(\frac{\left(7\,bcd^{3}-4\,ad^{4}\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^{2}}}\right)}{\left(b^{3}c^{5}-2\,ab^{2}c^{4}d+a^{2}bc^{3}d^{2}\right)\sqrt{bcd-ad^{2}}}+\frac{2\,ab^{3}c^{3}-2\,a^{2}b^{2}c^{2}d-\frac{3\,(ax+b)b^{3}c^{3}}{x}+\frac{7\,(ax+b)ab^{2}c^{2}d}{x}-\frac{3\,(ax+b)a^{2}bcd^{2}}{x}+\frac{2\,(ax+b)a^{3}d^{3}}{x}}{\left(a^{2}b^{2}c^{4}-2\,a^{3}bc^{3}d+a^{4}c^{2}d^{2}\right)\left(abc\sqrt{\frac{ax+b}{x}}-a^{2}d\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)bc\sqrt{\frac{ax+b}{x}}}{x}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + b/x)^(3/2)*(c + d/x)^2),x, algorithm="giac")
```

```
[Out] b*((7*b*c*d^3 - 4*a*d^4)*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d -
a*d^2))/((b^3*c^5 - 2*a*b^2*c^4*d + a^2*b*c^3*d^2)*sqrt(b*c*d - a
*d^2)) + (2*a*b^3*c^3 - 2*a^2*b^2*c^2*d - 3*(a*x + b)*b^3*c^3/x +
 7*(a*x + b)*a*b^2*c^2*d/x - 3*(a*x + b)*a^2*b*c*d^2/x + 2*(a*x +
 b)*a^3*d^3/x - 3*(a*x + b)^2*b^2*c^2*d/x^2 + 2*(a*x + b)^2*a*b*c
*d^2/x^2 - 2*(a*x + b)^2*a^2*d^3/x^2)/((a^2*b^2*c^4 - 2*a^3*b*c^3
*d + a^4*c^2*d^2)*(a*b*c*sqrt((a*x + b)/x) - a^2*d*sqrt((a*x + b)
/x) - (a*x + b)*b*c*sqrt((a*x + b)/x)/x + 2*(a*x + b)*a*d*sqrt((a
*x + b)/x)/x - (a*x + b)^2*d*sqrt((a*x + b)/x)/x^2)) + (3*b*c + 4
*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2*b*c^3))
```

## 3.159 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx$

**Optimal**. Leaf size=320

$$-\frac{3(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^4}+\frac{3d^{5/2}\left(8a^2d^2-24abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{7/2}}+\frac{d\left(12a^2d^2-21abcd+4b^2c^2\right)}{4ac^3\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{3b(2bc-ad)\left(4a^2d^2-abcd+2b^2c^2\right)}{4a^2c^3\sqrt{a+\frac{b}{x}}(bc-ad)^3}+\frac{d(2bc-3ad)}{2ac^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2(bc-ad)}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

```
[Out] (3*b*(2*b*c - a*d)*(2*b^2*c^2 - a*b*c*d + 4*a^2*d^2))/(4*a^2*c^3*
(b*c - a*d)^3*Sqrt[a + b/x]) + (d*(2*b*c - 3*a*d))/(2*a*c^2*(b*c
- a*d)*Sqrt[a + b/x]*(c + d/x)^2) + (d*(4*b^2*c^2 - 21*a*b*c*d +
12*a^2*d^2))/(4*a*c^3*(b*c - a*d)^2*Sqrt[a + b/x]*(c + d/x)) + x/
(a*c*Sqrt[a + b/x]*(c + d/x)^2) + (3*d^(5/2)*(21*b^2*c^2 - 24*a*b
*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]]
)/(4*c^4*(b*c - a*d)^(7/2)) - (3*(b*c + 2*a*d)*ArcTanh[Sqrt[a + b
/x]/Sqrt[a]])/(a^(5/2)*c^4)
```

---

**Rubi [A]** time = 1.52398, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$-\frac{3(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^4}+\frac{3d^{5/2}\left(8a^2d^2-24abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{7/2}}+\frac{d\left(12a^2d^2-21abcd+4b^2c^2\right)}{4ac^3\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{3b(2bc-ad)\left(4a^2d^2-abcd+2b^2c^2\right)}{4a^2c^3\sqrt{a+\frac{b}{x}}(bc-ad)^3}+\frac{d(2bc-3ad)}{2ac^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2(bc-ad)}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b/x)^(3/2)*(c + d/x)^3),x]
```


```
[Out] (3*b*(2*b*c - a*d)*(2*b^2*c^2 - a*b*c*d + 4*a^2*d^2))/(4*a^2*c^3*
(b*c - a*d)^3*Sqrt[a + b/x]) + (d*(2*b*c - 3*a*d))/(2*a*c^2*(b*c
- a*d)*Sqrt[a + b/x]*(c + d/x)^2) + (d*(4*b^2*c^2 - 21*a*b*c*d +
12*a^2*d^2))/(4*a*c^3*(b*c - a*d)^2*Sqrt[a + b/x]*(c + d/x)) + x/
(a*c*Sqrt[a + b/x]*(c + d/x)^2) + (3*d^(5/2)*(21*b^2*c^2 - 24*a*b
```

```
*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]]
)/(4*c^4*(b*c - a*d)^(7/2)) - (3*(b*c + 2*a*d)*ArcTanh[Sqrt[a + b
/x]/Sqrt[a]])/(a^(5/2)*c^4)
```

---

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b/x)**(3/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

---

**Mathematica [C]** time = 1.82433, size = 385, normalized size = 1.2

$$\frac{-\frac{12(2ad+bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{a^{5/2}}+\frac{3id^{5/2}\left(8a^2d^2-24abcd+21b^2c^2\right)\log\left(-\frac{8ic^5(bc-ad)^{5/2}\left(-2i\sqrt{d}x\sqrt{a+\frac{b}{x}}\sqrt{bc-ad}-2adx+bcx-bd\right)}{3d^{7/2}(cx+d)\left(8a^2d^2-24abcd+21b^2c^2\right)}\right)}{(bc-ad)^{7/2}}+\frac{2cx\sqrt{a+\frac{b}{x}}\left(2a^4d^3x\right.\cdots}{\cdots}}{8c^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b/x)^(3/2)*(c + d/x)^3),x]
```

```
[Out] ((2*c*Sqrt[a + b/x]*x*(-12*b^4*c^3*(d + c*x)^2 - 4*a*b^3*c^2*(-3*
d + c*x)*(d + c*x)^2 + 2*a^4*d^3*x*(6*d^2 + 9*c*d*x + 2*c^2*x^2)
+ a^3*b*d^2*(12*d^3 - 9*c*d^2*x - 37*c^2*d*x^2 - 12*c^3*x^3) + a^
2*b^2*c*d*(-27*d^3 - 29*c*d^2*x + 12*c^2*d*x^2 + 12*c^3*x^3)))/(a
^2*(-(b*c) + a*d)^3*(b + a*x)*(d + c*x)^2) - (12*(b*c + 2*a*d)*Lo
g[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x])/a^(5/2) + ((3*I)*d^(5/2
)*(21*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*Log[(((-8*I)/3)*c^5*(b*c
- a*d)^(5/2)*(-(b*d) + b*c*x - 2*a*d*x - (2*I)*Sqrt[d]*Sqrt[b*c -
 a*d]*Sqrt[a + b/x]*x))/(d^(7/2)*(21*b^2*c^2 - 24*a*b*c*d + 8*a^2
*d^2)*(d + c*x))])/(b*c - a*d)^(7/2))/(8*c^4)
```

---

**Maple [B]** time = 0.027, size = 5164, normalized size = 16.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b/x)^(3/2)/(c+d/x)^3,x)
```

```
[Out] result too large to display
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + b/x)^(3/2)*(c + d/x)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 2.24754, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + b/x)^(3/2)*(c + d/x)^3),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(21*a^2*b^2*c^2*d^4 - 24*a^3*b*c*d^5 + 8*a^4*d^6 + (21*a
^2*b^2*c^4*d^2 - 24*a^3*b*c^3*d^3 + 8*a^4*c^2*d^4)*x^2 + 2*(21*a^
2*b^2*c^3*d^3 - 24*a^3*b*c^2*d^4 + 8*a^4*c*d^5)*x)*sqrt(a)*sqrt(-
d/(b*c - a*d))*sqrt((a*x + b)/x)*log(-(2*(b*c - a*d)*x*sqrt(-d/(b
*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d))
- 12*(b^4*c^4*d^2 - a*b^3*c^3*d^3 - 3*a^2*b^2*c^2*d^4 + 5*a^3*b*c
*d^5 - 2*a^4*d^6 + (b^4*c^6 - a*b^3*c^5*d - 3*a^2*b^2*c^4*d^2 + 5
*a^3*b*c^3*d^3 - 2*a^4*c^2*d^4)*x^2 + 2*(b^4*c^5*d - a*b^3*c^4*d^
2 - 3*a^2*b^2*c^3*d^3 + 5*a^3*b*c^2*d^4 - 2*a^4*c*d^5)*x)*sqrt((a
*x + b)/x)*log(-2*a*x*sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(a)) -
2*(12*b^4*c^4*d^2 - 12*a*b^3*c^3*d^3 + 27*a^2*b^2*c^2*d^4 - 12*a^
3*b*c*d^5 + 4*(a*b^3*c^6 - 3*a^2*b^2*c^5*d + 3*a^3*b*c^4*d^2 - a^
4*c^3*d^3)*x^3 + (12*b^4*c^6 - 4*a*b^3*c^5*d - 12*a^2*b^2*c^4*d^2
 + 37*a^3*b*c^3*d^3 - 18*a^4*c^2*d^4)*x^2 + (24*b^4*c^5*d - 20*a*
b^3*c^4*d^2 + 29*a^2*b^2*c^3*d^3 + 9*a^3*b*c^2*d^4 - 12*a^4*c*d^5
)*x)*sqrt(a))/((a^2*b^3*c^7*d^2 - 3*a^3*b^2*c^6*d^3 + 3*a^4*b*c^5
*d^4 - a^5*c^4*d^5 + (a^2*b^3*c^9 - 3*a^3*b^2*c^8*d + 3*a^4*b*c^7
*d^2 - a^5*c^6*d^3)*x^2 + 2*(a^2*b^3*c^8*d - 3*a^3*b^2*c^7*d^2 +
3*a^4*b*c^6*d^3 - a^5*c^5*d^4)*x)*sqrt(a)*sqrt((a*x + b)/x)), -1/
8*(3*(21*a^2*b^2*c^2*d^4 - 24*a^3*b*c*d^5 + 8*a^4*d^6 + (21*a^2*b
^2*c^4*d^2 - 24*a^3*b*c^3*d^3 + 8*a^4*c^2*d^4)*x^2 + 2*(21*a^2*b^
2*c^3*d^3 - 24*a^3*b*c^2*d^4 + 8*a^4*c*d^5)*x)*sqrt(-a)*sqrt(-d/(
b*c - a*d))*sqrt((a*x + b)/x)*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c
- a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) - 2
4*(b^4*c^4*d^2 - a*b^3*c^3*d^3 - 3*a^2*b^2*c^2*d^4 + 5*a^3*b*c*d^
5 - 2*a^4*d^6 + (b^4*c^6 - a*b^3*c^5*d - 3*a^2*b^2*c^4*d^2 + 5*a^
3*b*c^3*d^3 - 2*a^4*c^2*d^4)*x^2 + 2*(b^4*c^5*d - a*b^3*c^4*d^2 -
```

```
 3*a^2*b^2*c^3*d^3 + 5*a^3*b*c^2*d^4 - 2*a^4*c*d^5)*x)*sqrt((a*x
+ b)/x)*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x))) - 2*(12*b^4*c^4*d^
2 - 12*a*b^3*c^3*d^3 + 27*a^2*b^2*c^2*d^4 - 12*a^3*b*c*d^5 + 4*(a
*b^3*c^6 - 3*a^2*b^2*c^5*d + 3*a^3*b*c^4*d^2 - a^4*c^3*d^3)*x^3 +
 (12*b^4*c^6 - 4*a*b^3*c^5*d - 12*a^2*b^2*c^4*d^2 + 37*a^3*b*c^3*
d^3 - 18*a^4*c^2*d^4)*x^2 + (24*b^4*c^5*d - 20*a*b^3*c^4*d^2 + 29
*a^2*b^2*c^3*d^3 + 9*a^3*b*c^2*d^4 - 12*a^4*c*d^5)*x)*sqrt(-a))/(
(a^2*b^3*c^7*d^2 - 3*a^3*b^2*c^6*d^3 + 3*a^4*b*c^5*d^4 - a^5*c^4*
d^5 + (a^2*b^3*c^9 - 3*a^3*b^2*c^8*d + 3*a^4*b*c^7*d^2 - a^5*c^6*
d^3)*x^2 + 2*(a^2*b^3*c^8*d - 3*a^3*b^2*c^7*d^2 + 3*a^4*b*c^6*d^3
 - a^5*c^5*d^4)*x)*sqrt(-a)*sqrt((a*x + b)/x)), 1/4*(3*(21*a^2*b^
2*c^2*d^4 - 24*a^3*b*c*d^5 + 8*a^4*d^6 + (21*a^2*b^2*c^4*d^2 - 24
*a^3*b*c^3*d^3 + 8*a^4*c^2*d^4)*x^2 + 2*(21*a^2*b^2*c^3*d^3 - 24*
a^3*b*c^2*d^4 + 8*a^4*c*d^5)*x)*sqrt(a)*sqrt(d/(b*c - a*d))*sqrt(
(a*x + b)/x)*arctan(-(b*c - a*d)*sqrt(d/(b*c - a*d))/(d*sqrt((a*x
 + b)/x))) + 6*(b^4*c^4*d^2 - a*b^3*c^3*d^3 - 3*a^2*b^2*c^2*d^4 +
 5*a^3*b*c*d^5 - 2*a^4*d^6 + (b^4*c^6 - a*b^3*c^5*d - 3*a^2*b^2*c
^4*d^2 + 5*a^3*b*c^3*d^3 - 2*a^4*c^2*d^4)*x^2 + 2*(b^4*c^5*d - a*
b^3*c^4*d^2 - 3*a^2*b^2*c^3*d^3 + 5*a^3*b*c^2*d^4 - 2*a^4*c*d^5)*
x)*sqrt((a*x + b)/x)*log(-2*a*x*sqrt((a*x + b)/x) + (2*a*x + b)*s
qrt(a)) + (12*b^4*c^4*d^2 - 12*a*b^3*c^3*d^3 + 27*a^2*b^2*c^2*d^4
 - 12*a^3*b*c*d^5 + 4*(a*b^3*c^6 - 3*a^2*b^2*c^5*d + 3*a^3*b*c^4*
d^2 - a^4*c^3*d^3)*x^3 + (12*b^4*c^6 - 4*a*b^3*c^5*d - 12*a^2*b^2
*c^4*d^2 + 37*a^3*b*c^3*d^3 - 18*a^4*c^2*d^4)*x^2 + (24*b^4*c^5*d
 - 20*a*b^3*c^4*d^2 + 29*a^2*b^2*c^3*d^3 + 9*a^3*b*c^2*d^4 - 12*a
^4*c*d^5)*x)*sqrt(a))/((a^2*b^3*c^7*d^2 - 3*a^3*b^2*c^6*d^3 + 3*a
^4*b*c^5*d^4 - a^5*c^4*d^5 + (a^2*b^3*c^9 - 3*a^3*b^2*c^8*d + 3*a
^4*b*c^7*d^2 - a^5*c^6*d^3)*x^2 + 2*(a^2*b^3*c^8*d - 3*a^3*b^2*c^
7*d^2 + 3*a^4*b*c^6*d^3 - a^5*c^5*d^4)*x)*sqrt(a)*sqrt((a*x + b)/
x)), 1/4*(3*(21*a^2*b^2*c^2*d^4 - 24*a^3*b*c*d^5 + 8*a^4*d^6 + (2
1*a^2*b^2*c^4*d^2 - 24*a^3*b*c^3*d^3 + 8*a^4*c^2*d^4)*x^2 + 2*(21
*a^2*b^2*c^3*d^3 - 24*a^3*b*c^2*d^4 + 8*a^4*c*d^5)*x)*sqrt(-a)*sq
rt(d/(b*c - a*d))*sqrt((a*x + b)/x)*arctan(-(b*c - a*d)*sqrt(d/(b
*c - a*d))/(d*sqrt((a*x + b)/x))) + 12*(b^4*c^4*d^2 - a*b^3*c^3*d
^3 - 3*a^2*b^2*c^2*d^4 + 5*a^3*b*c*d^5 - 2*a^4*d^6 + (b^4*c^6 - a
*b^3*c^5*d - 3*a^2*b^2*c^4*d^2 + 5*a^3*b*c^3*d^3 - 2*a^4*c^2*d^4)
*x^2 + 2*(b^4*c^5*d - a*b^3*c^4*d^2 - 3*a^2*b^2*c^3*d^3 + 5*a^3*b
*c^2*d^4 - 2*a^4*c*d^5)*x)*sqrt((a*x + b)/x)*arctan(a/(sqrt(-a)*s
qrt((a*x + b)/x))) + (12*b^4*c^4*d^2 - 12*a*b^3*c^3*d^3 + 27*a^2*
b^2*c^2*d^4 - 12*a^3*b*c*d^5 + 4*(a*b^3*c^6 - 3*a^2*b^2*c^5*d + 3
*a^3*b*c^4*d^2 - a^4*c^3*d^3)*x^3 + (12*b^4*c^6 - 4*a*b^3*c^5*d -
 12*a^2*b^2*c^4*d^2 + 37*a^3*b*c^3*d^3 - 18*a^4*c^2*d^4)*x^2 + (2
4*b^4*c^5*d - 20*a*b^3*c^4*d^2 + 29*a^2*b^2*c^3*d^3 + 9*a^3*b*c^2
*d^4 - 12*a^4*c*d^5)*x)*sqrt(-a))/((a^2*b^3*c^7*d^2 - 3*a^3*b^2*c
^6*d^3 + 3*a^4*b*c^5*d^4 - a^5*c^4*d^5 + (a^2*b^3*c^9 - 3*a^3*b^2
*c^8*d + 3*a^4*b*c^7*d^2 - a^5*c^6*d^3)*x^2 + 2*(a^2*b^3*c^8*d -
3*a^3*b^2*c^7*d^2 + 3*a^4*b*c^6*d^3 - a^5*c^5*d^4)*x)*sqrt(-a)*sq
rt((a*x + b)/x))]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b/x)**(3/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [A]** time = 0.261271, size = 678, normalized size = 2.12

$$\frac{1}{4}\,b\left(\frac{3\left(21\,b^2c^2d^3-24\,abcd^4+8\,a^2d^5\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^4c^7-3\,ab^3c^6d+3\,a^2b^2c^5d^2-a^3bc^4d^3\right)\sqrt{bcd-ad^2}}+\frac{4\left(2\,ab^3c^3-\frac{3\,(ax+b)b^3c^3}{x}+\frac{3\,(ax+b)ab^2c^2d}{x}-\frac{3\,(ax+b)a^2bcd^2}{x}+\frac{(ax+b}{x}\right.}{\left(a^2b^3c^6-3\,a^3b^2c^5d+3\,a^4bc^4d^2-a^5c^3d^3\right)\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{}}{x}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + b/x)^(3/2)*(c + d/x)^3),x, algorithm="giac")
```

```
[Out] 1/4*b*(3*(21*b^2*c^2*d^3 - 24*a*b*c*d^4 + 8*a^2*d^5)*arctan(d*sqr
t((a*x + b)/x)/sqrt(b*c*d - a*d^2))/((b^4*c^7 - 3*a*b^3*c^6*d + 3
*a^2*b^2*c^5*d^2 - a^3*b*c^4*d^3)*sqrt(b*c*d - a*d^2)) + 4*(2*a*b
^3*c^3 - 3*(a*x + b)*b^3*c^3/x + 3*(a*x + b)*a*b^2*c^2*d/x - 3*(a
*x + b)*a^2*b*c*d^2/x + (a*x + b)*a^3*d^3/x)/((a^2*b^3*c^6 - 3*a^
3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*(a*sqrt((a*x + b)/x)
 - (a*x + b)*sqrt((a*x + b)/x)/x)) + (17*b^2*c^2*d^3*sqrt((a*x +
b)/x) - 25*a*b*c*d^4*sqrt((a*x + b)/x) + 8*a^2*d^5*sqrt((a*x + b)
/x) + 15*(a*x + b)*b*c*d^4*sqrt((a*x + b)/x)/x - 8*(a*x + b)*a*d^
5*sqrt((a*x + b)/x)/x)/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^
2 - a^3*c^3*d^3)*(b*c - a*d + (a*x + b)*d/x)^2) + 12*(b*c + 2*a*d
)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2*b*c^4))
```

## 3.160 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

**Optimal**. Leaf size=143

$$-\frac{c^2(5bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{(bc-ad)\left(-4a^3d^2x-2a^2bd(5cx+3d)+ab^2c(20cx-3d)+15b^3c^2\right)}{3a^3b^2x\left(a+\frac{b}{x}\right)^{3/2}}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] (c*(c + d/x)^2*x)/(a*(a + b/x)^(3/2)) + ((b*c - a*d)*(15*b^3*c^2
- 4*a^3*d^2*x - 2*a^2*b*d*(3*d + 5*c*x) + a*b^2*c*(-3*d + 20*c*x)
))/(3*a^3*b^2*(a + b/x)^(3/2)*x) - (c^2*(5*b*c - 6*a*d)*ArcTanh[S
qrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

---

**Rubi [A]** time = 0.428991, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$-\frac{c^2(5bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{(bc-ad)\left(-4a^3d^2x-2a^2bd(5cx+3d)-ab^2c(3d-20cx)+15b^3c^2\right)}{3a^3b^2x\left(a+\frac{b}{x}\right)^{3/2}}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d/x)^3/(a + b/x)^(5/2),x]
```

```
[Out] (c*(c + d/x)^2*x)/(a*(a + b/x)^(3/2)) + ((b*c - a*d)*(15*b^3*c^2
- 4*a^3*d^2*x - a*b^2*c*(3*d - 20*c*x) - 2*a^2*b*d*(3*d + 5*c*x))
)/(3*a^3*b^2*(a + b/x)^(3/2)*x) - (c^2*(5*b*c - 6*a*d)*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/a^(7/2)
```

---

**Rubi in Sympy [A]** time = 27.8366, size = 138, normalized size = 0.97

$$\frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}+\frac{4\,(ad-bc)\left(\frac{a\left(2a^2d^2+5abcd-10b^2c^2\right)}{2}+\frac{3b\left(2a^2d^2+abcd-5b^2c^2\right)}{4x}\right)}{3a^3b^2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}+\frac{c^2\,(6ad-5bc)\,\text{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d/x)**3/(a+b/x)**(5/2),x)
```

```
[Out] c*x*(c + d/x)**2/(a*(a + b/x)**(3/2)) + 4*(a*d - b*c)*(a*(2*a**2*
d**2 + 5*a*b*c*d - 10*b**2*c**2)/2 + 3*b*(2*a**2*d**2 + a*b*c*d -
 5*b**2*c**2)/(4*x))/(3*a**3*b**2*(a + b/x)**(3/2)) + c**2*(6*a*d
 - 5*b*c)*atanh(sqrt(a + b/x)/sqrt(a))/a**(7/2)
```

---

**Mathematica [A]** time = 0.242433, size = 145, normalized size = 1.01

$$\frac{c^2(6ad-5bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{2a^{7/2}}+\frac{x\sqrt{a+\frac{b}{x}}\left(4a^4d^3x+6a^3bd^2(cx+d)+3a^2b^2c^2x(cx-8d)+2ab^3c^2(10cx-9d)+15b^4c^3\right)}{3a^3b^2(ax+b)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d/x)^3/(a + b/x)^(5/2),x]
```

```
[Out] (Sqrt[a + b/x]*x*(15*b^4*c^3 + 4*a^4*d^3*x + 3*a^2*b^2*c^2*x*(-8*
d + c*x) + 6*a^3*b*d^2*(d + c*x) + 2*a*b^3*c^2*(-9*d + 10*c*x)))/
(3*a^3*b^2*(b + a*x)^2) + (c^2*(-5*b*c + 6*a*d)*Log[b + 2*a*x + 2
*Sqrt[a]*Sqrt[a + b/x]*x])/(2*a^(7/2))
```

---

**Maple [B]** time = 0.023, size = 1157, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d/x)^3/(a+b/x)^(5/2),x)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)*x/a^(13/2)*(90*a^(11/2)*(x*(a*x+b))^(1/2)*x
^2*b^4*c^3+9*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)
)*a^7*x*b^3*d^3-9*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^
(1/2))*a^7*x*b^3*d^3+18*a^(15/2)*(a*x^2+b*x)^(1/2)*x*b^2*d^3+12*a
^(13/2)*(x*(a*x+b))^(3/2)*b^2*c*d^2+24*a^(11/2)*(x*(a*x+b))^(3/2)
*b^3*c^2*d+18*a^(15/2)*(x*(a*x+b))^(1/2)*x*b^2*d^3+90*a^(9/2)*(x*
(a*x+b))^(1/2)*x*b^5*c^3-36*a^(9/2)*(x*(a*x+b))^(1/2)*b^5*c^2*d+1
8*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*b^6*c
^2*d-15*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^3
*a^6*b^4*c^3-45*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1
```

```
/2))*x^2*a^5*b^5*c^3-45*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x
+b)/a^(1/2))*x*a^4*b^6*c^3+3*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+
2*a*x+b)/a^(1/2))*a^9*x^3*b*d^3-3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(
1/2)+2*a*x+b)/a^(1/2))*a^9*x^3*b*d^3+30*a^(13/2)*(x*(a*x+b))^(1/2
)*x^3*b^3*c^3+9*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1
/2))*a^8*x^2*b^2*d^3-9*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+
b)/a^(1/2))*a^8*x^2*b^2*d^3+18*a^(17/2)*(a*x^2+b*x)^(1/2)*x^2*b*d
^3-24*a^(11/2)*(x*(a*x+b))^(3/2)*x*b^3*c^3+18*a^(17/2)*(x*(a*x+b)
)^(1/2)*x^2*b*d^3+30*a^(7/2)*(x*(a*x+b))^(1/2)*b^6*c^3-15*ln(1/2*
(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*b^7*c^3+6*a^(1
9/2)*(a*x^2+b*x)^(1/2)*x^3*d^3+6*a^(19/2)*(x*(a*x+b))^(1/2)*x^3*d
^3-12*a^(17/2)*(x*(a*x+b))^(3/2)*x*d^3-16*a^(15/2)*(x*(a*x+b))^(3
/2)*b*d^3-20*a^(9/2)*(x*(a*x+b))^(3/2)*b^4*c^3+3*ln(1/2*(2*(a*x^2
+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^6*b^4*d^3-3*ln(1/2*(2*(x*
(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^6*b^4*d^3+6*a^(13/2)*(
a*x^2+b*x)^(1/2)*b^3*d^3+6*a^(13/2)*(x*(a*x+b))^(1/2)*b^3*d^3+54*
ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5*x*b^5*c
^2*d-36*a^(15/2)*(x*(a*x+b))^(1/2)*x^3*b^2*c^2*d+36*a^(13/2)*(x*(
a*x+b))^(3/2)*x*b^2*c^2*d-108*a^(13/2)*(x*(a*x+b))^(1/2)*x^2*b^3*
c^2*d+18*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^
7*x^3*b^3*c^2*d+54*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a
^(1/2))*a^6*x^2*b^4*c^2*d-108*a^(11/2)*(x*(a*x+b))^(1/2)*x*b^4*c^
2*d)/(x*(a*x+b))^(1/2)/b^3/(a*x+b)^3
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)^3/(a + b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.254988, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(5\,b^4c^3-6\,ab^3c^2d+\left(5\,ab^3c^3-6\,a^2b^2c^2d\right)x\right)\sqrt{\frac{ax+b}{x}}\log\left(2\,ax\sqrt{\frac{ax+b}{x}}+(2\,ax+b)\sqrt{a}\right)-2\left(3\,a^2b^2c^3x^2+15\,b^4c^3-18\,ab\right.}{6\left(a^4b^2x+a^3b^3\right)\sqrt{a}\sqrt{\frac{ax+b}{x}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)^3/(a + b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(5*b^4*c^3 - 6*a*b^3*c^2*d + (5*a*b^3*c^3 - 6*a^2*b^2*c^
2*d)*x)*sqrt((a*x + b)/x)*log(2*a*x*sqrt((a*x + b)/x) + (2*a*x +
```

```
b)*sqrt(a)) - 2*(3*a^2*b^2*c^3*x^2 + 15*b^4*c^3 - 18*a*b^3*c^2*d
+ 6*a^3*b*d^3 + 2*(10*a*b^3*c^3 - 12*a^2*b^2*c^2*d + 3*a^3*b*c*d^
2 + 2*a^4*d^3)*x)*sqrt(a))/((a^4*b^2*x + a^3*b^3)*sqrt(a)*sqrt((a
*x + b)/x)), 1/3*(3*(5*b^4*c^3 - 6*a*b^3*c^2*d + (5*a*b^3*c^3 - 6
*a^2*b^2*c^2*d)*x)*sqrt((a*x + b)/x)*arctan(a/(sqrt(-a)*sqrt((a*x
 + b)/x))) + (3*a^2*b^2*c^3*x^2 + 15*b^4*c^3 - 18*a*b^3*c^2*d + 6
*a^3*b*d^3 + 2*(10*a*b^3*c^3 - 12*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 +
 2*a^4*d^3)*x)*sqrt(-a))/((a^4*b^2*x + a^3*b^3)*sqrt(-a)*sqrt((a*
x + b)/x))]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx+d)^3}{x^3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d/x)**3/(a+b/x)**(5/2),x)
```

```
[Out] Integral((c*x + d)**3/(x**3*(a + b/x)**(5/2)), x)
```

---

**GIAC/XCAS [A]** time = 0.259834, size = 267, normalized size = 1.87

$$-\frac{1}{3}\,b\left(\frac{3\,c^3\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^3}-\frac{3\left(5\,bc^3-6\,ac^2d\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3b}-\frac{2\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3+\frac{6\,(ax+b)b^3c^3}{x}-\frac{9\,(ax+b)}{x}\cdots\right.}{(ax+b)a^3b^3\sqrt{\frac{ax+b}{x}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)^3/(a + b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*b*(3*c^3*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3) - 3*(5*b*
c^3 - 6*a*c^2*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3
*b) - 2*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3 +
6*(a*x + b)*b^3*c^3/x - 9*(a*x + b)*a*b^2*c^2*d/x + 3*(a*x + b)*a
^3*d^3/x)*x/((a*x + b)*a^3*b^3*sqrt((a*x + b)/x)))
```

## 3.161 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

**Optimal**. Leaf size=122

$$-\frac{c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}}+\frac{2a^2d^2+bc(5bc-4ad)}{3a^2b\left(a+\frac{b}{x}\right)^{3/2}}+\frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] (2*a^2*d^2 + b*c*(5*b*c - 4*a*d))/(3*a^2*b*(a + b/x)^(3/2)) + (c*
(5*b*c - 4*a*d))/(a^3*Sqrt[a + b/x]) + (c^2*x)/(a*(a + b/x)^(3/2)
) - (c*(5*b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

---

**Rubi [A]** time = 0.296017, antiderivative size = 118, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$-\frac{c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}}+\frac{\frac{c(5bc-4ad)}{a^2}+\frac{2d^2}{b}}{3\left(a+\frac{b}{x}\right)^{3/2}}+\frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d/x)^2/(a + b/x)^(5/2),x]
```

```
[Out] ((2*d^2)/b + (c*(5*b*c - 4*a*d))/a^2)/(3*(a + b/x)^(3/2)) + (c*(5
*b*c - 4*a*d))/(a^3*Sqrt[a + b/x]) + (c^2*x)/(a*(a + b/x)^(3/2))
- (c*(5*b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

---

**Rubi in Sympy [A]** time = 22.7247, size = 107, normalized size = 0.88

$$\frac{c^2x}{a\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}+\frac{2\left(a^2d^2-\frac{bc(4ad-5bc)}{2}\right)}{3a^2b\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}-\frac{c\left(4ad-5bc\right)}{a^3\sqrt{a+\frac{b}{x}}}+\frac{c\left(4ad-5bc\right)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d/x)**2/(a+b/x)**(5/2),x)
```

```
[Out] c**2*x/(a*(a + b/x)**(3/2)) + 2*(a**2*d**2 - b*c*(4*a*d - 5*b*c)/
2)/(3*a**2*b*(a + b/x)**(3/2)) - c*(4*a*d - 5*b*c)/(a**3*sqrt(a +
 b/x)) + c*(4*a*d - 5*b*c)*atanh(sqrt(a + b/x)/sqrt(a))/a**(7/2)
```

---

**Mathematica [A]** time = 0.189269, size = 123, normalized size = 1.01

$$\frac{c(4ad-5bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{2a^{7/2}}+\frac{x\sqrt{a+\frac{b}{x}}\left(2a^3d^2x+a^2bcx(3cx-16d)+4ab^2c(5cx-3d)+15b^3c^2\right)}{3a^3b(ax+b)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d/x)^2/(a + b/x)^(5/2),x]
```

```
[Out] (Sqrt[a + b/x]*x*(15*b^3*c^2 + 2*a^3*d^2*x + a^2*b*c*x*(-16*d + 3
*c*x) + 4*a*b^2*c*(-3*d + 5*c*x)))/(3*a^3*b*(b + a*x)^2) + (c*(-5
*b*c + 4*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x])/(2*a^(7
/2))
```

---

**Maple [B]** time = 0.02, size = 595, normalized size = 4.9

$$-\frac{x}{6\,b\,(ax+b)^3}\sqrt{\frac{ax+b}{x}}\left(24\,a^{15/2}\sqrt{x\,(ax+b)}x^3cd-30\,a^{13/2}\sqrt{x\,(ax+b)}x^3bc^2-24\,a^{13/2}\,(x\,(ax+b))^{3/2}\,xcd+72\,a^{13/2}\sqrt{x\,(ax\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d/x)^2/(a+b/x)^(5/2),x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)*x*(24*a^(15/2)*(x*(a*x+b))^(1/2)*x^3*c*d-3
0*a^(13/2)*(x*(a*x+b))^(1/2)*x^3*b*c^2-24*a^(13/2)*(x*(a*x+b))^(3
/2)*x*c*d+72*a^(13/2)*(x*(a*x+b))^(1/2)*x^2*b*c*d-4*a^(13/2)*(x*(
a*x+b))^(3/2)*d^2+24*a^(11/2)*(x*(a*x+b))^(3/2)*x*b*c^2-90*a^(11/
2)*(x*(a*x+b))^(1/2)*x^2*b^2*c^2-16*a^(11/2)*(x*(a*x+b))^(3/2)*b*
c*d+72*a^(11/2)*(x*(a*x+b))^(1/2)*x*b^2*c*d+20*a^(9/2)*(x*(a*x+b)
)^(3/2)*b^2*c^2-90*a^(9/2)*(x*(a*x+b))^(1/2)*x*b^3*c^2+24*a^(9/2)
*(x*(a*x+b))^(1/2)*b^3*c*d-30*a^(7/2)*(x*(a*x+b))^(1/2)*b^4*c^2-1
2*a^7*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^3*b
*c*d+15*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^3
*a^6*b^2*c^2-36*a^6*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/
a^(1/2))*x^2*b^2*c*d+45*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x
+b)/a^(1/2))*x^2*a^5*b^3*c^2-36*a^5*ln(1/2*(2*(x*(a*x+b))^(1/2)*a
^(1/2)+2*a*x+b)/a^(1/2))*x*b^3*c*d+45*ln(1/2*(2*(x*(a*x+b))^(1/2)
*a^(1/2)+2*a*x+b)/a^(1/2))*x*a^4*b^4*c^2-12*a^4*ln(1/2*(2*(x*(a*x
```

```
+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^4*c*d+15*ln(1/2*(2*(x*(a*x
+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*b^5*c^2)/a^(13/2)/(x*(a*
x+b))^(1/2)/b/(a*x+b)^3
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)^2/(a + b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.254442, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(5\,b^3c^2-4\,ab^2cd+\left(5\,ab^2c^2-4\,a^2bcd\right)x\right)\sqrt{\frac{ax+b}{x}}\log\left(2\,ax\sqrt{\frac{ax+b}{x}}+(2\,ax+b)\sqrt{a}\right)-2\left(3\,a^2bc^2x^2+15\,b^3c^2-12\,ab^2cd\right.}{6\left(a^4bx+a^3b^2\right)\sqrt{a}\sqrt{\frac{ax+b}{x}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)^2/(a + b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(5*b^3*c^2 - 4*a*b^2*c*d + (5*a*b^2*c^2 - 4*a^2*b*c*d)*x
)*sqrt((a*x + b)/x)*log(2*a*x*sqrt((a*x + b)/x) + (2*a*x + b)*sqr
t(a)) - 2*(3*a^2*b*c^2*x^2 + 15*b^3*c^2 - 12*a*b^2*c*d + 2*(10*a*
b^2*c^2 - 8*a^2*b*c*d + a^3*d^2)*x)*sqrt(a))/((a^4*b*x + a^3*b^2)
*sqrt(a)*sqrt((a*x + b)/x)), 1/3*(3*(5*b^3*c^2 - 4*a*b^2*c*d + (5
*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt((a*x + b)/x)*arctan(a/(sqrt(-a)
*sqrt((a*x + b)/x))) + (3*a^2*b*c^2*x^2 + 15*b^3*c^2 - 12*a*b^2*c
*d + 2*(10*a*b^2*c^2 - 8*a^2*b*c*d + a^3*d^2)*x)*sqrt(-a))/((a^4*
b*x + a^3*b^2)*sqrt(-a)*sqrt((a*x + b)/x))]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx+d)^2}{x^2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d/x)**2/(a+b/x)**(5/2),x)
```

```
[Out] Integral((c*x + d)**2/(x**2*(a + b/x)**(5/2)), x)
```

---

**GIAC/XCAS [A]** time = 0.257682, size = 217, normalized size = 1.78

$$-\frac{1}{3}\,b\left(\frac{3\,c^{2}\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^{3}}-\frac{3\,\left(5\,bc^{2}-4\,acd\right)\,\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^{3}b}-\frac{2\,\left(ab^{2}c^{2}-2\,a^{2}bcd+a^{3}d^{2}+\frac{6\,(ax+b)b^{2}c^{2}}{x}-\frac{6\,(ax+b)abcd}{x}\right)x}{(ax+b)a^{3}b^{2}\sqrt{\frac{ax+b}{x}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)^2/(a + b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*b*(3*c^2*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3) - 3*(5*b*
c^2 - 4*a*c*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3*b
) - 2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + 6*(a*x + b)*b^2*c^2/x
- 6*(a*x + b)*a*b*c*d/x)*x/((a*x + b)*a^3*b^2*sqrt((a*x + b)/x)))
```

## 3.162 $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

**Optimal**. Leaf size=103

$$-\frac{(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}}+\frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] (5*b*c - 2*a*d)/(3*a^2*(a + b/x)^(3/2)) + (5*b*c - 2*a*d)/(a^3*Sq
rt[a + b/x]) + (c*x)/(a*(a + b/x)^(3/2)) - ((5*b*c - 2*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

---

**Rubi [A]** time = 0.198709, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$-\frac{(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}}+\frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d/x)/(a + b/x)^(5/2),x]
```

```
[Out] (5*b*c - 2*a*d)/(3*a^2*(a + b/x)^(3/2)) + (5*b*c - 2*a*d)/(a^3*Sq
rt[a + b/x]) + (c*x)/(a*(a + b/x)^(3/2)) - ((5*b*c - 2*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

---

**Rubi in Sympy [A]** time = 16.3615, size = 90, normalized size = 0.87

$$\frac{cx}{a\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}-\frac{2\left(ad-\frac{5bc}{2}\right)}{3a^2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}-\frac{2\left(ad-\frac{5bc}{2}\right)}{a^3\sqrt{a+\frac{b}{x}}}+\frac{2\left(ad-\frac{5bc}{2}\right)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d/x)/(a+b/x)**(5/2),x)
```

```
[Out] c*x/(a*(a + b/x)**(3/2)) - 2*(a*d - 5*b*c/2)/(3*a**2*(a + b/x)**(
3/2)) - 2*(a*d - 5*b*c/2)/(a**3*sqrt(a + b/x)) + 2*(a*d - 5*b*c/2
)*atanh(sqrt(a + b/x)/sqrt(a))/a**(7/2)
```

---

**Mathematica [A]** time = 0.140561, size = 102, normalized size = 0.99

$$\frac{(2ad-5bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{2a^{7/2}}+\frac{x\sqrt{a+\frac{b}{x}}\left(a^2x(3cx-8d)+ab(20cx-6d)+15b^2c\right)}{3a^3(ax+b)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d/x)/(a + b/x)^(5/2),x]
```

```
[Out] (Sqrt[a + b/x]*x*(15*b^2*c + a^2*x*(-8*d + 3*c*x) + a*b*(-6*d + 2
0*c*x)))/(3*a^3*(b + a*x)^2) + ((-5*b*c + 2*a*d)*Log[b + 2*a*x +
2*Sqrt[a]*Sqrt[a + b/x]*x])/(2*a^(7/2))
```

---

**Maple [B]** time = 0.019, size = 548, normalized size = 5.3

$$-\frac{x}{6b(ax+b)^3}\sqrt{\frac{ax+b}{x}}\left(12a^{15/2}\sqrt{x(ax+b)}x^3d-30a^{13/2}\sqrt{x(ax+b)}x^3bc-12a^{13/2}(x(ax+b))^{3/2}xd+36a^{13/2}\sqrt{x(ax+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d/x)/(a+b/x)^(5/2),x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)*x*(12*a^(15/2)*(x*(a*x+b))^(1/2)*x^3*d-30*
a^(13/2)*(x*(a*x+b))^(1/2)*x^3*b*c-12*a^(13/2)*(x*(a*x+b))^(3/2)*
x*d+36*a^(13/2)*(x*(a*x+b))^(1/2)*x^2*b*d+24*a^(11/2)*(x*(a*x+b))
^(3/2)*x*b*c-90*a^(11/2)*(x*(a*x+b))^(1/2)*x^2*b^2*c-8*a^(11/2)*(
x*(a*x+b))^(3/2)*b*d+36*a^(11/2)*(x*(a*x+b))^(1/2)*x*b^2*d+20*a^(
9/2)*(x*(a*x+b))^(3/2)*b^2*c-90*a^(9/2)*(x*(a*x+b))^(1/2)*x*b^3*c
+12*a^(9/2)*(x*(a*x+b))^(1/2)*b^3*d-30*a^(7/2)*(x*(a*x+b))^(1/2)*
b^4*c-6*a^7*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))
*x^3*b*d+15*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))
*x^3*a^6*b^2*c-18*a^6*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b
)/a^(1/2))*x^2*b^2*d+45*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x
+b)/a^(1/2))*x^2*a^5*b^3*c-18*a^5*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(
1/2)+2*a*x+b)/a^(1/2))*x*b^3*d+45*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(
1/2)+2*a*x+b)/a^(1/2))*x*a^4*b^4*c-6*a^4*ln(1/2*(2*(x*(a*x+b))^(1
/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^4*d+15*ln(1/2*(2*(x*(a*x+b))^(1/2
)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*b^5*c)/a^(13/2)/(x*(a*x+b))^(1/2)
/b/(a*x+b)^3
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)/(a + b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.241657, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(5\,b^2c-2\,abd+\left(5\,abc-2\,a^2d\right)x\right)\sqrt{\frac{ax+b}{x}}\log\left(2\,ax\sqrt{\frac{ax+b}{x}}+(2\,ax+b)\sqrt{a}\right)-2\left(3\,a^2cx^2+15\,b^2c-6\,abd+4\left(5\,abc-2\right.\right.}{6\left(a^4x+a^3b\right)\sqrt{a}\sqrt{\frac{ax+b}{x}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)/(a + b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(5*b^2*c - 2*a*b*d + (5*a*b*c - 2*a^2*d)*x)*sqrt((a*x +
b)/x)*log(2*a*x*sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(a)) - 2*(3*a
^2*c*x^2 + 15*b^2*c - 6*a*b*d + 4*(5*a*b*c - 2*a^2*d)*x)*sqrt(a))
/((a^4*x + a^3*b)*sqrt(a)*sqrt((a*x + b)/x)), 1/3*(3*(5*b^2*c - 2
*a*b*d + (5*a*b*c - 2*a^2*d)*x)*sqrt((a*x + b)/x)*arctan(a/(sqrt(
-a)*sqrt((a*x + b)/x))) + (3*a^2*c*x^2 + 15*b^2*c - 6*a*b*d + 4*(
5*a*b*c - 2*a^2*d)*x)*sqrt(-a))/((a^4*x + a^3*b)*sqrt(-a)*sqrt((a
*x + b)/x))]
```

---

**Sympy [A]** time = 39.1088, size = 1479, normalized size = 14.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d/x)/(a+b/x)**(5/2),x)
```

```
[Out] c*(6*a**17*x**4*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2
)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 46*a**16*b*x
**3*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 1
8*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 15*a**16*b*x**3*log(b/(a
*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*
```

```
x + 6*a**(33/2)*b**3) - 30*a**16*b*x**3*log(sqrt(1 + b/(a*x)) + 1
)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x +
 6*a**(33/2)*b**3) + 70*a**15*b**2*x**2*sqrt(1 + b/(a*x))/(6*a**(
39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33
/2)*b**3) + 45*a**15*b**2*x**2*log(b/(a*x))/(6*a**(39/2)*x**3 + 1
8*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 90
*a**15*b**2*x**2*log(sqrt(1 + b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 1
8*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 30
*a**14*b**3*x*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*
b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**14*b**3*
x*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(3
5/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**14*b**3*x*log(sqrt(1 + b/
(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2
)*b**2*x + 6*a**(33/2)*b**3) + 15*a**13*b**4*log(b/(a*x))/(6*a**(
39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33
/2)*b**3) - 30*a**13*b**4*log(sqrt(1 + b/(a*x)) + 1)/(6*a**(39/2)
*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b
**3)) + d*(-8*a**7*x**3*sqrt(1 + b/(a*x))/(3*a**(19/2)*x**3 + 9*a
**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 3*a**7
*x**3*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**
(15/2)*b**2*x + 3*a**(13/2)*b**3) + 6*a**7*x**3*log(sqrt(1 + b/(a
*x)) + 1)/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b*
*2*x + 3*a**(13/2)*b**3) - 14*a**6*b*x**2*sqrt(1 + b/(a*x))/(3*a*
*(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13
/2)*b**3) - 9*a**6*b*x**2*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(
17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) + 18*a**6*b
*x**2*log(sqrt(1 + b/(a*x)) + 1)/(3*a**(19/2)*x**3 + 9*a**(17/2)*
b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 6*a**5*b**2*x*s
qrt(1 + b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(1
5/2)*b**2*x + 3*a**(13/2)*b**3) - 9*a**5*b**2*x*log(b/(a*x))/(3*a
**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(1
3/2)*b**3) + 18*a**5*b**2*x*log(sqrt(1 + b/(a*x)) + 1)/(3*a**(19/
2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b
**3) - 3*a**4*b**3*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b
*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) + 6*a**4*b**3*log(
sqrt(1 + b/(a*x)) + 1)/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9
*a**(15/2)*b**2*x + 3*a**(13/2)*b**3))
```

---

**GIAC/XCAS [A]** time = 0.255601, size = 185, normalized size = 1.8

$$-\frac{1}{3}\,b\left(\frac{3\,c\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^3}-\frac{3\,(5\,bc-2\,ad)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3b}-\frac{2\left(abc-a^2d+\frac{6\,(ax+b)bc}{x}-\frac{3\,(ax+b)ad}{x}\right)x}{(ax+b)a^3b\sqrt{\frac{ax+b}{x}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + d/x)/(a + b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*b*(3*c*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3) - 3*(5*b*c
```

```
- 2*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3*b) - 2*
(a*b*c - a^2*d + 6*(a*x + b)*b*c/x - 3*(a*x + b)*a*d/x)*x/((a*x +
 b)*a^3*b*sqrt((a*x + b)/x)))
```

## 3.163 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

**Optimal**. Leaf size=82

$$-\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5x\sqrt{a+\frac{b}{x}}}{a^3}-\frac{10x}{3a^2\sqrt{a+\frac{b}{x}}}-\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] (-2*x)/(3*a*(a + b/x)^(3/2)) - (10*x)/(3*a^2*Sqrt[a + b/x]) + (5*
Sqrt[a + b/x]*x)/a^3 - (5*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/
2)
```

---

**Rubi [A]** time = 0.112652, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$-\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5x\sqrt{a+\frac{b}{x}}}{a^3}-\frac{10x}{3a^2\sqrt{a+\frac{b}{x}}}-\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^(-5/2),x]
```

```
[Out] (-2*x)/(3*a*(a + b/x)^(3/2)) - (10*x)/(3*a^2*Sqrt[a + b/x]) + (5*
Sqrt[a + b/x]*x)/a^3 - (5*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/
2)
```

---

**Rubi in Sympy [A]** time = 11.5393, size = 70, normalized size = 0.85

$$-\frac{2x}{3a\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}-\frac{10x}{3a^2\sqrt{a+\frac{b}{x}}}+\frac{5x\sqrt{a+\frac{b}{x}}}{a^3}-\frac{5b\,\text{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b/x)**(5/2),x)
```

```
[Out] -2*x/(3*a*(a + b/x)**(3/2)) - 10*x/(3*a**2*sqrt(a + b/x)) + 5*x*s
qrt(a + b/x)/a**3 - 5*b*atanh(sqrt(a + b/x)/sqrt(a))/a**(7/2)
```

---

**Mathematica [A]** time = 0.133915, size = 82, normalized size = 1.

$$\frac{x\sqrt{a+\frac{b}{x}}\left(3a^2x^2+20abx+15b^2\right)}{3a^3(ax+b)^2}-\frac{5b\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{2a^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x)^(-5/2),x]
```

```
[Out] (Sqrt[a + b/x]*x*(15*b^2 + 20*a*b*x + 3*a^2*x^2))/(3*a^3*(b + a*x
)^2) - (5*b*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x])/(2*a^(7/2
))
```

---

**Maple [B]** time = 0.006, size = 276, normalized size = 3.4

$$-\frac{x}{6\,(ax+b)^3}\sqrt{\frac{ax+b}{x}}\left(-30\,a^{13/2}\sqrt{x\,(ax+b)}x^3+24\,a^{11/2}\,(x\,(ax+b))^{3/2}\,x-90\,a^{11/2}\sqrt{x\,(ax+b)}x^2b+20\,ba^{9/2}\,(x\,(ax+b))^{3/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b/x)^(5/2),x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)*x/a^(13/2)*(-30*a^(13/2)*(x*(a*x+b))^(1/2)
*x^3+24*a^(11/2)*(x*(a*x+b))^(3/2)*x-90*a^(11/2)*(x*(a*x+b))^(1/2
)*x^2*b+20*b*a^(9/2)*(x*(a*x+b))^(3/2)-90*a^(9/2)*(x*(a*x+b))^(1/
2)*x*b^2+15*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))
*x^3*a^6*b-30*a^(7/2)*(x*(a*x+b))^(1/2)*b^3+45*ln(1/2*(2*(x*(a*x+
b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^2*a^5*b^2+45*ln(1/2*(2*(x*(
a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x*a^4*b^3+15*ln(1/2*(2*(x
*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*b^4)/(x*(a*x+b))^(1
/2)/(a*x+b)^3
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(-5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.242169, size = 1, normalized size = 0.01

$$\left[\frac{15\left(abx+b^{2}\right)\sqrt{\frac{ax+b}{x}}\log\left(-2\,ax\sqrt{\frac{ax+b}{x}}+(2\,ax+b)\sqrt{a}\right)+2\left(3\,a^{2}x^{2}+20\,abx+15\,b^{2}\right)\sqrt{a}}{6\left(a^{4}x+a^{3}b\right)\sqrt{a}\sqrt{\frac{ax+b}{x}}},\frac{15\left(abx+b^{2}\right)\sqrt{\frac{ax+b}{x}}\arctan\left(\frac{\ }{\sqrt{-}}\right.}{3\left(a^{4}x+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(-5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b*x + b^2)*sqrt((a*x + b)/x)*log(-2*a*x*sqrt((a*x + b
)/x) + (2*a*x + b)*sqrt(a)) + 2*(3*a^2*x^2 + 20*a*b*x + 15*b^2)*s
qrt(a))/((a^4*x + a^3*b)*sqrt(a)*sqrt((a*x + b)/x)), 1/3*(15*(a*b
*x + b^2)*sqrt((a*x + b)/x)*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x))
) + (3*a^2*x^2 + 20*a*b*x + 15*b^2)*sqrt(-a))/((a^4*x + a^3*b)*sq
rt(-a)*sqrt((a*x + b)/x))]
```

---

**Sympy [A]** time = 17.8996, size = 774, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b/x)**(5/2),x)
```

```
[Out] 6*a**17*x**4*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b
*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 46*a**16*b*x**3
*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a
**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 15*a**16*b*x**3*log(b/(a*x)
)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x +
 6*a**(33/2)*b**3) - 30*a**16*b*x**3*log(sqrt(1 + b/(a*x)) + 1)/(
6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*
a**(33/2)*b**3) + 70*a**15*b**2*x**2*sqrt(1 + b/(a*x))/(6*a**(39/
2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)
*b**3) + 45*a**15*b**2*x**2*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a
**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a*
*15*b**2*x**2*log(sqrt(1 + b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a
**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 30*a*
*14*b**3*x*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x
**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**14*b**3*x*l
```

```
og(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2
)*b**2*x + 6*a**(33/2)*b**3) - 90*a**14*b**3*x*log(sqrt(1 + b/(a*
x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b
**2*x + 6*a**(33/2)*b**3) + 15*a**13*b**4*log(b/(a*x))/(6*a**(39/
2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)
*b**3) - 30*a**13*b**4*log(sqrt(1 + b/(a*x)) + 1)/(6*a**(39/2)*x*
*3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3
)
```

---

**GIAC/XCAS [A]** time = 0.254127, size = 132, normalized size = 1.61

$$\frac{1}{3}\, b\left(\frac{2\left(a + \frac{6(ax+b)}{x}\right)x}{(ax+b)a^3\sqrt{\frac{ax+b}{x}}} + \frac{15\,\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3} - \frac{3\sqrt{\frac{ax+b}{x}}}{\left(a - \frac{ax+b}{x}\right)a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^(-5/2),x, algorithm="giac")
```

```
[Out] 1/3*b*(2*(a + 6*(a*x + b)/x)*x/((a*x + b)*a^3*sqrt((a*x + b)/x))
+ 15*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3) - 3*sqrt((
a*x + b)/x)/((a - (a*x + b)/x)*a^3))
```

## 3.164 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx$

**Optimal**. Leaf size=201

$$-\frac{(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^2}+\frac{b(5bc-3ad)}{3a^2c\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}+\frac{b\left(a^2d^2-8abcd+5b^2c^2\right)}{a^3c\sqrt{a+\frac{b}{x}}(bc-ad)^2}-\frac{2d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{5/2}}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] (b*(5*b*c - 3*a*d))/(3*a^2*c*(b*c - a*d)*(a + b/x)^(3/2)) + (b*(5
*b^2*c^2 - 8*a*b*c*d + a^2*d^2))/(a^3*c*(b*c - a*d)^2*Sqrt[a + b/
x]) + x/(a*c*(a + b/x)^(3/2)) - (2*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[a
 + b/x])/Sqrt[b*c - a*d]])/(c^2*(b*c - a*d)^(5/2)) - ((5*b*c + 2*
a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(7/2)*c^2)
```

---

**Rubi [A]** time = 0.946317, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^2}+\frac{b(5bc-3ad)}{3a^2c\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}+\frac{b\left(a^2d^2-8abcd+5b^2c^2\right)}{a^3c\sqrt{a+\frac{b}{x}}(bc-ad)^2}-\frac{2d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{5/2}}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b/x)^(5/2)*(c + d/x)),x]
```

```
[Out] (b*(5*b*c - 3*a*d))/(3*a^2*c*(b*c - a*d)*(a + b/x)^(3/2)) + (b*(5
*b^2*c^2 - 8*a*b*c*d + a^2*d^2))/(a^3*c*(b*c - a*d)^2*Sqrt[a + b/
x]) + x/(a*c*(a + b/x)^(3/2)) - (2*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[a
 + b/x])/Sqrt[b*c - a*d]])/(c^2*(b*c - a*d)^(5/2)) - ((5*b*c + 2*
a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(7/2)*c^2)
```

---

**Rubi in Sympy [A]** time = 106.153, size = 173, normalized size = 0.86

$$\frac{2d^{\frac{7}{2}}\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{c^{2}\left(ad-bc\right)^{\frac{5}{2}}}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}+\frac{b\left(3ad-5bc\right)}{3a^{2}c\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(ad-bc\right)}+\frac{b\left(a^{2}d^{2}-8abcd+5b^{2}c^{2}\right)}{a^{3}c\sqrt{a+\frac{b}{x}}\left(ad-bc\right)^{2}}-\frac{\left(2ad+5bc\right)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b/x)**(5/2)/(c+d/x),x)
```

```
[Out] 2*d**(7/2)*atanh(sqrt(d)*sqrt(a + b/x)/sqrt(a*d - b*c))/(c**2*(a*
d - b*c)**(5/2)) + x/(a*c*(a + b/x)**(3/2)) + b*(3*a*d - 5*b*c)/(
3*a**2*c*(a + b/x)**(3/2)*(a*d - b*c)) + b*(a**2*d**2 - 8*a*b*c*d
 + 5*b**2*c**2)/(a**3*c*sqrt(a + b/x)*(a*d - b*c)**2) - (2*a*d +
5*b*c)*atanh(sqrt(a + b/x)/sqrt(a))/(a**(7/2)*c**2)
```

---

**Mathematica [A]** time = 0.963246, size = 250, normalized size = 1.24

$$\frac{-\frac{3(2ad+5bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{a^{7/2}}+\frac{2cx\sqrt{a+\frac{b}{x}}\left(3a^4d^2x^2+6a^3bdx(d-cx)+a^2b^2\left(3c^2x^2-32cdx+3d^2\right)+4ab^3c(5cx-6d)+15b^4c^2\right)}{a^3(ax+b)^2(bc-ad)^2}+\frac{6d^{7/2}\log(cx+d)}{(ad-bc)^{5/2}}-}{6c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b/x)^(5/2)*(c + d/x)),x]
```

```
[Out] ((2*c*Sqrt[a + b/x]*x*(15*b^4*c^2 + 3*a^4*d^2*x^2 + 6*a^3*b*d*x*(
d - c*x) + 4*a*b^3*c*(-6*d + 5*c*x) + a^2*b^2*(3*d^2 - 32*c*d*x +
 3*c^2*x^2)))/(a^3*(b*c - a*d)^2*(b + a*x)^2) + (6*d^(7/2)*Log[d
+ c*x])/(-(b*c) + a*d)^(5/2) - (3*(5*b*c + 2*a*d)*Log[b + 2*a*x +
 2*Sqrt[a]*Sqrt[a + b/x]*x])/a^(7/2) - (6*d^(7/2)*Log[-(b*d) + b*
c*x - 2*a*d*x + 2*Sqrt[d]*Sqrt[-(b*c) + a*d]*Sqrt[a + b/x]*x])/(-
(b*c) + a*d)^(5/2))/(6*c^2)
```

---

**Maple [B]** time = 0.024, size = 2490, normalized size = 12.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b/x)^(5/2)/(c+d/x),x)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)*x/a^(13/2)*(-54*(x*(a*x+b))^(1/2)*a^(17/2)*
((a*d-b*c)*d/c^2)^(1/2)*x^3*b*c^3*d^2+78*(x*(a*x+b))^(1/2)*a^(15/
2)*((a*d-b*c)*d/c^2)^(1/2)*x^3*b^2*c^4*d-18*a^9*ln(1/2*(2*(x*(a*x
+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^2*
b*c*d^4+3*a^9*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2
))*((a*d-b*c)*d/c^2)^(1/2)*x^3*b*c^2*d^3+36*(x*(a*x+b))^(3/2)*a^(
15/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b*c^3*d^2-60*(x*(a*x+b))^(3/2)*a^
(13/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b^2*c^4*d+18*(x*(a*x+b))^(1/2)*a
^(17/2)*((a*d-b*c)*d/c^2)^(1/2)*x^2*b*c^2*d^3+3*a^6*ln(1/2*(2*(x*
(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*
b^4*c^2*d^3+27*a^5*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a
^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*b^5*c^3*d^2-39*a^4*ln(1/2*(2*(x*(
a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*b
^6*c^4*d-6*a^10*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1
/2))*((a*d-b*c)*d/c^2)^(1/2)*x^3*c*d^4+6*(x*(a*x+b))^(1/2)*a^(19/
2)*((a*d-b*c)*d/c^2)^(1/2)*x^3*c^2*d^3-30*(x*(a*x+b))^(1/2)*a^(13
/2)*((a*d-b*c)*d/c^2)^(1/2)*x^3*b^3*c^5+6*a^(19/2)*ln((2*(x*(a*x+
b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x
^3*b*c*d^4+24*(x*(a*x+b))^(3/2)*a^(11/2)*((a*d-b*c)*d/c^2)^(1/2)*
x*b^3*c^5-90*(x*(a*x+b))^(1/2)*a^(11/2)*((a*d-b*c)*d/c^2)^(1/2)*x
^2*b^4*c^5+15*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2
))*((a*d-b*c)*d/c^2)^(1/2)*x^3*a^6*b^4*c^5+45*ln(1/2*(2*(x*(a*x+b
))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^2*a^
5*b^5*c^5+45*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)
)*((a*d-b*c)*d/c^2)^(1/2)*x*a^4*b^6*c^5-162*(x*(a*x+b))^(1/2)*a^(
13/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b^3*c^3*d^2+234*(x*(a*x+b))^(1/2)
*a^(11/2)*((a*d-b*c)*d/c^2)^(1/2)*x*b^4*c^4*d+18*a^(17/2)*ln((2*(
x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*
x+d))*x^2*b^2*c*d^4+32*(x*(a*x+b))^(3/2)*a^(13/2)*((a*d-b*c)*d/c^
2)^(1/2)*b^2*c^3*d^2-52*(x*(a*x+b))^(3/2)*a^(11/2)*((a*d-b*c)*d/c
^2)^(1/2)*b^3*c^4*d-90*(x*(a*x+b))^(1/2)*a^(9/2)*((a*d-b*c)*d/c^2
)^(1/2)*x*b^5*c^5-6*a^7*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x
+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*b^3*c*d^4+18*a^(15/2)*ln((2*
(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c
*x+d))*x*b^3*c*d^4+6*(x*(a*x+b))^(1/2)*a^(13/2)*((a*d-b*c)*d/c^2)
^(1/2)*b^3*c^2*d^3-54*(x*(a*x+b))^(1/2)*a^(11/2)*((a*d-b*c)*d/c^2
)^(1/2)*b^4*c^3*d^2+78*(x*(a*x+b))^(1/2)*a^(9/2)*((a*d-b*c)*d/c^2
)^(1/2)*b^5*c^4*d-6*a^(21/2)*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d
/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^3*d^5-6*a^(15/2)*ln((
2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/
(c*x+d))*b^3*d^5+9*a^7*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+
b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x*b^3*c^2*d^3+81*a^7*ln(1/2*(
2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(
1/2)*x^2*b^3*c^3*d^2-39*a^7*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2
*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^3*b^3*c^4*d+81*a^6*ln(
1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c
^2)^(1/2)*x*b^4*c^3*d^2-117*a^6*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/
2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^4*c^4*d-117*a^
5*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c
)*d/c^2)^(1/2)*x*b^5*c^4*d-162*(x*(a*x+b))^(1/2)*a^(15/2)*((a*d-b
*c)*d/c^2)^(1/2)*x^2*b^2*c^3*d^2+234*(x*(a*x+b))^(1/2)*a^(13/2)*(
(a*d-b*c)*d/c^2)^(1/2)*x^2*b^3*c^4*d-18*a^8*ln(1/2*(2*(x*(a*x+b))
^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x*b^2*c*
```

```
d^4+9*a^8*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(
(a*d-b*c)*d/c^2)^(1/2)*x^2*b^2*c^2*d^3+27*a^8*ln(1/2*(2*(x*(a*x+b
))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^3*b^
2*c^3*d^2+18*(x*(a*x+b))^(1/2)*a^(15/2)*((a*d-b*c)*d/c^2)^(1/2)*x
*b^2*c^2*d^3-18*a^(19/2)*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2
)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*b*d^5+20*(x*(a*x+b))^(3
/2)*a^(9/2)*((a*d-b*c)*d/c^2)^(1/2)*b^4*c^5-18*a^(17/2)*ln((2*(x*
(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+
d))*x*b^2*d^5+6*a^(13/2)*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2
)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^4*c*d^4-30*(x*(a*x+b))^(1
/2)*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*b^6*c^5+15*ln(1/2*(2*(x*(a*x+
b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^3*b
^7*c^5)/(x*(a*x+b))^(1/2)/(a*d-b*c)^3/c^3/((a*d-b*c)*d/c^2)^(1/2)
/(a*x+b)^3
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + b/x)^(5/2)*(c + d/x)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.903203, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + b/x)^(5/2)*(c + d/x)),x, algorithm="fricas")

[Out] [1/6*(6*(a^4*d^3*x + a^3*b*d^3)*sqrt(a)*sqrt(-d/(b*c - a*d))*sqrt
((a*x + b)/x)*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*
x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 3*(5*b^4*c^3 - 8*
a*b^3*c^2*d + a^2*b^2*c*d^2 + 2*a^3*b*d^3 + (5*a*b^3*c^3 - 8*a^2*
b^2*c^2*d + a^3*b*c*d^2 + 2*a^4*d^3)*x)*sqrt((a*x + b)/x)*log(-2*
a*x*sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(a)) + 2*(15*b^4*c^3 - 24
*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 + 3*(a^2*b^2*c^3 - 2*a^3*b*c^2*d +
 a^4*c*d^2)*x^2 + 2*(10*a*b^3*c^3 - 16*a^2*b^2*c^2*d + 3*a^3*b*c*
d^2)*x)*sqrt(a))/((a^3*b^3*c^4 - 2*a^4*b^2*c^3*d + a^5*b*c^2*d^2
+ (a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^2*d^2)*x)*sqrt(a)*sqrt((a*
x + b)/x)), 1/3*(3*(a^4*d^3*x + a^3*b*d^3)*sqrt(-a)*sqrt(-d/(b*c
- a*d))*sqrt((a*x + b)/x)*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*
d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 3*(5*
b^4*c^3 - 8*a*b^3*c^2*d + a^2*b^2*c*d^2 + 2*a^3*b*d^3 + (5*a*b^3*
```

```
c^3 - 8*a^2*b^2*c^2*d + a^3*b*c*d^2 + 2*a^4*d^3)*x)*sqrt((a*x + b
)/x)*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x))) + (15*b^4*c^3 - 24*a*
b^3*c^2*d + 3*a^2*b^2*c*d^2 + 3*(a^2*b^2*c^3 - 2*a^3*b*c^2*d + a^
4*c*d^2)*x^2 + 2*(10*a*b^3*c^3 - 16*a^2*b^2*c^2*d + 3*a^3*b*c*d^2
)*x)*sqrt(-a))/((a^3*b^3*c^4 - 2*a^4*b^2*c^3*d + a^5*b*c^2*d^2 +
(a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^2*d^2)*x)*sqrt(-a)*sqrt((a*x
 + b)/x)), -1/6*(12*(a^4*d^3*x + a^3*b*d^3)*sqrt(a)*sqrt(d/(b*c -
 a*d))*sqrt((a*x + b)/x)*arctan(-(b*c - a*d)*sqrt(d/(b*c - a*d))/
(d*sqrt((a*x + b)/x))) - 3*(5*b^4*c^3 - 8*a*b^3*c^2*d + a^2*b^2*c
*d^2 + 2*a^3*b*d^3 + (5*a*b^3*c^3 - 8*a^2*b^2*c^2*d + a^3*b*c*d^2
 + 2*a^4*d^3)*x)*sqrt((a*x + b)/x)*log(-2*a*x*sqrt((a*x + b)/x) +
 (2*a*x + b)*sqrt(a)) - 2*(15*b^4*c^3 - 24*a*b^3*c^2*d + 3*a^2*b^
2*c*d^2 + 3*(a^2*b^2*c^3 - 2*a^3*b*c^2*d + a^4*c*d^2)*x^2 + 2*(10
*a*b^3*c^3 - 16*a^2*b^2*c^2*d + 3*a^3*b*c*d^2)*x)*sqrt(a))/((a^3*
b^3*c^4 - 2*a^4*b^2*c^3*d + a^5*b*c^2*d^2 + (a^4*b^2*c^4 - 2*a^5*
b*c^3*d + a^6*c^2*d^2)*x)*sqrt(a)*sqrt((a*x + b)/x)), -1/3*(6*(a^
4*d^3*x + a^3*b*d^3)*sqrt(-a)*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/
x)*arctan(-(b*c - a*d)*sqrt(d/(b*c - a*d))/(d*sqrt((a*x + b)/x)))
 - 3*(5*b^4*c^3 - 8*a*b^3*c^2*d + a^2*b^2*c*d^2 + 2*a^3*b*d^3 + (
5*a*b^3*c^3 - 8*a^2*b^2*c^2*d + a^3*b*c*d^2 + 2*a^4*d^3)*x)*sqrt(
(a*x + b)/x)*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x))) - (15*b^4*c^3
 - 24*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 + 3*(a^2*b^2*c^3 - 2*a^3*b*c^
2*d + a^4*c*d^2)*x^2 + 2*(10*a*b^3*c^3 - 16*a^2*b^2*c^2*d + 3*a^3
*b*c*d^2)*x)*sqrt(-a))/((a^3*b^3*c^4 - 2*a^4*b^2*c^3*d + a^5*b*c^
2*d^2 + (a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^2*d^2)*x)*sqrt(-a)*s
qrt((a*x + b)/x))]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}(cx+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b/x)**(5/2)/(c+d/x),x)
```

```
[Out] Integral(x/((a + b/x)**(5/2)*(c*x + d)), x)
```

---

**GIAC/XCAS [A]** time = 0.258445, size = 332, normalized size = 1.65

$$-\frac{1}{3}\left(\frac{6\,d^4\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{(b^3c^4-2\,ab^2c^3d+a^2bc^2d^2)\sqrt{bcd-ad^2}}-\frac{2\left(ab^2c-a^2bd+\frac{6\,(ax+b)b^2c}{x}-\frac{9\,(ax+b)abd}{x}\right)x}{(a^3b^2c^2-2\,a^4bcd+a^5d^2)(ax+b)\sqrt{\frac{ax+b}{x}}}+\frac{3\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^3c}-\frac{3\,(5\,bc+2\,\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + b/x)^(5/2)*(c + d/x)),x, algorithm="giac")
```

```
[Out] -1/3*(6*d^4*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))/((b^3
*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2)*sqrt(b*c*d - a*d^2)) - 2*(a
*b^2*c - a^2*b*d + 6*(a*x + b)*b^2*c/x - 9*(a*x + b)*a*b*d/x)*x/(
(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*(a*x + b)*sqrt((a*x + b)/x)
) + 3*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3*c) - 3*(5*b*c + 2*
a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3*b*c^2))*b
```

## 3.165 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx$

**Optimal**. Leaf size=287

$$-\frac{(4ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^3}+\frac{b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a^2c^2\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)^2}+\frac{b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a^3c^2\sqrt{a+\frac{b}{x}}(bc-ad)^3}-\frac{d^{7/2}(9bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{7/2}}+\frac{d(bc-2ad)}{ac^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

```
[Out] (b*(5*b^2*c^2 - 6*a*b*c*d + 6*a^2*d^2))/(3*a^2*c^2*(b*c - a*d)^2*
(a + b/x)^(3/2)) + (b*(b*c - 2*a*d)*(5*b^2*c^2 - a*b*c*d + a^2*d^
2))/(a^3*c^2*(b*c - a*d)^3*Sqrt[a + b/x]) + (d*(b*c - 2*a*d))/(a*
c^2*(b*c - a*d)*(a + b/x)^(3/2)*(c + d/x)) + x/(a*c*(a + b/x)^(3/
2)*(c + d/x)) - (d^(7/2)*(9*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a +
 b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(7/2)) - ((5*b*c + 4*a*
d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(7/2)*c^3)
```

---

**Rubi [A]** time = 1.40881, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$


$$-\frac{(4ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^3}+\frac{b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a^2c^2\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)^2}+\frac{b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a^3c^2\sqrt{a+\frac{b}{x}}(bc-ad)^3}-\frac{d^{7/2}(9bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{7/2}}+\frac{d(bc-2ad)}{ac^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$


Antiderivative was successfully verified.

```
[In]  Int[1/((a + b/x)^(5/2)*(c + d/x)^2),x]
```



```
[Out] (b*(5*b^2*c^2 - 6*a*b*c*d + 6*a^2*d^2))/(3*a^2*c^2*(b*c - a*d)^2*
(a + b/x)^(3/2)) + (b*(b*c - 2*a*d)*(5*b^2*c^2 - a*b*c*d + a^2*d^
2))/(a^3*c^2*(b*c - a*d)^3*Sqrt[a + b/x]) + (d*(b*c - 2*a*d))/(a*
c^2*(b*c - a*d)*(a + b/x)^(3/2)*(c + d/x)) + x/(a*c*(a + b/x)^(3/
2)*(c + d/x)) - (d^(7/2)*(9*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a +
 b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(7/2)) - ((5*b*c + 4*a*
d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(7/2)*c^3)
```

**Rubi in Sympy [A]** time = 160.074, size = 252, normalized size = 0.88

$$-\frac{dx}{c\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(c+\frac{d}{x}\right)(ad-bc)}+\frac{d^{\frac{7}{2}}(4ad-9bc)\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{ad-bc}}\right)}{c^3(ad-bc)^{\frac{7}{2}}}+\frac{x(2ad-bc)}{ac^2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}(ad-bc)}+\frac{b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a^2c^2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}(ad-bc)^2}+\frac{b(2ad-bc)\left(a^2d^2-abcd+5b^2c^2\right)}{a^3c^2\sqrt{a+\frac{b}{x}}(ad-bc)^3}-\frac{(4ad+5bc)\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b/x)**(5/2)/(c+d/x)**2,x)
```

```
[Out] -d*x/(c*(a + b/x)**(3/2)*(c + d/x)*(a*d - b*c)) + d**(7/2)*(4*a*d
 - 9*b*c)*atanh(sqrt(d)*sqrt(a + b/x)/sqrt(a*d - b*c))/(c**3*(a*d
 - b*c)**(7/2)) + x*(2*a*d - b*c)/(a*c**2*(a + b/x)**(3/2)*(a*d -
 b*c)) + b*(6*a**2*d**2 - 6*a*b*c*d + 5*b**2*c**2)/(3*a**2*c**2*(
a + b/x)**(3/2)*(a*d - b*c)**2) + b*(2*a*d - b*c)*(a**2*d**2 - a*
b*c*d + 5*b**2*c**2)/(a**3*c**2*sqrt(a + b/x)*(a*d - b*c)**3) - (
4*a*d + 5*b*c)*atanh(sqrt(a + b/x)/sqrt(a))/(a**(7/2)*c**3)
```

**Mathematica [C]** time = 1.84408, size = 364, normalized size = 1.27

$$\frac{-\frac{3(4ad+5bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{a^{7/2}}+\frac{2\sqrt{a+\frac{b}{x}}\left(-3a^4d^4(ax+b)^2(cx+d)+3a^4d^5(ax+b)^2+2b^5c^3(cx+d)(bc-ad)+14b^4c^4(ax+b)^2(cx+d)-4b^4c^3(ax+b)(cx+d)\right.}{a^4(ax+b)^2(cx+d)(bc-ad)^3}}{6c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b/x)^(5/2)*(c + d/x)^2),x]
```

```
[Out] ((2*Sqrt[a + b/x]*(3*a^4*d^5*(b + a*x)^2 + 2*b^5*c^3*(b*c - a*d)*
(d + c*x) - 4*b^4*c^3*(4*b*c - 7*a*d)*(b + a*x)*(d + c*x) + 14*b^
4*c^4*(b + a*x)^2*(d + c*x) - 26*a*b^3*c^3*d*(b + a*x)^2*(d + c*x
) - 3*a^4*d^4*(b + a*x)^2*(d + c*x) + 3*a*c*(b*c - a*d)^3*x*(b +
a*x)^2*(d + c*x)))/(a^4*(b*c - a*d)^3*(b + a*x)^2*(d + c*x)) - (3
*(5*b*c + 4*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x])/a^(7
/2) + ((3*I)*d^(7/2)*(-9*b*c + 4*a*d)*Log[(2*c^4*(b*c - a*d)^(5/2
)*((-2*I)*a*d*x + 2*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[a + b/x]*x - I*b
*(d - c*x)))/(d^(9/2)*(9*b*c - 4*a*d)*(d + c*x))])/(b*c - a*d)^(7
/2))/(6*c^3)
```

**Maple [B]** time = 0.028, size = 4648, normalized size = 16.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b/x)^(5/2)/(c+d/x)^2,x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)*x/a^(13/2)*(27*ln((2*(x*(a*x+b))^(1/2)*((a
*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(19/2)*x^4*b
^2*c^3*d^4-6*((a*d-b*c)*d/c^2)^(1/2)*a^(19/2)*(x*(a*x+b))^(3/2)*x
^3*c^4*d^3-6*((a*d-b*c)*d/c^2)^(1/2)*a^(21/2)*(x*(a*x+b))^(1/2)*x
^4*c^3*d^4-30*((a*d-b*c)*d/c^2)^(1/2)*a^(13/2)*(x*(a*x+b))^(1/2)*
x^4*b^4*c^7-3*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2
*a*d*x+b*c*x-b*d)/(c*x+d))*a^(21/2)*x^3*b*c*d^6-90*ln((2*(x*(a*x+
b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a
^(19/2)*x^3*b^2*c^2*d^5+81*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c
^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(17/2)*x^3*b^3*c^3*d^4+
24*((a*d-b*c)*d/c^2)^(1/2)*a^(11/2)*(x*(a*x+b))^(3/2)*x^2*b^4*c^7
-12*((a*d-b*c)*d/c^2)^(1/2)*a^(21/2)*(x*(a*x+b))^(1/2)*x^3*c^2*d^
5-90*((a*d-b*c)*d/c^2)^(1/2)*a^(11/2)*(x*(a*x+b))^(1/2)*x^3*b^5*c
^7+12*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d
-b*c)*d/c^2)^(1/2)*a^8*b^3*c*d^6-81*ln((2*(x*(a*x+b))^(1/2)*((a*d
-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(19/2)*x^2*b^2
*c*d^6-36*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d
*x+b*c*x-b*d)/(c*x+d))*a^(17/2)*x^2*b^3*c^2*d^5+81*ln((2*(x*(a*x+
b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a
^(15/2)*x^2*b^4*c^3*d^4+20*((a*d-b*c)*d/c^2)^(1/2)*a^(9/2)*(x*(a*
x+b))^(3/2)*x*b^5*c^7-90*((a*d-b*c)*d/c^2)^(1/2)*a^(9/2)*(x*(a*x+
b))^(1/2)*x^2*b^6*c^7-48*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*
x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^4*b^7*c^5*d^2+15*ln(1/2*(
2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(
1/2)*x^4*a^6*b^5*c^7-39*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)
^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(15/2)*b^4*c*d^6+27*ln((2*
(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c
*x+d))*a^(13/2)*b^5*c^2*d^5+12*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)
*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(23/2)*x^4*c*d^6+36
*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-
b*d)/(c*x+d))*a^(21/2)*x^2*b*d^7+36*ln((2*(x*(a*x+b))^(1/2)*((a*d
-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(19/2)*x*b^2*d
^7-30*((a*d-b*c)*d/c^2)^(1/2)*a^(7/2)*(x*(a*x+b))^(1/2)*x*b^7*c^7
-84*((a*d-b*c)*d/c^2)^(1/2)*a^(11/2)*(x*(a*x+b))^(1/2)*b^5*c^4*d^
3+96*((a*d-b*c)*d/c^2)^(1/2)*a^(9/2)*(x*(a*x+b))^(1/2)*b^6*c^5*d^
2+42*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-
b*c)*d/c^2)^(1/2)*a^5*b^6*c^4*d^3+12*ln(1/2*(2*(x*(a*x+b))^(1/2)*
a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^11*x^4*c^2*d^
5+12*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-
b*c)*d/c^2)^(1/2)*a^11*x^3*c*d^6+6*((a*d-b*c)*d/c^2)^(1/2)*a^(21/
2)*(x*(a*x+b))^(1/2)*x^5*c^4*d^3-39*ln((2*(x*(a*x+b))^(1/2)*((a*d
-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(21/2)*x^4*b*c
^2*d^5-156*((a*d-b*c)*d/c^2)^(1/2)*a^(15/2)*(x*(a*x+b))^(1/2)*x^3
*b^3*c^5*d^2+12*((a*d-b*c)*d/c^2)^(1/2)*a^(19/2)*(x*(a*x+b))^(1/2
)*x^3*b*c^3*d^4+258*((a*d-b*c)*d/c^2)^(1/2)*a^(13/2)*(x*(a*x+b))^
```

(1/2)*x^3*b^4*c^6*d+3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b
)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^7*x*b^4*c^3*d^4-48*ln(1/2*(2
*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1
/2)*a^7*x^4*b^4*c^6*d+30*((a*d-b*c)*d/c^2)^(1/2)*a^(15/2)*(x*(a*x
+b))^(3/2)*x*b^2*c^4*d^3-28*((a*d-b*c)*d/c^2)^(1/2)*a^(13/2)*(x*(
a*x+b))^(3/2)*x*b^3*c^5*d^2-40*((a*d-b*c)*d/c^2)^(1/2)*a^(11/2)*(
x*(a*x+b))^(3/2)*x*b^4*c^6*d+96*((a*d-b*c)*d/c^2)^(1/2)*a^(15/2)*
(x*(a*x+b))^(1/2)*x^4*b^3*c^6*d-84*((a*d-b*c)*d/c^2)^(1/2)*a^(17/
2)*(x*(a*x+b))^(1/2)*x^4*b^2*c^5*d^2+42*ln(1/2*(2*(x*(a*x+b))^(1/
2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^8*x^4*b^3*
c^5*d^2-63*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*
((a*d-b*c)*d/c^2)^(1/2)*a^8*x^2*b^3*c^3*d^4+78*ln(1/2*(2*(x*(a*x+
b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^8*x
^3*b^3*c^4*d^3+36*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^
(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^9*x*b^2*c*d^6-87*ln(1/2*(2*(x*(a
*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^
8*x*b^3*c^2*d^5+36*((a*d-b*c)*d/c^2)^(1/2)*a^(13/2)*(x*(a*x+b))^(
1/2)*x^2*b^4*c^5*d^2+12*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x
+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^9*x^4*b^2*c^4*d^3+48*((a*d
-b*c)*d/c^2)^(1/2)*a^(19/2)*(x*(a*x+b))^(1/2)*x^4*b*c^4*d^3+12*ln
((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d
)/(c*x+d))*a^(23/2)*x^3*d^7+12*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)
*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(17/2)*b^3*d^7+204*
((a*d-b*c)*d/c^2)^(1/2)*a^(11/2)*(x*(a*x+b))^(1/2)*x*b^5*c^5*d^2+
6*((a*d-b*c)*d/c^2)^(1/2)*a^(9/2)*(x*(a*x+b))^(1/2)*x*b^6*c^6*d+4
5*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c
)*d/c^2)^(1/2)*x^3*a^5*b^6*c^7-30*((a*d-b*c)*d/c^2)^(1/2)*a^(7/2)
*(x*(a*x+b))^(1/2)*b^7*c^6*d+45*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/
2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^2*a^4*b^7*c^7+15*l
n(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d
/c^2)^(1/2)*x*a^3*b^8*c^7+15*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+
2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^3*b^8*c^6*d-33*ln(1/2
*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)
^(1/2)*a^7*b^4*c^2*d^5-105*ln((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c
^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(17/2)*x*b^3*c*d^6+42*l
n((2*(x*(a*x+b))^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*
d)/(c*x+d))*a^(15/2)*x*b^4*c^2*d^5+27*ln((2*(x*(a*x+b))^(1/2)*((a
*d-b*c)*d/c^2)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(13/2)*x*b^5
*c^3*d^4+38*((a*d-b*c)*d/c^2)^(1/2)*a^(13/2)*(x*(a*x+b))^(3/2)*b^
3*c^4*d^3-64*((a*d-b*c)*d/c^2)^(1/2)*a^(11/2)*(x*(a*x+b))^(3/2)*b
^4*c^5*d^2+20*((a*d-b*c)*d/c^2)^(1/2)*a^(9/2)*(x*(a*x+b))^(3/2)*b
^5*c^6*d+12*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))
*((a*d-b*c)*d/c^2)^(1/2)*a^6*b^5*c^3*d^4-12*((a*d-b*c)*d/c^2)^(1/
2)*a^(15/2)*(x*(a*x+b))^(1/2)*b^3*c^2*d^5+30*((a*d-b*c)*d/c^2)^(1
/2)*a^(13/2)*(x*(a*x+b))^(1/2)*b^4*c^3*d^4-36*((a*d-b*c)*d/c^2)^(
1/2)*a^(17/2)*(x*(a*x+b))^(1/2)*x*b^2*c^2*d^5+84*((a*d-b*c)*d/c^2
)^(1/2)*a^(15/2)*(x*(a*x+b))^(1/2)*x*b^3*c^3*d^4-18*ln(1/2*(2*(x*
(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*
a^6*x^2*b^5*c^5*d^2-129*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x
+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^6*x^3*b^5*c^6*d-3*ln(1/2*(
2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(
1/2)*a^4*x*b^7*c^6*d+78*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x
+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^7*x^3*b^4*c^5*d^2-72*((a*d
-b*c)*d/c^2)^(1/2)*a^(13/2)*(x*(a*x+b))^(3/2)*x^2*b^3*c^6*d-87*ln
(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/
c^2)^(1/2)*a^9*x^3*b^2*c^3*d^4-99*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(

```
1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^5*x^2*b^6*c^6*d+
198*((a*d-b*c)*d/c^2)^(1/2)*a^(11/2)*(x*(a*x+b))^(1/2)*x^2*b^5*c^
6*d+138*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a
*d-b*c)*d/c^2)^(1/2)*a^6*x*b^5*c^4*d^3-156*((a*d-b*c)*d/c^2)^(1/2
)*a^(15/2)*(x*(a*x+b))^(1/2)*x^2*b^3*c^4*d^3-102*ln(1/2*(2*(x*(a*
x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^5
*x*b^6*c^5*d^2+3*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(
1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^10*x^3*b*c^2*d^5-33*ln(1/2*(2*(x*
(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*
a^10*x^4*b*c^3*d^4-36*((a*d-b*c)*d/c^2)^(1/2)*a^(19/2)*(x*(a*x+b)
)^(1/2)*x^2*b*c^2*d^5+72*((a*d-b*c)*d/c^2)^(1/2)*a^(17/2)*(x*(a*x
+b))^(1/2)*x^2*b^2*c^3*d^4+36*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)
+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^10*x^2*b*c*d^6-63*ln
(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/
c^2)^(1/2)*a^9*x^2*b^2*c^2*d^5-222*((a*d-b*c)*d/c^2)^(1/2)*a^(13/
2)*(x*(a*x+b))^(1/2)*x*b^4*c^4*d^3-18*((a*d-b*c)*d/c^2)^(1/2)*a^(
17/2)*(x*(a*x+b))^(3/2)*x^2*b*c^4*d^3+48*((a*d-b*c)*d/c^2)^(1/2)*
a^(15/2)*(x*(a*x+b))^(3/2)*x^2*b^2*c^5*d^2+162*ln(1/2*(2*(x*(a*x+
b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^7*x
^2*b^4*c^4*d^3+24*((a*d-b*c)*d/c^2)^(1/2)*a^(17/2)*(x*(a*x+b))^(1
/2)*x^3*b^2*c^4*d^3)/(x*(a*x+b))^(1/2)/(a*d-b*c)^4/(a*x+b)^3/c^4/
((a*d-b*c)*d/c^2)^(1/2)/(c*x+d)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + b/x)^(5/2)*(c + d/x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 2.54242, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + b/x)^(5/2)*(c + d/x)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(9*a^3*b^2*c*d^4 - 4*a^4*b*d^5 + (9*a^4*b*c^2*d^3 - 4*a^5
*c*d^4)*x^2 + (9*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - 4*a^5*d^5)*x)*
sqrt(a)*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x)*log(-(2*(b*c - a*d
)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*
x)/(c*x + d)) + 3*(5*b^5*c^4*d - 11*a*b^4*c^3*d^2 + 3*a^2*b^3*c^2
*d^3 + 7*a^3*b^2*c*d^4 - 4*a^4*b*d^5 + (5*a*b^4*c^5 - 11*a^2*b^3*
```

c^4*d + 3*a^3*b^2*c^3*d^2 + 7*a^4*b*c^2*d^3 - 4*a^5*c*d^4)*x^2 +
(5*b^5*c^5 - 6*a*b^4*c^4*d - 8*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d
^3 + 3*a^4*b*c*d^4 - 4*a^5*d^5)*x)*sqrt((a*x + b)/x)*log(-2*a*x*s
qrt((a*x + b)/x) + (2*a*x + b)*sqrt(a)) + 2*(15*b^5*c^4*d - 33*a*
b^4*c^3*d^2 + 9*a^2*b^3*c^2*d^3 - 6*a^3*b^2*c*d^4 + 3*(a^2*b^3*c^
5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3)*x^3 + (20*a*
b^4*c^5 - 41*a^2*b^3*c^4*d + 9*a^3*b^2*c^3*d^2 + 3*a^4*b*c^2*d^3
- 6*a^5*c*d^4)*x^2 + (15*b^5*c^5 - 13*a*b^4*c^4*d - 35*a^2*b^3*c^
3*d^2 + 15*a^3*b^2*c^2*d^3 - 12*a^4*b*c*d^4)*x)*sqrt(a))/((a^3*b^
4*c^6*d - 3*a^4*b^3*c^5*d^2 + 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*d^4 +
(a^4*b^3*c^7 - 3*a^5*b^2*c^6*d + 3*a^6*b*c^5*d^2 - a^7*c^4*d^3)*
x^2 + (a^3*b^4*c^7 - 2*a^4*b^3*c^6*d + 2*a^6*b*c^4*d^3 - a^7*c^3*
d^4)*x)*sqrt(a)*sqrt((a*x + b)/x)), -1/6*(6*(9*a^3*b^2*c*d^4 - 4*
a^4*b*d^5 + (9*a^4*b*c^2*d^3 - 4*a^5*c*d^4)*x^2 + (9*a^3*b^2*c^2*
d^3 + 5*a^4*b*c*d^4 - 4*a^5*d^5)*x)*sqrt(a)*sqrt(d/(b*c - a*d))*s
qrt((a*x + b)/x)*arctan(-(b*c - a*d)*sqrt(d/(b*c - a*d))/(d*sqrt(
(a*x + b)/x))) - 3*(5*b^5*c^4*d - 11*a*b^4*c^3*d^2 + 3*a^2*b^3*c^
2*d^3 + 7*a^3*b^2*c*d^4 - 4*a^4*b*d^5 + (5*a*b^4*c^5 - 11*a^2*b^3
*c^4*d + 3*a^3*b^2*c^3*d^2 + 7*a^4*b*c^2*d^3 - 4*a^5*c*d^4)*x^2 +
(5*b^5*c^5 - 6*a*b^4*c^4*d - 8*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*
d^3 + 3*a^4*b*c*d^4 - 4*a^5*d^5)*x)*sqrt((a*x + b)/x)*log(-2*a*x*
sqrt((a*x + b)/x) + (2*a*x + b)*sqrt(a)) - 2*(15*b^5*c^4*d - 33*a
*b^4*c^3*d^2 + 9*a^2*b^3*c^2*d^3 - 6*a^3*b^2*c*d^4 + 3*(a^2*b^3*c
^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3)*x^3 + (20*a
*b^4*c^5 - 41*a^2*b^3*c^4*d + 9*a^3*b^2*c^3*d^2 + 3*a^4*b*c^2*d^3
- 6*a^5*c*d^4)*x^2 + (15*b^5*c^5 - 13*a*b^4*c^4*d - 35*a^2*b^3*c
^3*d^2 + 15*a^3*b^2*c^2*d^3 - 12*a^4*b*c*d^4)*x)*sqrt(a))/((a^3*b
^4*c^6*d - 3*a^4*b^3*c^5*d^2 + 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*d^4
+ (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d + 3*a^6*b*c^5*d^2 - a^7*c^4*d^3)
*x^2 + (a^3*b^4*c^7 - 2*a^4*b^3*c^6*d + 2*a^6*b*c^4*d^3 - a^7*c^3
*d^4)*x)*sqrt(a)*sqrt((a*x + b)/x)), 1/6*(3*(9*a^3*b^2*c*d^4 - 4*
a^4*b*d^5 + (9*a^4*b*c^2*d^3 - 4*a^5*c*d^4)*x^2 + (9*a^3*b^2*c^2*
d^3 + 5*a^4*b*c*d^4 - 4*a^5*d^5)*x)*sqrt(-a)*sqrt(-d/(b*c - a*d))
*sqrt((a*x + b)/x)*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqr
t((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 6*(5*b^5*c^4
*d - 11*a*b^4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 + 7*a^3*b^2*c*d^4 - 4*a
^4*b*d^5 + (5*a*b^4*c^5 - 11*a^2*b^3*c^4*d + 3*a^3*b^2*c^3*d^2 +
7*a^4*b*c^2*d^3 - 4*a^5*c*d^4)*x^2 + (5*b^5*c^5 - 6*a*b^4*c^4*d -
8*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 3*a^4*b*c*d^4 - 4*a^5*d
^5)*x)*sqrt((a*x + b)/x)*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x))) +
2*(15*b^5*c^4*d - 33*a*b^4*c^3*d^2 + 9*a^2*b^3*c^2*d^3 - 6*a^3*b
^2*c*d^4 + 3*(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a
^5*c^2*d^3)*x^3 + (20*a*b^4*c^5 - 41*a^2*b^3*c^4*d + 9*a^3*b^2*c^
3*d^2 + 3*a^4*b*c^2*d^3 - 6*a^5*c*d^4)*x^2 + (15*b^5*c^5 - 13*a*b
^4*c^4*d - 35*a^2*b^3*c^3*d^2 + 15*a^3*b^2*c^2*d^3 - 12*a^4*b*c*d
^4)*x)*sqrt(-a))/((a^3*b^4*c^6*d - 3*a^4*b^3*c^5*d^2 + 3*a^5*b^2*
c^4*d^3 - a^6*b*c^3*d^4 + (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d + 3*a^6*
b*c^5*d^2 - a^7*c^4*d^3)*x^2 + (a^3*b^4*c^7 - 2*a^4*b^3*c^6*d + 2
*a^6*b*c^4*d^3 - a^7*c^3*d^4)*x)*sqrt(-a)*sqrt((a*x + b)/x)), -1/
3*(3*(9*a^3*b^2*c*d^4 - 4*a^4*b*d^5 + (9*a^4*b*c^2*d^3 - 4*a^5*c*
d^4)*x^2 + (9*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - 4*a^5*d^5)*x)*sqr
t(-a)*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)*arctan(-(b*c - a*d)*s
qrt(d/(b*c - a*d))/(d*sqrt((a*x + b)/x))) - 3*(5*b^5*c^4*d - 11*a
*b^4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 + 7*a^3*b^2*c*d^4 - 4*a^4*b*d^5
+ (5*a*b^4*c^5 - 11*a^2*b^3*c^4*d + 3*a^3*b^2*c^3*d^2 + 7*a^4*b*c
^2*d^3 - 4*a^5*c*d^4)*x^2 + (5*b^5*c^5 - 6*a*b^4*c^4*d - 8*a^2*b^

```
3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 3*a^4*b*c*d^4 - 4*a^5*d^5)*x)*sq
rt((a*x + b)/x)*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x))) - (15*b^5*
c^4*d - 33*a*b^4*c^3*d^2 + 9*a^2*b^3*c^2*d^3 - 6*a^3*b^2*c*d^4 +
3*(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3)
*x^3 + (20*a*b^4*c^5 - 41*a^2*b^3*c^4*d + 9*a^3*b^2*c^3*d^2 + 3*a
^4*b*c^2*d^3 - 6*a^5*c*d^4)*x^2 + (15*b^5*c^5 - 13*a*b^4*c^4*d -
35*a^2*b^3*c^3*d^2 + 15*a^3*b^2*c^2*d^3 - 12*a^4*b*c*d^4)*x)*sqrt
(-a))/((a^3*b^4*c^6*d - 3*a^4*b^3*c^5*d^2 + 3*a^5*b^2*c^4*d^3 - a
^6*b*c^3*d^4 + (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d + 3*a^6*b*c^5*d^2 -
 a^7*c^4*d^3)*x^2 + (a^3*b^4*c^7 - 2*a^4*b^3*c^6*d + 2*a^6*b*c^4*
d^3 - a^7*c^3*d^4)*x)*sqrt(-a)*sqrt((a*x + b)/x))]
```

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b/x)**(5/2)/(c+d/x)**2,x)
```

```
[Out] Exception raised: ValueError
```

---

**GIAC/XCAS [A]** time = 0.280938, size = 779, normalized size = 2.71

$$-\frac{1}{3}\, b\left(\frac{3\,\left(9\,bcd^{4} - 4\,ad^{5}\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^{2}}}\right)}{\left(b^{4}c^{6} - 3\,ab^{3}c^{5}d + 3\,a^{2}b^{2}c^{4}d^{2} - a^{3}bc^{3}d^{3}\right)\sqrt{bcd-ad^{2}}} - \frac{2\left(ab^{3}c - a^{2}b^{2}d + \frac{6\,(ax+b)b^{3}c}{x} - \frac{12\,(ax+b)ab^{2}d}{x}\right)x}{\left(a^{3}b^{3}c^{3} - 3\,a^{4}b^{2}c^{2}d + 3\,a^{5}bcd^{2} - a^{6}d^{3}\right)(ax+b)\sqrt{\frac{ax+b}{x}}} + \frac{3\left(b^{4} \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + b/x)^(5/2)*(c + d/x)^2),x, algorithm="giac")
```

```
[Out] -1/3*b*(3*(9*b*c*d^4 - 4*a*d^5)*arctan(d*sqrt((a*x + b)/x)/sqrt(b
*c*d - a*d^2))/((b^4*c^6 - 3*a*b^3*c^5*d + 3*a^2*b^2*c^4*d^2 - a^
3*b*c^3*d^3)*sqrt(b*c*d - a*d^2)) - 2*(a*b^3*c - a^2*b^2*d + 6*(a
*x + b)*b^3*c/x - 12*(a*x + b)*a*b^2*d/x)*x/((a^3*b^3*c^3 - 3*a^4
*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*(a*x + b)*sqrt((a*x + b)/x)
) + 3*(b^4*c^4*sqrt((a*x + b)/x) - 4*a*b^3*c^3*d*sqrt((a*x + b)/x
) + 6*a^2*b^2*c^2*d^2*sqrt((a*x + b)/x) - 4*a^3*b*c*d^3*sqrt((a*x
 + b)/x) + 2*a^4*d^4*sqrt((a*x + b)/x) + (a*x + b)*b^3*c^3*d*sqrt
((a*x + b)/x)/x - 3*(a*x + b)*a*b^2*c^2*d^2*sqrt((a*x + b)/x)/x +
 3*(a*x + b)*a^2*b*c*d^3*sqrt((a*x + b)/x)/x - 2*(a*x + b)*a^3*d^
4*sqrt((a*x + b)/x)/x)/((a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*
c^3*d^2 - a^6*c^2*d^3)*(a*b*c - a^2*d - (a*x + b)*b*c/x + 2*(a*x
```

```
+ b)*a*d/x - (a*x + b)^2*d/x^2)) - 3*(5*b*c + 4*a*d)*arctan(sqrt(
(a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3*b*c^3))
```

## 3.166 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx$

**Optimal**. Leaf size=409

$$-\frac{(6ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^4}-\frac{d^{7/2}\left(24a^2d^2-88abcd+99b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{9/2}}+\frac{d\left(12a^2d^2-23abcd+4b^2c^2\right)}{4ac^3\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{b\left(-36a^3d^3+87a^2bcd^2-36ab^2c^2d+20b^3c^3\right)}{12a^2c^3\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)^3}+\frac{b\left(12a^4d^4-35a^3bcd^3+24a^2b^2c^2d^2-56ab^3c^3d+20b^4c^4\right)}{4a^3c^3\sqrt{a+\frac{b}{x}}(bc-ad)^4}+\frac{d(2bc-3ad)}{2ac^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2(bc-ad)}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

```
[Out] (b*(20*b^3*c^3 - 36*a*b^2*c^2*d + 87*a^2*b*c*d^2 - 36*a^3*d^3))/(
12*a^2*c^3*(b*c - a*d)^3*(a + b/x)^(3/2)) + (b*(20*b^4*c^4 - 56*a
*b^3*c^3*d + 24*a^2*b^2*c^2*d^2 - 35*a^3*b*c*d^3 + 12*a^4*d^4))/(
4*a^3*c^3*(b*c - a*d)^4*Sqrt[a + b/x]) + (d*(2*b*c - 3*a*d))/(2*a
*c^2*(b*c - a*d)*(a + b/x)^(3/2)*(c + d/x)^2) + (d*(4*b^2*c^2 - 2
3*a*b*c*d + 12*a^2*d^2))/(4*a*c^3*(b*c - a*d)^2*(a + b/x)^(3/2)*(
c + d/x)) + x/(a*c*(a + b/x)^(3/2)*(c + d/x)^2) - (d^(7/2)*(99*b^
2*c^2 - 88*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/S
qrt[b*c - a*d]])/(4*c^4*(b*c - a*d)^(9/2)) - ((5*b*c + 6*a*d)*Arc
Tanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(7/2)*c^4)
```

---

**Rubi [A]** time = 2.07224, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$-\frac{(6ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^4}-\frac{d^{7/2}\left(24a^2d^2-88abcd+99b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{9/2}}+\frac{d\left(12a^2d^2-23abcd+4b^2c^2\right)}{4ac^3\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{b\left(-36a^3d^3+87a^2bcd^2-36ab^2c^2d+20b^3c^3\right)}{12a^2c^3\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)^3}+\frac{b\left(12a^4d^4-35a^3bcd^3+24a^2b^2c^2d^2-56ab^3c^3d+20b^4c^4\right)}{4a^3c^3\sqrt{a+\frac{b}{x}}(bc-ad)^4}+\frac{d(2bc-3ad)}{2ac^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2(bc-ad)}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b/x)^(5/2)*(c + d/x)^3),x]
```

```
[Out] (b*(20*b^3*c^3 - 36*a*b^2*c^2*d + 87*a^2*b*c*d^2 - 36*a^3*d^3))/(
12*a^2*c^3*(b*c - a*d)^3*(a + b/x)^(3/2)) + (b*(20*b^4*c^4 - 56*a
*b^3*c^3*d + 24*a^2*b^2*c^2*d^2 - 35*a^3*b*c*d^3 + 12*a^4*d^4))/(
4*a^3*c^3*(b*c - a*d)^4*Sqrt[a + b/x]) + (d*(2*b*c - 3*a*d))/(2*a
*c^2*(b*c - a*d)*(a + b/x)^(3/2)*(c + d/x)^2) + (d*(4*b^2*c^2 - 2
3*a*b*c*d + 12*a^2*d^2))/(4*a*c^3*(b*c - a*d)^2*(a + b/x)^(3/2)*(
c + d/x)) + x/(a*c*(a + b/x)^(3/2)*(c + d/x)^2) - (d^(7/2)*(99*b^
2*c^2 - 88*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/S
qrt[b*c - a*d]])/(4*c^4*(b*c - a*d)^(9/2)) - ((5*b*c + 6*a*d)*Arc
Tanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(7/2)*c^4)
```

---

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b/x)**(5/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

---

**Mathematica [C]** time = 3.81985, size = 465, normalized size = 1.14

$$-\frac{\frac{12(6ad+5bc)\log\left(2\sqrt{a}x\sqrt{a+\frac{b}{x}}+2ax+b\right)}{a^{7/2}}+\frac{3id^{7/2}\left(24a^2d^2-88abcd+99b^2c^2\right)\log\left(\frac{8c^5(bc-ad)^{7/2}\left(2\sqrt{d}x\sqrt{a+\frac{b}{x}}\sqrt{bc-ad}-2iadx-ib(d-cx)\right)}{d^{9/2}(cx+d)\left(24a^2d^2-88abcd+99b^2c^2\right)}\right)}{(bc-ad)^{9/2}}+\frac{2\sqrt{a+\frac{b}{x}}\left(-30a^5d^5(a\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b/x)^(5/2)*(c + d/x)^3),x]
```

```
[Out] -((2*Sqrt[a + b/x]*(6*a^4*d^6*(b*c - a*d)*(b + a*x)^2 + 3*a^4*d^5
*(-23*b*c + 12*a*d)*(b + a*x)^2*(d + c*x) - 8*b^6*c^4*(b*c - a*d)
*(d + c*x)^2 + 8*b^5*c^4*(8*b*c - 17*a*d)*(b + a*x)*(d + c*x)^2 -
 56*b^5*c^5*(b + a*x)^2*(d + c*x)^2 + 128*a*b^4*c^4*d*(b + a*x)^2
*(d + c*x)^2 + 63*a^4*b*c*d^4*(b + a*x)^2*(d + c*x)^2 - 30*a^5*d^
5*(b + a*x)^2*(d + c*x)^2 - 12*a*c*(b*c - a*d)^4*x*(b + a*x)^2*(d
 + c*x)^2))/(a^4*(b*c - a*d)^4*(b + a*x)^2*(d + c*x)^2) + (12*(5*
b*c + 6*a*d)*Log[b + 2*a*x + 2*Sqrt[a]*Sqrt[a + b/x]*x])/a^(7/2)
+ ((3*I)*d^(7/2)*(99*b^2*c^2 - 88*a*b*c*d + 24*a^2*d^2)*Log[(8*c^
5*(b*c - a*d)^(7/2)*((-2*I)*a*d*x + 2*Sqrt[d]*Sqrt[b*c - a*d]*Sqr
t[a + b/x]*x - I*b*(d - c*x)))/(d^(9/2)*(99*b^2*c^2 - 88*a*b*c*d
+ 24*a^2*d^2)*(d + c*x))])/(b*c - a*d)^(9/2))/(24*c^4)
```

**Maple [B]** time = 0.031, size = 7306, normalized size = 17.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b/x)^(5/2)/(c+d/x)^3,x)
```

```
[Out] result too large to display
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + b/x)^(5/2)*(c + d/x)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 6.63689, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + b/x)^(5/2)*(c + d/x)^3),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(99*a^3*b^3*c^2*d^5 - 88*a^4*b^2*c*d^6 + 24*a^5*b*d^7 +
(99*a^4*b^2*c^4*d^3 - 88*a^5*b*c^3*d^4 + 24*a^6*c^2*d^5)*x^3 + (9
9*a^3*b^3*c^4*d^3 + 110*a^4*b^2*c^3*d^4 - 152*a^5*b*c^2*d^5 + 48*
a^6*c*d^6)*x^2 + (198*a^3*b^3*c^3*d^4 - 77*a^4*b^2*c^2*d^5 - 40*a
^5*b*c*d^6 + 24*a^6*d^7)*x)*sqrt(a)*sqrt(-d/(b*c - a*d))*sqrt((a*
x + b)/x)*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x +
b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 12*(5*b^6*c^5*d^2 - 1
4*a*b^5*c^4*d^3 + 6*a^2*b^4*c^3*d^4 + 16*a^3*b^3*c^2*d^5 - 19*a^4
*b^2*c*d^6 + 6*a^5*b*d^7 + (5*a*b^5*c^7 - 14*a^2*b^4*c^6*d + 6*a^
3*b^3*c^5*d^2 + 16*a^4*b^2*c^4*d^3 - 19*a^5*b*c^3*d^4 + 6*a^6*c^2
*d^5)*x^3 + (5*b^6*c^7 - 4*a*b^5*c^6*d - 22*a^2*b^4*c^5*d^2 + 28*
a^3*b^3*c^4*d^3 + 13*a^4*b^2*c^3*d^4 - 32*a^5*b*c^2*d^5 + 12*a^6*
c*d^6)*x^2 + (10*b^6*c^6*d - 23*a*b^5*c^5*d^2 - 2*a^2*b^4*c^4*d^3
 + 38*a^3*b^3*c^3*d^4 - 22*a^4*b^2*c^2*d^5 - 7*a^5*b*c*d^6 + 6*a^
6*d^7)*x)*sqrt((a*x + b)/x)*log(-2*a*x*sqrt((a*x + b)/x) + (2*a*x
```

+ b)*sqrt(a)) + 2*(60*b^6*c^5*d^2 - 168*a*b^5*c^4*d^3 + 72*a^2*b
^4*c^3*d^4 - 105*a^3*b^3*c^2*d^5 + 36*a^4*b^2*c*d^6 + 12*(a^2*b^4
*c^7 - 4*a^3*b^3*c^6*d + 6*a^4*b^2*c^5*d^2 - 4*a^5*b*c^4*d^3 + a^
6*c^3*d^4)*x^4 + (80*a*b^5*c^7 - 200*a^2*b^4*c^6*d + 48*a^3*b^3*c
^5*d^2 + 48*a^4*b^2*c^4*d^3 - 135*a^5*b*c^3*d^4 + 54*a^6*c^2*d^5)
*x^3 + (60*b^6*c^7 - 8*a*b^5*c^6*d - 364*a^2*b^4*c^5*d^2 + 192*a^
3*b^3*c^4*d^3 - 234*a^4*b^2*c^3*d^4 + 3*a^5*b*c^2*d^5 + 36*a^6*c*
d^6)*x^2 + (120*b^6*c^6*d - 256*a*b^5*c^5*d^2 - 80*a^2*b^4*c^4*d^
3 - 15*a^3*b^3*c^3*d^4 - 156*a^4*b^2*c^2*d^5 + 72*a^5*b*c*d^6)*x)
*sqrt(a))/((a^3*b^5*c^8*d^2 - 4*a^4*b^4*c^7*d^3 + 6*a^5*b^3*c^6*d
^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6 + (a^4*b^4*c^10 - 4*a^5*b^
3*c^9*d + 6*a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^3 + a^8*c^6*d^4)*x^3
+ (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d - 2*a^5*b^3*c^8*d^2 + 8*a^6*b^2
*c^7*d^3 - 7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*x^2 + (2*a^3*b^5*c^9*
d - 7*a^4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*d^4 - 2
*a^7*b*c^5*d^5 + a^8*c^4*d^6)*x)*sqrt(a)*sqrt((a*x + b)/x)), 1/24
*(3*(99*a^3*b^3*c^2*d^5 - 88*a^4*b^2*c*d^6 + 24*a^5*b*d^7 + (99*a
^4*b^2*c^4*d^3 - 88*a^5*b*c^3*d^4 + 24*a^6*c^2*d^5)*x^3 + (99*a^3
*b^3*c^4*d^3 + 110*a^4*b^2*c^3*d^4 - 152*a^5*b*c^2*d^5 + 48*a^6*c
*d^6)*x^2 + (198*a^3*b^3*c^3*d^4 - 77*a^4*b^2*c^2*d^5 - 40*a^5*b*
c*d^6 + 24*a^6*d^7)*x)*sqrt(-a)*sqrt(-d/(b*c - a*d))*sqrt((a*x +
b)/x)*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x
) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 24*(5*b^6*c^5*d^2 - 14*a*
b^5*c^4*d^3 + 6*a^2*b^4*c^3*d^4 + 16*a^3*b^3*c^2*d^5 - 19*a^4*b^2
*c*d^6 + 6*a^5*b*d^7 + (5*a*b^5*c^7 - 14*a^2*b^4*c^6*d + 6*a^3*b^
3*c^5*d^2 + 16*a^4*b^2*c^4*d^3 - 19*a^5*b*c^3*d^4 + 6*a^6*c^2*d^5
)*x^3 + (5*b^6*c^7 - 4*a*b^5*c^6*d - 22*a^2*b^4*c^5*d^2 + 28*a^3*
b^3*c^4*d^3 + 13*a^4*b^2*c^3*d^4 - 32*a^5*b*c^2*d^5 + 12*a^6*c*d^
6)*x^2 + (10*b^6*c^6*d - 23*a*b^5*c^5*d^2 - 2*a^2*b^4*c^4*d^3 + 3
8*a^3*b^3*c^3*d^4 - 22*a^4*b^2*c^2*d^5 - 7*a^5*b*c*d^6 + 6*a^6*d^
7)*x)*sqrt((a*x + b)/x)*arctan(a/(sqrt(-a)*sqrt((a*x + b)/x))) +
2*(60*b^6*c^5*d^2 - 168*a*b^5*c^4*d^3 + 72*a^2*b^4*c^3*d^4 - 105*
a^3*b^3*c^2*d^5 + 36*a^4*b^2*c*d^6 + 12*(a^2*b^4*c^7 - 4*a^3*b^3*
c^6*d + 6*a^4*b^2*c^5*d^2 - 4*a^5*b*c^4*d^3 + a^6*c^3*d^4)*x^4 +
(80*a*b^5*c^7 - 200*a^2*b^4*c^6*d + 48*a^3*b^3*c^5*d^2 + 48*a^4*b
^2*c^4*d^3 - 135*a^5*b*c^3*d^4 + 54*a^6*c^2*d^5)*x^3 + (60*b^6*c^
7 - 8*a*b^5*c^6*d - 364*a^2*b^4*c^5*d^2 + 192*a^3*b^3*c^4*d^3 - 2
34*a^4*b^2*c^3*d^4 + 3*a^5*b*c^2*d^5 + 36*a^6*c*d^6)*x^2 + (120*b
^6*c^6*d - 256*a*b^5*c^5*d^2 - 80*a^2*b^4*c^4*d^3 - 15*a^3*b^3*c^
3*d^4 - 156*a^4*b^2*c^2*d^5 + 72*a^5*b*c*d^6)*x)*sqrt(-a))/((a^3*
b^5*c^8*d^2 - 4*a^4*b^4*c^7*d^3 + 6*a^5*b^3*c^6*d^4 - 4*a^6*b^2*c
^5*d^5 + a^7*b*c^4*d^6 + (a^4*b^4*c^10 - 4*a^5*b^3*c^9*d + 6*a^6*
b^2*c^8*d^2 - 4*a^7*b*c^7*d^3 + a^8*c^6*d^4)*x^3 + (a^3*b^5*c^10
- 2*a^4*b^4*c^9*d - 2*a^5*b^3*c^8*d^2 + 8*a^6*b^2*c^7*d^3 - 7*a^7
*b*c^6*d^4 + 2*a^8*c^5*d^5)*x^2 + (2*a^3*b^5*c^9*d - 7*a^4*b^4*c^
8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*d^4 - 2*a^7*b*c^5*d^5 +
a^8*c^4*d^6)*x)*sqrt(-a)*sqrt((a*x + b)/x)), -1/12*(3*(99*a^3*b^
3*c^2*d^5 - 88*a^4*b^2*c*d^6 + 24*a^5*b*d^7 + (99*a^4*b^2*c^4*d^3
- 88*a^5*b*c^3*d^4 + 24*a^6*c^2*d^5)*x^3 + (99*a^3*b^3*c^4*d^3 +
110*a^4*b^2*c^3*d^4 - 152*a^5*b*c^2*d^5 + 48*a^6*c*d^6)*x^2 + (1
98*a^3*b^3*c^3*d^4 - 77*a^4*b^2*c^2*d^5 - 40*a^5*b*c*d^6 + 24*a^6
*d^7)*x)*sqrt(a)*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)*arctan(-(b
*c - a*d)*sqrt(d/(b*c - a*d))/(d*sqrt((a*x + b)/x))) - 6*(5*b^6*c
^5*d^2 - 14*a*b^5*c^4*d^3 + 6*a^2*b^4*c^3*d^4 + 16*a^3*b^3*c^2*d^
5 - 19*a^4*b^2*c*d^6 + 6*a^5*b*d^7 + (5*a*b^5*c^7 - 14*a^2*b^4*c^
6*d + 6*a^3*b^3*c^5*d^2 + 16*a^4*b^2*c^4*d^3 - 19*a^5*b*c^3*d^4 +

```
 6*a^6*c^2*d^5)*x^3 + (5*b^6*c^7 - 4*a*b^5*c^6*d - 22*a^2*b^4*c^5
*d^2 + 28*a^3*b^3*c^4*d^3 + 13*a^4*b^2*c^3*d^4 - 32*a^5*b*c^2*d^5
 + 12*a^6*c*d^6)*x^2 + (10*b^6*c^6*d - 23*a*b^5*c^5*d^2 - 2*a^2*b
^4*c^4*d^3 + 38*a^3*b^3*c^3*d^4 - 22*a^4*b^2*c^2*d^5 - 7*a^5*b*c*
d^6 + 6*a^6*d^7)*x)*sqrt((a*x + b)/x)*log(-2*a*x*sqrt((a*x + b)/x
) + (2*a*x + b)*sqrt(a)) - (60*b^6*c^5*d^2 - 168*a*b^5*c^4*d^3 +
72*a^2*b^4*c^3*d^4 - 105*a^3*b^3*c^2*d^5 + 36*a^4*b^2*c*d^6 + 12*
(a^2*b^4*c^7 - 4*a^3*b^3*c^6*d + 6*a^4*b^2*c^5*d^2 - 4*a^5*b*c^4*
d^3 + a^6*c^3*d^4)*x^4 + (80*a*b^5*c^7 - 200*a^2*b^4*c^6*d + 48*a
^3*b^3*c^5*d^2 + 48*a^4*b^2*c^4*d^3 - 135*a^5*b*c^3*d^4 + 54*a^6*
c^2*d^5)*x^3 + (60*b^6*c^7 - 8*a*b^5*c^6*d - 364*a^2*b^4*c^5*d^2
+ 192*a^3*b^3*c^4*d^3 - 234*a^4*b^2*c^3*d^4 + 3*a^5*b*c^2*d^5 + 3
6*a^6*c*d^6)*x^2 + (120*b^6*c^6*d - 256*a*b^5*c^5*d^2 - 80*a^2*b^
4*c^4*d^3 - 15*a^3*b^3*c^3*d^4 - 156*a^4*b^2*c^2*d^5 + 72*a^5*b*c
*d^6)*x)*sqrt(a))/((a^3*b^5*c^8*d^2 - 4*a^4*b^4*c^7*d^3 + 6*a^5*b
^3*c^6*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6 + (a^4*b^4*c^10 -
4*a^5*b^3*c^9*d + 6*a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^3 + a^8*c^6*d
^4)*x^3 + (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d - 2*a^5*b^3*c^8*d^2 + 8
*a^6*b^2*c^7*d^3 - 7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*x^2 + (2*a^3*
b^5*c^9*d - 7*a^4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6
*d^4 - 2*a^7*b*c^5*d^5 + a^8*c^4*d^6)*x)*sqrt(a)*sqrt((a*x + b)/x
)), -1/12*(3*(99*a^3*b^3*c^2*d^5 - 88*a^4*b^2*c*d^6 + 24*a^5*b*d^
7 + (99*a^4*b^2*c^4*d^3 - 88*a^5*b*c^3*d^4 + 24*a^6*c^2*d^5)*x^3
+ (99*a^3*b^3*c^4*d^3 + 110*a^4*b^2*c^3*d^4 - 152*a^5*b*c^2*d^5 +
 48*a^6*c*d^6)*x^2 + (198*a^3*b^3*c^3*d^4 - 77*a^4*b^2*c^2*d^5 -
40*a^5*b*c*d^6 + 24*a^6*d^7)*x)*sqrt(-a)*sqrt(d/(b*c - a*d))*sqrt
((a*x + b)/x)*arctan(-(b*c - a*d)*sqrt(d/(b*c - a*d))/(d*sqrt((a*
x + b)/x))) - 12*(5*b^6*c^5*d^2 - 14*a*b^5*c^4*d^3 + 6*a^2*b^4*c^
3*d^4 + 16*a^3*b^3*c^2*d^5 - 19*a^4*b^2*c*d^6 + 6*a^5*b*d^7 + (5*
a*b^5*c^7 - 14*a^2*b^4*c^6*d + 6*a^3*b^3*c^5*d^2 + 16*a^4*b^2*c^4
*d^3 - 19*a^5*b*c^3*d^4 + 6*a^6*c^2*d^5)*x^3 + (5*b^6*c^7 - 4*a*b
^5*c^6*d - 22*a^2*b^4*c^5*d^2 + 28*a^3*b^3*c^4*d^3 + 13*a^4*b^2*c
^3*d^4 - 32*a^5*b*c^2*d^5 + 12*a^6*c*d^6)*x^2 + (10*b^6*c^6*d - 2
3*a*b^5*c^5*d^2 - 2*a^2*b^4*c^4*d^3 + 38*a^3*b^3*c^3*d^4 - 22*a^4
*b^2*c^2*d^5 - 7*a^5*b*c*d^6 + 6*a^6*d^7)*x)*sqrt((a*x + b)/x)*ar
ctan(a/(sqrt(-a)*sqrt((a*x + b)/x))) - (60*b^6*c^5*d^2 - 168*a*b^
5*c^4*d^3 + 72*a^2*b^4*c^3*d^4 - 105*a^3*b^3*c^2*d^5 + 36*a^4*b^2
*c*d^6 + 12*(a^2*b^4*c^7 - 4*a^3*b^3*c^6*d + 6*a^4*b^2*c^5*d^2 -
4*a^5*b*c^4*d^3 + a^6*c^3*d^4)*x^4 + (80*a*b^5*c^7 - 200*a^2*b^4*
c^6*d + 48*a^3*b^3*c^5*d^2 + 48*a^4*b^2*c^4*d^3 - 135*a^5*b*c^3*d
^4 + 54*a^6*c^2*d^5)*x^3 + (60*b^6*c^7 - 8*a*b^5*c^6*d - 364*a^2*
b^4*c^5*d^2 + 192*a^3*b^3*c^4*d^3 - 234*a^4*b^2*c^3*d^4 + 3*a^5*b
*c^2*d^5 + 36*a^6*c*d^6)*x^2 + (120*b^6*c^6*d - 256*a*b^5*c^5*d^2
 - 80*a^2*b^4*c^4*d^3 - 15*a^3*b^3*c^3*d^4 - 156*a^4*b^2*c^2*d^5
+ 72*a^5*b*c*d^6)*x)*sqrt(-a))/((a^3*b^5*c^8*d^2 - 4*a^4*b^4*c^7*
d^3 + 6*a^5*b^3*c^6*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6 + (a^
4*b^4*c^10 - 4*a^5*b^3*c^9*d + 6*a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^
3 + a^8*c^6*d^4)*x^3 + (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d - 2*a^5*b^
3*c^8*d^2 + 8*a^6*b^2*c^7*d^3 - 7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*
x^2 + (2*a^3*b^5*c^9*d - 7*a^4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 -
2*a^6*b^2*c^6*d^4 - 2*a^7*b*c^5*d^5 + a^8*c^4*d^6)*x)*sqrt(-a)*sq
rt((a*x + b)/x))]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b/x)**(5/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [A]** time = 0.266349, size = 703, normalized size = 1.72

$$-\frac{1}{12}\,b\left(\frac{3\left(99\,b^2c^2d^4-88\,abcd^5+24\,a^2d^6\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^5c^8-4\,ab^4c^7d+6\,a^2b^3c^6d^2-4\,a^3b^2c^5d^3+a^4bc^4d^4\right)\sqrt{bcd-ad^2}}-\frac{8\left(ab^4c-a^2b^3d+\frac{6\,(ax+b)b^4c}{x}-\frac{15\,(ax\cdots}{\cdots}\right.}{\left(a^3b^4c^4-4\,a^4b^3c^3d+6\,a^5b^2c^2d^2-4\,a^6bcd^3+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + b/x)^(5/2)*(c + d/x)^3),x, algorithm="giac")
```

```
[Out] -1/12*b*(3*(99*b^2*c^2*d^4 - 88*a*b*c*d^5 + 24*a^2*d^6)*arctan(d*
sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))/((b^5*c^8 - 4*a*b^4*c^7*d
+ 6*a^2*b^3*c^6*d^2 - 4*a^3*b^2*c^5*d^3 + a^4*b*c^4*d^4)*sqrt(b*c
*d - a*d^2)) - 8*(a*b^4*c - a^2*b^3*d + 6*(a*x + b)*b^4*c/x - 15*
(a*x + b)*a*b^3*d/x)*x/((a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^
2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)*(a*x + b)*sqrt((a*x + b)/x))
 + 3*(21*b^2*c^2*d^4*sqrt((a*x + b)/x) - 29*a*b*c*d^5*sqrt((a*x +
 b)/x) + 8*a^2*d^6*sqrt((a*x + b)/x) + 19*(a*x + b)*b*c*d^5*sqrt(
(a*x + b)/x)/x - 8*(a*x + b)*a*d^6*sqrt((a*x + b)/x)/x)/((b^4*c^7
 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*
d^4)*(b*c - a*d + (a*x + b)*d/x)^2) + 12*sqrt((a*x + b)/x)/((a -
(a*x + b)/x)*a^3*c^3) - 12*(5*b*c + 6*a*d)*arctan(sqrt((a*x + b)/
x)/sqrt(-a))/(sqrt(-a)*a^3*b*c^4))
```

## 3.167 $\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$

**Optimal**. Leaf size=123

$$x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}+\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}-2\sqrt{b}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)$$

```
[Out] Sqrt[a + b/x]*Sqrt[c + d/x]*x + ((b*c + a*d)*ArcTanh[(Sqrt[c]*Sqr
t[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*Sqrt[c]) - 2*Sqrt[
b]*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b/x])/(Sqrt[b]*Sqrt[c + d/x]
)]
```

---

**Rubi [A]** time = 0.353574, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261

$$x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}+\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}-2\sqrt{b}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b/x]*Sqrt[c + d/x],x]
```

```
[Out] Sqrt[a + b/x]*Sqrt[c + d/x]*x + ((b*c + a*d)*ArcTanh[(Sqrt[c]*Sqr
t[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*Sqrt[c]) - 2*Sqrt[
b]*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b/x])/(Sqrt[b]*Sqrt[c + d/x]
)]
```

---

**Rubi in Sympy [A]** time = 32.876, size = 104, normalized size = 0.85

$$-2\sqrt{b}\sqrt{d}\,\mathrm{atanh}\left(\frac{\sqrt{b}\sqrt{c+\frac{d}{x}}}{\sqrt{d}\sqrt{a+\frac{b}{x}}}\right)+x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}+\frac{(ad+bc)\,\mathrm{atanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d/x)**(1/2)*(a+b/x)**(1/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(d)*atanh(sqrt(b)*sqrt(c + d/x)/(sqrt(d)*sqrt(a +
b/x))) + x*sqrt(a + b/x)*sqrt(c + d/x) + (a*d + b*c)*atanh(sqrt(c
)*sqrt(a + b/x)/(sqrt(a)*sqrt(c + d/x)))/(sqrt(a)*sqrt(c))
```

---

**Mathematica [A]** time = 0.355393, size = 168, normalized size = 1.37

$$x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}-\sqrt{b}\sqrt{d}\log\left(2\sqrt{b}\sqrt{d}x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}+adx+bcx+2bd\right)+\frac{(ad+bc)\log\left(2\sqrt{a}\sqrt{c}x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}+a(2cx+d)+bc\right)}{2\sqrt{a}\sqrt{c}}+\sqrt{b}\sqrt{d}\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b/x]*Sqrt[c + d/x],x]
```

```
[Out] Sqrt[a + b/x]*Sqrt[c + d/x]*x + Sqrt[b]*Sqrt[d]*Log[x] - Sqrt[b]*
Sqrt[d]*Log[2*b*d + b*c*x + a*d*x + 2*Sqrt[b]*Sqrt[d]*Sqrt[a + b/
x]*Sqrt[c + d/x]*x] + ((b*c + a*d)*Log[b*c + 2*Sqrt[a]*Sqrt[c]*Sq
rt[a + b/x]*Sqrt[c + d/x]*x + a*(d + 2*c*x)])/(2*Sqrt[a]*Sqrt[c])
```

---

**Maple [B]** time = 0.033, size = 253, normalized size = 2.1

$$\frac{x}{2}\sqrt{\frac{cx+d}{x}}\sqrt{\frac{ax+b}{x}}\left(\sqrt{bd}\ln\left(\frac{1}{2}\left(2\,acx+2\,\sqrt{ac{x}^{2}+adx+bcx+bd}\sqrt{ac}+ad+bc\right)\frac{1}{\sqrt{ac}}\right)ad+\sqrt{bd}\ln\left(\frac{1}{2}\left(2\,acx+2\,\sqrt{ac{x}^{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d/x)^(1/2)*(a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*((b*d)^(1/2)*ln(1/2*(2*
a*c*x+2*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c
)^(1/2))*a*d+(b*d)^(1/2)*ln(1/2*(2*a*c*x+2*(a*c*x^2+a*d*x+b*c*x+b
*d)^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b*c-2*b*d*ln((a*d*x+b
*c*x+2*(b*d)^(1/2)*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)+2*b*d)/x)*(a*c
)^(1/2)+2*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*(a*c)^(1/2)*(b*d)^(1/2)
)/(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)/(a*c)^(1/2)/(b*d)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)*sqrt(c + d/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.728648, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)*sqrt(c + d/x),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + (b*c +
a*d)*log(-4*(2*a^2*c^2*x^2 + (a*b*c^2 + a^2*c*d)*x)*sqrt((a*x + b
)/x)*sqrt((c*x + d)/x) - (8*a^2*c^2*x^2 + b^2*c^2 + 6*a*b*c*d + a
^2*d^2 + 8*(a*b*c^2 + a^2*c*d)*x)*sqrt(a*c)) + 2*sqrt(a*c)*sqrt(b
*d)*log(-(8*b^2*d^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^2 - 4*(2*
b*d*x + (b*c + a*d)*x^2)*sqrt(b*d)*sqrt((a*x + b)/x)*sqrt((c*x +
d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2))/sqrt(a*c), 1/4*(4*sqrt(a*c
)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 4*sqrt(a*c)*sqrt(-b*d)*
arctan(1/2*(2*b*d + (b*c + a*d)*x)*sqrt(-b*d)/(b*d*x*sqrt((a*x +
b)/x)*sqrt((c*x + d)/x))) + (b*c + a*d)*log(-4*(2*a^2*c^2*x^2 + (
a*b*c^2 + a^2*c*d)*x)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - (8*a^
2*c^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(a*b*c^2 + a^2*c*d)
*x)*sqrt(a*c)))/sqrt(a*c), 1/2*(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*
sqrt((c*x + d)/x) + (b*c + a*d)*arctan(2*sqrt(-a*c)*x*sqrt((a*x +
 b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c + a*d)) + sqrt(-a*c)*sqrt
(b*d)*log(-(8*b^2*d^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^2 - 4*(
2*b*d*x + (b*c + a*d)*x^2)*sqrt(b*d)*sqrt((a*x + b)/x)*sqrt((c*x
+ d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2))/sqrt(-a*c), 1/2*(2*sqrt(
-a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + (b*c + a*d)*arctan(
2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c
 + a*d)) + 2*sqrt(-a*c)*sqrt(-b*d)*arctan(1/2*(2*b*d + (b*c + a*d
)*x)*sqrt(-b*d)/(b*d*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x))))/sqr
t(-a*c)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d/x)**(1/2)*(a+b/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x)*sqrt(c + d/x), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + \frac{b}{x}}\sqrt{c + \frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)*sqrt(c + d/x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x)*sqrt(c + d/x), x)
```

## 3.168 $\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$

**Optimal**. Leaf size=81

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{3/2}}+\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}$$

```
[Out] (Sqrt[a + b/x]*Sqrt[c + d/x]*x)/c + ((b*c - a*d)*ArcTanh[(Sqrt[c]
*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(3/2))
```

---

**Rubi [A]** time = 0.192844, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{3/2}}+\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b/x]/Sqrt[c + d/x],x]
```

```
[Out] (Sqrt[a + b/x]*Sqrt[c + d/x]*x)/c + ((b*c - a*d)*ArcTanh[(Sqrt[c]
*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(3/2))
```

---

**Rubi in Sympy [A]** time = 14.8743, size = 65, normalized size = 0.8

$$\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}-\frac{(ad-bc)\operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**(1/2)/(c+d/x)**(1/2),x)
```

```
[Out] x*sqrt(a + b/x)*sqrt(c + d/x)/c - (a*d - b*c)*atanh(sqrt(c)*sqrt(
a + b/x)/(sqrt(a)*sqrt(c + d/x)))/(sqrt(a)*c**(3/2))
```

---

**Mathematica [A]** time = 0.19538, size = 98, normalized size = 1.21

$$\frac{(bc-ad)\log\left(2\sqrt{a}\sqrt{c}x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}+2acx+ad+bc\right)}{2\sqrt{a}c^{3/2}}+\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b/x]/Sqrt[c + d/x],x]
```

```
[Out] (Sqrt[a + b/x]*Sqrt[c + d/x]*x)/c + ((b*c - a*d)*Log[b*c + a*d +
2*a*c*x + 2*Sqrt[a]*Sqrt[c]*Sqrt[a + b/x]*Sqrt[c + d/x]*x])/(2*Sq
rt[a]*c^(3/2))
```

---

**Maple [B]** time = 0.034, size = 155, normalized size = 1.9

$$-\frac{x}{2\,c}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}\left(\ln\left(\frac{1}{2}\left(2\,acx+2\,\sqrt{(cx+d)\,(ax+b)}\sqrt{ac}+ad+bc\right)\frac{1}{\sqrt{ac}}\right)ad-\ln\left(\frac{1}{2}\left(2\,acx+2\,\sqrt{(cx+d)\,(ax+b)}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^(1/2)/(c+d/x)^(1/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(ln(1/2*(2*a*c*x+2*((c
*x+d)*(a*x+b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*d-ln(1/2
*(2*a*c*x+2*((c*x+d)*(a*x+b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1
/2))*b*c-2*((c*x+d)*(a*x+b))^(1/2)*(a*c)^(1/2))/((c*x+d)*(a*x+b))
^(1/2)/c/(a*c)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)/sqrt(c + d/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.327013, size = 1, normalized size = 0.01

$$\left[\frac{4\sqrt{ac}x\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}} - (bc-ad)\log\left(4\left(2a^2c^2x^2 + \left(abc^2 + a^2cd\right)x\right)\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}} - \left(8a^2c^2x^2 + b^2c^2 + 6abcd + a^2d^2 + 8\right.\right.}{4\sqrt{ac}c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)/sqrt(c + d/x),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - (b*c -
a*d)*log(4*(2*a^2*c^2*x^2 + (a*b*c^2 + a^2*c*d)*x)*sqrt((a*x + b)
/x)*sqrt((c*x + d)/x) - (8*a^2*c^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^
2*d^2 + 8*(a*b*c^2 + a^2*c*d)*x)*sqrt(a*c)))/(sqrt(a*c)*c), 1/2*(
2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + (b*c - a*d)*
arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*
x + b*c + a*d)))/(sqrt(-a*c)*c)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{c + \frac{d}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**(1/2)/(c+d/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x)/sqrt(c + d/x), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{c + \frac{d}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)/sqrt(c + d/x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x)/sqrt(c + d/x), x)
```

## 3.169 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=122

$$\frac{(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{5/2}}-\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{ac^2\sqrt{c+\frac{d}{x}}}+\frac{x\left(a+\frac{b}{x}\right)^{3/2}}{ac\sqrt{c+\frac{d}{x}}}$$

```
[Out] -(((b*c - 3*a*d)*Sqrt[a + b/x])/(a*c^2*Sqrt[c + d/x])) + ((a + b/
x)^(3/2)*x)/(a*c*Sqrt[c + d/x]) + ((b*c - 3*a*d)*ArcTanh[(Sqrt[c]
*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(5/2))
```

---

**Rubi [A]** time = 0.300756, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$\frac{(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{5/2}}-\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{ac^2\sqrt{c+\frac{d}{x}}}+\frac{x\left(a+\frac{b}{x}\right)^{3/2}}{ac\sqrt{c+\frac{d}{x}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b/x]/(c + d/x)^(3/2),x]
```

```
[Out] -(((b*c - 3*a*d)*Sqrt[a + b/x])/(a*c^2*Sqrt[c + d/x])) + ((a + b/
x)^(3/2)*x)/(a*c*Sqrt[c + d/x]) + ((b*c - 3*a*d)*ArcTanh[(Sqrt[c]
*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(5/2))
```

---

**Rubi in Sympy [A]** time = 24.1821, size = 114, normalized size = 0.93

$$-\frac{2dx\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{c\sqrt{c+\frac{d}{x}}\left(ad-bc\right)}+\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\left(3ad-bc\right)}{c^2\left(ad-bc\right)}-\frac{(3ad-bc)\operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**(1/2)/(c+d/x)**(3/2),x)
```

```
[Out] -2*d*x*(a + b/x)**(3/2)/(c*sqrt(c + d/x)*(a*d - b*c)) + x*sqrt(a
+ b/x)*sqrt(c + d/x)*(3*a*d - b*c)/(c**2*(a*d - b*c)) - (3*a*d -
b*c)*atanh(sqrt(c)*sqrt(a + b/x)/(sqrt(a)*sqrt(c + d/x)))/(sqrt(a
)*c**(5/2))
```

---

**Mathematica [A]** time = 0.214013, size = 112, normalized size = 0.92

$$\frac{(bc-3ad)\log\left(2\sqrt{a}\sqrt{c}x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}+2acx+ad+bc\right)}{2\sqrt{a}c^{5/2}}+\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}(cx+3d)}{c^2(cx+d)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b/x]/(c + d/x)^(3/2),x]
```

```
[Out] (Sqrt[a + b/x]*Sqrt[c + d/x]*x*(3*d + c*x))/(c^2*(d + c*x)) + ((b
*c - 3*a*d)*Log[b*c + a*d + 2*a*c*x + 2*Sqrt[a]*Sqrt[c]*Sqrt[a +
b/x]*Sqrt[c + d/x]*x])/(2*Sqrt[a]*c^(5/2))
```

---

**Maple [B]** time = 0.049, size = 280, normalized size = 2.3

$$\frac{x}{(2\,cx+2\,d)\,c^2}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}\left(-3\,\ln\left(1/2\,\frac{2\,acx+2\,\sqrt{(cx+d)\,(ax+b)}\sqrt{ac}+ad+bc}{\sqrt{ac}}\right)xacd+\ln\left(\frac{1}{2}\left(2\,acx+2\,\sqrt{(cx+d)}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^(1/2)/(c+d/x)^(3/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(-3*ln(1/2*(2*a*c*x+2*(
(c*x+d)*(a*x+b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*x*a*c*d+
ln(1/2*(2*a*c*x+2*((c*x+d)*(a*x+b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a
*c)^(1/2))*x*b*c^2+2*x*c*((c*x+d)*(a*x+b))^(1/2)*(a*c)^(1/2)-3*ln
(1/2*(2*a*c*x+2*((c*x+d)*(a*x+b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c
)^(1/2))*a*d^2+ln(1/2*(2*a*c*x+2*((c*x+d)*(a*x+b))^(1/2)*(a*c)^(1
/2)+a*d+b*c)/(a*c)^(1/2))*b*c*d+6*d*((c*x+d)*(a*x+b))^(1/2)*(a*c)
^(1/2))/(c*x+d)/(a*c)^(1/2)/((c*x+d)*(a*x+b))^(1/2)/c^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)/(c + d/x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.328971, size = 1, normalized size = 0.01

$$\left[\frac{4\,\left(cx^2+3\,dx\right)\sqrt{ac}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}-\left(bcd-3\,ad^2+\left(bc^2-3\,acd\right)x\right)\log\left(4\,\left(2\,a^2c^2x^2+\left(abc^2+a^2cd\right)x\right)\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}-\left(8\,a\right.\right.}{4\,(c^3x+c^2d)\sqrt{ac}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)/(c + d/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(c*x^2 + 3*d*x)*sqrt(a*c)*sqrt((a*x + b)/x)*sqrt((c*x + d
)/x) - (b*c*d - 3*a*d^2 + (b*c^2 - 3*a*c*d)*x)*log(4*(2*a^2*c^2*x
^2 + (a*b*c^2 + a^2*c*d)*x)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) -
 (8*a^2*c^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(a*b*c^2 + a^
2*c*d)*x)*sqrt(a*c)))/((c^3*x + c^2*d)*sqrt(a*c)), 1/2*(2*(c*x^2
+ 3*d*x)*sqrt(-a*c)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + (b*c*d
- 3*a*d^2 + (b*c^2 - 3*a*c*d)*x)*arctan(2*sqrt(-a*c)*x*sqrt((a*x
+ b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c + a*d)))/((c^3*x + c^2*d
)*sqrt(-a*c))]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**(1/2)/(c+d/x)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b/x)/(c + d/x)**(3/2), x)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x)/(c + d/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.170 $\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q \, dx$

**Optimal**. Leaf size=96

$$-\frac{b\left(a+\frac{b}{x}\right)^{p+1}\left(c+\frac{d}{x}\right)^q\left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,2;p+2;-\frac{d\left(a+\frac{b}{x}\right)}{bc-ad},\frac{a+\frac{b}{x}}{a}\right)}{a^2(p+1)}$$

```
[Out] -((b*(a + b/x)^(1 + p)*(c + d/x)^q*AppellF1[1 + p, -q, 2, 2 + p,
-((d*(a + b/x))/(b*c - a*d)), (a + b/x)/a])/(a^2*(1 + p)*((b*(c +
 d/x))/(b*c - a*d))^q))
```

---

**Rubi [A]** time = 0.180777, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$-\frac{b\left(a+\frac{b}{x}\right)^{p+1}\left(c+\frac{d}{x}\right)^q\left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,2;p+2;-\frac{d\left(a+\frac{b}{x}\right)}{bc-ad},\frac{a+\frac{b}{x}}{a}\right)}{a^2(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x)^p*(c + d/x)^q,x]
```

```
[Out] -((b*(a + b/x)^(1 + p)*(c + d/x)^q*AppellF1[1 + p, -q, 2, 2 + p,
-((d*(a + b/x))/(b*c - a*d)), (a + b/x)/a])/(a^2*(1 + p)*((b*(c +
 d/x))/(b*c - a*d))^q))
```

---

**Rubi in Sympy [A]** time = 23.2342, size = 66, normalized size = 0.69

$$-\frac{b\left(\frac{b\left(-c-\frac{d}{x}\right)}{ad-bc}\right)^{-q}\left(a+\frac{b}{x}\right)^{p+1}\left(c+\frac{d}{x}\right)^q \operatorname{appellf}_1\left(p+1,2,-q,p+2,\frac{a+\frac{b}{x}}{a},\frac{d\left(a+\frac{b}{x}\right)}{ad-bc}\right)}{a^2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x)**p*(c+d/x)**q,x)
```

```
[Out] -b*(b*(-c - d/x)/(a*d - b*c))**(-q)*(a + b/x)**(p + 1)*(c + d/x)*
*q*appellf1(p + 1, 2, -q, p + 2, (a + b/x)/a, d*(a + b/x)/(a*d -
b*c))/(a**2*(p + 1))
```

**Mathematica [B]** time = 0.493105, size = 206, normalized size = 2.15

$$\frac{bdx(p+q-2)\left(a+\frac{b}{x}\right)^p\left(c+\frac{d}{x}\right)^q F_1\left(-p-q+1;-p,-q;-p-q+2;-\right.}{(p+q-1)\left(x\left(adpF_1\left(-p-q+2;1-p,-q;-p-q+3;-\frac{ax}{b},-\frac{cx}{d}\right)+bcqF_1\left(-p-q+2;-p,1-q;-p-q+3;-\frac{ax}{b},-\frac{cx}{d}\right)\right)\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b/x)^p*(c + d/x)^q,x]
```

```
[Out] (b*d*(-2 + p + q)*(a + b/x)^p*(c + d/x)^q*x*AppellF1[1 - p - q, -
p, -q, 2 - p - q, -((a*x)/b), -((c*x)/d)])/((-1 + p + q)*(-(b*d*(
-2 + p + q)*AppellF1[1 - p - q, -p, -q, 2 - p - q, -((a*x)/b), -(
(c*x)/d)]) + x*(a*d*p*AppellF1[2 - p - q, 1 - p, -q, 3 - p - q, -
((a*x)/b), -((c*x)/d)] + b*c*q*AppellF1[2 - p - q, -p, 1 - q, 3 -
 p - q, -((a*x)/b), -((c*x)/d)])))
```

**Maple [F]** time = 0.142, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x)^p*(c+d/x)^q,x)
```

```
[Out] int((a+b/x)^p*(c+d/x)^q,x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^p*(c + d/x)^q,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^p*(c + d/x)^q, x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{ax+b}{x}\right)^p\left(\frac{cx+d}{x}\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^p*(c + d/x)^q,x, algorithm="fricas")

[Out] integral(((a*x + b)/x)^p*((c*x + d)/x)^q, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\left(a+\frac{b}{x}\right)^p\left(c+\frac{d}{x}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x)**p*(c+d/x)**q,x)

[Out] Integral((a + b/x)**p*(c + d/x)**q, x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int\left(a+\frac{b}{x}\right)^p\left(c+\frac{d}{x}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x)^p*(c + d/x)^q,x, algorithm="giac")

[Out] integrate((a + b/x)^p*(c + d/x)^q, x)
```

# 3.171 $\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx$

**Optimal**. Leaf size=39

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}+\frac{ax}{c}$$

```
[Out] (a*x)/c + ((b*c - a*d)*ArcTan[(Sqrt[c]*x)/Sqrt[d]])/(c^(3/2)*Sqrt
[d])
```

---

**Rubi [A]** time = 0.0718176, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}+\frac{ax}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x^2)/(c + d/x^2),x]
```

```
[Out] (a*x)/c + ((b*c - a*d)*ArcTan[(Sqrt[c]*x)/Sqrt[d]])/(c^(3/2)*Sqrt
[d])
```

---

**Rubi in Sympy [A]** time = 11.336, size = 34, normalized size = 0.87

$$\frac{ax}{c}-\frac{(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{c^{\frac{3}{2}}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x**2)/(c+d/x**2),x)
```

```
[Out] a*x/c - (a*d - b*c)*atan(sqrt(c)*x/sqrt(d))/(c**(3/2)*sqrt(d))
```

---

**Mathematica [A]** time = 0.0442703, size = 40, normalized size = 1.03

$$\frac{ax}{c}-\frac{(ad-bc)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x^2)/(c + d/x^2),x]
```

```
[Out] (a*x)/c - ((-(b*c) + a*d)*ArcTan[(Sqrt[c]*x)/Sqrt[d]])/(c^(3/2)*S
qrt[d])
```

---

**Maple [A]** time = 0.007, size = 45, normalized size = 1.2

$$\frac{ax}{c} - \frac{ad}{c}\arctan\left(cx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} + b\arctan\left(cx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x^2)/(c+d/x^2),x)
```

```
[Out] a*x/c-1/c/(c*d)^(1/2)*arctan(c*x/(c*d)^(1/2))*a*d+1/(c*d)^(1/2)*a
rctan(c*x/(c*d)^(1/2))*b
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x^2)/(c + d/x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.234813, size = 1, normalized size = 0.03

$$\left[\frac{2\sqrt{-cd}ax - (bc - ad)\log\left(-\frac{2\,cdx - (cx^2 - d)\sqrt{-cd}}{cx^2 + d}\right)}{2\sqrt{-cd}c}, \frac{\sqrt{cd}ax + (bc - ad)\arctan\left(\frac{\sqrt{cd}x}{d}\right)}{\sqrt{cd}c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x^2)/(c + d/x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(-c*d)*a*x - (b*c - a*d)*log(-(2*c*d*x - (c*x^2 - d)*
sqrt(-c*d))/(c*x^2 + d)))/(sqrt(-c*d)*c), (sqrt(c*d)*a*x + (b*c -
 a*d)*arctan(sqrt(c*d)*x/d))/(sqrt(c*d)*c)]
```

---

**Sympy [A]** time = 1.66471, size = 82, normalized size = 2.1

$$\frac{ax}{c} + \frac{\sqrt{-\frac{1}{c^3d}}\left(ad - bc\right)\log\left(-cd\sqrt{-\frac{1}{c^3d}} + x\right)}{2} - \frac{\sqrt{-\frac{1}{c^3d}}\left(ad - bc\right)\log\left(cd\sqrt{-\frac{1}{c^3d}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x**2)/(c+d/x**2),x)
```

```
[Out] a*x/c + sqrt(-1/(c**3*d))*(a*d - b*c)*log(-c*d*sqrt(-1/(c**3*d))
+ x)/2 - sqrt(-1/(c**3*d))*(a*d - b*c)*log(c*d*sqrt(-1/(c**3*d))
+ x)/2
```

---

**GIAC/XCAS [A]** time = 0.213993, size = 45, normalized size = 1.15

$$\frac{ax}{c} + \frac{(bc - ad)\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x^2)/(c + d/x^2),x, algorithm="giac")
```

```
[Out] a*x/c + (b*c - a*d)*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c)
```

## 3.172 $\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$

**Optimal**. Leaf size=233

$$-\frac{2d\sqrt{a+\frac{b}{x^2}}}{x\sqrt{c+\frac{d}{x^2}}}+x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}-\frac{\sqrt{c}\sqrt{a+\frac{b}{x^2}}(ad+bc)F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}+\frac{2\sqrt{c}\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

```
[Out] (-2*d*Sqrt[a + b/x^2])/(Sqrt[c + d/x^2]*x) + Sqrt[a + b/x^2]*Sqrt
[c + d/x^2]*x + (2*Sqrt[c]*Sqrt[d]*Sqrt[a + b/x^2]*EllipticE[ArcC
ot[(Sqrt[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)])/(Sqrt[(c*(a + b/x^2))/
(a*(c + d/x^2))]*Sqrt[c + d/x^2]) - (Sqrt[c]*(b*c + a*d)*Sqrt[a +
 b/x^2]*EllipticF[ArcCot[(Sqrt[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)])/
(a*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2])
```

---

**Rubi [A]** time = 0.633792, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261

$$-\frac{2d\sqrt{a+\frac{b}{x^2}}}{x\sqrt{c+\frac{d}{x^2}}}+x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}-\frac{\sqrt{c}\sqrt{a+\frac{b}{x^2}}(ad+bc)F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}+\frac{2\sqrt{c}\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b/x^2]*Sqrt[c + d/x^2],x]
```

```
[Out] (-2*d*Sqrt[a + b/x^2])/(Sqrt[c + d/x^2]*x) + Sqrt[a + b/x^2]*Sqrt
[c + d/x^2]*x + (2*Sqrt[c]*Sqrt[d]*Sqrt[a + b/x^2]*EllipticE[ArcC
ot[(Sqrt[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)])/(Sqrt[(c*(a + b/x^2))/
(a*(c + d/x^2))]*Sqrt[c + d/x^2]) - (Sqrt[c]*(b*c + a*d)*Sqrt[a +
 b/x^2]*EllipticF[ArcCot[(Sqrt[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)])/
(a*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2])
```

---

**Rubi in Sympy [A]** time = 62.9457, size = 202, normalized size = 0.87

$$-\frac{\sqrt{a}\sqrt{c+\frac{d}{x^2}}\,(ad+bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)\middle|-\frac{ad}{bc}+1\right)}{\sqrt{b}c\sqrt{\frac{a\left(c+\frac{d}{x^2}\right)}{c\left(a+\frac{b}{x^2}\right)}}\sqrt{a+\frac{b}{x^2}}}+\frac{2\sqrt{c}\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\operatorname{atan}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)\middle|1-\frac{bc}{ad}\right)}{\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}}-\frac{2d\sqrt{a+\frac{b}{x^2}}}{x\sqrt{c+\frac{d}{x^2}}}+x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d/x**2)**(1/2)*(a+b/x**2)**(1/2),x)
```

```
[Out] -sqrt(a)*sqrt(c + d/x**2)*(a*d + b*c)*elliptic_f(atan(sqrt(b)/(sq
rt(a)*x)), -a*d/(b*c) + 1)/(sqrt(b)*c*sqrt(a*(c + d/x**2)/(c*(a +
 b/x**2)))*sqrt(a + b/x**2)) + 2*sqrt(c)*sqrt(d)*sqrt(a + b/x**2)
*elliptic_e(atan(sqrt(d)/(sqrt(c)*x)), 1 - b*c/(a*d))/(sqrt(c*(a
+ b/x**2)/(a*(c + d/x**2)))*sqrt(c + d/x**2)) - 2*d*sqrt(a + b/x*
*2)/(x*sqrt(c + d/x**2)) + x*sqrt(a + b/x**2)*sqrt(c + d/x**2)
```

---

**Mathematica [C]** time = 0.519844, size = 205, normalized size = 0.88

$$-\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\left(\sqrt{\frac{a}{b}}\left(ax^2+b\right)\left(cx^2+d\right)+ix\sqrt{\frac{ax^2}{b}+1}\sqrt{\frac{cx^2}{d}+1}(bc-ad)F\left(i\sinh^{-1}\left(\sqrt{\frac{a}{b}}x\right)\middle|\frac{bc}{ad}\right)+2iadx\sqrt{\frac{ax^2}{b}+1}\sqrt{c}\right.}{\sqrt{\frac{a}{b}}\left(ax^2+b\right)\left(cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b/x^2]*Sqrt[c + d/x^2],x]
```

```
[Out] -((Sqrt[a + b/x^2]*Sqrt[c + d/x^2]*x*(Sqrt[a/b]*(b + a*x^2)*(d +
c*x^2) + (2*I)*a*d*x*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c*x^2)/d]*Elli
pticE[I*ArcSinh[Sqrt[a/b]*x], (b*c)/(a*d)] + I*(b*c - a*d)*x*Sqrt
[1 + (a*x^2)/b]*Sqrt[1 + (c*x^2)/d]*EllipticF[I*ArcSinh[Sqrt[a/b]
*x], (b*c)/(a*d)]))/(Sqrt[a/b]*(b + a*x^2)*(d + c*x^2)))
```

---

**Maple [A]** time = 0.05, size = 277, normalized size = 1.2

$$\frac{x}{acx^4+adx^2+bcx^2+bd}\sqrt{\frac{cx^2+d}{x^2}}\sqrt{\frac{ax^2+b}{x^2}}\left(-\sqrt{-\frac{c}{d}}x^4ac+\sqrt{\frac{cx^2+d}{d}}\sqrt{\frac{ax^2+b}{b}}EllipticF\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{ad}{bc}}\right)xad-cb\sqrt{\frac{cx^2}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d/x^2)^(1/2)*(a+b/x^2)^(1/2),x)
```

```
[Out] ((a*x^2+b)/x^2)^(1/2)*x*((c*x^2+d)/x^2)^(1/2)*(-(-c/d)^(1/2)*x^4*
a*c+((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*EllipticF(x*(-c/d)^(1
/2),(a*d/b/c)^(1/2))*x*a*d-c*b*((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^
(1/2)*x*EllipticF(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))+2*c*b*((c*x^2+d
)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*x*EllipticE(x*(-c/d)^(1/2),(a*d/b/
c)^(1/2))-(-c/d)^(1/2)*x^2*a*d-(-c/d)^(1/2)*x^2*b*c-(-c/d)^(1/2)*
b*d)/(a*c*x^4+a*d*x^2+b*c*x^2+b*d)/(-c/d)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + \frac{b}{x^2}}\sqrt{c + \frac{d}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x^2)*sqrt(c + d/x^2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^2)*sqrt(c + d/x^2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{\frac{ax^2 + b}{x^2}}\sqrt{\frac{cx^2 + d}{x^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x^2)*sqrt(c + d/x^2),x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^2 + b)/x^2)*sqrt((c*x^2 + d)/x^2), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + \frac{b}{x^2}}\sqrt{c + \frac{d}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d/x**2)**(1/2)*(a+b/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x**2)*sqrt(c + d/x**2), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x^2)*sqrt(c + d/x^2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^2)*sqrt(c + d/x^2), x)
```

## 3.173 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$

**Optimal**. Leaf size=232

$$-\frac{d\sqrt{a+\frac{b}{x^2}}}{cx\sqrt{c+\frac{d}{x^2}}}+\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\sqrt{c}\sqrt{a+\frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}+\frac{\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

```
[Out] -((d*Sqrt[a + b/x^2])/(c*Sqrt[c + d/x^2]*x)) + (Sqrt[a + b/x^2]*S
qrt[c + d/x^2]*x)/c + (Sqrt[d]*Sqrt[a + b/x^2]*EllipticE[ArcCot[(
Sqrt[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[(c*(a + b/x^
2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2]) - (b*Sqrt[c]*Sqrt[a + b/x^2
]*EllipticF[ArcCot[(Sqrt[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)])/(a*Sqr
t[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2])
```

---

**Rubi [A]** time = 0.587952, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304

$$-\frac{d\sqrt{a+\frac{b}{x^2}}}{cx\sqrt{c+\frac{d}{x^2}}}+\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\sqrt{c}\sqrt{a+\frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}+\frac{\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b/x^2]/Sqrt[c + d/x^2],x]
```


```
[Out] -((d*Sqrt[a + b/x^2])/(c*Sqrt[c + d/x^2]*x)) + (Sqrt[a + b/x^2]*S
qrt[c + d/x^2]*x)/c + (Sqrt[d]*Sqrt[a + b/x^2]*EllipticE[ArcCot[(
Sqrt[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[(c*(a + b/x^
2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2]) - (b*Sqrt[c]*Sqrt[a + b/x^2
]*EllipticF[ArcCot[(Sqrt[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)])/(a*Sqr
t[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2])
```

**Rubi in Sympy [A]** time = 56.6809, size = 199, normalized size = 0.86

$$\frac{\sqrt{a}\sqrt{b}\sqrt{c+\frac{d}{x^2}}E\left(\operatorname{atan}\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)\middle|-\frac{ad}{bc}+1\right)}{c\sqrt{\frac{a\left(c+\frac{d}{x^2}\right)}{c\left(a+\frac{b}{x^2}\right)}}\sqrt{a+\frac{b}{x^2}}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{cx\sqrt{a+\frac{b}{x^2}}}+\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\sqrt{c}\sqrt{a+\frac{b}{x^2}}F\left(\operatorname{atan}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)\middle|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x**2)**(1/2)/(c+d/x**2)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(b)*sqrt(c + d/x**2)*elliptic_e(atan(sqrt(b)/(sqrt(a)
*x)), -a*d/(b*c) + 1)/(c*sqrt(a*(c + d/x**2)/(c*(a + b/x**2)))*sq
rt(a + b/x**2)) - b*sqrt(c + d/x**2)/(c*x*sqrt(a + b/x**2)) + x*s
qrt(a + b/x**2)*sqrt(c + d/x**2)/c - b*sqrt(c)*sqrt(a + b/x**2)*e
lliptic_f(atan(sqrt(d)/(sqrt(c)*x)), 1 - b*c/(a*d))/(a*sqrt(d)*sq
rt(c*(a + b/x**2)/(a*(c + d/x**2)))*sqrt(c + d/x**2))
```

**Mathematica [A]** time = 0.101855, size = 86, normalized size = 0.37

$$\frac{\sqrt{a+\frac{b}{x^2}}\sqrt{\frac{cx^2+d}{d}}E\left(\sin^{-1}\left(\sqrt{-\frac{c}{d}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{c}{d}}\sqrt{\frac{ax^2+b}{b}}\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b/x^2]/Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[a + b/x^2]*Sqrt[(d + c*x^2)/d]*EllipticE[ArcSin[Sqrt[-(c/d)
]*x], (a*d)/(b*c)])/(Sqrt[-(c/d)]*Sqrt[c + d/x^2]*Sqrt[(b + a*x^2
)/b])
```

**Maple [A]** time = 0.029, size = 94, normalized size = 0.4

$$\frac{b}{ax^2+b}\sqrt{\frac{ax^2+b}{x^2}}EllipticE\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{ad}{bc}}\right)\sqrt{\frac{ax^2+b}{b}}\sqrt{\frac{cx^2+d}{d}}\frac{1}{\sqrt{-\frac{c}{d}}}\frac{1}{\sqrt{\frac{cx^2+d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x)
```

```
[Out] ((a*x^2+b)/x^2)^(1/2)/(a*x^2+b)*EllipticE(x*(-c/d)^(1/2),(a*d/b/c
)^(1/2))*((a*x^2+b)/b)^(1/2)*((c*x^2+d)/d)^(1/2)*b/(-c/d)^(1/2)/(
(c*x^2+d)/x^2)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{c + \frac{d}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\frac{ax^2+b}{x^2}}}{\sqrt{\frac{cx^2+d}{x^2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2),x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^2 + b)/x^2)/sqrt((c*x^2 + d)/x^2), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{c + \frac{d}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x**2)**(1/2)/(c+d/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x**2)/sqrt(c + d/x**2), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{c + \frac{d}{x^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2), x)
```

## 3.174 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}} \, dx$

**Optimal**. Leaf size=262

$$\frac{2\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}-\frac{2d\sqrt{a+\frac{b}{x^2}}}{c^2x\sqrt{c+\frac{d}{x^2}}}+\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c^2}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{a+\frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

```
[Out] (-2*d*Sqrt[a + b/x^2])/(c^2*Sqrt[c + d/x^2]*x) - (Sqrt[a + b/x^2]
*x)/(c*Sqrt[c + d/x^2]) + (2*Sqrt[a + b/x^2]*Sqrt[c + d/x^2]*x)/c
^2 + (2*Sqrt[d]*Sqrt[a + b/x^2]*EllipticE[ArcCot[(Sqrt[c]*x)/Sqrt
[d]], 1 - (b*c)/(a*d)])/(c^(3/2)*Sqrt[(c*(a + b/x^2))/(a*(c + d/x
^2))]*Sqrt[c + d/x^2]) - (b*Sqrt[a + b/x^2]*EllipticF[ArcCot[(Sqr
t[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)])/(a*Sqrt[c]*Sqrt[d]*Sqrt[(c*(a
 + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2])
```

---

**Rubi [A]** time = 0.835678, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$

$$\frac{2\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}-\frac{2d\sqrt{a+\frac{b}{x^2}}}{c^2x\sqrt{c+\frac{d}{x^2}}}+\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c^2}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{a+\frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b/x^2]/(c + d/x^2)^(3/2),x]
```

```
[Out] (-2*d*Sqrt[a + b/x^2])/(c^2*Sqrt[c + d/x^2]*x) - (Sqrt[a + b/x^2]
*x)/(c*Sqrt[c + d/x^2]) + (2*Sqrt[a + b/x^2]*Sqrt[c + d/x^2]*x)/c
^2 + (2*Sqrt[d]*Sqrt[a + b/x^2]*EllipticE[ArcCot[(Sqrt[c]*x)/Sqrt
[d]], 1 - (b*c)/(a*d)])/(c^(3/2)*Sqrt[(c*(a + b/x^2))/(a*(c + d/x
^2))]*Sqrt[c + d/x^2]) - (b*Sqrt[a + b/x^2]*EllipticF[ArcCot[(Sqr
t[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)])/(a*Sqrt[c]*Sqrt[d]*Sqrt[(c*(a
```

```
 + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2])
```

---

**Rubi in Sympy [A]** time = 80.5002, size = 230, normalized size = 0.88

$$-\frac{\sqrt{a}\sqrt{b}\sqrt{c+\frac{d}{x^2}}F\left(\operatorname{atan}\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)\middle|-\frac{ad}{bc}+1\right)}{c^2\sqrt{\frac{a\left(c+\frac{d}{x^2}\right)}{c\left(a+\frac{b}{x^2}\right)}}\sqrt{a+\frac{b}{x^2}}}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}-\frac{2d\sqrt{a+\frac{b}{x^2}}}{c^2x\sqrt{c+\frac{d}{x^2}}}+\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c^2}+\frac{2\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\operatorname{atan}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)\middle|1-\frac{bc}{ad}\right)}{c^{\frac{3}{2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x**2)**(1/2)/(c+d/x**2)**(3/2),x)
```

```
[Out] -sqrt(a)*sqrt(b)*sqrt(c + d/x**2)*elliptic_f(atan(sqrt(b)/(sqrt(a
)*x)), -a*d/(b*c) + 1)/(c**2*sqrt(a*(c + d/x**2)/(c*(a + b/x**2))
)*sqrt(a + b/x**2)) - x*sqrt(a + b/x**2)/(c*sqrt(c + d/x**2)) - 2
*d*sqrt(a + b/x**2)/(c**2*x*sqrt(c + d/x**2)) + 2*x*sqrt(a + b/x*
*2)*sqrt(c + d/x**2)/c**2 + 2*sqrt(d)*sqrt(a + b/x**2)*elliptic_e
(atan(sqrt(d)/(sqrt(c)*x)), 1 - b*c/(a*d))/(c**(3/2)*sqrt(c*(a +
b/x**2)/(a*(c + d/x**2)))*sqrt(c + d/x**2))
```

---

**Mathematica [C]** time = 0.376759, size = 191, normalized size = 0.73

$$-\frac{\sqrt{a+\frac{b}{x^2}}\left(i\sqrt{\frac{ax^2}{b}+1}\sqrt{\frac{cx^2}{d}+1}(bc-2ad)F\left(i\sinh^{-1}\left(\sqrt{\frac{a}{b}}x\right)|\frac{bc}{ad}\right)+2iad\sqrt{\frac{ax^2}{b}+1}\sqrt{\frac{cx^2}{d}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{a}{b}}x\right)|\frac{bc}{ad}\right)+cx\sqrt{\cdots}\right)}{c^2\sqrt{\frac{a}{b}}\left(ax^2+b\right)\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b/x^2]/(c + d/x^2)^(3/2),x]
```

```
[Out] -((Sqrt[a + b/x^2]*(Sqrt[a/b]*c*x*(b + a*x^2) + (2*I)*a*d*Sqrt[1
+ (a*x^2)/b]*Sqrt[1 + (c*x^2)/d]*EllipticE[I*ArcSinh[Sqrt[a/b]*x]
, (b*c)/(a*d)] + I*(b*c - 2*a*d)*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c*
x^2)/d]*EllipticF[I*ArcSinh[Sqrt[a/b]*x], (b*c)/(a*d)]))/(Sqrt[a/
b]*c^2*Sqrt[c + d/x^2]*(b + a*x^2)))
```

---

**Maple [A]** time = 0.065, size = 185, normalized size = 0.7

$$-\frac{cx^2+d}{x^2c\,(ax^2+b)}\sqrt{\frac{ax^2+b}{x^2}}\left(x^3a\sqrt{-\frac{c}{d}}+EllipticF\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{ad}{bc}}\right)b\sqrt{\frac{cx^2+d}{d}}\sqrt{\frac{ax^2+b}{b}}-2\,EllipticE\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{ad}{bc}}\right)b\sqrt{\frac{cx}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x)
```

```
[Out] -((a*x^2+b)/x^2)^(1/2)/x^2/(a*x^2+b)*(x^3*a*(-c/d)^(1/2)+Elliptic
F(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))*b*((c*x^2+d)/d)^(1/2)*((a*x^2+b
)/b)^(1/2)-2*EllipticE(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))*b*((c*x^2+
d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)+x*b*(-c/d)^(1/2))*(c*x^2+d)/(-c/d
)^(1/2)/c/((c*x^2+d)/x^2)^(3/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2\sqrt{\frac{ax^2+b}{x^2}}}{(cx^2+d)\sqrt{\frac{cx^2+d}{x^2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^2*sqrt((a*x^2 + b)/x^2)/((c*x^2 + d)*sqrt((c*x^2 + d)/
x^2)), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x**2)**(1/2)/(c+d/x**2)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b/x**2)/(c + d/x**2)**(3/2), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2), x)
```

## 3.175 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx$

**Optimal**. Leaf size=79

$$x\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/2, -p, -q, 1/2, -(b/(a
*x^2)), -(d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

---

**Rubi [A]** time = 0.233549, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$x\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/2, -p, -q, 1/2, -(b/(a
*x^2)), -(d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

---

**Rubi in Sympy [A]** time = 31.4201, size = 63, normalized size = 0.8

$$x\left(1+\frac{b}{ax^2}\right)^{-p}\left(1+\frac{d}{cx^2}\right)^{-q}\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q \operatorname{appellf}_1\left(-\frac{1}{2},-p,-q,\frac{1}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x**2)**p*(c+d/x**2)**q,x)
```

```
[Out] x*(1 + b/(a*x**2))**(-p)*(1 + d/(c*x**2))**(-q)*(a + b/x**2)**p*(
c + d/x**2)**q*appellf1(-1/2, -p, -q, 1/2, -b/(a*x**2), -d/(c*x**
2))
```

---

**Mathematica [B]** time = 0.700548, size = 252, normalized size = 3.19

$$\frac{bdx(2p+2q-3)\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q F_1\left(-p-q+\frac{1}{2};-p,-q;-p\ \cdots\right.}{(2p+2q-1)\left(2x^2\left(adpF_1\left(-p-q+\frac{3}{2};1-p,-q;-p-q+\frac{5}{2};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)+bcqF_1\left(-p-q+\frac{3}{2};-p,1-q;-p-q+\frac{5}{2};-\frac{ax^2}{b}\cdots\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

```
[Out] (b*d*(-3 + 2*p + 2*q)*(a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[1/2
- p - q, -p, -q, 3/2 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((-1 +
 2*p + 2*q)*(b*d*(3 - 2*p - 2*q)*AppellF1[1/2 - p - q, -p, -q, 3/
2 - p - q, -((a*x^2)/b), -((c*x^2)/d)] + 2*x^2*(a*d*p*AppellF1[3/
2 - p - q, 1 - p, -q, 5/2 - p - q, -((a*x^2)/b), -((c*x^2)/d)] +
b*c*q*AppellF1[3/2 - p - q, -p, 1 - q, 5/2 - p - q, -((a*x^2)/b),
 -((c*x^2)/d)])))
```

---

**Maple [F]** time = 0.141, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x^2)^p*(c+d/x^2)^q,x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x^2)^p*(c + d/x^2)^q,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{ax^2+b}{x^2}\right)^p \left(\frac{cx^2+d}{x^2}\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x^2)^p*(c + d/x^2)^q,x, algorithm="fricas")
```

```
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x**2)**p*(c+d/x**2)**q,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x^2)^p*(c + d/x^2)^q,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

## 3.176 $\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx$

**Optimal**. Leaf size=145

$$-\frac{(bc-ad)\log\left(c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right)}{6c^{4/3}d^{2/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3c^{4/3}d^{2/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{c}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}c^{4/3}d^{2/3}}+\frac{ax}{c}$$

```
[Out] (a*x)/c - ((b*c - a*d)*ArcTan[(d^(1/3) - 2*c^(1/3)*x)/(Sqrt[3]*d^
(1/3))])/(Sqrt[3]*c^(4/3)*d^(2/3)) + ((b*c - a*d)*Log[d^(1/3) + c
^(1/3)*x])/(3*c^(4/3)*d^(2/3)) - ((b*c - a*d)*Log[d^(2/3) - c^(1/
3)*d^(1/3)*x + c^(2/3)*x^2])/(6*c^(4/3)*d^(2/3))
```

---

**Rubi [A]** time = 0.257555, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$

$$-\frac{(bc-ad)\log\left(c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right)}{6c^{4/3}d^{2/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3c^{4/3}d^{2/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{c}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}c^{4/3}d^{2/3}}+\frac{ax}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b/x^3)/(c + d/x^3),x]
```

```
[Out] (a*x)/c - ((b*c - a*d)*ArcTan[(d^(1/3) - 2*c^(1/3)*x)/(Sqrt[3]*d^
(1/3))])/(Sqrt[3]*c^(4/3)*d^(2/3)) + ((b*c - a*d)*Log[d^(1/3) + c
^(1/3)*x])/(3*c^(4/3)*d^(2/3)) - ((b*c - a*d)*Log[d^(2/3) - c^(1/
3)*d^(1/3)*x + c^(2/3)*x^2])/(6*c^(4/3)*d^(2/3))
```

---

**Rubi in Sympy [A]** time = 37.5772, size = 134, normalized size = 0.92

$$\frac{ax}{c}-\frac{(ad-bc)\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3c^{\frac{4}{3}}d^{\frac{2}{3}}}+\frac{(ad-bc)\log\left(c^{\frac{2}{3}}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{\frac{2}{3}}\right)}{6c^{\frac{4}{3}}d^{\frac{2}{3}}}+\frac{\sqrt{3}(ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{3}\left(-\frac{2\sqrt[3]{c}x}{3}+\frac{\sqrt[3]{d}}{3}\right)}{\sqrt[3]{d}}\right)}{3c^{\frac{4}{3}}d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b/x**3)/(c+d/x**3),x)
```

```
[Out] a*x/c - (a*d - b*c)*log(c**(1/3)*x + d**(1/3))/(3*c**(4/3)*d**(2/
3)) + (a*d - b*c)*log(c**(2/3)*x**2 - c**(1/3)*d**(1/3)*x + d**(2
```

```
/3))/(6*c**(4/3)*d**(2/3)) + sqrt(3)*(a*d - b*c)*atan(sqrt(3)*(-2
*c**(1/3)*x/3 + d**(1/3)/3)/d**(1/3))/(3*c**(4/3)*d**(2/3))
```

---

**Mathematica [A]** time = 0.165813, size = 129, normalized size = 0.89

$$\frac{-(bc-ad)\log\left(c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right)+2(bc-ad)\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)-2\sqrt{3}(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{c}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)+6a\sqrt[3]{c}d^{2/3}x}{6c^{4/3}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b/x^3)/(c + d/x^3),x]
```

```
[Out] (6*a*c^(1/3)*d^(2/3)*x - 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*c^(
1/3)*x)/d^(1/3))/Sqrt[3]] + 2*(b*c - a*d)*Log[d^(1/3) + c^(1/3)*x
] - (b*c - a*d)*Log[d^(2/3) - c^(1/3)*d^(1/3)*x + c^(2/3)*x^2])/(
6*c^(4/3)*d^(2/3))
```

---

**Maple [A]** time = 0.007, size = 195, normalized size = 1.3

$$\frac{ax}{c}-\frac{ad}{3\,c^2}\ln\left(x+\sqrt[3]{\frac{d}{c}}\right)\left(\frac{d}{c}\right)^{-\frac{2}{3}}+\frac{b}{3\,c}\ln\left(x+\sqrt[3]{\frac{d}{c}}\right)\left(\frac{d}{c}\right)^{-\frac{2}{3}}$$
$$+\frac{ad}{6\,c^2}\ln\left(x^2-x\sqrt[3]{\frac{d}{c}}+\left(\frac{d}{c}\right)^{\frac{2}{3}}\right)\left(\frac{d}{c}\right)^{-\frac{2}{3}}-\frac{b}{6\,c}\ln\left(x^2-x\sqrt[3]{\frac{d}{c}}+\left(\frac{d}{c}\right)^{\frac{2}{3}}\right)\left(\frac{d}{c}\right)^{-\frac{2}{3}}$$
$$-\frac{\sqrt{3}ad}{3\,c^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{d}{c}}}-1\right)\right)\left(\frac{d}{c}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}b}{3\,c}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{d}{c}}}-1\right)\right)\left(\frac{d}{c}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b/x^3)/(c+d/x^3),x)
```

```
[Out] a*x/c-1/3/c^2/(d/c)^(2/3)*ln(x+(d/c)^(1/3))*a*d+1/3/c/(d/c)^(2/3)
*ln(x+(d/c)^(1/3))*b+1/6/c^2/(d/c)^(2/3)*ln(x^2-x*(d/c)^(1/3)+(d/
c)^(2/3))*a*d-1/6/c/(d/c)^(2/3)*ln(x^2-x*(d/c)^(1/3)+(d/c)^(2/3))
*b-1/3/c^2/(d/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/c)^(1/3)*
x-1))*a*d+1/3/c/(d/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/c)^(
1/3)*x-1))*b
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x^3)/(c + d/x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.239295, size = 184, normalized size = 1.27

$$\frac{\sqrt{3}\Big(6\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}ax + \sqrt{3}(bc-ad)\log\left(\left(-cd^2\right)^{\frac{2}{3}}x^2 + \left(-cd^2\right)^{\frac{1}{3}}dx + d^2\right) - 2\sqrt{3}(bc-ad)\log\left(\left(-cd^2\right)^{\frac{1}{3}}x - d\right) + 6(bc-ad \cdots}{18\left(-cd^2\right)^{\frac{1}{3}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x^3)/(c + d/x^3),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(6*sqrt(3)*(-c*d^2)^(1/3)*a*x + sqrt(3)*(b*c - a*d)*
log((-c*d^2)^(2/3)*x^2 + (-c*d^2)^(1/3)*d*x + d^2) - 2*sqrt(3)*(b
*c - a*d)*log((-c*d^2)^(1/3)*x - d) + 6*(b*c - a*d)*arctan(1/3*(2
*sqrt(3)*(-c*d^2)^(1/3)*x + sqrt(3)*d)/d))/((-c*d^2)^(1/3)*c)
```

---

**Sympy [A]** time = 2.22337, size = 71, normalized size = 0.49

$$\frac{ax}{c} + \operatorname{RootSum}\left(27t^3c^4d^2 + a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3, \left(t \mapsto t\log\left(-\frac{3tcd}{ad-bc} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b/x**3)/(c+d/x**3),x)
```

```
[Out] a*x/c + RootSum(27*_t**3*c**4*d**2 + a**3*d**3 - 3*a**2*b*c*d**2
+ 3*a*b**2*c**2*d - b**3*c**3, Lambda(_t, _t*log(-3*_t*c*d/(a*d -
 b*c) + x)))
```

---

**GIAC/XCAS [A]** time = 0.219929, size = 217, normalized size = 1.5

$$\frac{ax}{c} - \frac{(bc-ad)\left(-\frac{d}{c}\right)^{\frac{1}{3}}\ln\left(\left|x-\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right|\right)}{3\,cd} + \frac{\sqrt{3}\left(\left(-c^2d\right)^{\frac{1}{3}}bc-\left(-c^2d\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{c}\right)^{\frac{1}{3}}}\right)}{3\,c^2d} + \frac{\left(\left(-c^2d\right)^{\frac{1}{3}}bc-\left(-c^2d\right)^{\frac{1}{3}}ad\right)\ln\left(x^2+x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\left(-\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + b/x^3)/(c + d/x^3),x, algorithm="giac")
```

```
[Out] a*x/c - 1/3*(b*c - a*d)*(-d/c)^(1/3)*ln(abs(x - (-d/c)^(1/3)))/(c
*d) + 1/3*sqrt(3)*((-c^2*d)^(1/3)*b*c - (-c^2*d)^(1/3)*a*d)*arcta
n(1/3*sqrt(3)*(2*x + (-d/c)^(1/3))/(-d/c)^(1/3))/(c^2*d) + 1/6*((
-c^2*d)^(1/3)*b*c - (-c^2*d)^(1/3)*a*d)*ln(x^2 + x*(-d/c)^(1/3) +
 (-d/c)^(2/3))/(c^2*d)
```

## 3.177 $\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx$

**Optimal**. Leaf size=49

$$\frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}-\frac{2\sqrt{x}(bc-ad)}{d^2}+\frac{bx}{d}$$

```
[Out] (-2*(b*c - a*d)*Sqrt[x])/d^2 + (b*x)/d + (2*c*(b*c - a*d)*Log[c +
 d*Sqrt[x]])/d^3
```

---

**Rubi [A]** time = 0.122266, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}-\frac{2\sqrt{x}(bc-ad)}{d^2}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*Sqrt[x])/(c + d*Sqrt[x]),x]
```

```
[Out] (-2*(b*c - a*d)*Sqrt[x])/d^2 + (b*x)/d + (2*c*(b*c - a*d)*Log[c +
 d*Sqrt[x]])/d^3
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2b\int^{\sqrt{x}} x\,dx}{d}-\frac{2c\,(ad-bc)\log\left(c+d\sqrt{x}\right)}{d^3}+(2ad-2bc)\int^{\sqrt{x}}\frac{1}{d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**(1/2))/(c+d*x**(1/2)),x)
```

```
[Out] 2*b*Integral(x, (x, sqrt(x)))/d - 2*c*(a*d - b*c)*log(c + d*sqrt(
x))/d**3 + (2*a*d - 2*b*c)*Integral(d**(-2), (x, sqrt(x)))
```

---

**Mathematica [A]** time = 0.0436009, size = 51, normalized size = 1.04

$$\frac{2\left(bc^2-acd\right)\log\left(c+d\sqrt{x}\right)}{d^3}+\frac{2\sqrt{x}(ad-bc)}{d^2}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*Sqrt[x])/(c + d*Sqrt[x]),x]
```

```
[Out] (2*(-(b*c) + a*d)*Sqrt[x])/d^2 + (b*x)/d + (2*(b*c^2 - a*c*d)*Log
[c + d*Sqrt[x]])/d^3
```

---

**Maple [A]** time = 0.007, size = 59, normalized size = 1.2

$$\frac{bx}{d} + 2\,\frac{a\sqrt{x}}{d} - 2\,\frac{b\sqrt{x}c}{d^2} - 2\,\frac{c\ln\left(c + d\sqrt{x}\right)a}{d^2} + 2\,\frac{c^2\ln\left(c + d\sqrt{x}\right)b}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^(1/2))/(c+d*x^(1/2)),x)
```

```
[Out] b*x/d+2/d*a*x^(1/2)-2/d^2*b*x^(1/2)*c-2*c/d^2*ln(c+d*x^(1/2))*a+2
*c^2/d^3*ln(c+d*x^(1/2))*b
```

---

**Maxima [A]** time = 1.70584, size = 63, normalized size = 1.29

$$\frac{bdx - 2\,(bc - ad)\sqrt{x}}{d^2} + \frac{2\left(bc^2 - acd\right)\log\left(d\sqrt{x} + c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*sqrt(x) + a)/(d*sqrt(x) + c),x, algorithm="maxima")
```

```
[Out] (b*d*x - 2*(b*c - a*d)*sqrt(x))/d^2 + 2*(b*c^2 - a*c*d)*log(d*sqr
t(x) + c)/d^3
```

---

**Fricas [A]** time = 0.239749, size = 65, normalized size = 1.33

$$\frac{bd^2x + 2\left(bc^2 - acd\right)\log\left(d\sqrt{x} + c\right) - 2\left(bcd - ad^2\right)\sqrt{x}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*sqrt(x) + a)/(d*sqrt(x) + c),x, algorithm="fricas")
```

```
[Out] (b*d^2*x + 2*(b*c^2 - a*c*d)*log(d*sqrt(x) + c) - 2*(b*c*d - a*d^
2)*sqrt(x))/d^3
```

**Sympy [A]** time = 0.817719, size = 82, normalized size = 1.67

$$\begin{cases} -\frac{2ac\log\left(\frac{c}{d}+\sqrt{x}\right)}{d^2} + \frac{2a\sqrt{x}}{d} + \frac{2bc^2\log\left(\frac{c}{d}+\sqrt{x}\right)}{d^3} - \frac{2bc\sqrt{x}}{d^2} + \frac{bx}{d} & \text{for } d \neq 0 \\ \frac{ax+\frac{2bx^{\frac{3}{2}}}{3}}{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**(1/2))/(c+d*x**(1/2)),x)
```

```
[Out] Piecewise((-2*a*c*log(c/d + sqrt(x))/d**2 + 2*a*sqrt(x)/d + 2*b*c
**2*log(c/d + sqrt(x))/d**3 - 2*b*c*sqrt(x)/d**2 + b*x/d, Ne(d, 0
)), ((a*x + 2*b*x**(3/2)/3)/c, True))
```

**GIAC/XCAS [A]** time = 0.215998, size = 66, normalized size = 1.35

$$\frac{bdx - 2\,bc\sqrt{x} + 2\,ad\sqrt{x}}{d^2} + \frac{2\,\left(bc^2 - acd\right)\ln\left(\left|d\sqrt{x} + c\right|\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*sqrt(x) + a)/(d*sqrt(x) + c),x, algorithm="giac")
```

```
[Out] (b*d*x - 2*b*c*sqrt(x) + 2*a*d*sqrt(x))/d^2 + 2*(b*c^2 - a*c*d)*l
n(abs(d*sqrt(x) + c))/d^3
```

## 3.178 $\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}} \, dx$

**Optimal**. Leaf size=26

$$-3x^{2/3} + x + 6\sqrt[3]{x} - 6\log\left(\sqrt[3]{x} + 1\right)$$

```
[Out] 6*x^(1/3) - 3*x^(2/3) + x - 6*Log[1 + x^(1/3)]
```

---

**Rubi [A]** time = 0.0462667, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$-3x^{2/3} + x + 6\sqrt[3]{x} - 6\log\left(\sqrt[3]{x} + 1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^(1/3))/(1 + x^(1/3)),x]
```

```
[Out] 6*x^(1/3) - 3*x^(2/3) + x - 6*Log[1 + x^(1/3)]
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$6\sqrt[3]{x} + x - 6\log\left(\sqrt[3]{x} + 1\right) - 6\int^{\sqrt[3]{x}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+x**(1/3))/(1+x**(1/3)),x)
```

```
[Out] 6*x**(1/3) + x - 6*log(x**(1/3) + 1) - 6*Integral(x, (x, x**(1/3)
))
```

---

**Mathematica [A]** time = 0.0100471, size = 26, normalized size = 1.

$$-3x^{2/3} + x + 6\sqrt[3]{x} - 6\log\left(\sqrt[3]{x} + 1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^(1/3))/(1 + x^(1/3)),x]
```

```
[Out] 6*x^(1/3) - 3*x^(2/3) + x - 6*Log[1 + x^(1/3)]
```

---

**Maple [A]** time = 0.004, size = 21, normalized size = 0.8

$$6\,\sqrt[3]{x}-3\,x^{2/3}+x-6\,\ln\left(1+\sqrt[3]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+x^(1/3))/(1+x^(1/3)),x)
```

```
[Out] 6*x^(1/3)-3*x^(2/3)+x-6*ln(1+x^(1/3))
```

---

**Maxima [A]** time = 1.40036, size = 27, normalized size = 1.04

$$x-3\,x^{\frac{2}{3}}+6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(1/3) - 1)/(x^(1/3) + 1),x, algorithm="maxima")
```

```
[Out] x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

---

**Fricas [A]** time = 0.229914, size = 27, normalized size = 1.04

$$x-3\,x^{\frac{2}{3}}+6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(1/3) - 1)/(x^(1/3) + 1),x, algorithm="fricas")
```

```
[Out] x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

---

**Sympy [A]** time = 0.471564, size = 24, normalized size = 0.92

$$-3x^{\frac{2}{3}}+6\sqrt[3]{x}+x-6\log\left(\sqrt[3]{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+x**(1/3))/(1+x**(1/3)),x)
```

```
[Out] -3*x**(2/3) + 6*x**(1/3) + x - 6*log(x**(1/3) + 1)
```

---

**GIAC/XCAS [A]** time = 0.215143, size = 27, normalized size = 1.04

$$x - 3x^{\frac{2}{3}} + 6x^{\frac{1}{3}} - 6\ln\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(1/3) - 1)/(x^(1/3) + 1),x, algorithm="giac")
```

```
[Out] x - 3*x^(2/3) + 6*x^(1/3) - 6*ln(x^(1/3) + 1)
```

## 3.179 $\int \frac{1+x^{2/3}}{-1+x^{2/3}} \, dx$

**Optimal**. Leaf size=17

$$x + 6\sqrt[3]{x} - 6\tanh^{-1}\left(\sqrt[3]{x}\right)$$

```
[Out] 6*x^(1/3) + x - 6*ArcTanh[x^(1/3)]
```

---

**Rubi [A]** time = 0.0500649, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$

$$x + 6\sqrt[3]{x} - 6\tanh^{-1}\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^(2/3))/(-1 + x^(2/3)),x]

[Out] 6*x^(1/3) + x - 6*ArcTanh[x^(1/3)]
```

---

**Rubi in Sympy [A]** time = 10.3831, size = 15, normalized size = 0.88

$$6\sqrt[3]{x} + x - 6\,\text{atanh}\left(\sqrt[3]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x**(2/3))/(-1+x**(2/3)),x)

[Out] 6*x**(1/3) + x - 6*atanh(x**(1/3))
```

---

**Mathematica [A]** time = 0.0102708, size = 31, normalized size = 1.82

$$x + 6\sqrt[3]{x} + 3\log\left(1 - \sqrt[3]{x}\right) - 3\log\left(\sqrt[3]{x} + 1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^(2/3))/(-1 + x^(2/3)),x]

[Out] 6*x^(1/3) + x + 3*Log[1 - x^(1/3)] - 3*Log[1 + x^(1/3)]
```

---

**Maple [A]** time = 0.005, size = 24, normalized size = 1.4

$$x+6\sqrt[3]{x}+3\ \ln\left(-1+\sqrt[3]{x}\right)-3\ \ln\left(1+\sqrt[3]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x^(2/3))/(-1+x^(2/3)),x)
```

```
[Out] x+6*x^(1/3)+3*ln(-1+x^(1/3))-3*ln(1+x^(1/3))
```

---

**Maxima [A]** time = 1.39064, size = 31, normalized size = 1.82

$$x+6\,x^{\frac{1}{3}}-3\ \log\left(x^{\frac{1}{3}}+1\right)+3\ \log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(2/3) + 1)/(x^(2/3) - 1),x, algorithm="maxima")
```

```
[Out] x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(x^(1/3) - 1)
```

---

**Fricas [A]** time = 0.226716, size = 31, normalized size = 1.82

$$x+6\,x^{\frac{1}{3}}-3\ \log\left(x^{\frac{1}{3}}+1\right)+3\ \log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(2/3) + 1)/(x^(2/3) - 1),x, algorithm="fricas")
```

```
[Out] x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(x^(1/3) - 1)
```

---

**Sympy [A]** time = 0.649011, size = 27, normalized size = 1.59

$$6\sqrt[3]{x}+x+3\log\left(\sqrt[3]{x}-1\right)-3\log\left(\sqrt[3]{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x**(2/3))/(-1+x**(2/3)),x)
```

```
[Out] 6*x**(1/3) + x + 3*log(x**(1/3) - 1) - 3*log(x**(1/3) + 1)
```

---

**GIAC/XCAS [A]** time = 0.216685, size = 32, normalized size = 1.88

$$x + 6\,x^{\frac{1}{3}} - 3\,\ln\left(x^{\frac{1}{3}} + 1\right) + 3\,\ln\left(\left|x^{\frac{1}{3}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(2/3) + 1)/(x^(2/3) - 1),x, algorithm="giac")
```

```
[Out] x + 6*x^(1/3) - 3*ln(x^(1/3) + 1) + 3*ln(abs(x^(1/3) - 1))
```

# 3.180 $\int \frac{-16+x^{3/4}}{16+x^{3/4}} \, dx$

**Optimal**. Leaf size=104

$$x-128\sqrt[4]{x}+\frac{256}{3}\sqrt[3]{2}\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}\right)-\frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -128*x^(1/4) + x - (256*2^(1/3)*ArcTan[(2^(1/3) - x^(1/4))/(2^(1/
3)*Sqrt[3])])/Sqrt[3] + (256*2^(1/3)*Log[2*2^(1/3) + x^(1/4)])/3
- (128*2^(1/3)*Log[4*2^(2/3) - 2*2^(1/3)*x^(1/4) + Sqrt[x]])/3
```

---

**Rubi [A]** time = 0.203454, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529

$$x-128\sqrt[4]{x}+\frac{256}{3}\sqrt[3]{2}\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}\right)-\frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-16 + x^(3/4))/(16 + x^(3/4)),x]
```

```
[Out] -128*x^(1/4) + x - (256*2^(1/3)*ArcTan[(2^(1/3) - x^(1/4))/(2^(1/
3)*Sqrt[3])])/Sqrt[3] + (256*2^(1/3)*Log[2*2^(1/3) + x^(1/4)])/3
- (128*2^(1/3)*Log[4*2^(2/3) - 2*2^(1/3)*x^(1/4) + Sqrt[x]])/3
```

---

**Rubi in Sympy [A]** time = 19.8801, size = 99, normalized size = 0.95

$$-128\sqrt[4]{x}+x+\frac{256\sqrt[3]{2}\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)}{3}-\frac{128\sqrt[3]{2}\log\left(-2\sqrt[3]{2}\sqrt[4]{x}+\sqrt{x}+4\cdot 2^{\frac{2}{3}}\right)}{3}-\frac{256\sqrt[3]{2}\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(-\frac{2^{\frac{2}{3}}\sqrt[4]{x}}{6}+\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-16+x**(3/4))/(16+x**(3/4)),x)
```

```
[Out] -128*x**(1/4) + x + 256*2**(1/3)*log(x**(1/4) + 2*2**(1/3))/3 - 1
28*2**(1/3)*log(-2*2**(1/3)*x**(1/4) + sqrt(x) + 4*2**(2/3))/3 -
256*2**(1/3)*sqrt(3)*atan(sqrt(3)*(-2**(2/3)*x**(1/4)/6 + 1/3))/3
```

**Mathematica [A]** time = 0.0496975, size = 102, normalized size = 0.98

$$x-128\sqrt[4]{x}+\frac{256}{3}\sqrt[3]{2}\log\left(2^{2/3}\sqrt[4]{x}+4\right)-\frac{128}{3}\sqrt[3]{2}\log\left(\sqrt[3]{2}\sqrt{x}-2\ 2^{2/3}\sqrt[4]{x}+8\right)+\frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{2^{2/3}\sqrt[4]{x}-2}{2\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-16 + x^(3/4))/(16 + x^(3/4)),x]
```

```
[Out] -128*x^(1/4) + x + (256*2^(1/3)*ArcTan[(-2 + 2^(2/3)*x^(1/4))/(2*
Sqrt[3])])/Sqrt[3] + (256*2^(1/3)*Log[4 + 2^(2/3)*x^(1/4)])/3 - (
128*2^(1/3)*Log[8 - 2*2^(2/3)*x^(1/4) + 2^(1/3)*Sqrt[x]])/3
```

**Maple [A]** time = 0.009, size = 66, normalized size = 0.6

$$x-128\sqrt[4]{x}+\frac{128\sqrt[3]{16}}{3}\ln\left(\sqrt[4]{x}+\sqrt[3]{16}\right)-\frac{64\sqrt[3]{16}}{3}\ln\left(\sqrt{x}-\sqrt[4]{x}\sqrt[3]{16}+16^{\frac{2}{3}}\right)+\frac{128\sqrt[3]{16}\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{16^{\frac{2}{3}}}{8}\sqrt[4]{x}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-16+x^(3/4))/(16+x^(3/4)),x)
```

```
[Out] x-128*x^(1/4)+128/3*16^(1/3)*ln(x^(1/4)+16^(1/3))-64/3*16^(1/3)*l
n(x^(1/2)-x^(1/4)*16^(1/3)+16^(2/3))+128/3*16^(1/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(1/8*16^(2/3)*x^(1/4)-1))
```

**Maxima [A]** time = 1.64799, size = 90, normalized size = 0.87

$$\frac{256}{3}\sqrt{3}2^{\frac{1}{3}}\arctan\left(-\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}-x^{\frac{1}{4}}\right)\right)-\frac{128}{3}\cdot 2^{\frac{1}{3}}\log\left(\left(4\cdot 2^{\frac{2}{3}}\right)-2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}}+\sqrt{x}\right)+\frac{256}{3}\cdot 2^{\frac{1}{3}}\log\left(\left(2\cdot 2^{\frac{1}{3}}\right)+x^{\frac{1}{4}}\right)+x-128\,x^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(3/4) - 16)/(x^(3/4) + 16),x, algorithm="maxima")
```

```
[Out] 256/3*sqrt(3)*2^(1/3)*arctan(-1/6*sqrt(3)*2^(2/3)*(2^(1/3) - x^(1
/4))) - 128/3*2^(1/3)*log((4*2^(2/3)) - 2*2^(1/3)*x^(1/4) + sqrt(
```

```
x)) + 256/3*2^(1/3)*log((2*2^(1/3)) + x^(1/4)) + x - 128*x^(1/4)
```

---

**Fricas [A]** time = 0.242223, size = 123, normalized size = 1.18

$$-\frac{1}{9}\sqrt{3}\left(128\sqrt{3}2^{\frac{1}{3}}\log\left(4\cdot 2^{\frac{2}{3}}-2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}}+\sqrt{x}\right)-256\sqrt{3}2^{\frac{1}{3}}\log\left(2\cdot 2^{\frac{1}{3}}+x^{\frac{1}{4}}\right)-3\sqrt{3}x-768\cdot 2^{\frac{1}{3}}\arctan\left(-\frac{1}{6}\cdot 2^{\frac{2}{3}}\left(\sqrt{3}2^{\frac{1}{3}}-\sqrt{\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(3/4) - 16)/(x^(3/4) + 16),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*(128*sqrt(3)*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/
4) + sqrt(x)) - 256*sqrt(3)*2^(1/3)*log(2*2^(1/3) + x^(1/4)) - 3*
sqrt(3)*x - 768*2^(1/3)*arctan(-1/6*2^(2/3)*(sqrt(3)*2^(1/3) - sq
rt(3)*x^(1/4))) + 384*sqrt(3)*x^(1/4))
```

---

**Sympy [A]** time = 20.2673, size = 102, normalized size = 0.98

$$-128\sqrt[4]{x}+x+\frac{256\sqrt[3]{2}\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)}{3}-\frac{128\sqrt[3]{2}\log\left(-2\sqrt[3]{2}\sqrt[4]{x}+\sqrt{x}+4\cdot 2^{\frac{2}{3}}\right)}{3}+\frac{256\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}\sqrt[4]{x}}{6}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-16+x**(3/4))/(16+x**(3/4)),x)
```

```
[Out] -128*x**(1/4) + x + 256*2**(1/3)*log(x**(1/4) + 2*2**(1/3))/3 - 1
28*2**(1/3)*log(-2*2**(1/3)*x**(1/4) + sqrt(x) + 4*2**(2/3))/3 +
256*2**(1/3)*sqrt(3)*atan(2**(2/3)*sqrt(3)*x**(1/4)/6 - sqrt(3)/3
)/3
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^(3/4) - 16)/(x^(3/4) + 16),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

## 3.181 $\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}} \, dx$

**Optimal**. Leaf size=30

$$-3x^{2/3}-x-6\sqrt[3]{x}-6\log\left(1-\sqrt[3]{x}\right)$$

```
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

---

**Rubi [A]** time = 0.0549251, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-3x^{2/3}-x-6\sqrt[3]{x}-6\log\left(1-\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
```

```
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-6\sqrt[3]{x}-x-6\log\left(-\sqrt[3]{x}+1\right)-6\int^{\sqrt[3]{x}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+1/x**(1/3))/(-1+1/x**(1/3)),x)
```

```
[Out] -6*x**(1/3) - x - 6*log(-x**(1/3) + 1) - 6*Integral(x, (x, x**(1/
3)))
```

---

**Mathematica [A]** time = 0.0105902, size = 30, normalized size = 1.

$$-3x^{2/3}-x-6\sqrt[3]{x}-6\log\left(1-\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
```

```
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

---

**Maple [A]** time = 0.004, size = 23, normalized size = 0.8

$$-x - 3\,x^{2/3} - 6\,\sqrt[3]{x} - 6\,\ln\left(-1 + \sqrt[3]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+1/x^(1/3))/(-1+1/x^(1/3)),x)

[Out] -x-3*x^(2/3)-6*x^(1/3)-6*ln(-1+x^(1/3))
```

---

**Maxima [A]** time = 1.37639, size = 30, normalized size = 1.

$$-x - 3\,x^{\frac{2}{3}} - 6\,x^{\frac{1}{3}} - 6\,\log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/x^(1/3) + 1)/(1/x^(1/3) - 1),x, algorithm="maxima")

[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

---

**Fricas [A]** time = 0.2368, size = 30, normalized size = 1.

$$-x - 3\,x^{\frac{2}{3}} - 6\,x^{\frac{1}{3}} - 6\,\log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/x^(1/3) + 1)/(1/x^(1/3) - 1),x, algorithm="fricas")

[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

---

**Sympy [A]** time = 0.405284, size = 26, normalized size = 0.87

$$-3x^{\frac{2}{3}} - 6\sqrt[3]{x} - x - 6\log\left(\sqrt[3]{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+1/x**(1/3))/(-1+1/x**(1/3)),x)

[Out] -3*x**(2/3) - 6*x**(1/3) - x - 6*log(x**(1/3) - 1)
```

---

**GIAC/XCAS [A]** time = 0.215655, size = 31, normalized size = 1.03

$$-x - 3\,x^{\frac{2}{3}} - 6\,x^{\frac{1}{3}} - 6\ln\left(\left|x^{\frac{1}{3}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/x^(1/3) + 1)/(1/x^(1/3) - 1),x, algorithm="giac")

[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*ln(abs(x^(1/3) - 1))
```

## 3.182 $\int (a-bx^n)^{3/2} (a+bx^n)^{3/2} \, dx$

**Optimal**. Leaf size=79

$$\frac{a^2x\sqrt{a-bx^n}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

```
[Out] (a^2*x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 1/
(2*n), (2 + n^(-1))/2, (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))
/a^2]
```

---

**Rubi [A]** time = 0.0826049, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{a^2x\sqrt{a-bx^n}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b*x^n)^(3/2)*(a + b*x^n)^(3/2),x]
```

```
[Out] (a^2*x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 1/
(2*n), (2 + n^(-1))/2, (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))
/a^2]
```

---

**Rubi in Sympy [A]** time = 19.6892, size = 66, normalized size = 0.84

$$\frac{a^2x\sqrt{a-bx^n}\sqrt{a+bx^n}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{1}{2n}\\ \frac{n+\frac{1}{2}}{n}\end{matrix}\middle|\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a-b*x**n)**(3/2)*(a+b*x**n)**(3/2),x)
```

```
[Out] a**2*x*sqrt(a - b*x**n)*sqrt(a + b*x**n)*hyper((-3/2, 1/(2*n)), (
(n + 1/2)/n,), b**2*x**(2*n)/a**2)/sqrt(1 - b**2*x**(2*n)/a**2)
```

---

**Mathematica [A]** time = 0.262048, size = 151, normalized size = 1.91

$$\frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\left(\left(a^2-b^2x^{2n}\right)\left(a^2(4n+1)-b^2(n+1)x^{2n}\right)+3a^4n^2\sqrt{1-\frac{b^2x^{2n}}{a^2}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2n};1+\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)\right)}{(n+1)(3n+1)\left(a^2-b^2x^{2n}\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a - b*x^n)^(3/2)*(a + b*x^n)^(3/2),x]
```

```
[Out] (x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*((a^2 - b^2*x^(2*n))*(a^2*(1 +
 4*n) - b^2*(1 + n)*x^(2*n)) + 3*a^4*n^2*Sqrt[1 - (b^2*x^(2*n))/a
^2]*Hypergeometric2F1[1/2, 1/(2*n), 1 + 1/(2*n), (b^2*x^(2*n))/a^
2]))/((1 + n)*(1 + 3*n)*(a^2 - b^2*x^(2*n)))
```

---

**Maple [F]** time = 0.118, size = 0, normalized size = 0.

$$\int (a-bx^n)^{\frac{3}{2}}\,(a+bx^n)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x)
```

```
[Out] int((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n+a)^{\frac{3}{2}}(-bx^n+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

---

**Sympy [F(-1)]**   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a-b*x**n)**(3/2)*(a+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]**   time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^{\frac{3}{2}}(-bx^n + a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2), x)
```

## 3.183 $\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx$

**Optimal**. Leaf size=76

$$\frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

```
[Out] (x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 1/(2*n
), (2 + n^(-1))/2, (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2
]
```

---

**Rubi [A]** time = 0.075931, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a - b*x^n]*Sqrt[a + b*x^n],x]
```

```
[Out] (x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 1/(2*n
), (2 + n^(-1))/2, (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2
]
```

---

**Rubi in Sympy [A]** time = 19.4373, size = 63, normalized size = 0.83

$$\frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{2n}\\ \frac{n+\frac{1}{2}}{n}\end{matrix}\middle|\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a-b*x**n)**(1/2)*(a+b*x**n)**(1/2),x)
```

```
[Out] x*sqrt(a - b*x**n)*sqrt(a + b*x**n)*hyper((-1/2, 1/(2*n)), ((n +
1/2)/n,), b**2*x**(2*n)/a**2)/sqrt(1 - b**2*x**(2*n)/a**2)
```

---

**Mathematica [A]** time = 0.147357, size = 116, normalized size = 1.53

$$\frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\left(a^2n\sqrt{1-\frac{b^2x^{2n}}{a^2}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2n};1+\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)+a^2-b^2x^{2n}\right)}{(n+1)\left(a^2-b^2x^{2n}\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a - b*x^n]*Sqrt[a + b*x^n],x]
```

```
[Out] (x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*(a^2 - b^2*x^(2*n) + a^2*n*Sqr
t[1 - (b^2*x^(2*n))/a^2]*Hypergeometric2F1[1/2, 1/(2*n), 1 + 1/(2
*n), (b^2*x^(2*n))/a^2]))/((1 + n)*(a^2 - b^2*x^(2*n)))
```

---

**Maple [F]** time = 0.068, size = 0, normalized size = 0.

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x)
```

```
[Out] int((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^n+a}\sqrt{-bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a - bx^n}\sqrt{a + bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a-b*x**n)**(1/2)*(a+b*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a - b*x**n)*sqrt(a + b*x**n), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^n + a}\sqrt{-bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a), x)
```

## 3.184 $\int (a-bx^n)^p (a+bx^n)^p \, dx$

**Optimal**. Leaf size=72

$$x(a-bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)$$

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, (2
+ n^(-1))/2, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

---

**Rubi [A]** time = 0.0733766, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$x(a-bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, (2
+ n^(-1))/2, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

---

**Rubi in Sympy [A]** time = 21.4, size = 56, normalized size = 0.78

$$x\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}(a-bx^n)^p(a+bx^n)^p{}_2F_1\left(\left.\begin{matrix}-p,\frac{1}{2n}\\ \frac{n+\frac{1}{2}}{n}\end{matrix}\right|\frac{b^2x^{2n}}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a-b*x**n)**p*(a+b*x**n)**p,x)
```

```
[Out] x*(1 - b**2*x**(2*n)/a**2)**(-p)*(a - b*x**n)**p*(a + b*x**n)**p*
hyper((-p, 1/(2*n)), ((n + 1/2)/n,), b**2*x**(2*n)/a**2)
```

---

**Mathematica [A]** time = 0.126143, size = 72, normalized size = 1.

$$x(a-bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;1+\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a - b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, 1 +
 1/(2*n), (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

---

**Maple [F]** time = 0.242, size = 0, normalized size = 0.

$$\int (a-bx^n)^p (a+bx^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a-b*x^n)^p*(a+b*x^n)^p,x)
```

```
[Out] int((a-b*x^n)^p*(a+b*x^n)^p,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n+a)^p(-bx^n+a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p*(-b*x^n + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^n+a)^p(-bx^n+a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p*(-b*x^n + a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a-bx^n)^p (a+bx^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a-b*x**n)**p*(a+b*x**n)**p,x)
```

```
[Out] Integral((a - b*x**n)**p*(a + b*x**n)**p, x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p (-bx^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p*(-b*x^n + a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

## 3.185 $\int (a+bx^n)(c+dx^n)^4\,dx$

**Optimal**. Leaf size=132

$$\frac{c^3x^{n+1}(4ad+bc)}{n+1}+\frac{2c^2dx^{2n+1}(3ad+2bc)}{2n+1}+\frac{d^3x^{4n+1}(ad+4bc)}{4n+1}+\frac{2cd^2x^{3n+1}(2ad+3bc)}{3n+1}+ac^4x+\frac{bd^4x^{5n+1}}{5n+1}$$

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^(1 + n))/(1 + n) + (2*c^2*d*(2*b*c
 + 3*a*d)*x^(1 + 2*n))/(1 + 2*n) + (2*c*d^2*(3*b*c + 2*a*d)*x^(1
+ 3*n))/(1 + 3*n) + (d^3*(4*b*c + a*d)*x^(1 + 4*n))/(1 + 4*n) + (
b*d^4*x^(1 + 5*n))/(1 + 5*n)
```

---

**Rubi [A]** time = 0.251411, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{c^3x^{n+1}(4ad+bc)}{n+1}+\frac{2c^2dx^{2n+1}(3ad+2bc)}{2n+1}+\frac{d^3x^{4n+1}(ad+4bc)}{4n+1}+\frac{2cd^2x^{3n+1}(2ad+3bc)}{3n+1}+ac^4x+\frac{bd^4x^{5n+1}}{5n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)*(c + d*x^n)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^(1 + n))/(1 + n) + (2*c^2*d*(2*b*c
 + 3*a*d)*x^(1 + 2*n))/(1 + 2*n) + (2*c*d^2*(3*b*c + 2*a*d)*x^(1
+ 3*n))/(1 + 3*n) + (d^3*(4*b*c + a*d)*x^(1 + 4*n))/(1 + 4*n) + (
b*d^4*x^(1 + 5*n))/(1 + 5*n)
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bd^4x^{5n+1}}{5n+1}+c^4\int a\,dx+\frac{c^3x^{n+1}(4ad+bc)}{n+1}+\frac{2c^2dx^{2n+1}(3ad+2bc)}{2n+1}+\frac{2cd^2x^{3n+1}(2ad+3bc)}{3n+1}+\frac{d^3x^{4n+1}(ad+4bc)}{4n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)*(c+d*x**n)**4,x)
```

```
[Out] b*d**4*x**(5*n + 1)/(5*n + 1) + c**4*Integral(a, x) + c**3*x**(n
+ 1)*(4*a*d + b*c)/(n + 1) + 2*c**2*d*x**(2*n + 1)*(3*a*d + 2*b*c
)/(2*n + 1) + 2*c*d**2*x**(3*n + 1)*(2*a*d + 3*b*c)/(3*n + 1) + d
**3*x**(4*n + 1)*(a*d + 4*b*c)/(4*n + 1)
```

---

**Mathematica [A]** time = 0.190337, size = 123, normalized size = 0.93

$$x\left(\frac{c^3x^n(4ad+bc)}{n+1}+\frac{2c^2dx^{2n}(3ad+2bc)}{2n+1}+\frac{d^3x^{4n}(ad+4bc)}{4n+1}+\frac{2cd^2x^{3n}(2ad+3bc)}{3n+1}+ac^4+\frac{bd^4x^{5n}}{5n+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)*(c + d*x^n)^4,x]
```

```
[Out] x*(a*c^4 + (c^3*(b*c + 4*a*d)*x^n)/(1 + n) + (2*c^2*d*(2*b*c + 3*
a*d)*x^(2*n))/(1 + 2*n) + (2*c*d^2*(3*b*c + 2*a*d)*x^(3*n))/(1 +
3*n) + (d^3*(4*b*c + a*d)*x^(4*n))/(1 + 4*n) + (b*d^4*x^(5*n))/(1
 + 5*n))
```

---

**Maple [A]** time = 0.018, size = 138, normalized size = 1.1

$$ac^4x+\frac{bd^4x\left(e^{n\ln(x)}\right)^5}{1+5n}+\frac{c^3(4ad+bc)xe^{n\ln(x)}}{1+n}+\frac{d^3(ad+4bc)x\left(e^{n\ln(x)}\right)^4}{1+4n}+2\frac{cd^2(2ad+3bc)x\left(e^{n\ln(x)}\right)^3}{1+3n}+2\frac{c^2d(3ad+2bc)x\left(e^{n\ln(x)}\right)^2}{1+2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)*(c+d*x^n)^4,x)
```

```
[Out] a*c^4*x+b*d^4/(1+5*n)*x*exp(n*ln(x))^5+c^3*(4*a*d+b*c)/(1+n)*x*ex
p(n*ln(x))+d^3*(a*d+4*b*c)/(1+4*n)*x*exp(n*ln(x))^4+2*c*d^2*(2*a*
d+3*b*c)/(1+3*n)*x*exp(n*ln(x))^3+2*c^2*d*(3*a*d+2*b*c)/(1+2*n)*x
*exp(n*ln(x))^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)*(d*x^n + c)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.262764, size = 711, normalized size = 5.39

$$\frac{\left(24\,bd^4n^4+50\,bd^4n^3+35\,bd^4n^2+10\,bd^4n+bd^4\right)xx^{5\,n}+\left(4\,bcd^3+ad^4+30\left(4\,bcd^3+ad^4\right)n^4+61\left(4\,bcd^3+ad^4\right)n^3+41\,(4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)*(d*x^n + c)^4,x, algorithm="fricas")
```

```
[Out] ((24*b*d^4*n^4 + 50*b*d^4*n^3 + 35*b*d^4*n^2 + 10*b*d^4*n + b*d^4
)*x*x^(5*n) + (4*b*c*d^3 + a*d^4 + 30*(4*b*c*d^3 + a*d^4)*n^4 + 6
1*(4*b*c*d^3 + a*d^4)*n^3 + 41*(4*b*c*d^3 + a*d^4)*n^2 + 11*(4*b*
c*d^3 + a*d^4)*n)*x*x^(4*n) + 2*(3*b*c^2*d^2 + 2*a*c*d^3 + 40*(3*
b*c^2*d^2 + 2*a*c*d^3)*n^4 + 78*(3*b*c^2*d^2 + 2*a*c*d^3)*n^3 + 4
9*(3*b*c^2*d^2 + 2*a*c*d^3)*n^2 + 12*(3*b*c^2*d^2 + 2*a*c*d^3)*n)
*x*x^(3*n) + 2*(2*b*c^3*d + 3*a*c^2*d^2 + 60*(2*b*c^3*d + 3*a*c^2
*d^2)*n^4 + 107*(2*b*c^3*d + 3*a*c^2*d^2)*n^3 + 59*(2*b*c^3*d + 3
*a*c^2*d^2)*n^2 + 13*(2*b*c^3*d + 3*a*c^2*d^2)*n)*x*x^(2*n) + (b*
c^4 + 4*a*c^3*d + 120*(b*c^4 + 4*a*c^3*d)*n^4 + 154*(b*c^4 + 4*a*
c^3*d)*n^3 + 71*(b*c^4 + 4*a*c^3*d)*n^2 + 14*(b*c^4 + 4*a*c^3*d)*
n)*x*x^n + (120*a*c^4*n^5 + 274*a*c^4*n^4 + 225*a*c^4*n^3 + 85*a*
c^4*n^2 + 15*a*c^4*n + a*c^4)*x)/(120*n^5 + 274*n^4 + 225*n^3 + 8
5*n^2 + 15*n + 1)
```

---

**Sympy [A]** time = 9.81876, size = 2744, normalized size = 20.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)*(c+d*x**n)**4,x)
```

```
[Out] Piecewise((a*c**4*x + 4*a*c**3*d*log(x) - 6*a*c**2*d**2/x - 2*a*c
*d**3/x**2 - a*d**4/(3*x**3) + b*c**4*log(x) - 4*b*c**3*d/x - 3*b
*c**2*d**2/x**2 - 4*b*c*d**3/(3*x**3) - b*d**4/(4*x**4), Eq(n, -1
)), (a*c**4*x + 8*a*c**3*d*sqrt(x) + 6*a*c**2*d**2*log(x) - 8*a*c
*d**3/sqrt(x) - a*d**4/x + 2*b*c**4*sqrt(x) + 4*b*c**3*d*log(x) -
 12*b*c**2*d**2/sqrt(x) - 4*b*c*d**3/x - 2*b*d**4/(3*x**(3/2)), E
q(n, -1/2)), (a*c**4*x + 6*a*c**3*d*x**(2/3) + 18*a*c**2*d**2*x**
(1/3) + 4*a*c*d**3*log(x) - 3*a*d**4/x**(1/3) + 3*b*c**4*x**(2/3)
/2 + 12*b*c**3*d*x**(1/3) + 6*b*c**2*d**2*log(x) - 12*b*c*d**3/x*
*(1/3) - 3*b*d**4/(2*x**(2/3)), Eq(n, -1/3)), (a*c**4*x + 16*a*c*
*3*d*x**(3/4)/3 + 12*a*c**2*d**2*sqrt(x) + 16*a*c*d**3*x**(1/4) +
 a*d**4*log(x) + 4*b*c**4*x**(3/4)/3 + 8*b*c**3*d*sqrt(x) + 24*b*
c**2*d**2*x**(1/4) + 4*b*c*d**3*log(x) - 4*b*d**4/x**(1/4), Eq(n,
 -1/4)), (a*c**4*x + 5*a*c**3*d*x**(4/5) + 10*a*c**2*d**2*x**(3/5
) + 10*a*c*d**3*x**(2/5) + 5*a*d**4*x**(1/5) + 5*b*c**4*x**(4/5)/
4 + 20*b*c**3*d*x**(3/5)/3 + 15*b*c**2*d**2*x**(2/5) + 20*b*c*d**
3*x**(1/5) + b*d**4*log(x), Eq(n, -1/5)), (120*a*c**4*n**5*x/(120
*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 274*a*c**4*n*
```

```
*4*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 225*
a*c**4*n**3*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 85*a*c**4*n**2*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 +
 15*n + 1) + 15*a*c**4*n*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n
**2 + 15*n + 1) + a*c**4*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n
**2 + 15*n + 1) + 480*a*c**3*d*n**4*x*x**n/(120*n**5 + 274*n**4 +
 225*n**3 + 85*n**2 + 15*n + 1) + 616*a*c**3*d*n**3*x*x**n/(120*n
**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 284*a*c**3*d*n*
*2*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
 56*a*c**3*d*n*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 +
 15*n + 1) + 4*a*c**3*d*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 +
85*n**2 + 15*n + 1) + 360*a*c**2*d**2*n**4*x*x**(2*n)/(120*n**5 +
 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 642*a*c**2*d**2*n**3
*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
 + 354*a*c**2*d**2*n**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**
3 + 85*n**2 + 15*n + 1) + 78*a*c**2*d**2*n*x*x**(2*n)/(120*n**5 +
 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 6*a*c**2*d**2*x*x**(
2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 160*
a*c*d**3*n**4*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**
2 + 15*n + 1) + 312*a*c*d**3*n**3*x*x**(3*n)/(120*n**5 + 274*n**4
 + 225*n**3 + 85*n**2 + 15*n + 1) + 196*a*c*d**3*n**2*x*x**(3*n)/
(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 48*a*c*d*
*3*n*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
+ 1) + 4*a*c*d**3*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85
*n**2 + 15*n + 1) + 30*a*d**4*n**4*x*x**(4*n)/(120*n**5 + 274*n**
4 + 225*n**3 + 85*n**2 + 15*n + 1) + 61*a*d**4*n**3*x*x**(4*n)/(1
20*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 41*a*d**4*n
**2*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
 1) + 11*a*d**4*n*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85
*n**2 + 15*n + 1) + a*d**4*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*
n**3 + 85*n**2 + 15*n + 1) + 120*b*c**4*n**4*x*x**n/(120*n**5 + 2
74*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 154*b*c**4*n**3*x*x**n
/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 71*b*c**
4*n**2*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 14*b*c**4*n*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2
 + 15*n + 1) + b*c**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85
*n**2 + 15*n + 1) + 240*b*c**3*d*n**4*x*x**(2*n)/(120*n**5 + 274*
n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 428*b*c**3*d*n**3*x*x**(2
*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 236*b
*c**3*d*n**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2
 + 15*n + 1) + 52*b*c**3*d*n*x*x**(2*n)/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + 4*b*c**3*d*x*x**(2*n)/(120*n**5 +
274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 240*b*c**2*d**2*n**4*
x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
+ 468*b*c**2*d**2*n**3*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3
 + 85*n**2 + 15*n + 1) + 294*b*c**2*d**2*n**2*x*x**(3*n)/(120*n**
5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 72*b*c**2*d**2*n*
x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
+ 6*b*c**2*d**2*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n
**2 + 15*n + 1) + 120*b*c*d**3*n**4*x*x**(4*n)/(120*n**5 + 274*n*
*4 + 225*n**3 + 85*n**2 + 15*n + 1) + 244*b*c*d**3*n**3*x*x**(4*n
)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 164*b*c
*d**3*n**2*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 +
 15*n + 1) + 44*b*c*d**3*n*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*
n**3 + 85*n**2 + 15*n + 1) + 4*b*c*d**3*x*x**(4*n)/(120*n**5 + 27
4*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 24*b*d**4*n**4*x*x**(5*
```

```
n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 50*b*d
**4*n**3*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 1
5*n + 1) + 35*b*d**4*n**2*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n
**3 + 85*n**2 + 15*n + 1) + 10*b*d**4*n*x*x**(5*n)/(120*n**5 + 27
4*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + b*d**4*x*x**(5*n)/(120*
n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1), True))
```

---

**GIAC/XCAS [A]** time = 0.228548, size = 1073, normalized size = 8.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)*(d*x^n + c)^4,x, algorithm="giac")
```

```
[Out] (120*a*c^4*n^5*x + 24*b*d^4*n^4*x*e^(5*n*ln(x)) + 120*b*c*d^3*n^4
*x*e^(4*n*ln(x)) + 30*a*d^4*n^4*x*e^(4*n*ln(x)) + 240*b*c^2*d^2*n
^4*x*e^(3*n*ln(x)) + 160*a*c*d^3*n^4*x*e^(3*n*ln(x)) + 240*b*c^3*
d*n^4*x*e^(2*n*ln(x)) + 360*a*c^2*d^2*n^4*x*e^(2*n*ln(x)) + 120*b
*c^4*n^4*x*e^(n*ln(x)) + 480*a*c^3*d*n^4*x*e^(n*ln(x)) + 274*a*c^
4*n^4*x + 50*b*d^4*n^3*x*e^(5*n*ln(x)) + 244*b*c*d^3*n^3*x*e^(4*n
*ln(x)) + 61*a*d^4*n^3*x*e^(4*n*ln(x)) + 468*b*c^2*d^2*n^3*x*e^(3
*n*ln(x)) + 312*a*c*d^3*n^3*x*e^(3*n*ln(x)) + 428*b*c^3*d*n^3*x*e
^(2*n*ln(x)) + 642*a*c^2*d^2*n^3*x*e^(2*n*ln(x)) + 154*b*c^4*n^3*
x*e^(n*ln(x)) + 616*a*c^3*d*n^3*x*e^(n*ln(x)) + 225*a*c^4*n^3*x +
 35*b*d^4*n^2*x*e^(5*n*ln(x)) + 164*b*c*d^3*n^2*x*e^(4*n*ln(x)) +
 41*a*d^4*n^2*x*e^(4*n*ln(x)) + 294*b*c^2*d^2*n^2*x*e^(3*n*ln(x))
 + 196*a*c*d^3*n^2*x*e^(3*n*ln(x)) + 236*b*c^3*d*n^2*x*e^(2*n*ln(
x)) + 354*a*c^2*d^2*n^2*x*e^(2*n*ln(x)) + 71*b*c^4*n^2*x*e^(n*ln(
x)) + 284*a*c^3*d*n^2*x*e^(n*ln(x)) + 85*a*c^4*n^2*x + 10*b*d^4*n
*x*e^(5*n*ln(x)) + 44*b*c*d^3*n*x*e^(4*n*ln(x)) + 11*a*d^4*n*x*e^
(4*n*ln(x)) + 72*b*c^2*d^2*n*x*e^(3*n*ln(x)) + 48*a*c*d^3*n*x*e^(
3*n*ln(x)) + 52*b*c^3*d*n*x*e^(2*n*ln(x)) + 78*a*c^2*d^2*n*x*e^(2
*n*ln(x)) + 14*b*c^4*n*x*e^(n*ln(x)) + 56*a*c^3*d*n*x*e^(n*ln(x))
 + 15*a*c^4*n*x + b*d^4*x*e^(5*n*ln(x)) + 4*b*c*d^3*x*e^(4*n*ln(x
)) + a*d^4*x*e^(4*n*ln(x)) + 6*b*c^2*d^2*x*e^(3*n*ln(x)) + 4*a*c*
d^3*x*e^(3*n*ln(x)) + 4*b*c^3*d*x*e^(2*n*ln(x)) + 6*a*c^2*d^2*x*e
^(2*n*ln(x)) + b*c^4*x*e^(n*ln(x)) + 4*a*c^3*d*x*e^(n*ln(x)) + a*
c^4*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

## 3.186 $\int (a+bx^n)(c+dx^n)^3\,dx$

**Optimal**. Leaf size=99

$$\frac{c^2x^{n+1}(3ad+bc)}{n+1}+\frac{d^2x^{3n+1}(ad+3bc)}{3n+1}+\frac{3cdx^{2n+1}(ad+bc)}{2n+1}+ac^3x+\frac{bd^3x^{4n+1}}{4n+1}$$

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^(1 + n))/(1 + n) + (3*c*d*(b*c + a
*d)*x^(1 + 2*n))/(1 + 2*n) + (d^2*(3*b*c + a*d)*x^(1 + 3*n))/(1 +
 3*n) + (b*d^3*x^(1 + 4*n))/(1 + 4*n)
```

---

**Rubi [A]** time = 0.167138, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{c^2x^{n+1}(3ad+bc)}{n+1}+\frac{d^2x^{3n+1}(ad+3bc)}{3n+1}+\frac{3cdx^{2n+1}(ad+bc)}{2n+1}+ac^3x+\frac{bd^3x^{4n+1}}{4n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)*(c + d*x^n)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^(1 + n))/(1 + n) + (3*c*d*(b*c + a
*d)*x^(1 + 2*n))/(1 + 2*n) + (d^2*(3*b*c + a*d)*x^(1 + 3*n))/(1 +
 3*n) + (b*d^3*x^(1 + 4*n))/(1 + 4*n)
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bd^3x^{4n+1}}{4n+1}+c^3\int a\,dx+\frac{c^2x^{n+1}\left(3ad+bc\right)}{n+1}+\frac{3cdx^{2n+1}\left(ad+bc\right)}{2n+1}+\frac{d^2x^{3n+1}\left(ad+3bc\right)}{3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)*(c+d*x**n)**3,x)
```

```
[Out] b*d**3*x**(4*n + 1)/(4*n + 1) + c**3*Integral(a, x) + c**2*x**(n
+ 1)*(3*a*d + b*c)/(n + 1) + 3*c*d*x**(2*n + 1)*(a*d + b*c)/(2*n
+ 1) + d**2*x**(3*n + 1)*(a*d + 3*b*c)/(3*n + 1)
```

---

**Mathematica [A]** time = 0.144884, size = 92, normalized size = 0.93

$$x\left(\frac{c^2x^n(3ad+bc)}{n+1}+\frac{d^2x^{3n}(ad+3bc)}{3n+1}+\frac{3cdx^{2n}(ad+bc)}{2n+1}+ac^3+\frac{bd^3x^{4n}}{4n+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)*(c + d*x^n)^3,x]
```

```
[Out] x*(a*c^3 + (c^2*(b*c + 3*a*d)*x^n)/(1 + n) + (3*c*d*(b*c + a*d)*x
^(2*n))/(1 + 2*n) + (d^2*(3*b*c + a*d)*x^(3*n))/(1 + 3*n) + (b*d^
3*x^(4*n))/(1 + 4*n))
```

---

**Maple [A]** time = 0.016, size = 104, normalized size = 1.1

$$ac^3x + \frac{bd^3x\left(e^{n\ln(x)}\right)^4}{1+4n} + \frac{c^2(3ad+bc)xe^{n\ln(x)}}{1+n} + \frac{d^2(ad+3bc)x\left(e^{n\ln(x)}\right)^3}{1+3n} + 3\frac{cd(ad+bc)x\left(e^{n\ln(x)}\right)^2}{1+2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)*(c+d*x^n)^3,x)
```

```
[Out] a*c^3*x+b*d^3/(1+4*n)*x*exp(n*ln(x))^4+c^2*(3*a*d+b*c)/(1+n)*x*ex
p(n*ln(x))+d^2*(a*d+3*b*c)/(1+3*n)*x*exp(n*ln(x))^3+3*c*d*(a*d+b*
c)/(1+2*n)*x*exp(n*ln(x))^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)*(d*x^n + c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.24791, size = 431, normalized size = 4.35

$$\frac{\left(6bd^3n^3 + 11bd^3n^2 + 6bd^3n + bd^3\right)xx^{4n} + \left(3bcd^2 + ad^3 + 8\left(3bcd^2 + ad^3\right)n^3 + 14\left(3bcd^2 + ad^3\right)n^2 + 7\left(3bcd^2 + ad^3\right)n\right)x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)*(d*x^n + c)^3,x, algorithm="fricas")
```

```
[Out] ((6*b*d^3*n^3 + 11*b*d^3*n^2 + 6*b*d^3*n + b*d^3)*x*x^(4*n) + (3*
b*c*d^2 + a*d^3 + 8*(3*b*c*d^2 + a*d^3)*n^3 + 14*(3*b*c*d^2 + a*d
^3)*n^2 + 7*(3*b*c*d^2 + a*d^3)*n)*x*x^(3*n) + 3*(b*c^2*d + a*c*d
^2 + 12*(b*c^2*d + a*c*d^2)*n^3 + 19*(b*c^2*d + a*c*d^2)*n^2 + 8*
(b*c^2*d + a*c*d^2)*n)*x*x^(2*n) + (b*c^3 + 3*a*c^2*d + 24*(b*c^3
 + 3*a*c^2*d)*n^3 + 26*(b*c^3 + 3*a*c^2*d)*n^2 + 9*(b*c^3 + 3*a*c
^2*d)*n)*x*x^n + (24*a*c^3*n^4 + 50*a*c^3*n^3 + 35*a*c^3*n^2 + 10
*a*c^3*n + a*c^3)*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

---

**Sympy [A]** time = 5.2599, size = 1540, normalized size = 15.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)*(c+d*x**n)**3,x)
```

```
[Out] Piecewise((a*c**3*x + 3*a*c**2*d*log(x) - 3*a*c*d**2/x - a*d**3/(
2*x**2) + b*c**3*log(x) - 3*b*c**2*d/x - 3*b*c*d**2/(2*x**2) - b*
d**3/(3*x**3), Eq(n, -1)), (a*c**3*x + 6*a*c**2*d*sqrt(x) + 3*a*c
*d**2*log(x) - 2*a*d**3/sqrt(x) + 2*b*c**3*sqrt(x) + 3*b*c**2*d*l
og(x) - 6*b*c*d**2/sqrt(x) - b*d**3/x, Eq(n, -1/2)), (a*c**3*x +
9*a*c**2*d*x**(2/3)/2 + 9*a*c*d**2*x**(1/3) + a*d**3*log(x) + 3*b
*c**3*x**(2/3)/2 + 9*b*c**2*d*x**(1/3) + 3*b*c*d**2*log(x) - 3*b*
d**3/x**(1/3), Eq(n, -1/3)), (a*c**3*x + 4*a*c**2*d*x**(3/4) + 6*
a*c*d**2*sqrt(x) + 4*a*d**3*x**(1/4) + 4*b*c**3*x**(3/4)/3 + 6*b*
c**2*d*sqrt(x) + 12*b*c*d**2*x**(1/4) + b*d**3*log(x), Eq(n, -1/4
)), (24*a*c**3*n**4*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) +
50*a*c**3*n**3*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 35*a*
c**3*n**2*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 10*a*c**3*
n*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + a*c**3*x/(24*n**4
+ 50*n**3 + 35*n**2 + 10*n + 1) + 72*a*c**2*d*n**3*x*x**n/(24*n**
4 + 50*n**3 + 35*n**2 + 10*n + 1) + 78*a*c**2*d*n**2*x*x**n/(24*n
**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 27*a*c**2*d*n*x*x**n/(24*n*
*4 + 50*n**3 + 35*n**2 + 10*n + 1) + 3*a*c**2*d*x*x**n/(24*n**4 +
 50*n**3 + 35*n**2 + 10*n + 1) + 36*a*c*d**2*n**3*x*x**(2*n)/(24*
n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 57*a*c*d**2*n**2*x*x**(2*n
)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 24*a*c*d**2*n*x*x**(
2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 3*a*c*d**2*x*x**(
2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8*a*d**3*n**3*x*x
**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 14*a*d**3*n**2
*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 7*a*d**3*n
*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + a*d**3*x*x
**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 24*b*c**3*n**3
*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 26*b*c**3*n**2
*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 9*b*c**3*n*x*x
**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b*c**3*x*x**n/(24*
n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 36*b*c**2*d*n**3*x*x**(2*n
)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 57*b*c**2*d*n**2*x*x
**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 24*b*c**2*d*n*
x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 3*b*c**2*d*
x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 24*b*c*d**2
```

```
*n**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 42*b*
c*d**2*n**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) +
 21*b*c*d**2*n*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1
) + 3*b*c*d**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1
) + 6*b*d**3*n**3*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n
+ 1) + 11*b*d**3*n**2*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 1
0*n + 1) + 6*b*d**3*n*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 1
0*n + 1) + b*d**3*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n
+ 1), True))
```

---

**GIAC/XCAS [A]** time = 0.224958, size = 656, normalized size = 6.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)*(d*x^n + c)^3,x, algorithm="giac")
```

```
[Out] (24*a*c^3*n^4*x + 6*b*d^3*n^3*x*e^(4*n*ln(x)) + 24*b*c*d^2*n^3*x*
e^(3*n*ln(x)) + 8*a*d^3*n^3*x*e^(3*n*ln(x)) + 36*b*c^2*d*n^3*x*e^
(2*n*ln(x)) + 36*a*c*d^2*n^3*x*e^(2*n*ln(x)) + 24*b*c^3*n^3*x*e^(
n*ln(x)) + 72*a*c^2*d*n^3*x*e^(n*ln(x)) + 50*a*c^3*n^3*x + 11*b*d
^3*n^2*x*e^(4*n*ln(x)) + 42*b*c*d^2*n^2*x*e^(3*n*ln(x)) + 14*a*d^
3*n^2*x*e^(3*n*ln(x)) + 57*b*c^2*d*n^2*x*e^(2*n*ln(x)) + 57*a*c*d
^2*n^2*x*e^(2*n*ln(x)) + 26*b*c^3*n^2*x*e^(n*ln(x)) + 78*a*c^2*d*
n^2*x*e^(n*ln(x)) + 35*a*c^3*n^2*x + 6*b*d^3*n*x*e^(4*n*ln(x)) +
21*b*c*d^2*n*x*e^(3*n*ln(x)) + 7*a*d^3*n*x*e^(3*n*ln(x)) + 24*b*c
^2*d*n*x*e^(2*n*ln(x)) + 24*a*c*d^2*n*x*e^(2*n*ln(x)) + 9*b*c^3*n
*x*e^(n*ln(x)) + 27*a*c^2*d*n*x*e^(n*ln(x)) + 10*a*c^3*n*x + b*d^
3*x*e^(4*n*ln(x)) + 3*b*c*d^2*x*e^(3*n*ln(x)) + a*d^3*x*e^(3*n*ln
(x)) + 3*b*c^2*d*x*e^(2*n*ln(x)) + 3*a*c*d^2*x*e^(2*n*ln(x)) + b*
c^3*x*e^(n*ln(x)) + 3*a*c^2*d*x*e^(n*ln(x)) + a*c^3*x)/(24*n^4 +
50*n^3 + 35*n^2 + 10*n + 1)
```

# 3.187 $\int (a+bx^n)(c+dx^n)^2\,dx$

**Optimal**. Leaf size=70

$$\frac{cx^{n+1}(2ad+bc)}{n+1}+\frac{dx^{2n+1}(ad+2bc)}{2n+1}+ac^2x+\frac{bd^2x^{3n+1}}{3n+1}$$

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^(1 + n))/(1 + n) + (d*(2*b*c + a*d)*
x^(1 + 2*n))/(1 + 2*n) + (b*d^2*x^(1 + 3*n))/(1 + 3*n)
```

---

**Rubi [A]** time = 0.114146, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{cx^{n+1}(2ad+bc)}{n+1}+\frac{dx^{2n+1}(ad+2bc)}{2n+1}+ac^2x+\frac{bd^2x^{3n+1}}{3n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)*(c + d*x^n)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^(1 + n))/(1 + n) + (d*(2*b*c + a*d)*
x^(1 + 2*n))/(1 + 2*n) + (b*d^2*x^(1 + 3*n))/(1 + 3*n)
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bd^2x^{3n+1}}{3n+1}+c^2\int a\,dx+\frac{cx^{n+1}\,(2ad+bc)}{n+1}+\frac{dx^{2n+1}\,(ad+2bc)}{2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)*(c+d*x**n)**2,x)
```

```
[Out] b*d**2*x**(3*n + 1)/(3*n + 1) + c**2*Integral(a, x) + c*x**(n + 1
)*(2*a*d + b*c)/(n + 1) + d*x**(2*n + 1)*(a*d + 2*b*c)/(2*n + 1)
```

---

**Mathematica [A]** time = 0.109143, size = 65, normalized size = 0.93

$$x\left(\frac{dx^{2n}(ad+2bc)}{2n+1}+\frac{cx^n(2ad+bc)}{n+1}+ac^2+\frac{bd^2x^{3n}}{3n+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)*(c + d*x^n)^2,x]
```

```
[Out] x*(a*c^2 + (c*(b*c + 2*a*d)*x^n)/(1 + n) + (d*(2*b*c + a*d)*x^(2*
n))/(1 + 2*n) + (b*d^2*x^(3*n))/(1 + 3*n))
```

---

**Maple [A]** time = 0.014, size = 74, normalized size = 1.1

$$ac^2x + \frac{bd^2x\left(e^{n\ln(x)}\right)^3}{1+3n} + \frac{c(2ad+bc)xe^{n\ln(x)}}{1+n} + \frac{d(ad+2bc)x\left(e^{n\ln(x)}\right)^2}{1+2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)*(c+d*x^n)^2,x)
```

```
[Out] a*c^2*x+b*d^2/(1+3*n)*x*exp(n*ln(x))^3+c*(2*a*d+b*c)/(1+n)*x*exp(
n*ln(x))+d*(a*d+2*b*c)/(1+2*n)*x*exp(n*ln(x))^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)*(d*x^n + c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.257042, size = 236, normalized size = 3.37

$$\frac{\left(2\,bd^2n^2 + 3\,bd^2n + bd^2\right)xx^{3\,n} + \left(2\,bcd + ad^2 + 3\left(2\,bcd + ad^2\right)n^2 + 4\left(2\,bcd + ad^2\right)n\right)xx^{2\,n} + \left(bc^2 + 2\,acd + 6\left(bc^2 + 2\,acd\right)\right.}{6\,n^3 + 11\,n^2 + 6\,n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)*(d*x^n + c)^2,x, algorithm="fricas")
```

```
[Out] ((2*b*d^2*n^2 + 3*b*d^2*n + b*d^2)*x*x^(3*n) + (2*b*c*d + a*d^2 +
 3*(2*b*c*d + a*d^2)*n^2 + 4*(2*b*c*d + a*d^2)*n)*x*x^(2*n) + (b*
c^2 + 2*a*c*d + 6*(b*c^2 + 2*a*c*d)*n^2 + 5*(b*c^2 + 2*a*c*d)*n)*
x*x^n + (6*a*c^2*n^3 + 11*a*c^2*n^2 + 6*a*c^2*n + a*c^2)*x)/(6*n^
```

```
3 + 11*n^2 + 6*n + 1)
```

**Sympy [A]** time = 2.79227, size = 726, normalized size = 10.37

$$\begin{cases} ac^2x + 2acd\log(x) - \frac{ad^2}{x} + bc^2\log(x) - \frac{2bcd}{x} - \frac{bd^2}{2x^2} \\ ac^2x + 4acd\sqrt{x} + ad^2\log(x) + 2bc^2\sqrt{x} + 2bcd\log(x) - \frac{2bd^2}{\sqrt{x}} \\ ac^2x + 3acdx^{\frac{2}{3}} + 3ad^2\sqrt[3]{x} + \frac{3bc^2x^{\frac{2}{3}}}{2} + 6bcd\sqrt[3]{x} + bd^2\log(x) \\ \frac{6ac^2n^3x}{6n^3+11n^2+6n+1} + \frac{11ac^2n^2x}{6n^3+11n^2+6n+1} + \frac{6ac^2nx}{6n^3+11n^2+6n+1} + \frac{ac^2x}{6n^3+11n^2+6n+1} + \frac{12acdn^2xx^n}{6n^3+11n^2+6n+1} + \frac{10acdnxx^n}{6n^3+11n^2+6n+1} + \frac{2acdxx^n}{6n^3+11n^2+6n+1} + \frac{3ad^2n^2xx^{2n}}{6n^3+11n^2+6n+1} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)*(c+d*x**n)**2,x)
```

```
[Out] Piecewise((a*c**2*x + 2*a*c*d*log(x) - a*d**2/x + b*c**2*log(x) -
 2*b*c*d/x - b*d**2/(2*x**2), Eq(n, -1)), (a*c**2*x + 4*a*c*d*sqr
t(x) + a*d**2*log(x) + 2*b*c**2*sqrt(x) + 2*b*c*d*log(x) - 2*b*d*
*2/sqrt(x), Eq(n, -1/2)), (a*c**2*x + 3*a*c*d*x**(2/3) + 3*a*d**2
*x**(1/3) + 3*b*c**2*x**(2/3)/2 + 6*b*c*d*x**(1/3) + b*d**2*log(x
), Eq(n, -1/3)), (6*a*c**2*n**3*x/(6*n**3 + 11*n**2 + 6*n + 1) +
11*a*c**2*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a*c**2*n*x/(6*n
**3 + 11*n**2 + 6*n + 1) + a*c**2*x/(6*n**3 + 11*n**2 + 6*n + 1)
+ 12*a*c*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 10*a*c*d*n*
x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 2*a*c*d*x*x**n/(6*n**3 + 11
*n**2 + 6*n + 1) + 3*a*d**2*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6
*n + 1) + 4*a*d**2*n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + a*
d**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 6*b*c**2*n**2*x*x*
*n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*b*c**2*n*x*x**n/(6*n**3 + 11*
n**2 + 6*n + 1) + b*c**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 6*
b*c*d*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 8*b*c*d*n*x*
x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 2*b*c*d*x*x**(2*n)/(6*n**
3 + 11*n**2 + 6*n + 1) + 2*b*d**2*n**2*x*x**(3*n)/(6*n**3 + 11*n*
*2 + 6*n + 1) + 3*b*d**2*n*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1
) + b*d**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1), True))
```

**GIAC/XCAS [A]** time = 0.221124, size = 342, normalized size = 4.89

$$\frac{6\,ac^2n^3x + 2\,bd^2n^2xe^{(3\,n\ln(x))} + 6\,bcdn^2xe^{(2\,n\ln(x))} + 3\,ad^2n^2xe^{(2\,n\ln(x))} + 6\,bc^2n^2xe^{(n\ln(x))} + 12\,acdn^2xe^{(n\ln(x))} + 11\,ac^2n^2x + 3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)*(d*x^n + c)^2,x, algorithm="giac")
```

```
[Out] (6*a*c^2*n^3*x + 2*b*d^2*n^2*x*e^(3*n*ln(x)) + 6*b*c*d*n^2*x*e^(2
*n*ln(x)) + 3*a*d^2*n^2*x*e^(2*n*ln(x)) + 6*b*c^2*n^2*x*e^(n*ln(x
```

```
)) + 12*a*c*d*n^2*x*e^(n*ln(x)) + 11*a*c^2*n^2*x + 3*b*d^2*n*x*e^
(3*n*ln(x)) + 8*b*c*d*n*x*e^(2*n*ln(x)) + 4*a*d^2*n*x*e^(2*n*ln(x
)) + 5*b*c^2*n*x*e^(n*ln(x)) + 10*a*c*d*n*x*e^(n*ln(x)) + 6*a*c^2
*n*x + b*d^2*x*e^(3*n*ln(x)) + 2*b*c*d*x*e^(2*n*ln(x)) + a*d^2*x*
e^(2*n*ln(x)) + b*c^2*x*e^(n*ln(x)) + 2*a*c*d*x*e^(n*ln(x)) + a*c
^2*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

## 3.188 $\int (a+bx^n)(c+dx^n)\,dx$

**Optimal**. Leaf size=40

$$\frac{x^{n+1}(ad+bc)}{n+1}+acx+\frac{bdx^{2n+1}}{2n+1}$$

```
[Out] a*c*x + ((b*c + a*d)*x^(1 + n))/(1 + n) + (b*d*x^(1 + 2*n))/(1 +
2*n)
```

---

**Rubi [A]** time = 0.05853, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{x^{n+1}(ad+bc)}{n+1}+acx+\frac{bdx^{2n+1}}{2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^(1 + n))/(1 + n) + (b*d*x^(1 + 2*n))/(1 +
2*n)
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bdx^{2n+1}}{2n+1}+c\int a\,dx+\frac{x^{n+1}\,(ad+bc)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a+b*x**n)*(c+d*x**n),x)
```

```
[Out] b*d*x**(2*n + 1)/(2*n + 1) + c*Integral(a, x) + x**(n + 1)*(a*d +
 b*c)/(n + 1)
```

---

**Mathematica [A]** time = 0.0829009, size = 37, normalized size = 0.92

$$x\left(\frac{x^n(ad+bc)}{n+1}+ac+\frac{bdx^{2n}}{2n+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)*(c + d*x^n),x]

[Out] x*(a*c + ((b*c + a*d)*x^n)/(1 + n) + (b*d*x^(2*n))/(1 + 2*n))
```

---

**Maple [A]** time = 0.012, size = 43, normalized size = 1.1

$$acx + \frac{(ad+bc)\,x\mathrm{e}^{n\ln(x)}}{1+n} + \frac{bdx\left(\mathrm{e}^{n\ln(x)}\right)^2}{1+2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)*(c+d*x^n),x)

[Out] a*c*x+(a*d+b*c)/(1+n)*x*exp(n*ln(x))+b*d/(1+2*n)*x*exp(n*ln(x))^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)*(d*x^n + c),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.249835, size = 93, normalized size = 2.32

$$\frac{(bdn+bd)xx^{2\,n} + (bc+ad+2\,(bc+ad)n)xx^{n} + \left(2\,acn^2+3\,acn+ac\right)x}{2\,n^2+3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)*(d*x^n + c),x, algorithm="fricas")

[Out] ((b*d*n + b*d)*x*x^(2*n) + (b*c + a*d + 2*(b*c + a*d)*n)*x*x^n +
(2*a*c*n^2 + 3*a*c*n + a*c)*x)/(2*n^2 + 3*n + 1)
```

---

**Sympy [A]** time = 1.31153, size = 236, normalized size = 5.9

$$\begin{cases} acx + ad\log(x) + bc\log(x) - \frac{bd}{x} & \text{for } n = -1 \\ acx + 2ad\sqrt{x} + 2bc\sqrt{x} + bd\log(x) & \text{for } n = -\frac{1}{2} \\ \frac{2acn^2x}{2n^2+3n+1} + \frac{3acnx}{2n^2+3n+1} + \frac{acx}{2n^2+3n+1} + \frac{2adnxx^n}{2n^2+3n+1} + \frac{adxx^n}{2n^2+3n+1} + \frac{2bcnxx^n}{2n^2+3n+1} + \frac{bcxx^n}{2n^2+3n+1} + \frac{bdnxx^{2n}}{2n^2+3n+1} + \frac{bdxx^{2n}}{2n^2+3n+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)*(c+d*x**n),x)
```

```
[Out] Piecewise((a*c*x + a*d*log(x) + b*c*log(x) - b*d/x, Eq(n, -1)), (
a*c*x + 2*a*d*sqrt(x) + 2*b*c*sqrt(x) + b*d*log(x), Eq(n, -1/2)),
 (2*a*c*n**2*x/(2*n**2 + 3*n + 1) + 3*a*c*n*x/(2*n**2 + 3*n + 1)
+ a*c*x/(2*n**2 + 3*n + 1) + 2*a*d*n*x*x**n/(2*n**2 + 3*n + 1) +
a*d*x*x**n/(2*n**2 + 3*n + 1) + 2*b*c*n*x*x**n/(2*n**2 + 3*n + 1)
 + b*c*x*x**n/(2*n**2 + 3*n + 1) + b*d*n*x*x**(2*n)/(2*n**2 + 3*n
 + 1) + b*d*x*x**(2*n)/(2*n**2 + 3*n + 1), True))
```

---

**GIAC/XCAS [A]** time = 0.213836, size = 126, normalized size = 3.15

$$\frac{2\,acn^2x + bdnxe^{(2\,n\ln(x))} + 2\,bcnxe^{(n\ln(x))} + 2\,adnxe^{(n\ln(x))} + 3\,acnx + bdxe^{(2\,n\ln(x))} + bcxe^{(n\ln(x))} + adxe^{(n\ln(x))} + acx}{2\,n^2 + 3\,n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)*(d*x^n + c),x, algorithm="giac")
```

```
[Out] (2*a*c*n^2*x + b*d*n*x*e^(2*n*ln(x)) + 2*b*c*n*x*e^(n*ln(x)) + 2*
a*d*n*x*e^(n*ln(x)) + 3*a*c*n*x + b*d*x*e^(2*n*ln(x)) + b*c*x*e^(
n*ln(x)) + a*d*x*e^(n*ln(x)) + a*c*x)/(2*n^2 + 3*n + 1)
```

# 3.189 $\int \frac{a+bx^n}{c+dx^n} dx$

**Optimal**. Leaf size=43

$$\frac{bx}{d} - \frac{x(bc-ad)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd}$$

```
[Out] (b*x)/d - ((b*c - a*d)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
 -((d*x^n)/c)])/(c*d)
```

---

**Rubi [A]** time = 0.0551257, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{bx}{d} - \frac{x(bc-ad)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)/(c + d*x^n),x]
```

```
[Out] (b*x)/d - ((b*c - a*d)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
 -((d*x^n)/c)])/(c*d)
```

---

**Rubi in Sympy [A]** time = 7.20277, size = 31, normalized size = 0.72

$$\frac{bx}{d} + \frac{x\,(ad-bc)\,{}_2F_1\left(\begin{matrix} 1,\frac{1}{n} \\ 1+\frac{1}{n} \end{matrix}\middle| -\frac{dx^n}{c}\right)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)/(c+d*x**n),x)
```

```
[Out] b*x/d + x*(a*d - b*c)*hyper((1, 1/n), (1 + 1/n,), -d*x**n/c)/(c*d
)
```

---

**Mathematica [A]** time = 0.0369414, size = 40, normalized size = 0.93

$$\frac{x\left((ad-bc)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+bc\right)}{cd}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)/(c + d*x^n),x]
```

```
[Out] (x*(b*c + (-(b*c) + a*d)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
 -((d*x^n)/c)]))/(c*d)
```

---

**Maple [F]** time = 0.083, size = 0, normalized size = 0.

$$\int \frac{a + bx^n}{c + dx^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)/(c+d*x^n),x)
```

```
[Out] int((a+b*x^n)/(c+d*x^n),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-(bc - ad) \int \frac{1}{d^2 x^n + cd}\, dx + \frac{bx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)/(d*x^n + c),x, algorithm="maxima")
```

```
[Out] -(b*c - a*d)*integrate(1/(d^2*x^n + c*d), x) + b*x/d
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^n + a}{dx^n + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)/(d*x^n + c),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)/(d*x^n + c), x)
```

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)/(c+d*x**n),x)

[Out] Exception raised: TypeError
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^n + a}{dx^n + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)/(d*x^n + c),x, algorithm="giac")

[Out] integrate((b*x^n + a)/(d*x^n + c), x)
```

## 3.190 $\int \frac{a+bx^n}{(c+dx^n)^2}\,dx$

**Optimal**. Leaf size=73

$$\frac{x(bc-ad(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2dn}-\frac{x(bc-ad)}{cdn\,(c+dx^n)}$$

```
[Out] -(((b*c - a*d)*x)/(c*d*n*(c + d*x^n))) + ((b*c - a*d*(1 - n))*x*H
ypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*d*n)
```

---

**Rubi [A]** time = 0.0910764, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{x(bc-ad(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2dn}-\frac{x(bc-ad)}{cdn\,(c+dx^n)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)/(c + d*x^n)^2,x]
```

```
[Out] -(((b*c - a*d)*x)/(c*d*n*(c + d*x^n))) + ((b*c - a*d*(1 - n))*x*H
ypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*d*n)
```

---

**Rubi in Sympy [A]** time = 9.66263, size = 53, normalized size = 0.73

$$\frac{x\,(ad-bc)}{cdn\,(c+dx^n)}+\frac{x\,(-ad\,(-n+1)+bc)\,{}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{dx^n}{c}\right)}{c^2dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)/(c+d*x**n)**2,x)
```

```
[Out] x*(a*d - b*c)/(c*d*n*(c + d*x**n)) + x*(-a*d*(-n + 1) + b*c)*hype
r((1, 1/n), (1 + 1/n,), -d*x**n/c)/(c**2*d*n)
```

---

**Mathematica [A]** time = 0.0790748, size = 68, normalized size = 0.93

$$\frac{x\left((c+dx^n)\,(ad(n-1)+bc)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+c(ad-bc)\right)}{c^2dn\,(c+dx^n)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)/(c + d*x^n)^2,x]
```

```
[Out] (x*(c*(-(b*c) + a*d) + (b*c + a*d*(-1 + n))*(c + d*x^n)*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(c^2*d*n*(c + d*x
^n))
```

---

**Maple [F]** time = 0.064, size = 0, normalized size = 0.

$$\int \frac{a + bx^n}{(c + dx^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)/(c+d*x^n)^2,x)
```

```
[Out] int((a+b*x^n)/(c+d*x^n)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$(ad(n-1) + bc)\int \frac{1}{cd^2nx^n + c^2dn}\, dx - \frac{(bc - ad)x}{cd^2nx^n + c^2dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)/(d*x^n + c)^2,x, algorithm="maxima")
```

```
[Out] (a*d*(n - 1) + b*c)*integrate(1/(c*d^2*n*x^n + c^2*d*n), x) - (b*
c - a*d)*x/(c*d^2*n*x^n + c^2*d*n)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^n + a}{d^2x^{2n} + 2\,cdx^n + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)/(d*x^n + c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)
```

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)/(c+d*x**n)**2,x)
```

```
[Out] Exception raised: TypeError
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^n + a}{(dx^n + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)/(d*x^n + c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)/(d*x^n + c)^2, x)
```

## 3.191 $\int \frac{a+bx^n}{(c+dx^n)^3}\,dx$

**Optimal**. Leaf size=78

$$\frac{x(bc-ad(1-2n))\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3dn}-\frac{x(bc-ad)}{2cdn\,(c+dx^n)^2}$$

```
[Out] -((b*c - a*d)*x)/(2*c*d*n*(c + d*x^n)^2) + ((b*c - a*d*(1 - 2*n))
*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3
*d*n)
```

---

**Rubi [A]** time = 0.092494, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{x(bc-ad(1-2n))\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3dn}-\frac{x(bc-ad)}{2cdn\,(c+dx^n)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)/(c + d*x^n)^3,x]
```

```
[Out] -((b*c - a*d)*x)/(2*c*d*n*(c + d*x^n)^2) + ((b*c - a*d*(1 - 2*n))
*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3
*d*n)
```

---

**Rubi in Sympy [A]** time = 10.1555, size = 60, normalized size = 0.77

$$\frac{x\,(ad-bc)}{2cdn\,(c+dx^n)^2}+\frac{x\,(-ad\,(-2n+1)+bc)\,{}_2F_1\left(\begin{matrix}2,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{dx^n}{c}\right)}{2c^3dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)/(c+d*x**n)**3,x)
```

```
[Out] x*(a*d - b*c)/(2*c*d*n*(c + d*x**n)**2) + x*(-a*d*(-2*n + 1) + b*
c)*hyper((2, 1/n), (1 + 1/n,), -d*x**n/c)/(2*c**3*d*n)
```

---

**Mathematica [A]** time = 0.107669, size = 96, normalized size = 1.23

$$\frac{x\left(-\frac{c^2n(bc-ad)}{(c+dx^n)^2}+(n-1)(ad(2n-1)+bc)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+\frac{c(ad(2n-1)+bc)}{c+dx^n}\right)}{2c^3dn^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)/(c + d*x^n)^3,x]
```

```
[Out] (x*(-((c^2*(b*c - a*d)*n)/(c + d*x^n)^2) + (c*(b*c + a*d*(-1 + 2*
n)))/(c + d*x^n) + (-1 + n)*(b*c + a*d*(-1 + 2*n))*Hypergeometric
2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(2*c^3*d*n^2)
```

---

**Maple [F]** time = 0.076, size = 0, normalized size = 0.

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)/(c+d*x^n)^3,x)
```

```
[Out] int((a+b*x^n)/(c+d*x^n)^3,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left(\left(2\,n^2-3\,n+1\right)ad+bc(n-1)\right)\int \frac{1}{2\left(c^2d^2n^2x^n+c^3dn^2\right)}\,dx + \frac{\left(ad^2(2\,n-1)+bcd\right)xx^n+\left(acd(3\,n-1)-bc^2(n-1)\right)x}{2\left(c^2d^3n^2x^{2\,n}+2\,c^3d^2n^2x^n+c^4dn^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)/(d*x^n + c)^3,x, algorithm="maxima")
```

```
[Out] ((2*n^2 - 3*n + 1)*a*d + b*c*(n - 1))*integrate(1/2/(c^2*d^2*n^2*
x^n + c^3*d*n^2), x) + 1/2*((a*d^2*(2*n - 1) + b*c*d)*x*x^n + (a*
c*d*(3*n - 1) - b*c^2*(n - 1))*x)/(c^2*d^3*n^2*x^(2*n) + 2*c^3*d^
2*n^2*x^n + c^4*d*n^2)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^n + a}{d^3x^{3n} + 3\,cd^2x^{2n} + 3\,c^2dx^n + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)/(d*x^n + c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n
 + c^3), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)/(c+d*x**n)**3,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^n + a}{(dx^n + c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)/(d*x^n + c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)/(d*x^n + c)^3, x)
```

## 3.192 $\int \frac{a+bx^n}{(c+dx^n)^4} dx$

**Optimal**. Leaf size=78

$$\frac{x(bc-ad(1-3n))\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{3c^4dn}-\frac{x(bc-ad)}{3cdn\,(c+dx^n)^3}$$

```
[Out] -((b*c - a*d)*x)/(3*c*d*n*(c + d*x^n)^3) + ((b*c - a*d*(1 - 3*n))
*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(3*c^4
*d*n)
```

---

**Rubi [A]** time = 0.0904342, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{x(bc-ad(1-3n))\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{3c^4dn}-\frac{x(bc-ad)}{3cdn\,(c+dx^n)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)/(c + d*x^n)^4,x]
```

```
[Out] -((b*c - a*d)*x)/(3*c*d*n*(c + d*x^n)^3) + ((b*c - a*d*(1 - 3*n))
*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(3*c^4
*d*n)
```

---

**Rubi in Sympy [A]** time = 10.1421, size = 60, normalized size = 0.77

$$\frac{x\,(ad-bc)}{3cdn\,(c+dx^n)^3}+\frac{x\,(-ad\,(-3n+1)+bc)\,{}_2F_1\left(\begin{matrix}3,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{dx^n}{c}\right)}{3c^4dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)/(c+d*x**n)**4,x)
```

```
[Out] x*(a*d - b*c)/(3*c*d*n*(c + d*x**n)**3) + x*(-a*d*(-3*n + 1) + b*
c)*hyper((3, 1/n), (1 + 1/n,), -d*x**n/c)/(3*c**4*d*n)
```

---

**Mathematica [A]** time = 0.165302, size = 136, normalized size = 1.74

$$\frac{x\left(-\frac{2c^3n^2(bc-ad)}{(c+dx^n)^3}+\frac{c^2n(ad(3n-1)+bc)}{(c+dx^n)^2}+\left(2n^2-3n+1\right)(ad(3n-1)+bc)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+\frac{c(2n-1)(ad(3n-1)+bc)}{c+dx^n}\right)}{6c^4dn^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)/(c + d*x^n)^4,x]
```

```
[Out] (x*((-2*c^3*(b*c - a*d)*n^2)/(c + d*x^n)^3 + (c^2*n*(b*c + a*d*(-
1 + 3*n)))/(c + d*x^n)^2 + (c*(-1 + 2*n)*(b*c + a*d*(-1 + 3*n)))/
(c + d*x^n) + (1 - 3*n + 2*n^2)*(b*c + a*d*(-1 + 3*n))*Hypergeome
tric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(6*c^4*d*n^3)
```

---

**Maple [F]** time = 0.093, size = 0, normalized size = 0.

$$\int \frac{a+bx^n}{(c+dx^n)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)/(c+d*x^n)^4,x)
```

```
[Out] int((a+b*x^n)/(c+d*x^n)^4,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left(\left(2\,n^2-3\,n+1\right)bc+\left(6\,n^3-11\,n^2+6\,n-1\right)ad\right)\int \frac{1}{6\,(c^3d^2n^3x^n+c^4dn^3)}\,dx$$
$$+\frac{\left(\left(6\,n^2-5\,n+1\right)ad^3+bcd^2(2\,n-1)\right)xx^{2\,n}+\left(\left(15\,n^2-11\,n+2\right)acd^2+bc^2d(5\,n-2)\right)xx^n-\left(\left(2\,n^2-3\,n+1\right)bc^3-\left(11\,n^2-\right.\right.}{6\,(c^3d^4n^3x^{3\,n}+3\,c^4d^3n^3x^{2\,n}+3\,c^5d^2n^3x^n+c^6dn^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)/(d*x^n + c)^4,x, algorithm="maxima")
```

```
[Out] ((2*n^2 - 3*n + 1)*b*c + (6*n^3 - 11*n^2 + 6*n - 1)*a*d)*integrat
e(1/6/(c^3*d^2*n^3*x^n + c^4*d*n^3), x) + 1/6*(((6*n^2 - 5*n + 1)
*a*d^3 + b*c*d^2*(2*n - 1))*x*x^(2*n) + ((15*n^2 - 11*n + 2)*a*c*
d^2 + b*c^2*d*(5*n - 2))*x*x^n - ((2*n^2 - 3*n + 1)*b*c^3 - (11*n
^2 - 6*n + 1)*a*c^2*d)*x)/(c^3*d^4*n^3*x^(3*n) + 3*c^4*d^3*n^3*x^
(2*n) + 3*c^5*d^2*n^3*x^n + c^6*d*n^3)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^n + a}{d^4x^{4\,n} + 4\,cd^3x^{3\,n} + 6\,c^2d^2x^{2\,n} + 4\,c^3dx^n + c^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)/(d*x^n + c)^4,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)/(d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x
^(2*n) + 4*c^3*d*x^n + c^4), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)/(c+d*x**n)**4,x)
```

```
[Out] Timed out
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^n + a}{(dx^n + c)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)/(d*x^n + c)^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)/(d*x^n + c)^4, x)
```

# 3.193 $\int (a+bx^n)^2 (d+ex^n)^3 \, dx$

**Optimal**. Leaf size=158

$$\frac{dx^{2n+1}\left(3a^2e^2+6abde+b^2d^2\right)}{2n+1}+\frac{ex^{3n+1}\left(a^2e^2+6abde+3b^2d^2\right)}{3n+1}+a^2d^3x+\frac{ad^2x^{n+1}(3ae+2bd)}{n+1}+\frac{be^2x^{4n+1}(2ae+3bd)}{4n+1}+\frac{b^2e^3x^{5n+1}}{5n+1}$$

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^(1 + n))/(1 + n) + (d*(b^2*d
^2 + 6*a*b*d*e + 3*a^2*e^2)*x^(1 + 2*n))/(1 + 2*n) + (e*(3*b^2*d^
2 + 6*a*b*d*e + a^2*e^2)*x^(1 + 3*n))/(1 + 3*n) + (b*e^2*(3*b*d +
 2*a*e)*x^(1 + 4*n))/(1 + 4*n) + (b^2*e^3*x^(1 + 5*n))/(1 + 5*n)
```

---

**Rubi [A]** time = 0.291011, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{dx^{2n+1}\left(3a^2e^2+6abde+b^2d^2\right)}{2n+1}+\frac{ex^{3n+1}\left(a^2e^2+6abde+3b^2d^2\right)}{3n+1}+a^2d^3x+\frac{ad^2x^{n+1}(3ae+2bd)}{n+1}+\frac{be^2x^{4n+1}(2ae+3bd)}{4n+1}+\frac{b^2e^3x^{5n+1}}{5n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)^2*(d + e*x^n)^3,x]
```

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^(1 + n))/(1 + n) + (d*(b^2*d
^2 + 6*a*b*d*e + 3*a^2*e^2)*x^(1 + 2*n))/(1 + 2*n) + (e*(3*b^2*d^
2 + 6*a*b*d*e + a^2*e^2)*x^(1 + 3*n))/(1 + 3*n) + (b*e^2*(3*b*d +
 2*a*e)*x^(1 + 4*n))/(1 + 4*n) + (b^2*e^3*x^(1 + 5*n))/(1 + 5*n)
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{ad^2x^{n+1}\left(3ae+2bd\right)}{n+1}+\frac{b^2e^3x^{5n+1}}{5n+1}+\frac{be^2x^{4n+1}\left(2ae+3bd\right)}{4n+1}+d^3\int a^2\,dx+\frac{dx^{2n+1}\left(3ae\left(ae+2bd\right)+b^2d^2\right)}{2n+1}+\frac{ex^{3n+1}\left(a^2e^2+3bd\left(2ae+bd\right)\right)}{3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)**2*(d+e*x**n)**3,x)
```

```
[Out] a*d**2*x**(n + 1)*(3*a*e + 2*b*d)/(n + 1) + b**2*e**3*x**(5*n + 1
)/(5*n + 1) + b*e**2*x**(4*n + 1)*(2*a*e + 3*b*d)/(4*n + 1) + d**
```

```
3*Integral(a**2, x) + d*x**(2*n + 1)*(3*a*e*(a*e + 2*b*d) + b**2*
d**2)/(2*n + 1) + e*x**(3*n + 1)*(a**2*e**2 + 3*b*d*(2*a*e + b*d)
)/(3*n + 1)
```

---

**Mathematica [A]** time = 0.251017, size = 149, normalized size = 0.94

$$x\left(\frac{dx^{2n}\left(3a^2e^2+6abde+b^2d^2\right)}{2n+1}+\frac{ex^{3n}\left(a^2e^2+6abde+3b^2d^2\right)}{3n+1}+a^2d^3+\frac{ad^2x^n(3ae+2bd)}{n+1}+\frac{be^2x^{4n}(2ae+3bd)}{4n+1}+\frac{b^2e^3x^{5n}}{5n+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)^2*(d + e*x^n)^3,x]
```

```
[Out] x*(a^2*d^3 + (a*d^2*(2*b*d + 3*a*e)*x^n)/(1 + n) + (d*(b^2*d^2 +
6*a*b*d*e + 3*a^2*e^2)*x^(2*n))/(1 + 2*n) + (e*(3*b^2*d^2 + 6*a*b
*d*e + a^2*e^2)*x^(3*n))/(1 + 3*n) + (b*e^2*(3*b*d + 2*a*e)*x^(4*
n))/(1 + 4*n) + (b^2*e^3*x^(5*n))/(1 + 5*n))
```

---

**Maple [A]** time = 0.019, size = 164, normalized size = 1.

$$a^2d^3x+\frac{b^2e^3x\left(e^{n\ln(x)}\right)^5}{1+5n}+\frac{d\left(3a^2e^2+6abde+b^2d^2\right)x\left(e^{n\ln(x)}\right)^2}{1+2n}+\frac{e\left(a^2e^2+6abde+3b^2d^2\right)x\left(e^{n\ln(x)}\right)^3}{1+3n}+\frac{ad^2\left(3ae+2bd\right)xe^{n\ln(x)}}{1+n}+\frac{be^2\left(2ae+3bd\right)x\left(e^{n\ln(x)}\right)^4}{1+4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)^2*(d+e*x^n)^3,x)
```

```
[Out] a^2*d^3*x+b^2*e^3/(1+5*n)*x*exp(n*ln(x))^5+d*(3*a^2*e^2+6*a*b*d*e
+b^2*d^2)/(1+2*n)*x*exp(n*ln(x))^2+e*(a^2*e^2+6*a*b*d*e+3*b^2*d^2
)/(1+3*n)*x*exp(n*ln(x))^3+a*d^2*(3*a*e+2*b*d)/(1+n)*x*exp(n*ln(x
))+b*e^2*(2*a*e+3*b*d)/(1+4*n)*x*exp(n*ln(x))^4
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2*(e*x^n + d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.253386, size = 900, normalized size = 5.7

$$\frac{\left(24\,b^2e^3n^4 + 50\,b^2e^3n^3 + 35\,b^2e^3n^2 + 10\,b^2e^3n + b^2e^3\right)xx^{5\,n} + \left(3\,b^2de^2 + 2\,abe^3 + 30\,\left(3\,b^2de^2 + 2\,abe^3\right)n^4 + 61\,\left(3\,b^2de^2 + 2\,a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2*(e*x^n + d)^3,x, algorithm="fricas")
```

```
[Out] ((24*b^2*e^3*n^4 + 50*b^2*e^3*n^3 + 35*b^2*e^3*n^2 + 10*b^2*e^3*n
 + b^2*e^3)*x*x^(5*n) + (3*b^2*d*e^2 + 2*a*b*e^3 + 30*(3*b^2*d*e^
2 + 2*a*b*e^3)*n^4 + 61*(3*b^2*d*e^2 + 2*a*b*e^3)*n^3 + 41*(3*b^2
*d*e^2 + 2*a*b*e^3)*n^2 + 11*(3*b^2*d*e^2 + 2*a*b*e^3)*n)*x*x^(4*
n) + (3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3 + 40*(3*b^2*d^2*e + 6*a
*b*d*e^2 + a^2*e^3)*n^4 + 78*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3
)*n^3 + 49*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n^2 + 12*(3*b^2*
d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n)*x*x^(3*n) + (b^2*d^3 + 6*a*b*d^
2*e + 3*a^2*d*e^2 + 60*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*n^4
+ 107*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*n^3 + 59*(b^2*d^3 + 6
*a*b*d^2*e + 3*a^2*d*e^2)*n^2 + 13*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2
*d*e^2)*n)*x*x^(2*n) + (2*a*b*d^3 + 3*a^2*d^2*e + 120*(2*a*b*d^3
+ 3*a^2*d^2*e)*n^4 + 154*(2*a*b*d^3 + 3*a^2*d^2*e)*n^3 + 71*(2*a*
b*d^3 + 3*a^2*d^2*e)*n^2 + 14*(2*a*b*d^3 + 3*a^2*d^2*e)*n)*x*x^n
+ (120*a^2*d^3*n^5 + 274*a^2*d^3*n^4 + 225*a^2*d^3*n^3 + 85*a^2*d
^3*n^2 + 15*a^2*d^3*n + a^2*d^3)*x)/(120*n^5 + 274*n^4 + 225*n^3
+ 85*n^2 + 15*n + 1)
```

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)**2*(d+e*x**n)**3,x)
```

```
[Out] Exception raised: TypeError
```

---

**GIAC/XCAS [A]** time = 0.226763, size = 1, normalized size = 0.01

*Done*

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2*(e*x^n + d)^3,x, algorithm="giac")
```

```
[Out] Done
```

## 3.194 $\int (a+bx^n)^2 (d+ex^n)^2 \, dx$

**Optimal**. Leaf size=112

$$\frac{x^{2n+1}\left(a^2e^2+4abde+b^2d^2\right)}{2n+1}+a^2d^2x+\frac{2adx^{n+1}(ae+bd)}{n+1}+\frac{2bex^{3n+1}(ae+bd)}{3n+1}+\frac{b^2e^2x^{4n+1}}{4n+1}$$

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^(1 + n))/(1 + n) + ((b^2*d^2 + 4
*a*b*d*e + a^2*e^2)*x^(1 + 2*n))/(1 + 2*n) + (2*b*e*(b*d + a*e)*x
^(1 + 3*n))/(1 + 3*n) + (b^2*e^2*x^(1 + 4*n))/(1 + 4*n)
```

---

**Rubi [A]** time = 0.187095, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{x^{2n+1}\left(a^2e^2+4abde+b^2d^2\right)}{2n+1}+a^2d^2x+\frac{2adx^{n+1}(ae+bd)}{n+1}+\frac{2bex^{3n+1}(ae+bd)}{3n+1}+\frac{b^2e^2x^{4n+1}}{4n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)^2*(d + e*x^n)^2,x]
```

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^(1 + n))/(1 + n) + ((b^2*d^2 + 4
*a*b*d*e + a^2*e^2)*x^(1 + 2*n))/(1 + 2*n) + (2*b*e*(b*d + a*e)*x
^(1 + 3*n))/(1 + 3*n) + (b^2*e^2*x^(1 + 4*n))/(1 + 4*n)
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2adx^{n+1}\left(ae+bd\right)}{n+1}+\frac{b^2e^2x^{4n+1}}{4n+1}+\frac{2bex^{3n+1}\left(ae+bd\right)}{3n+1}+d^2\int a^2\,dx+\frac{x^{2n+1}\left(a^2e^2+bd\left(4ae+bd\right)\right)}{2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)**2*(d+e*x**n)**2,x)
```

```
[Out] 2*a*d*x**(n + 1)*(a*e + b*d)/(n + 1) + b**2*e**2*x**(4*n + 1)/(4*
n + 1) + 2*b*e*x**(3*n + 1)*(a*e + b*d)/(3*n + 1) + d**2*Integral
(a**2, x) + x**(2*n + 1)*(a**2*e**2 + b*d*(4*a*e + b*d))/(2*n + 1
)
```

---

**Mathematica [A]** time = 0.25921, size = 105, normalized size = 0.94

$$x\left(\frac{x^{2n}\left(a^2e^2+4abde+b^2d^2\right)}{2n+1}+a^2d^2+\frac{2bex^{3n}(ae+bd)}{3n+1}+\frac{2adx^n(ae+bd)}{n+1}+\frac{b^2e^2x^{4n}}{4n+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)^2*(d + e*x^n)^2,x]
```

```
[Out] x*(a^2*d^2 + (2*a*d*(b*d + a*e)*x^n)/(1 + n) + ((b^2*d^2 + 4*a*b*
d*e + a^2*e^2)*x^(2*n))/(1 + 2*n) + (2*b*e*(b*d + a*e)*x^(3*n))/(
1 + 3*n) + (b^2*e^2*x^(4*n))/(1 + 4*n))
```

---

**Maple [A]** time = 0.017, size = 117, normalized size = 1.

$$a^2d^2x + \frac{\left(a^2e^2 + 4\,abde + b^2d^2\right)x\left(e^{n\ln(x)}\right)^2}{1+2\,n} + \frac{b^2e^2x\left(e^{n\ln(x)}\right)^4}{1+4\,n} + 2\,\frac{ad\,(ae+bd)\,xe^{n\ln(x)}}{1+n} + 2\,\frac{be\,(ae+bd)\,x\left(e^{n\ln(x)}\right)^3}{1+3\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)^2*(d+e*x^n)^2,x)
```

```
[Out] a^2*d^2*x+(a^2*e^2+4*a*b*d*e+b^2*d^2)/(1+2*n)*x*exp(n*ln(x))^2+b^
2*e^2/(1+4*n)*x*exp(n*ln(x))^4+2*a*d*(a*e+b*d)/(1+n)*x*exp(n*ln(x
))+2*b*e*(a*e+b*d)/(1+3*n)*x*exp(n*ln(x))^3
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2*(e*x^n + d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.255879, size = 500, normalized size = 4.46

$$\frac{\left(6\,b^2e^2n^3 + 11\,b^2e^2n^2 + 6\,b^2e^2n + b^2e^2\right)xx^{4\,n} + 2\left(b^2de + abe^2 + 8\left(b^2de + abe^2\right)n^3 + 14\left(b^2de + abe^2\right)n^2 + 7\left(b^2de + abe^2\right)n\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2*(e*x^n + d)^2,x, algorithm="fricas")
```

```
[Out] ((6*b^2*e^2*n^3 + 11*b^2*e^2*n^2 + 6*b^2*e^2*n + b^2*e^2)*x*x^(4*
n) + 2*(b^2*d*e + a*b*e^2 + 8*(b^2*d*e + a*b*e^2)*n^3 + 14*(b^2*d
*e + a*b*e^2)*n^2 + 7*(b^2*d*e + a*b*e^2)*n)*x*x^(3*n) + (b^2*d^2
 + 4*a*b*d*e + a^2*e^2 + 12*(b^2*d^2 + 4*a*b*d*e + a^2*e^2)*n^3 +
 19*(b^2*d^2 + 4*a*b*d*e + a^2*e^2)*n^2 + 8*(b^2*d^2 + 4*a*b*d*e
+ a^2*e^2)*n)*x*x^(2*n) + 2*(a*b*d^2 + a^2*d*e + 24*(a*b*d^2 + a^
2*d*e)*n^3 + 26*(a*b*d^2 + a^2*d*e)*n^2 + 9*(a*b*d^2 + a^2*d*e)*n
)*x*x^n + (24*a^2*d^2*n^4 + 50*a^2*d^2*n^3 + 35*a^2*d^2*n^2 + 10*
a^2*d^2*n + a^2*d^2)*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)**2*(d+e*x**n)**2,x)
```

```
[Out] Exception raised: TypeError
```

---

**GIAC/XCAS [A]** time = 0.227152, size = 782, normalized size = 6.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2*(e*x^n + d)^2,x, algorithm="giac")
```

```
[Out] (24*a^2*d^2*n^4*x + 12*b^2*d^2*n^3*x*e^(2*n*ln(x)) + 48*a*b*d^2*n
^3*x*e^(n*ln(x)) + 50*a^2*d^2*n^3*x + 19*b^2*d^2*n^2*x*e^(2*n*ln(
x)) + 16*b^2*d*n^3*x*e^(3*n*ln(x) + 1) + 48*a*b*d*n^3*x*e^(2*n*ln
(x) + 1) + 48*a^2*d*n^3*x*e^(n*ln(x) + 1) + 52*a*b*d^2*n^2*x*e^(n
*ln(x)) + 35*a^2*d^2*n^2*x + 8*b^2*d^2*n*x*e^(2*n*ln(x)) + 6*b^2*
n^3*x*e^(4*n*ln(x) + 2) + 16*a*b*n^3*x*e^(3*n*ln(x) + 2) + 28*b^2
*d*n^2*x*e^(3*n*ln(x) + 1) + 12*a^2*n^3*x*e^(2*n*ln(x) + 2) + 76*
a*b*d*n^2*x*e^(2*n*ln(x) + 1) + 52*a^2*d*n^2*x*e^(n*ln(x) + 1) +
18*a*b*d^2*n*x*e^(n*ln(x)) + 10*a^2*d^2*n*x + b^2*d^2*x*e^(2*n*ln
(x)) + 11*b^2*n^2*x*e^(4*n*ln(x) + 2) + 28*a*b*n^2*x*e^(3*n*ln(x)
 + 2) + 14*b^2*d*n*x*e^(3*n*ln(x) + 1) + 19*a^2*n^2*x*e^(2*n*ln(x
) + 2) + 32*a*b*d*n*x*e^(2*n*ln(x) + 1) + 18*a^2*d*n*x*e^(n*ln(x)
 + 1) + 2*a*b*d^2*x*e^(n*ln(x)) + a^2*d^2*x + 6*b^2*n*x*e^(4*n*ln
(x) + 2) + 14*a*b*n*x*e^(3*n*ln(x) + 2) + 2*b^2*d*x*e^(3*n*ln(x)
+ 1) + 8*a^2*n*x*e^(2*n*ln(x) + 2) + 4*a*b*d*x*e^(2*n*ln(x) + 1)
+ 2*a^2*d*x*e^(n*ln(x) + 1) + b^2*x*e^(4*n*ln(x) + 2) + 2*a*b*x*e
```

```
^(3*n*ln(x) + 2) + a^2*x*e^(2*n*ln(x) + 2))/(24*n^4 + 50*n^3 + 35
*n^2 + 10*n + 1)
```

# 3.195 $\int (a + bx^n)^2 (c + dx^n) \, dx$

**Optimal**. Leaf size=70

$$a^2cx + \frac{ax^{n+1}(ad + 2bc)}{n + 1} + \frac{bx^{2n+1}(2ad + bc)}{2n + 1} + \frac{b^2dx^{3n+1}}{3n + 1}$$

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^(1 + n))/(1 + n) + (b*(b*c + 2*a*d)*
x^(1 + 2*n))/(1 + 2*n) + (b^2*d*x^(1 + 3*n))/(1 + 3*n)
```

---

**Rubi [A]** time = 0.114034, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$a^2cx + \frac{ax^{n+1}(ad + 2bc)}{n + 1} + \frac{bx^{2n+1}(2ad + bc)}{2n + 1} + \frac{b^2dx^{3n+1}}{3n + 1}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)^2*(c + d*x^n),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^(1 + n))/(1 + n) + (b*(b*c + 2*a*d)*
x^(1 + 2*n))/(1 + 2*n) + (b^2*d*x^(1 + 3*n))/(1 + 3*n)
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$a^2 \int c \, dx + \frac{ax^{n+1} (ad + 2bc)}{n + 1} + \frac{b^2dx^{3n+1}}{3n + 1} + \frac{bx^{2n+1} (2ad + bc)}{2n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)**2*(c+d*x**n),x)
```

```
[Out] a**2*Integral(c, x) + a*x**(n + 1)*(a*d + 2*b*c)/(n + 1) + b**2*d
*x**(3*n + 1)/(3*n + 1) + b*x**(2*n + 1)*(2*a*d + b*c)/(2*n + 1)
```

---

**Mathematica [A]** time = 0.110819, size = 65, normalized size = 0.93

$$x \left(a^2c + \frac{bx^{2n}(2ad + bc)}{2n + 1} + \frac{ax^n(ad + 2bc)}{n + 1} + \frac{b^2dx^{3n}}{3n + 1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)^2*(c + d*x^n),x]
```

```
[Out] x*(a^2*c + (a*(2*b*c + a*d)*x^n)/(1 + n) + (b*(b*c + 2*a*d)*x^(2*
n))/(1 + 2*n) + (b^2*d*x^(3*n))/(1 + 3*n))
```

---

**Maple [A]** time = 0.014, size = 74, normalized size = 1.1

$$a^{2}cx+\frac{a\left(ad+2\,bc\right)x\mathrm{e}^{n\ln\left(x\right)}}{1+n}+\frac{b\left(2\,ad+bc\right)x\left(\mathrm{e}^{n\ln\left(x\right)}\right)^{2}}{1+2\,n}+\frac{b^{2}dx\left(\mathrm{e}^{n\ln\left(x\right)}\right)^{3}}{1+3\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)^2*(c+d*x^n),x)
```

```
[Out] a^2*c*x+a*(a*d+2*b*c)/(1+n)*x*exp(n*ln(x))+b*(2*a*d+b*c)/(1+2*n)*
x*exp(n*ln(x))^2+b^2*d/(1+3*n)*x*exp(n*ln(x))^3
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2*(d*x^n + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.254831, size = 236, normalized size = 3.37

$$\frac{\left(2\,b^{2}dn^{2}+3\,b^{2}dn+b^{2}d\right)xx^{3\,n}+\left(b^{2}c+2\,abd+3\left(b^{2}c+2\,abd\right)n^{2}+4\left(b^{2}c+2\,abd\right)n\right)xx^{2\,n}+\left(2\,abc+a^{2}d+6\left(2\,abc+a^{2}d\right)\right.}{6\,n^{3}+11\,n^{2}+6\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2*(d*x^n + c),x, algorithm="fricas")
```

```
[Out] ((2*b^2*d*n^2 + 3*b^2*d*n + b^2*d)*x*x^(3*n) + (b^2*c + 2*a*b*d +
 3*(b^2*c + 2*a*b*d)*n^2 + 4*(b^2*c + 2*a*b*d)*n)*x*x^(2*n) + (2*
a*b*c + a^2*d + 6*(2*a*b*c + a^2*d)*n^2 + 5*(2*a*b*c + a^2*d)*n)*
x*x^n + (6*a^2*c*n^3 + 11*a^2*c*n^2 + 6*a^2*c*n + a^2*c)*x)/(6*n^
```

```
3 + 11*n^2 + 6*n + 1)
```

---

**Sympy [A]** time = 2.77067, size = 726, normalized size = 10.37

$$\begin{cases} a^2cx + a^2d\log(x) + 2abc\log(x) - \frac{2abd}{x} - \frac{b^2c}{x} - \frac{b^2d}{2x^2} \\ a^2cx + 2a^2d\sqrt{x} + 4abc\sqrt{x} + 2abd\log(x) + b^2c\log(x) - \frac{2b^2d}{\sqrt{x}} \\ a^2cx + \frac{3a^2dx^{\frac{2}{3}}}{2} + 3abcx^{\frac{2}{3}} + 6abd\sqrt[3]{x} + 3b^2c\sqrt[3]{x} + b^2d\log(x) \\ \frac{6a^2cn^3x}{6n^3+11n^2+6n+1} + \frac{11a^2cn^2x}{6n^3+11n^2+6n+1} + \frac{6a^2cnx}{6n^3+11n^2+6n+1} + \frac{a^2cx}{6n^3+11n^2+6n+1} + \frac{6a^2dn^2xx^n}{6n^3+11n^2+6n+1} + \frac{5a^2dnxx^n}{6n^3+11n^2+6n+1} + \frac{a^2dxx^n}{6n^3+11n^2+6n+1} + \frac{12abcn^2xx^n}{6n^3+11n^2+6n+1} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)**2*(c+d*x**n),x)
```

```
[Out] Piecewise((a**2*c*x + a**2*d*log(x) + 2*a*b*c*log(x) - 2*a*b*d/x
- b**2*c/x - b**2*d/(2*x**2), Eq(n, -1)), (a**2*c*x + 2*a**2*d*sq
rt(x) + 4*a*b*c*sqrt(x) + 2*a*b*d*log(x) + b**2*c*log(x) - 2*b**2
*d/sqrt(x), Eq(n, -1/2)), (a**2*c*x + 3*a**2*d*x**(2/3)/2 + 3*a*b
*c*x**(2/3) + 6*a*b*d*x**(1/3) + 3*b**2*c*x**(1/3) + b**2*d*log(x
), Eq(n, -1/3)), (6*a**2*c*n**3*x/(6*n**3 + 11*n**2 + 6*n + 1) +
11*a**2*c*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a**2*c*n*x/(6*n
**3 + 11*n**2 + 6*n + 1) + a**2*c*x/(6*n**3 + 11*n**2 + 6*n + 1)
+ 6*a**2*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*a**2*d*n*
x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + a**2*d*x*x**n/(6*n**3 + 11*
n**2 + 6*n + 1) + 12*a*b*c*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n +
1) + 10*a*b*c*n*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 2*a*b*c*x*x
**n/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a*b*d*n**2*x*x**(2*n)/(6*n**
3 + 11*n**2 + 6*n + 1) + 8*a*b*d*n*x*x**(2*n)/(6*n**3 + 11*n**2 +
 6*n + 1) + 2*a*b*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b
**2*c*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*b**2*c*n*x
*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b**2*c*x*x**(2*n)/(6*n**
3 + 11*n**2 + 6*n + 1) + 2*b**2*d*n**2*x*x**(3*n)/(6*n**3 + 11*n*
*2 + 6*n + 1) + 3*b**2*d*n*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1
) + b**2*d*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1), True))
```

---

**GIAC/XCAS [A]** time = 0.219823, size = 342, normalized size = 4.89

$$\frac{6\,a^2cn^3x + 2\,b^2dn^2xe^{(3\,n\ln(x))} + 3\,b^2cn^2xe^{(2\,n\ln(x))} + 6\,abdn^2xe^{(2\,n\ln(x))} + 12\,abcn^2xe^{(n\ln(x))} + 6\,a^2dn^2xe^{(n\ln(x))} + 11\,a^2cn^2x + 3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2*(d*x^n + c),x, algorithm="giac")
```

```
[Out] (6*a^2*c*n^3*x + 2*b^2*d*n^2*x*e^(3*n*ln(x)) + 3*b^2*c*n^2*x*e^(2
*n*ln(x)) + 6*a*b*d*n^2*x*e^(2*n*ln(x)) + 12*a*b*c*n^2*x*e^(n*ln(
```

```
x)) + 6*a^2*d*n^2*x*e^(n*ln(x)) + 11*a^2*c*n^2*x + 3*b^2*d*n*x*e^
(3*n*ln(x)) + 4*b^2*c*n*x*e^(2*n*ln(x)) + 8*a*b*d*n*x*e^(2*n*ln(x
)) + 10*a*b*c*n*x*e^(n*ln(x)) + 5*a^2*d*n*x*e^(n*ln(x)) + 6*a^2*c
*n*x + b^2*d*x*e^(3*n*ln(x)) + b^2*c*x*e^(2*n*ln(x)) + 2*a*b*d*x*
e^(2*n*ln(x)) + 2*a*b*c*x*e^(n*ln(x)) + a^2*d*x*e^(n*ln(x)) + a^2
*c*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

## 3.196 $\int \frac{(a+bx^n)^2}{c+dx^n} \, dx$

**Optimal**. Leaf size=84

$$\frac{x(bc-ad)^2 \, {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd^2}-\frac{bx(bc(n+1)-ad(2n+1))}{d^2(n+1)}+\frac{bx\,(a+bx^n)}{d(n+1)}$$

```
[Out] -((b*(b*c*(1 + n) - a*d*(1 + 2*n))*x)/(d^2*(1 + n))) + (b*x*(a +
b*x^n))/(d*(1 + n)) + ((b*c - a*d)^2*x*Hypergeometric2F1[1, n^(-1
), 1 + n^(-1), -((d*x^n)/c)])/(c*d^2)
```

---

**Rubi [A]** time = 0.21881, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{x(bc-ad)^2 \, {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd^2}-\frac{bx(bc(n+1)-ad(2n+1))}{d^2(n+1)}+\frac{bx\,(a+bx^n)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)^2/(c + d*x^n),x]
```

```
[Out] -((b*(b*c*(1 + n) - a*d*(1 + 2*n))*x)/(d^2*(1 + n))) + (b*x*(a +
b*x^n))/(d*(1 + n)) + ((b*c - a*d)^2*x*Hypergeometric2F1[1, n^(-1
), 1 + n^(-1), -((d*x^n)/c)])/(c*d^2)
```

---

**Rubi in Sympy [A]** time = 22.8338, size = 70, normalized size = 0.83

$$\frac{bx\,(a+bx^n)}{d\,(n+1)}+\frac{bx\,(ad\,(2n+1)-bc\,(n+1))}{d^2\,(n+1)}+\frac{x\,(ad-bc)^2 \, {}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{dx^n}{c}\right)}{cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] b*x*(a + b*x**n)/(d*(n + 1)) + b*x*(a*d*(2*n + 1) - b*c*(n + 1))/
(d**2*(n + 1)) + x*(a*d - b*c)**2*hyper((1, 1/n), (1 + 1/n,), -d*
x**n/c)/(c*d**2)
```

---

**Mathematica [A]** time = 0.069443, size = 82, normalized size = 0.98

$$\frac{a^2x}{c} + \frac{x(ad-bc)^2\, {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{dx^n}{c}\right)}{cd^2} - \frac{x(bc-ad)^2}{cd^2} + \frac{b^2x^{n+1}}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)^2/(c + d*x^n),x]
```

```
[Out] (a^2*x)/c - ((b*c - a*d)^2*x)/(c*d^2) + (b^2*x^(1 + n))/(d*(1 + n
)) + ((-(b*c) + a*d)^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
 -((d*x^n)/c)])/(c*d^2)
```

---

**Maple [F]** time = 0.062, size = 0, normalized size = 0.

$$\int \frac{(a+bx^n)^2}{c+dx^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] int((a+b*x^n)^2/(c+d*x^n),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left(b^2c^2 - 2\,abcd + a^2d^2\right)\int \frac{1}{d^3x^n + cd^2}\, dx + \frac{b^2dxx^n - \left(b^2c(n+1) - 2\,abd(n+1)\right)x}{d^2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2/(d*x^n + c),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*integrate(1/(d^3*x^n + c*d^2), x)
 + (b^2*d*x*x^n - (b^2*c*(n + 1) - 2*a*b*d*(n + 1))*x)/(d^2*(n +
1))
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^{2\,n} + 2\,abx^n + a^2}{dx^n + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2/(d*x^n + c),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d*x^n + c), x)
```

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: TypeError
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^2}{dx^n + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2/(d*x^n + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2/(d*x^n + c), x)
```

## 3.197 $\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx$

**Optimal**. Leaf size=115

$$\frac{x(bc-ad)(ad(1-n)-bc(n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2d^2n}-\frac{bx(ad-bc(n+1))}{cd^2n}-\frac{x(bc-ad)\,(a+bx^n)}{cdn\,(c+dx^n)}$$

```
[Out] -((b*(a*d - b*c*(1 + n))*x)/(c*d^2*n)) - ((b*c - a*d)*x*(a + b*x^
n))/(c*d*n*(c + d*x^n)) + ((b*c - a*d)*(a*d*(1 - n) - b*c*(1 + n)
)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*
d^2*n)
```

---

**Rubi [A]** time = 0.244695, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{x(bc-ad)(ad(1-n)-bc(n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2d^2n}-\frac{bx(ad-bc(n+1))}{cd^2n}-\frac{x(bc-ad)\,(a+bx^n)}{cdn\,(c+dx^n)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)^2/(c + d*x^n)^2,x]
```

```
[Out] -((b*(a*d - b*c*(1 + n))*x)/(c*d^2*n)) - ((b*c - a*d)*x*(a + b*x^
n))/(c*d*n*(c + d*x^n)) + ((b*c - a*d)*(a*d*(1 - n) - b*c*(1 + n)
)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*
d^2*n)
```

---

**Rubi in Sympy [A]** time = 23.0974, size = 95, normalized size = 0.83

$$-\frac{bx\,(ad-bc\,(n+1))}{cd^2n}+\frac{x\,(a+bx^n)\,(ad-bc)}{cdn\,(c+dx^n)}-\frac{x\,(ad-bc)\,(-adn+ad-bcn-bc)\,{}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{dx^n}{c}\right)}{c^2d^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)**2/(c+d*x**n)**2,x)
```

```
[Out] -b*x*(a*d - b*c*(n + 1))/(c*d**2*n) + x*(a + b*x**n)*(a*d - b*c)/
(c*d*n*(c + d*x**n)) - x*(a*d - b*c)*(-a*d*n + a*d - b*c*n - b*c)
*hyper((1, 1/n), (1 + 1/n,), -d*x**n/c)/(c**2*d**2*n)
```

---

**Mathematica [A]** time = 0.190573, size = 95, normalized size = 0.83

$$\frac{x\left(\frac{c\left(a^2d^2-2abcd+b^2c(cn+c+dnx^n)\right)}{c+dx^n}-(bc-ad)(ad(n-1)+bc(n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)\right)}{c^2d^2n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)^2/(c + d*x^n)^2,x]
```

```
[Out] (x*((c*(-2*a*b*c*d + a^2*d^2 + b^2*c*(c + c*n + d*n*x^n)))/(c + d
*x^n) - (b*c - a*d)*(a*d*(-1 + n) + b*c*(1 + n))*Hypergeometric2F
1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(c^2*d^2*n)
```

---

**Maple [F]** time = 0.067, size = 0, normalized size = 0.

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)^2/(c+d*x^n)^2,x)
```

```
[Out] int((a+b*x^n)^2/(c+d*x^n)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\left(b^2c^2(n+1)-a^2d^2(n-1)-2\,abcd\right)\int \frac{1}{cd^3nx^n+c^2d^2n}\,dx+\frac{b^2cdnxx^n+\left(b^2c^2(n+1)-2\,abcd+a^2d^2\right)x}{cd^3nx^n+c^2d^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2/(d*x^n + c)^2,x, algorithm="maxima")
```

```
[Out] -(b^2*c^2*(n + 1) - a^2*d^2*(n - 1) - 2*a*b*c*d)*integrate(1/(c*d
^3*n*x^n + c^2*d^2*n), x) + (b^2*c*d*n*x*x^n + (b^2*c^2*(n + 1) -
 2*a*b*c*d + a^2*d^2)*x)/(c*d^3*n*x^n + c^2*d^2*n)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^{2\,n}+2\,abx^n+a^2}{d^2x^{2\,n}+2\,cdx^n+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2/(d*x^n + c)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d^2*x^(2*n) + 2*c*d*x^n
 + c^2), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)**2/(c+d*x**n)**2,x)
```

```
[Out] Integral((a + b*x**n)**2/(c + d*x**n)**2, x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n+a)^2}{(dx^n+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2/(d*x^n + c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2/(d*x^n + c)^2, x)
```

## 3.198 $\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx$

**Optimal**. Leaf size=160

$$-\frac{x\left(-a^2d^2\left(2n^2-3n+1\right)+2abcd(1-n)-b^2c^2(n+1)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3d^2n^2}+\frac{x(bc-ad)(ad(1-2n)-bc(n+1))}{2c^2d^2n^2\left(c+dx^n\right)}-\frac{x(bc-ad)\left(a+bx^n\right)}{2cdn\left(c+dx^n\right)^2}$$

```
[Out] -((b*c - a*d)*x*(a + b*x^n))/(2*c*d*n*(c + d*x^n)^2) + ((b*c - a*
d)*(a*d*(1 - 2*n) - b*c*(1 + n))*x)/(2*c^2*d^2*n^2*(c + d*x^n)) -
 ((2*a*b*c*d*(1 - n) - b^2*c^2*(1 + n) - a^2*d^2*(1 - 3*n + 2*n^2
))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(2*c
^3*d^2*n^2)
```

---

**Rubi [A]** time = 0.359242, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{x\left(-a^2d^2\left(2n^2-3n+1\right)+2abcd(1-n)-b^2c^2(n+1)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3d^2n^2}+\frac{x(bc-ad)(ad(1-2n)-bc(n+1))}{2c^2d^2n^2\left(c+dx^n\right)}-\frac{x(bc-ad)\left(a+bx^n\right)}{2cdn\left(c+dx^n\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)^2/(c + d*x^n)^3,x]
```


```
[Out] -((b*c - a*d)*x*(a + b*x^n))/(2*c*d*n*(c + d*x^n)^2) + ((b*c - a*
d)*(a*d*(1 - 2*n) - b*c*(1 + n))*x)/(2*c^2*d^2*n^2*(c + d*x^n)) -
 ((2*a*b*c*d*(1 - n) - b^2*c^2*(1 + n) - a^2*d^2*(1 - 3*n + 2*n^2
))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(2*c
^3*d^2*n^2)
```


---

**Rubi in Sympy [A]** time = 27.9249, size = 141, normalized size = 0.88

$$\frac{x\left(a+bx^n\right)(ad-bc)}{2cdn\left(c+dx^n\right)^2}-\frac{x(ad-bc)(-2adn+ad-bcn-bc)}{2c^2d^2n^2\left(c+dx^n\right)}-\frac{x\left(-ad(-n+1)(-2adn+ad-bc)+bc(ad(-n+1)-bc(n+1))\right){}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{dx^n}{c}\right)}{2c^3d^2n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)**2/(c+d*x**n)**3,x)
```

```
[Out] x*(a + b*x**n)*(a*d - b*c)/(2*c*d*n*(c + d*x**n)**2) - x*(a*d - b
*c)*(-2*a*d*n + a*d - b*c*n - b*c)/(2*c**2*d**2*n**2*(c + d*x**n)
) - x*(-a*d*(-n + 1)*(-2*a*d*n + a*d - b*c) + b*c*(a*d*(-n + 1) -
 b*c*(n + 1)))*hyper((1, 1/n), (1 + 1/n,), -d*x**n/c)/(2*c**3*d**
2*n**2)
```

---

**Mathematica [A]** time = 0.173422, size = 133, normalized size = 0.83

$$\frac{x\left(\left(a^2d^2\left(2n^2-3n+1\right)+2abcd(n-1)+b^2c^2(n+1)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+\frac{c^2n(bc-ad)^2}{(c+dx^n)^2}-\frac{c(bc-ad)(ad(2n-1)+b(2cn+c))}{c+dx^n}\right)}{2c^3d^2n^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)^2/(c + d*x^n)^3,x]
```

```
[Out] (x*((c^2*(b*c - a*d)^2*n)/(c + d*x^n)^2 - (c*(b*c - a*d)*(a*d*(-1
 + 2*n) + b*(c + 2*c*n)))/(c + d*x^n) + (2*a*b*c*d*(-1 + n) + b^2
*c^2*(1 + n) + a^2*d^2*(1 - 3*n + 2*n^2))*Hypergeometric2F1[1, n^
(-1), 1 + n^(-1), -((d*x^n)/c)]))/(2*c^3*d^2*n^2)
```

---

**Maple [F]** time = 0.079, size = 0, normalized size = 0.

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)^2/(c+d*x^n)^3,x)
```

```
[Out] int((a+b*x^n)^2/(c+d*x^n)^3,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left(\left(2\,n^2-3\,n+1\right)a^2d^2+b^2c^2(n+1)+2\,abcd(n-1)\right)\int \frac{1}{2\left(c^2d^3n^2x^n+c^3d^2n^2\right)}\,dx - \frac{\left(b^2c^2d(2\,n+1)-a^2d^3(2\,n-1)-2\,abcd^2\right)xx^n-\left(a^2cd^2(3\,n-1)-b^2c^3(n+1)-2\,abc^2d(n-1)\right)x}{2\left(c^2d^4n^2x^{2\,n}+2\,c^3d^3n^2x^n+c^4d^2n^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2/(d*x^n + c)^3,x, algorithm="maxima")
```

```
[Out] ((2*n^2 - 3*n + 1)*a^2*d^2 + b^2*c^2*(n + 1) + 2*a*b*c*d*(n - 1))
*integrate(1/2/(c^2*d^3*n^2*x^n + c^3*d^2*n^2), x) - 1/2*((b^2*c^
2*d*(2*n + 1) - a^2*d^3*(2*n - 1) - 2*a*b*c*d^2)*x*x^n - (a^2*c*d
^2*(3*n - 1) - b^2*c^3*(n + 1) - 2*a*b*c^2*d*(n - 1))*x)/(c^2*d^4
*n^2*x^(2*n) + 2*c^3*d^3*n^2*x^n + c^4*d^2*n^2)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^{2\,n} + 2\,abx^n + a^2}{d^3x^{3\,n} + 3\,cd^2x^{2\,n} + 3\,c^2dx^n + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2/(d*x^n + c)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d^3*x^(3*n) + 3*c*d^2*x
^(2*n) + 3*c^2*d*x^n + c^3), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)**2/(c+d*x**n)**3,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^2}{(dx^n + c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2/(d*x^n + c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2/(d*x^n + c)^3, x)
```

## 3.199 $\int \frac{(c+dx^n)^4}{a+bx^n}\,dx$

**Optimal**. Leaf size=310

$$-\frac{dx\,(c+dx^n)\left(-a^2d^2\left(6n^2+5n+1\right)+2abcd(3n+1)^2-b^2c^2\left(18n^2+7n+1\right)\right)}{b^3(n+1)(2n+1)(3n+1)}-\frac{dx\left(a^3d^3\left(6n^3+11n^2+6n+1\right)-a^2bcd^2\left(24n^3+38n^2+19n+3\right)+ab^2c^2d\left(36n^3+45n^2+20n+3\right)-b^3c^3\left(24n^3+18n^2+\right.\right.}{b^4(n+1)(2n+1)(3n+1)}$$

$$+\frac{x(bc-ad)^4\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^4}-\frac{dx\,(c+dx^n)^2\,(ad(3n+1)-b(6cn+c))}{b^2\left(6n^2+5n+1\right)}+\frac{dx\,(c+dx^n)^3}{b(3n+1)}$$

```
[Out] -((d*(a^3*d^3*(1 + 6*n + 11*n^2 + 6*n^3) - b^3*c^3*(1 + 7*n + 18*
n^2 + 24*n^3) - a^2*b*c*d^2*(3 + 19*n + 38*n^2 + 24*n^3) + a*b^2*
c^2*d*(3 + 20*n + 45*n^2 + 36*n^3))*x)/(b^4*(1 + n)*(1 + 2*n)*(1
+ 3*n))) - (d*(2*a*b*c*d*(1 + 3*n)^2 - a^2*d^2*(1 + 5*n + 6*n^2)
- b^2*c^2*(1 + 7*n + 18*n^2))*x*(c + d*x^n))/(b^3*(1 + n)*(1 + 2*
n)*(1 + 3*n)) - (d*(a*d*(1 + 3*n) - b*(c + 6*c*n))*x*(c + d*x^n)^
2)/(b^2*(1 + 5*n + 6*n^2)) + (d*x*(c + d*x^n)^3)/(b*(1 + 3*n)) +
((b*c - a*d)^4*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^
n)/a)])/(a*b^4)
```

---

**Rubi [A]** time = 1.12801, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$-\frac{dx\,(c+dx^n)\left(-a^2d^2\left(6n^2+5n+1\right)+2abcd(3n+1)^2-b^2c^2\left(18n^2+7n+1\right)\right)}{b^3(n+1)(2n+1)(3n+1)}-\frac{dx\left(a^3d^3\left(6n^3+11n^2+6n+1\right)-a^2bcd^2\left(24n^3+38n^2+19n+3\right)+ab^2c^2d\left(36n^3+45n^2+20n+3\right)-b^3c^3\left(24n^3+18n^2+\right.\right.}{b^4(n+1)(2n+1)(3n+1)}$$

$$+\frac{x(bc-ad)^4\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^4}-\frac{dx\,(c+dx^n)^2\,(ad(3n+1)-b(6cn+c))}{b^2\left(6n^2+5n+1\right)}+\frac{dx\,(c+dx^n)^3}{b(3n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^n)^4/(a + b*x^n),x]
```

```
[Out] -((d*(a^3*d^3*(1 + 6*n + 11*n^2 + 6*n^3) - b^3*c^3*(1 + 7*n + 18*
n^2 + 24*n^3) - a^2*b*c*d^2*(3 + 19*n + 38*n^2 + 24*n^3) + a*b^2*
c^2*d*(3 + 20*n + 45*n^2 + 36*n^3))*x)/(b^4*(1 + n)*(1 + 2*n)*(1
+ 3*n))) - (d*(2*a*b*c*d*(1 + 3*n)^2 - a^2*d^2*(1 + 5*n + 6*n^2)
- b^2*c^2*(1 + 7*n + 18*n^2))*x*(c + d*x^n))/(b^3*(1 + n)*(1 + 2*
n)*(1 + 3*n)) - (d*(a*d*(1 + 3*n) - b*(c + 6*c*n))*x*(c + d*x^n)^
2)/(b^2*(1 + 5*n + 6*n^2)) + (d*x*(c + d*x^n)^3)/(b*(1 + 3*n)) +
((b*c - a*d)^4*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^
n)/a)])/(a*b^4)
```

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**n)**4/(a+b*x**n),x)
```

```
[Out] Timed out
```

**Mathematica [A]** time = 0.185418, size = 146, normalized size = 0.47

$$x\left(\frac{d^2x^n\left(a^2d^2-4abcd+6b^2c^2\right)}{b^3(n+1)}+\frac{(bc-ad)^4\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^4}-\frac{(bc-ad)^4}{ab^4}+\frac{d^3x^{2n}(4bc-ad)}{b^2(2n+1)}+\frac{c^4}{a}+\frac{d^4x^{3n}}{3bn+b}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^n)^4/(a + b*x^n),x]
```

```
[Out] x*(c^4/a - (b*c - a*d)^4/(a*b^4) + (d^2*(6*b^2*c^2 - 4*a*b*c*d +
a^2*d^2)*x^n)/(b^3*(1 + n)) + (d^3*(4*b*c - a*d)*x^(2*n))/(b^2*(1
 + 2*n)) + (d^4*x^(3*n))/(b + 3*b*n) + ((b*c - a*d)^4*Hypergeomet
ric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*b^4))
```

**Maple [F]** time = 0.072, size = 0, normalized size = 0.

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^n)^4/(a+b*x^n),x)
```

```
[Out] int((c+d*x^n)^4/(a+b*x^n),x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\right)\int \frac{1}{b^5x^n + ab^4}\,dx + \frac{\left(2\,n^2 + 3\,n + 1\right)b^3d^4xx^{3\,n} + \left(4\left(3\,n^2 + 4\,n + 1\right)b^3cd^3 - \left(3\,n^2 + 4\,n + 1\right)ab^2d^4\right)xx^{2\,n} + \left(6\left(6\,n^2 + 5\,n + 1\right)b^3c^2d^2 - 4\left(6\,n^2 + 5\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^4/(b*x^n + a),x, algorithm="maxima")
```

```
[Out] (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^
4*d^4)*integrate(1/(b^5*x^n + a*b^4), x) + ((2*n^2 + 3*n + 1)*b^3
*d^4*x*x^(3*n) + (4*(3*n^2 + 4*n + 1)*b^3*c*d^3 - (3*n^2 + 4*n +
1)*a*b^2*d^4)*x*x^(2*n) + (6*(6*n^2 + 5*n + 1)*b^3*c^2*d^2 - 4*(6
*n^2 + 5*n + 1)*a*b^2*c*d^3 + (6*n^2 + 5*n + 1)*a^2*b*d^4)*x*x^n
+ (4*(6*n^3 + 11*n^2 + 6*n + 1)*b^3*c^3*d - 6*(6*n^3 + 11*n^2 + 6
*n + 1)*a*b^2*c^2*d^2 + 4*(6*n^3 + 11*n^2 + 6*n + 1)*a^2*b*c*d^3
- (6*n^3 + 11*n^2 + 6*n + 1)*a^3*d^4)*x)/((6*n^3 + 11*n^2 + 6*n +
 1)*b^4)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^4x^{4\,n} + 4\,cd^3x^{3\,n} + 6\,c^2d^2x^{2\,n} + 4\,c^3dx^n + c^4}{bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^4/(b*x^n + a),x, algorithm="fricas")
```

```
[Out] integral((d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) + 4*c
^3*d*x^n + c^4)/(b*x^n + a), x)
```

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**n)**4/(a+b*x**n),x)
```

```
[Out] Exception raised: TypeError
```

---

**<u>GIAC/XCAS</u> [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^4}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^4/(b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^4/(b*x^n + a), x)
```

## 3.200 $\int \frac{(c+dx^n)^3}{a+bx^n} dx$

**Optimal**. Leaf size=173

$$\frac{dx\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(6n^2+7n+2\right)+b^2c^2\left(6n^2+4n+1\right)\right)}{b^3(n+1)(2n+1)} + \frac{x(bc-ad)^3\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^3} - \frac{dx\left(c+dx^n\right)\left(ad(2n+1)-b(4cn+c)\right)}{b^2(n+1)(2n+1)} + \frac{dx\left(c+dx^n\right)^2}{b(2n+1)}$$

```
[Out] (d*(a^2*d^2*(1 + 3*n + 2*n^2) + b^2*c^2*(1 + 4*n + 6*n^2) - a*b*c
*d*(2 + 7*n + 6*n^2))*x)/(b^3*(1 + n)*(1 + 2*n)) - (d*(a*d*(1 + 2
*n) - b*(c + 4*c*n))*x*(c + d*x^n))/(b^2*(1 + n)*(1 + 2*n)) + (d*
x*(c + d*x^n)^2)/(b*(1 + 2*n)) + ((b*c - a*d)^3*x*Hypergeometric2
F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*b^3)
```

---

**Rubi [A]** time = 0.614258, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{dx\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(6n^2+7n+2\right)+b^2c^2\left(6n^2+4n+1\right)\right)}{b^3(n+1)(2n+1)} + \frac{x(bc-ad)^3\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^3} - \frac{dx\left(c+dx^n\right)\left(ad(2n+1)-b(4cn+c)\right)}{b^2(n+1)(2n+1)} + \frac{dx\left(c+dx^n\right)^2}{b(2n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^n)^3/(a + b*x^n),x]
```

```
[Out] (d*(a^2*d^2*(1 + 3*n + 2*n^2) + b^2*c^2*(1 + 4*n + 6*n^2) - a*b*c
*d*(2 + 7*n + 6*n^2))*x)/(b^3*(1 + n)*(1 + 2*n)) - (d*(a*d*(1 + 2
*n) - b*(c + 4*c*n))*x*(c + d*x^n))/(b^2*(1 + n)*(1 + 2*n)) + (d*
x*(c + d*x^n)^2)/(b*(1 + 2*n)) + ((b*c - a*d)^3*x*Hypergeometric2
F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*b^3)
```

---

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**n)**3/(a+b*x**n),x)
```

```
[Out] Timed out
```

**Mathematica [A]** time = 0.50718, size = 104, normalized size = 0.6

$$x\left(\frac{(bc-ad)^3\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^3}+\frac{(ad-bc)^3}{ab^3}+\frac{d^2x^n(3bc-ad)}{b^2(n+1)}+\frac{c^3}{a}+\frac{d^3x^{2n}}{2bn+b}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^n)^3/(a + b*x^n),x]
```

```
[Out] x*(c^3/a + (-(b*c) + a*d)^3/(a*b^3) + (d^2*(3*b*c - a*d)*x^n)/(b^
2*(1 + n)) + (d^3*x^(2*n))/(b + 2*b*n) + ((b*c - a*d)^3*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*b^3))
```

**Maple [F]** time = 0.068, size = 0, normalized size = 0.

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^n)^3/(a+b*x^n),x)
```

```
[Out] int((c+d*x^n)^3/(a+b*x^n),x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\int \frac{1}{b^4x^n+ab^3}\,dx+\frac{b^2d^3(n+1)xx^{2\,n}+\left(3\,b^2cd^2(2\,n+1)-abd^3(2\,n+1)\right)xx^n+\left(3\left(2\,n^2+3\,n+1\right)b^2c^2d-3\left(2\,n^2+3\,n+1\right)abcd^2+\left(2\,n^2+3\,n\right.\right.}{(2\,n^2+3\,n+1)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^3/(b*x^n + a),x, algorithm="maxima")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*integrate(1/(
b^4*x^n + a*b^3), x) + (b^2*d^3*(n + 1)*x*x^(2*n) + (3*b^2*c*d^2*
(2*n + 1) - a*b*d^3*(2*n + 1))*x*x^n + (3*(2*n^2 + 3*n + 1)*b^2*c
^2*d - 3*(2*n^2 + 3*n + 1)*a*b*c*d^2 + (2*n^2 + 3*n + 1)*a^2*d^3)
*x)/((2*n^2 + 3*n + 1)*b^3)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^3x^{3\,n}+3\,cd^2x^{2\,n}+3\,c^2dx^n+c^3}{bx^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^3/(b*x^n + a),x, algorithm="fricas")
```

```
[Out] integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)/(b*x
^n + a), x)
```

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**n)**3/(a+b*x**n),x)
```

```
[Out] Exception raised: TypeError
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n+c)^3}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^3/(b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^3/(b*x^n + a), x)
```

# 3.201 $\int \frac{(c+dx^n)^2}{a+bx^n}\,dx$

**Optimal**. Leaf size=84

$$\frac{x(bc-ad)^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^2}-\frac{dx(ad(n+1)-b(2cn+c))}{b^2(n+1)}+\frac{dx\,(c+dx^n)}{b(n+1)}$$

```
[Out] -((d*(a*d*(1 + n) - b*(c + 2*c*n))*x)/(b^2*(1 + n))) + (d*x*(c +
d*x^n))/(b*(1 + n)) + ((b*c - a*d)^2*x*Hypergeometric2F1[1, n^(-1
), 1 + n^(-1), -((b*x^n)/a)])/(a*b^2)
```

---

**Rubi [A]** time = 0.236371, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{x(bc-ad)^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^2}-\frac{dx(ad(n+1)-b(2cn+c))}{b^2(n+1)}+\frac{dx\,(c+dx^n)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^n)^2/(a + b*x^n),x]
```

```
[Out] -((d*(a*d*(1 + n) - b*(c + 2*c*n))*x)/(b^2*(1 + n))) + (d*x*(c +
d*x^n))/(b*(1 + n)) + ((b*c - a*d)^2*x*Hypergeometric2F1[1, n^(-1
), 1 + n^(-1), -((b*x^n)/a)])/(a*b^2)
```

---

**Rubi in Sympy [A]** time = 22.1612, size = 70, normalized size = 0.83

$$\frac{dx\,(c+dx^n)}{b\,(n+1)}-\frac{dx\,(ad\,(n+1)-bc\,(2n+1))}{b^2\,(n+1)}+\frac{x\,(ad-bc)^2\,{}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**n)**2/(a+b*x**n),x)
```

```
[Out] d*x*(c + d*x**n)/(b*(n + 1)) - d*x*(a*d*(n + 1) - b*c*(2*n + 1))/
(b**2*(n + 1)) + x*(a*d - b*c)**2*hyper((1, 1/n), (1 + 1/n,), -b*
x**n/a)/(a*b**2)
```

---

**Mathematica [A]** time = 0.0705425, size = 82, normalized size = 0.98

$$\frac{x(bc-ad)^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^2}-\frac{x(ad-bc)^2}{ab^2}+\frac{c^2x}{a}+\frac{d^2x^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^n)^2/(a + b*x^n),x]
```

```
[Out] (c^2*x)/a - ((-(b*c) + a*d)^2*x)/(a*b^2) + (d^2*x^(1 + n))/(b*(1
+ n)) + ((b*c - a*d)^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
 -((b*x^n)/a)])/(a*b^2)
```

---

**Maple [F]** time = 0.062, size = 0, normalized size = 0.

$$\int \frac{(c+dx^n)^2}{a+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^n)^2/(a+b*x^n),x)
```

```
[Out] int((c+d*x^n)^2/(a+b*x^n),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left(b^2c^2-2\,abcd+a^2d^2\right)\int\frac{1}{b^3x^n+ab^2}\,dx+\frac{bd^2xx^n+\left(2\,bcd(n+1)-ad^2(n+1)\right)x}{b^2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^2/(b*x^n + a),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*integrate(1/(b^3*x^n + a*b^2), x)
 + (b*d^2*x*x^n + (2*b*c*d*(n + 1) - a*d^2*(n + 1))*x)/(b^2*(n +
1))
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^2x^{2\,n}+2\,cdx^n+c^2}{bx^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^2/(b*x^n + a),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)/(b*x^n + a), x)
```

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**n)**2/(a+b*x**n),x)
```

```
[Out] Exception raised: TypeError
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^2}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^2/(b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^2/(b*x^n + a), x)
```

## 3.202 $\int \frac{c+dx^n}{a+bx^n}\,dx$

**Optimal**. Leaf size=42

$$\frac{x(bc-ad)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab}+\frac{dx}{b}$$

```
[Out] (d*x)/b + ((b*c - a*d)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
 -((b*x^n)/a)])/(a*b)
```

---

**Rubi [A]** time = 0.0525895, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{x(bc-ad)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^n)/(a + b*x^n),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
 -((b*x^n)/a)])/(a*b)
```

---

**Rubi in Sympy [A]** time = 6.35457, size = 31, normalized size = 0.74

$$\frac{dx}{b}-\frac{x\,(ad-bc)\,{}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**n)/(a+b*x**n),x)
```

```
[Out] d*x/b - x*(a*d - b*c)*hyper((1, 1/n), (1 + 1/n,), -b*x**n/a)/(a*b
)
```

---

**Mathematica [A]** time = 0.0332002, size = 40, normalized size = 0.95

$$\frac{x\left((bc-ad)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)+ad\right)}{ab}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^n)/(a + b*x^n),x]
```

```
[Out] (x*(a*d + (b*c - a*d)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(
(b*x^n)/a)]))/(a*b)
```

---

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int \frac{c + dx^n}{a + bx^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^n)/(a+b*x^n),x)
```

```
[Out] int((c+d*x^n)/(a+b*x^n),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$(bc - ad)\int \frac{1}{b^2x^n + ab}\, dx + \frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)/(b*x^n + a),x, algorithm="maxima")
```

```
[Out] (b*c - a*d)*integrate(1/(b^2*x^n + a*b), x) + d*x/b
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^n + c}{bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)/(b*x^n + a),x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)/(b*x^n + a), x)
```

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**n)/(a+b*x**n),x)
```

```
[Out] Exception raised: TypeError
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^n + c}{bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)/(b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)/(b*x^n + a), x)
```

## 3.203 $\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx$

**Optimal**. Leaf size=72

$$\frac{bx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

```
[Out] (b*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(
b*c - a*d)) - (d*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*
x^n)/c)])/(c*(b*c - a*d))
```

---

**Rubi [A]** time = 0.0794435, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{bx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(
b*c - a*d)) - (d*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*
x^n)/c)])/(c*(b*c - a*d))
```

---

**Rubi in Sympy [A]** time = 10.9658, size = 53, normalized size = 0.74

$$\frac{dx{}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{dx^n}{c}\right)}{c\,(ad-bc)}-\frac{bx{}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{a\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] d*x*hyper((1, 1/n), (1 + 1/n,), -d*x**n/c)/(c*(a*d - b*c)) - b*x*
hyper((1, 1/n), (1 + 1/n,), -b*x**n/a)/(a*(a*d - b*c))
```

---

**Mathematica [A]** time = 0.0545357, size = 64, normalized size = 0.89

$$\frac{x\left(ad\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)-bc\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)\right)}{ac(ad-bc)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (x*(-(b*c*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])
 + a*d*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(
a*c*(-(b*c) + a*d))
```

---

**Maple [F]** time = 0.091, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*x^n)/(c+d*x^n),x)
```

```
[Out] int(1/(a+b*x^n)/(c+d*x^n),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)*(d*x^n + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{bdx^{2n}+ac+(bc+ad)x^n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)*(d*x^n + c)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)\,(c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Integral(1/((a + b*x**n)*(c + d*x**n)), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)*(d*x^n + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

## 3.204 $\int \frac{1}{(a+bx^n)(c+dx^n)^2} dx$

**Optimal**. Leaf size=123

$$\frac{b^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)^2}+\frac{dx(bc(1-2n)-ad(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2n(bc-ad)^2}-\frac{dx}{cn(bc-ad)\left(c+dx^n\right)}$$

```
[Out] -((d*x)/(c*(b*c - a*d)*n*(c + d*x^n))) + (b^2*x*Hypergeometric2F1
[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)^2) + (d*(b*
c*(1 - 2*n) - a*d*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(
-1), -((d*x^n)/c)])/(c^2*(b*c - a*d)^2*n)
```

---

**Rubi [A]** time = 0.359577, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{b^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)^2}-\frac{dx(ad(1-n)-b(c-2cn))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2n(bc-ad)^2}-\frac{dx}{cn(bc-ad)\left(c+dx^n\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^n)*(c + d*x^n)^2),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*n*(c + d*x^n))) + (b^2*x*Hypergeometric2F1
[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)^2) - (d*(a*
d*(1 - n) - b*(c - 2*c*n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(
-1), -((d*x^n)/c)])/(c^2*(b*c - a*d)^2*n)
```

---

**Rubi in Sympy [A]** time = 48.4865, size = 100, normalized size = 0.81

$$\frac{dx}{cn\left(c+dx^n\right)(ad-bc)}-\frac{dx\left(-adn+ad+2bcn-bc\right){}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{dx^n}{c}\right)}{c^2n\,(ad-bc)^2}+\frac{b^2x{}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{a\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*x**n)/(c+d*x**n)**2,x)
```

```
[Out] d*x/(c*n*(c + d*x**n)*(a*d - b*c)) - d*x*(-a*d*n + a*d + 2*b*c*n
- b*c)*hyper((1, 1/n), (1 + 1/n,), -d*x**n/c)/(c**2*n*(a*d - b*c)
**2) + b**2*x*hyper((1, 1/n), (1 + 1/n,), -b*x**n/a)/(a*(a*d - b*
c)**2)
```

**Mathematica [A]** time = 0.217176, size = 121, normalized size = 0.98

$$\frac{x\left(b^2c^2n\left(c+dx^n\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)+ad\left(\left(c+dx^n\right)\left(ad(n-1)+b(c-2cn)\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+c(ad-bc)\right)\right)}{ac^2n(bc-ad)^2\left(c+dx^n\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^n)*(c + d*x^n)^2),x]
```

```
[Out] (x*(b^2*c^2*n*(c + d*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1)
, -((b*x^n)/a)] + a*d*(c*(-(b*c) + a*d) + (a*d*(-1 + n) + b*(c -
2*c*n))*(c + d*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d
*x^n)/c)])))/(a*c^2*(b*c - a*d)^2*n*(c + d*x^n))
```

**Maple [F]** time = 0.165, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)\,(c+dx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*x^n)/(c+d*x^n)^2,x)
```

```
[Out] int(1/(a+b*x^n)/(c+d*x^n)^2,x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$b^2\int \frac{1}{ab^2c^2-2\,a^2bcd+a^3d^2+(b^3c^2-2\,ab^2cd+a^2bd^2)x^n}\,dx$$
$$-\left(bcd(2\,n-1)-ad^2(n-1)\right)\int \frac{1}{b^2c^4n-2\,abc^3dn+a^2c^2d^2n+(b^2c^3dn-2\,abc^2d^2n+a^2cd^3n)x^n}\,dx$$
$$-\frac{dx}{bc^3n-ac^2dn+(bc^2dn-acd^2n)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)*(d*x^n + c)^2),x, algorithm="maxima")
```

```
[Out] b^2*integrate(1/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)*x^n), x) - (b*c*d*(2*n - 1) - a*d^2*(n -
1))*integrate(1/(b^2*c^4*n - 2*a*b*c^3*d*n + a^2*c^2*d^2*n + (b^2
```

```
*c^3*d*n - 2*a*b*c^2*d^2*n + a^2*c*d^3*n)*x^n), x) - d*x/(b*c^3*n
 - a*c^2*d*n + (b*c^2*d*n - a*c*d^2*n)*x^n)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bd^2x^{3\,n} + ac^2 + (2\,bcd + ad^2)x^{2\,n} + (bc^2 + 2\,acd)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)*(d*x^n + c)^2),x, algorithm="fricas")
```

```
[Out] integral(1/(b*d^2*x^(3*n) + a*c^2 + (2*b*c*d + a*d^2)*x^(2*n) + (
b*c^2 + 2*a*c*d)*x^n), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*x**n)/(c+d*x**n)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)(dx^n + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)*(d*x^n + c)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)^2), x)
```

## 3.205 $\int \frac{1}{(a+bx^n)(c+dx^n)^3}\,dx$

**Optimal**. Leaf size=210

$$-\frac{dx\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(3n^2-4n+1\right)+b^2c^2\left(6n^2-5n+1\right)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3n^2(bc-ad)^3}+\frac{b^3x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)^3}-\frac{dx(ad(1-2n)-b(c-4cn))}{2c^2n^2(bc-ad)^2\left(c+dx^n\right)}-\frac{dx}{2cn(bc-ad)\left(c+dx^n\right)^2}$$

```
[Out] -(d*x)/(2*c*(b*c - a*d)*n*(c + d*x^n)^2) - (d*(a*d*(1 - 2*n) - b*
(c - 4*c*n))*x)/(2*c^2*(b*c - a*d)^2*n^2*(c + d*x^n)) + (b^3*x*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a
*d)^3) - (d*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d*(1 - 4*n + 3*n
^2) + b^2*c^2*(1 - 5*n + 6*n^2))*x*Hypergeometric2F1[1, n^(-1), 1
 + n^(-1), -((d*x^n)/c)])/(2*c^3*(b*c - a*d)^3*n^2)
```

---

**Rubi [A]** time = 0.818794, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$-\frac{dx\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(3n^2-4n+1\right)+b^2c^2\left(6n^2-5n+1\right)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3n^2(bc-ad)^3}+\frac{b^3x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)^3}+\frac{dx(bc(1-4n)-ad(1-2n))}{2c^2n^2(bc-ad)^2\left(c+dx^n\right)}-\frac{dx}{2cn(bc-ad)\left(c+dx^n\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^n)*(c + d*x^n)^3),x]
```

```
[Out] -(d*x)/(2*c*(b*c - a*d)*n*(c + d*x^n)^2) + (d*(b*c*(1 - 4*n) - a*
d*(1 - 2*n))*x)/(2*c^2*(b*c - a*d)^2*n^2*(c + d*x^n)) + (b^3*x*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a
*d)^3) - (d*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d*(1 - 4*n + 3*n
^2) + b^2*c^2*(1 - 5*n + 6*n^2))*x*Hypergeometric2F1[1, n^(-1), 1
 + n^(-1), -((d*x^n)/c)])/(2*c^3*(b*c - a*d)^3*n^2)
```

---

**Rubi in Sympy [A]** time = 112.827, size = 218, normalized size = 1.04

$$\frac{dx}{2cn\left(c+dx^{n}\right)^{2}\left(ad-bc\right)}-\frac{dx\left(-2adn+ad+4bcn-bc\right)}{2c^{2}n^{2}\left(c+dx^{n}\right)\left(ad-bc\right)^{2}}$$

$$+\frac{dx\left(ad\left(-2adn+ad+2bcn-bc\left(-2n+1\right)\right)-bc\left(-n+1\right)\left(-2adn+ad+4bcn-bc\right)-n\left(ad-bc\right)\left(-2adn+ad+2bcn\right)\right){}_2F_1}{2c^{3}n^{2}\left(ad-bc\right)^{3}}$$

$$-\frac{b^{3}x\,{}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{bx^{n}}{a}\right)}{a\left(ad-bc\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*x**n)/(c+d*x**n)**3,x)
```

```
[Out] d*x/(2*c*n*(c + d*x**n)**2*(a*d - b*c)) - d*x*(-2*a*d*n + a*d + 4
*b*c*n - b*c)/(2*c**2*n**2*(c + d*x**n)*(a*d - b*c)**2) + d*x*(a*
d*(-2*a*d*n + a*d + 2*b*c*n - b*c*(-2*n + 1)) - b*c*(-n + 1)*(-2*
a*d*n + a*d + 4*b*c*n - b*c) - n*(a*d - b*c)*(-2*a*d*n + a*d + 2*
b*c*n))*hyper((1, 1/n), (1 + 1/n,), -d*x**n/c)/(2*c**3*n**2*(a*d
- b*c)**3) - b**3*x*hyper((1, 1/n), (1 + 1/n,), -b*x**n/a)/(a*(a*
d - b*c)**3)
```

---

**Mathematica [A]** time = 0.333671, size = 210, normalized size = 1.

$$\frac{x\left(-ad\left(c+dx^{n}\right)^{2}\left(a^{2}d^{2}\left(2n^{2}-3n+1\right)-2abcd\left(3n^{2}-4n+1\right)+b^{2}c^{2}\left(6n^{2}-5n+1\right)\right)\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^{n}}{c}\right)+2b^{3}c^{3}n^{2}\left(c\right.}{2ac^{3}n^{2}(bc-ad)^{3}\left(c+dx\right.}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^n)*(c + d*x^n)^3),x]
```

```
[Out] (x*(-(a*c^2*d*(b*c - a*d)^2*n) + a*c*d*(b*c - a*d)*(a*d*(-1 + 2*n
) + b*(c - 4*c*n))*(c + d*x^n) + 2*b^3*c^3*n^2*(c + d*x^n)^2*Hype
rgeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)] - a*d*(a^2*d^2
*(1 - 3*n + 2*n^2) - 2*a*b*c*d*(1 - 4*n + 3*n^2) + b^2*c^2*(1 - 5
*n + 6*n^2))*(c + d*x^n)^2*Hypergeometric2F1[1, n^(-1), 1 + n^(-1
), -((d*x^n)/c)]))/(2*a*c^3*(b*c - a*d)^3*n^2*(c + d*x^n)^2)
```

---

**Maple [F]** time = 0.175, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^{n}\right)\left(c+dx^{n}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*x^n)/(c+d*x^n)^3,x)
```

```
[Out] int(1/(a+b*x^n)/(c+d*x^n)^3,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-b^3 \int -\frac{1}{ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3 + (b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3)x^n}\,dx + \left(\left(6\,n^2 - 5\,n + 1\right)b^2c^2d - 2\left(3\,n^2 - 4\,n + 1\right)abcd^2 + \left(2\,n^2 - 3\,n + 1\right)a^2d^3\right)\int -\frac{1}{2\,(b^3c^6n^2 - 3\,ab^2c^5dn^2 + 3\,a^2bc^4d^2n^2 - a^3c^3d^3n^2 + (b^3c^5dn^2 - 3\,ab^2c^4d^2n^2 + 3\,a^2bc^3d^3n^2 - a^3c^2d^4n^2)x^n)}\,dx - \frac{\left(bcd^2(4\,n - 1) - ad^3(2\,n - 1)\right)xx^n + \left(bc^2d(5\,n - 1) - acd^2(3\,n - 1)\right)x}{2\,(b^2c^6n^2 - 2\,abc^5dn^2 + a^2c^4d^2n^2 + (b^2c^4d^2n^2 - 2\,abc^3d^3n^2 + a^2c^2d^4n^2)x^{2\,n} + 2\,(b^2c^5dn^2 - 2\,abc^4d^2n^2 + a^2c^3d^3n^2)x^n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)*(d*x^n + c)^3),x, algorithm="maxima")
```

```
[Out] -b^3*integrate(-1/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 -
a^4*d^3 + (b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)
*x^n), x) + ((6*n^2 - 5*n + 1)*b^2*c^2*d - 2*(3*n^2 - 4*n + 1)*a*
b*c*d^2 + (2*n^2 - 3*n + 1)*a^2*d^3)*integrate(-1/2/(b^3*c^6*n^2
- 3*a*b^2*c^5*d*n^2 + 3*a^2*b*c^4*d^2*n^2 - a^3*c^3*d^3*n^2 + (b^
3*c^5*d*n^2 - 3*a*b^2*c^4*d^2*n^2 + 3*a^2*b*c^3*d^3*n^2 - a^3*c^2
*d^4*n^2)*x^n), x) - 1/2*((b*c*d^2*(4*n - 1) - a*d^3*(2*n - 1))*x
*x^n + (b*c^2*d*(5*n - 1) - a*c*d^2*(3*n - 1))*x)/(b^2*c^6*n^2 -
2*a*b*c^5*d*n^2 + a^2*c^4*d^2*n^2 + (b^2*c^4*d^2*n^2 - 2*a*b*c^3*
d^3*n^2 + a^2*c^2*d^4*n^2)*x^(2*n) + 2*(b^2*c^5*d*n^2 - 2*a*b*c^4
*d^2*n^2 + a^2*c^3*d^3*n^2)*x^n)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bd^3x^{4\,n} + ac^3 + (3\,bcd^2 + ad^3)x^{3\,n} + 3\,(bc^2d + acd^2)x^{2\,n} + (bc^3 + 3\,ac^2d)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)*(d*x^n + c)^3),x, algorithm="fricas")
```

```
[Out] integral(1/(b*d^3*x^(4*n) + a*c^3 + (3*b*c*d^2 + a*d^3)*x^(3*n) +
 3*(b*c^2*d + a*c*d^2)*x^(2*n) + (b*c^3 + 3*a*c^2*d)*x^n), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*x**n)/(c+d*x**n)**3,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)(dx^n + c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)*(d*x^n + c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)^3), x)
```

# 3.206 $\int \frac{(c+dx^n)^4}{(a+bx^n)^2} dx$

**Optimal**. Leaf size=341

$$-\frac{x(bc-ad)^3(bc(1-n)-ad(3n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2b^4n}$$
$$-\frac{dx\,(c+dx^n)\left(a^2d^2\left(6n^2+5n+1\right)-2abcd\left(5n^2+4n+1\right)+b^2c^2\left(2n^2+3n+1\right)\right)}{ab^3n(n+1)(2n+1)}$$
$$-\frac{dx\left(-a^3d^3\left(6n^3+11n^2+6n+1\right)+a^2bcd^2\left(16n^3+26n^2+15n+3\right)-ab^2c^2d\left(12n^3+17n^2+12n+3\right)+b^3c^3\left(2n^2+3n+1\right.\right.}{ab^4n(n+1)(2n+1)}$$
$$+\frac{dx\,(c+dx^n)^2\,(ad(3n+1)-b(2cn+c))}{ab^2n(2n+1)}+\frac{x(bc-ad)\,(c+dx^n)^3}{abn\,(a+bx^n)}$$

```
[Out] -((d*(b^3*c^3*(1 + 3*n + 2*n^2) - a^3*d^3*(1 + 6*n + 11*n^2 + 6*n
^3) - a*b^2*c^2*d*(3 + 12*n + 17*n^2 + 12*n^3) + a^2*b*c*d^2*(3 +
 15*n + 26*n^2 + 16*n^3))*x)/(a*b^4*n*(1 + n)*(1 + 2*n))) - (d*(b
^2*c^2*(1 + 3*n + 2*n^2) - 2*a*b*c*d*(1 + 4*n + 5*n^2) + a^2*d^2*
(1 + 5*n + 6*n^2))*x*(c + d*x^n))/(a*b^3*n*(1 + n)*(1 + 2*n)) + (
d*(a*d*(1 + 3*n) - b*(c + 2*c*n))*x*(c + d*x^n)^2)/(a*b^2*n*(1 +
2*n)) + ((b*c - a*d)*x*(c + d*x^n)^3)/(a*b*n*(a + b*x^n)) - ((b*c
 - a*d)^3*(b*c*(1 - n) - a*d*(1 + 3*n))*x*Hypergeometric2F1[1, n^
(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b^4*n)
```

---

**Rubi [A]** time = 1.2872, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$-\frac{x(bc-ad)^3(bc(1-n)-ad(3n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2b^4n}$$
$$-\frac{dx\,(c+dx^n)\left(a^2d^2\left(6n^2+5n+1\right)-2abcd\left(5n^2+4n+1\right)+b^2c^2\left(2n^2+3n+1\right)\right)}{ab^3n(n+1)(2n+1)}$$
$$-\frac{dx\left(-a^3d^3\left(6n^3+11n^2+6n+1\right)+a^2bcd^2\left(16n^3+26n^2+15n+3\right)-ab^2c^2d\left(12n^3+17n^2+12n+3\right)+b^3c^3\left(2n^2+3n+1\right.\right.}{ab^4n(n+1)(2n+1)}$$
$$+\frac{dx\,(c+dx^n)^2\,(ad(3n+1)-b(2cn+c))}{ab^2n(2n+1)}+\frac{x(bc-ad)\,(c+dx^n)^3}{abn\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^n)^4/(a + b*x^n)^2,x]
```

```
[Out] -((d*(b^3*c^3*(1 + 3*n + 2*n^2) - a^3*d^3*(1 + 6*n + 11*n^2 + 6*n
^3) - a*b^2*c^2*d*(3 + 12*n + 17*n^2 + 12*n^3) + a^2*b*c*d^2*(3 +
 15*n + 26*n^2 + 16*n^3))*x)/(a*b^4*n*(1 + n)*(1 + 2*n))) - (d*(b
^2*c^2*(1 + 3*n + 2*n^2) - 2*a*b*c*d*(1 + 4*n + 5*n^2) + a^2*d^2*
(1 + 5*n + 6*n^2))*x*(c + d*x^n))/(a*b^3*n*(1 + n)*(1 + 2*n)) + (
d*(a*d*(1 + 3*n) - b*(c + 2*c*n))*x*(c + d*x^n)^2)/(a*b^2*n*(1 +
```

```
2*n)) + ((b*c - a*d)*x*(c + d*x^n)^3)/(a*b*n*(a + b*x^n)) - ((b*c
 - a*d)^3*(b*c*(1 - n) - a*d*(1 + 3*n))*x*Hypergeometric2F1[1, n^
(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b^4*n)
```

---

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**n)**4/(a+b*x**n)**2,x)
```

```
[Out] Timed out
```

---

**Mathematica [A]** time = 0.337356, size = 217, normalized size = 0.64

$$\frac{x\left(\frac{(bc-ad)^3(ad(3n+1)+bc(n-1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n}+\frac{(ad-bc)^3(ad(3n+1)+bc(n-1))}{a^2n}+\frac{-a^4d^4+4a^3bcd^3-6a^2b^2c^2d^2+4ab^3c^3d+b^4c^4(n-1)}{a^2n}+\frac{2bd^3x^n(2b}{n+1}\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^n)^4/(a + b*x^n)^2,x]
```

```
[Out] (x*((4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4
+ b^4*c^4*(-1 + n))/(a^2*n) + ((-(b*c) + a*d)^3*(b*c*(-1 + n) + a
*d*(1 + 3*n)))/(a^2*n) + (2*b*d^3*(2*b*c - a*d)*x^n)/(1 + n) + (b
^2*d^4*x^(2*n))/(1 + 2*n) + (b*c - a*d)^4/(a*n*(a + b*x^n)) + ((b
*c - a*d)^3*(b*c*(-1 + n) + a*d*(1 + 3*n))*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n)))/b^4
```

---

**Maple [F]** time = 0.078, size = 0, normalized size = 0.

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^n)^4/(a+b*x^n)^2,x)
```

```
[Out] int((c+d*x^n)^4/(a+b*x^n)^2,x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\left(a^4d^4(3\,n+1) - 4\,a^3bcd^3(2\,n+1) + 6\,a^2b^2c^2d^2(n+1) - b^4c^4(n-1) - 4\,ab^3c^3d\right)\int \frac{1}{ab^5nx^n + a^2b^4n}\,dx$$
$$+ \frac{\left(n^2+n\right)ab^3d^4xx^{3\,n} + \left(4\left(2\,n^2+n\right)ab^3cd^3 - \left(3\,n^2+n\right)a^2b^2d^4\right)xx^{2\,n} + \left(6\left(2\,n^3+3\,n^2+n\right)ab^3c^2d^2 - 4\left(4\,n^3+4\,n^2+n\right)a^2b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^4/(b*x^n + a)^2,x, algorithm="maxima")
```

```
[Out] -(a^4*d^4*(3*n + 1) - 4*a^3*b*c*d^3*(2*n + 1) + 6*a^2*b^2*c^2*d^2
*(n + 1) - b^4*c^4*(n - 1) - 4*a*b^3*c^3*d)*integrate(1/(a*b^5*n*
x^n + a^2*b^4*n), x) + ((n^2 + n)*a*b^3*d^4*x*x^(3*n) + (4*(2*n^2
 + n)*a*b^3*c*d^3 - (3*n^2 + n)*a^2*b^2*d^4)*x*x^(2*n) + (6*(2*n^
3 + 3*n^2 + n)*a*b^3*c^2*d^2 - 4*(4*n^3 + 4*n^2 + n)*a^2*b^2*c*d^
3 + (6*n^3 + 5*n^2 + n)*a^3*b*d^4)*x*x^n + ((2*n^2 + 3*n + 1)*b^4
*c^4 - 4*(2*n^2 + 3*n + 1)*a*b^3*c^3*d + 6*(2*n^3 + 5*n^2 + 4*n +
 1)*a^2*b^2*c^2*d^2 - 4*(4*n^3 + 8*n^2 + 5*n + 1)*a^3*b*c*d^3 + (
6*n^3 + 11*n^2 + 6*n + 1)*a^4*d^4)*x)/((2*n^3 + 3*n^2 + n)*a*b^5*
x^n + (2*n^3 + 3*n^2 + n)*a^2*b^4)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^4x^{4\,n} + 4\,cd^3x^{3\,n} + 6\,c^2d^2x^{2\,n} + 4\,c^3dx^n + c^4}{b^2x^{2\,n} + 2\,abx^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^4/(b*x^n + a)^2,x, algorithm="fricas")
```

```
[Out] integral((d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) + 4*c
^3*d*x^n + c^4)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**n)**4/(a+b*x**n)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^4}{(bx^n + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^4/(b*x^n + a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^4/(b*x^n + a)^2, x)
```

# 3.207 $\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx$

**Optimal**. Leaf size=200

$$-\frac{x(bc-ad)^2(bc(1-n)-ad(2n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2b^3n}-\frac{dx\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(3n^2+4n+2\right)+b^2c^2(n+1)\right)}{ab^3n(n+1)}-\frac{dx\,(c+dx^n)\,(bc(n+1)-ad(2n+1))}{ab^2n(n+1)}+\frac{x(bc-ad)\,(c+dx^n)^2}{abn\,(a+bx^n)}$$

```
[Out] -((d*(b^2*c^2*(1 + n) + a^2*d^2*(1 + 3*n + 2*n^2) - a*b*c*d*(2 +
4*n + 3*n^2))*x)/(a*b^3*n*(1 + n))) - (d*(b*c*(1 + n) - a*d*(1 +
2*n))*x*(c + d*x^n))/(a*b^2*n*(1 + n)) + ((b*c - a*d)*x*(c + d*x^
n)^2)/(a*b*n*(a + b*x^n)) - ((b*c - a*d)^2*(b*c*(1 - n) - a*d*(1
+ 2*n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])
/(a^2*b^3*n)
```

---

**Rubi [A]** time = 0.609973, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$-\frac{x(bc-ad)^2(bc(1-n)-ad(2n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2b^3n}-\frac{dx\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(3n^2+4n+2\right)+b^2c^2(n+1)\right)}{ab^3n(n+1)}-\frac{dx\,(c+dx^n)\,(bc(n+1)-ad(2n+1))}{ab^2n(n+1)}+\frac{x(bc-ad)\,(c+dx^n)^2}{abn\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^n)^3/(a + b*x^n)^2,x]
```

```
[Out] -((d*(b^2*c^2*(1 + n) + a^2*d^2*(1 + 3*n + 2*n^2) - a*b*c*d*(2 +
4*n + 3*n^2))*x)/(a*b^3*n*(1 + n))) - (d*(b*c*(1 + n) - a*d*(1 +
2*n))*x*(c + d*x^n))/(a*b^2*n*(1 + n)) + ((b*c - a*d)*x*(c + d*x^
n)^2)/(a*b*n*(a + b*x^n)) - ((b*c - a*d)^2*(b*c*(1 - n) - a*d*(1
+ 2*n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])
/(a^2*b^3*n)
```

---

**Rubi in Sympy [A]** time = 65.6107, size = 172, normalized size = 0.86

$$-\frac{x\left(c+dx^{n}\right)^{2}\left(ad-bc\right)}{abn\left(a+bx^{n}\right)}+\frac{dx\left(c\left(ad-bc\left(-n+1\right)\right)+dx^{n}\left(ad\left(2n+1\right)-bc\left(n+1\right)\right)\right)}{ab^{2}n\left(n+1\right)}-\frac{dx\left(adn+ad-bc\right)\left(2adn+ad-3bcn-bc\right)}{ab^{3}n\left(n+1\right)}+\frac{x\left(ad-bc\right)^{2}\left(2adn+ad+bcn-bc\right){}_{2}F_{1}\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{bx^{n}}{a}\right)}{a^{2}b^{3}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**n)**3/(a+b*x**n)**2,x)
```

```
[Out] -x*(c + d*x**n)**2*(a*d - b*c)/(a*b*n*(a + b*x**n)) + d*x*(c*(a*d
 - b*c*(-n + 1)) + d*x**n*(a*d*(2*n + 1) - b*c*(n + 1)))/(a*b**2*
n*(n + 1)) - d*x*(a*d*n + a*d - b*c)*(2*a*d*n + a*d - 3*b*c*n - b
*c)/(a*b**3*n*(n + 1)) + x*(a*d - b*c)**2*(2*a*d*n + a*d + b*c*n
- b*c)*hyper((1, 1/n), (1 + 1/n,), -b*x**n/a)/(a**2*b**3*n)
```

---

**Mathematica [A]** time = 0.457563, size = 167, normalized size = 0.84

$$\frac{x\left(\frac{a\left(-a^{3}d^{3}\left(2n^{2}+3n+1\right)+a^{2}bd^{2}\left(3c(n+1)^{2}-dn(2n+1)x^{n}\right)+ab^{2}d\left(-3c^{2}(n+1)+3cdn(n+1)x^{n}+d^{2}nx^{2n}\right)+b^{3}c^{3}(n+1)\right)}{(n+1)(a+bx^{n})}+(bc-ad)^{2}(ad(2n+1)+bc(n-1)\right.}{a^{2}b^{3}n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^n)^3/(a + b*x^n)^2,x]
```

```
[Out] (x*((a*(b^3*c^3*(1 + n) - a^3*d^3*(1 + 3*n + 2*n^2) + a^2*b*d^2*(
3*c*(1 + n)^2 - d*n*(1 + 2*n)*x^n) + a*b^2*d*(-3*c^2*(1 + n) + 3*
c*d*n*(1 + n)*x^n + d^2*n*x^(2*n))))/((1 + n)*(a + b*x^n)) + (b*c
 - a*d)^2*(b*c*(-1 + n) + a*d*(1 + 2*n))*Hypergeometric2F1[1, n^(
-1), 1 + n^(-1), -((b*x^n)/a)]))/(a^2*b^3*n)
```

---

**Maple [F]** time = 0.073, size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^{n}\right)^{3}}{\left(a+bx^{n}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^n)^3/(a+b*x^n)^2,x)
```

```
[Out] int((c+d*x^n)^3/(a+b*x^n)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left(a^3d^3(2\,n+1) - 3\,a^2bcd^2(n+1) + b^3c^3(n-1) + 3\,ab^2c^2d\right)\int \frac{1}{ab^4nx^n + a^2b^3n}\,dx + \frac{ab^2d^3nxx^{2\,n} + \left(3\,(n^2+n)\,ab^2cd^2 - (2\,n^2+n)\,a^2bd^3\right)xx^n + \left(3\,(n^2+2\,n+1)\,a^2bcd^2 - (2\,n^2+3\,n+1)\,a^3d^3 + b^3c^3(n+1) - 3\right.}{(n^2+n)ab^4x^n + (n^2+n)a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^3/(b*x^n + a)^2,x, algorithm="maxima")
```

```
[Out] (a^3*d^3*(2*n + 1) - 3*a^2*b*c*d^2*(n + 1) + b^3*c^3*(n - 1) + 3*
a*b^2*c^2*d)*integrate(1/(a*b^4*n*x^n + a^2*b^3*n), x) + (a*b^2*d
^3*n*x*x^(2*n) + (3*(n^2 + n)*a*b^2*c*d^2 - (2*n^2 + n)*a^2*b*d^3
)*x*x^n + (3*(n^2 + 2*n + 1)*a^2*b*c*d^2 - (2*n^2 + 3*n + 1)*a^3*
d^3 + b^3*c^3*(n + 1) - 3*a*b^2*c^2*d*(n + 1))*x)/((n^2 + n)*a*b^
4*x^n + (n^2 + n)*a^2*b^3)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^3x^{3\,n} + 3\,cd^2x^{2\,n} + 3\,c^2dx^n + c^3}{b^2x^{2\,n} + 2\,abx^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^3/(b*x^n + a)^2,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)/(b^2
*x^(2*n) + 2*a*b*x^n + a^2), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**n)**3/(a+b*x**n)**2,x)
```

```
[Out] Timed out
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^3}{(bx^n + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^3/(b*x^n + a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^3/(b*x^n + a)^2, x)
```

# 3.208 $\int \frac{(c+dx^n)^2}{(a+bx^n)^2} \, dx$

**Optimal**. Leaf size=115

$$-\frac{x(bc-ad)(bc(1-n)-ad(n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2b^2n}-\frac{dx(bc-ad(n+1))}{ab^2n}+\frac{x(bc-ad)\left(c+dx^n\right)}{abn\left(a+bx^n\right)}$$

```
[Out] -((d*(b*c - a*d*(1 + n))*x)/(a*b^2*n)) + ((b*c - a*d)*x*(c + d*x^
n))/(a*b*n*(a + b*x^n)) - ((b*c - a*d)*(b*c*(1 - n) - a*d*(1 + n)
)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*
b^2*n)
```

---

**Rubi [A]** time = 0.236724, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$-\frac{x(bc-ad)(bc(1-n)-ad(n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2b^2n}-\frac{dx(bc-ad(n+1))}{ab^2n}+\frac{x(bc-ad)\left(c+dx^n\right)}{abn\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^n)^2/(a + b*x^n)^2,x]
```

```
[Out] -((d*(b*c - a*d*(1 + n))*x)/(a*b^2*n)) + ((b*c - a*d)*x*(c + d*x^
n))/(a*b*n*(a + b*x^n)) - ((b*c - a*d)*(b*c*(1 - n) - a*d*(1 + n)
)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*
b^2*n)
```

---

**Rubi in Sympy [A]** time = 22.5983, size = 97, normalized size = 0.84

$$-\frac{x\left(c+dx^n\right)(ad-bc)}{abn\left(a+bx^n\right)}-\frac{dx\left(-ad\left(n+1\right)+bc\right)}{ab^2n}-\frac{x\left(ad-bc\right)\left(adn+ad+bcn-bc\right){}_2F_1\left(\left.\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\right|-\frac{bx^n}{a}\right)}{a^2b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**n)**2/(a+b*x**n)**2,x)
```

```
[Out] -x*(c + d*x**n)*(a*d - b*c)/(a*b*n*(a + b*x**n)) - d*x*(-a*d*(n +
 1) + b*c)/(a*b**2*n) - x*(a*d - b*c)*(a*d*n + a*d + b*c*n - b*c)
*hyper((1, 1/n), (1 + 1/n,), -b*x**n/a)/(a**2*b**2*n)
```

**Mathematica [A]** time = 0.199505, size = 96, normalized size = 0.83

$$\frac{x\left(\frac{a\left(a^2d^2(n+1)+abd(dnx^n-2c)+b^2c^2\right)}{a+bx^n}+(bc-ad)(ad(n+1)+bc(n-1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)\right)}{a^2b^2n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^n)^2/(a + b*x^n)^2,x]
```

```
[Out] (x*((a*(b^2*c^2 + a^2*d^2*(1 + n) + a*b*d*(-2*c + d*n*x^n)))/(a +
 b*x^n) + (b*c - a*d)*(b*c*(-1 + n) + a*d*(1 + n))*Hypergeometric
2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]))/(a^2*b^2*n)
```

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^n)^2/(a+b*x^n)^2,x)
```

```
[Out] int((c+d*x^n)^2/(a+b*x^n)^2,x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\left(a^2d^2(n+1)-b^2c^2(n-1)-2\,abcd\right)\int \frac{1}{ab^3nx^n+a^2b^2n}\,dx+\frac{abd^2nxx^n+\left(a^2d^2(n+1)+b^2c^2-2\,abcd\right)x}{ab^3nx^n+a^2b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^2/(b*x^n + a)^2,x, algorithm="maxima")
```

```
[Out] -(a^2*d^2*(n + 1) - b^2*c^2*(n - 1) - 2*a*b*c*d)*integrate(1/(a*b
^3*n*x^n + a^2*b^2*n), x) + (a*b*d^2*n*x*x^n + (a^2*d^2*(n + 1) +
 b^2*c^2 - 2*a*b*c*d)*x)/(a*b^3*n*x^n + a^2*b^2*n)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^2x^{2\,n}+2\,cdx^n+c^2}{b^2x^{2\,n}+2\,abx^n+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^2/(b*x^n + a)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)/(b^2*x^(2*n) + 2*a*b*x^n
 + a^2), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(c + dx^n)^2}{(a + bx^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**n)**2/(a+b*x**n)**2,x)
```

```
[Out] Integral((c + d*x**n)**2/(a + b*x**n)**2, x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^2}{(bx^n + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^2/(b*x^n + a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^2/(b*x^n + a)^2, x)
```

## 3.209 $\int \frac{c+dx^n}{(a+bx^n)^2}\,dx$

**Optimal**. Leaf size=72

$$\frac{x(ad-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2bn}+\frac{x(bc-ad)}{abn\,(a+bx^n)}$$

```
[Out] ((b*c - a*d)*x)/(a*b*n*(a + b*x^n)) + ((a*d - b*c*(1 - n))*x*Hype
rgeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b*n)
```

---

**Rubi [A]** time = 0.0844173, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{x(ad-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2bn}+\frac{x(bc-ad)}{abn\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^n)/(a + b*x^n)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(a*b*n*(a + b*x^n)) + ((a*d - b*c*(1 - n))*x*Hype
rgeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b*n)
```

---

**Rubi in Sympy [A]** time = 9.0002, size = 53, normalized size = 0.74

$$-\frac{x\,(ad-bc)}{abn\,(a+bx^n)}+\frac{x\,(ad-bc\,(-n+1))\,{}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{a^2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**n)/(a+b*x**n)**2,x)
```

```
[Out] -x*(a*d - b*c)/(a*b*n*(a + b*x**n)) + x*(a*d - b*c*(-n + 1))*hype
r((1, 1/n), (1 + 1/n,), -b*x**n/a)/(a**2*b*n)
```

---

**Mathematica [A]** time = 0.0768954, size = 68, normalized size = 0.94

$$\frac{x\left((a+bx^n)\,(ad+bc(n-1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)+a(bc-ad)\right)}{a^2bn\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^n)/(a + b*x^n)^2,x]
```

```
[Out] (x*(a*(b*c - a*d) + (a*d + b*c*(-1 + n))*(a + b*x^n)*Hypergeometr
ic2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]))/(a^2*b*n*(a + b*x^n)
)
```

---

**Maple [F]** time = 0.065, size = 0, normalized size = 0.

$$\int \frac{c + dx^n}{(a + bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^n)/(a+b*x^n)^2,x)
```

```
[Out] int((c+d*x^n)/(a+b*x^n)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$(bc(n-1) + ad)\int \frac{1}{ab^2nx^n + a^2bn}\,dx + \frac{(bc - ad)x}{ab^2nx^n + a^2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)/(b*x^n + a)^2,x, algorithm="maxima")
```

```
[Out] (b*c*(n - 1) + a*d)*integrate(1/(a*b^2*n*x^n + a^2*b*n), x) + (b*
c - a*d)*x/(a*b^2*n*x^n + a^2*b*n)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^n + c}{b^2x^{2\,n} + 2\,abx^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)/(b*x^n + a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**n)/(a+b*x**n)**2,x)
```

```
[Out] Exception raised: TypeError
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^n + c}{(bx^n + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)/(b*x^n + a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)/(b*x^n + a)^2, x)
```

# 3.210 $\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx$

**Optimal**. Leaf size=122

$$\frac{bx(ad(1-2n)-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^2}+\frac{d^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)^2}+\frac{bx}{an(bc-ad)\,(a+bx^n)}$$

```
[Out] (b*x)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(a*d*(1 - 2*n) - b*c*(1
- n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(
a^2*(b*c - a*d)^2*n) + (d^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^
(-1), -((d*x^n)/c)])/(c*(b*c - a*d)^2)
```

---

**Rubi [A]** time = 0.343957, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{bx(ad(1-2n)-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^2}+\frac{d^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)^2}+\frac{bx}{an(bc-ad)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(a*d*(1 - 2*n) - b*c*(1
- n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(
a^2*(b*c - a*d)^2*n) + (d^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^
(-1), -((d*x^n)/c)])/(c*(b*c - a*d)^2)
```

---

**Rubi in Sympy [A]** time = 48.6244, size = 100, normalized size = 0.82

$$\frac{d^2x\,{}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{dx^n}{c}\right)}{c\,(ad-bc)^2}-\frac{bx}{an\,(a+bx^n)\,(ad-bc)}+\frac{bx\,(-2adn+ad+bcn-bc)\,{}_2F_1\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{a^2n\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] d**2*x*hyper((1, 1/n), (1 + 1/n,), -d*x**n/c)/(c*(a*d - b*c)**2)
- b*x/(a*n*(a + b*x**n)*(a*d - b*c)) + b*x*(-2*a*d*n + a*d + b*c*
n - b*c)*hyper((1, 1/n), (1 + 1/n,), -b*x**n/a)/(a**2*n*(a*d - b*
c)**2)
```

---

**Mathematica [A]** time = 0.254896, size = 108, normalized size = 0.89

$$\frac{x\left(\frac{b^2c-abd}{a^2n+abnx^n}+\frac{b(ad(1-2n)+bc(n-1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n}+\frac{d^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c}\right)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (x*((b^2*c - a*b*d)/(a^2*n + a*b*n*x^n) + (b*(a*d*(1 - 2*n) + b*c
*(-1 + n))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]
)/(a^2*n) + (d^2*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^
n)/c)])/c))/(b*c - a*d)^2
```

---

**Maple [F]** time = 0.154, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$d^2\int \frac{1}{b^2c^3-2\,abc^2d+a^2cd^2+(b^2c^2d-2\,abcd^2+a^2d^3)x^n}\,dx - \left(abd(2\,n-1)-b^2c(n-1)\right)\int \frac{1}{a^2b^2c^2n-2\,a^3bcdn+a^4d^2n+(ab^3c^2n-2\,a^2b^2cdn+a^3bd^2n)x^n}\,dx + \frac{bx}{a^2bcn-a^3dn+(ab^2cn-a^2bdn)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)^2*(d*x^n + c)),x, algorithm="maxima")
```

```
[Out] d^2*integrate(1/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)*x^n), x) - (a*b*d*(2*n - 1) - b^2*c*(n -
```

```
1))*integrate(1/(a^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b
^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^n), x) + b*x/(a^2*b*c
*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x^n)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2dx^{3\,n} + a^2c + (b^2c + 2\,abd)x^{2\,n} + (2\,abc + a^2d)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)^2*(d*x^n + c)),x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (
2*a*b*c + a^2*d)*x^n), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)^2(dx^n + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)^2*(d*x^n + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)), x)
```

## 3.211 $\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx$

**Optimal**. Leaf size=193

$$\frac{b^2x(ad(1-3n)-b(c-cn))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^3}-\frac{d^2x(bc(1-3n)-ad(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2n(bc-ad)^3}+\frac{bx}{an(bc-ad)(a+bx^n)(c+dx^n)}+\frac{dx(ad+bc)}{acn(bc-ad)^2(c+dx^n)}$$

```
[Out] (d*(b*c + a*d)*x)/(a*c*(b*c - a*d)^2*n*(c + d*x^n)) + (b*x)/(a*(b
*c - a*d)*n*(a + b*x^n)*(c + d*x^n)) + (b^2*(a*d*(1 - 3*n) - b*(c
 - c*n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]
)/(a^2*(b*c - a*d)^3*n) - (d^2*(b*c*(1 - 3*n) - a*d*(1 - n))*x*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*(b*c -
 a*d)^3*n)
```

---

**Rubi [A]** time = 0.694641, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{b^2x(ad(1-3n)-b(c-cn))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^3}+\frac{d^2x(ad(1-n)-b(c-3cn))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2n(bc-ad)^3}+\frac{bx}{an(bc-ad)(a+bx^n)(c+dx^n)}+\frac{dx(ad+bc)}{acn(bc-ad)^2(c+dx^n)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^n)^2*(c + d*x^n)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(a*c*(b*c - a*d)^2*n*(c + d*x^n)) + (b*x)/(a*(b
*c - a*d)*n*(a + b*x^n)*(c + d*x^n)) + (b^2*(a*d*(1 - 3*n) - b*(c
 - c*n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]
)/(a^2*(b*c - a*d)^3*n) + (d^2*(a*d*(1 - n) - b*(c - 3*c*n))*x*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*(b*c -
 a*d)^3*n)
```

---

**Rubi in Sympy [A]** time = 105.36, size = 162, normalized size = 0.84

$$\frac{dx}{cn\left(a+bx^{n}\right)\left(c+dx^{n}\right)\left(ad-bc\right)}-\frac{d^{2}x\left(-adn+ad+3bcn-bc\right){}_{2}F_{1}\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{dx^{n}}{c}\right)}{c^{2}n\left(ad-bc\right)^{3}}+\frac{bx\left(ad+bc\right)}{acn\left(a+bx^{n}\right)\left(ad-bc\right)^{2}}-\frac{b^{2}x\left(-3adn+ad+bcn-bc\right){}_{2}F_{1}\left(\begin{matrix}1,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{bx^{n}}{a}\right)}{a^{2}n\left(ad-bc\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*x**n)**2/(c+d*x**n)**2,x)
```

```
[Out] d*x/(c*n*(a + b*x**n)*(c + d*x**n)*(a*d - b*c)) - d**2*x*(-a*d*n
+ a*d + 3*b*c*n - b*c)*hyper((1, 1/n), (1 + 1/n,), -d*x**n/c)/(c*
*2*n*(a*d - b*c)**3) + b*x*(a*d + b*c)/(a*c*n*(a + b*x**n)*(a*d -
 b*c)**2) - b**2*x*(-3*a*d*n + a*d + b*c*n - b*c)*hyper((1, 1/n),
 (1 + 1/n,), -b*x**n/a)/(a**2*n*(a*d - b*c)**3)
```

---

**Mathematica [A]** time = 0.344232, size = 147, normalized size = 0.76

$$\frac{x\left(\frac{b^{2}(ad(1-3n)+bc(n-1))\,{}_{2}F_{1}\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^{n}}{a}\right)}{a^{2}}+\frac{b^{2}(bc-ad)}{a(a+bx^{n})}+\frac{d^{2}(bc(3n-1)-ad(n-1))\,{}_{2}F_{1}\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^{n}}{c}\right)}{c^{2}}+\frac{d^{2}(bc-ad)}{c(c+dx^{n})}\right)}{n(bc-ad)^{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^n)^2*(c + d*x^n)^2),x]
```

```
[Out] (x*((b^2*(b*c - a*d))/(a*(a + b*x^n)) + (d^2*(b*c - a*d))/(c*(c +
 d*x^n)) + (b^2*(a*d*(1 - 3*n) + b*c*(-1 + n))*Hypergeometric2F1[
1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a^2 + (d^2*(-(a*d*(-1 + n))
 + b*c*(-1 + 3*n))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*
x^n)/c)])/c^2))/((b*c - a*d)^3*n)
```

---

**Maple [F]** time = 0.153, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^{n}\right)^{2}\left(c+dx^{n}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*x^n)^2/(c+d*x^n)^2,x)
```

```
[Out] int(1/(a+b*x^n)^2/(c+d*x^n)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left(ab^2d(3\,n-1)-b^3c(n-1)\right)\int -\frac{1}{a^2b^3c^3n-3\,a^3b^2c^2dn+3\,a^4bcd^2n-a^5d^3n+(ab^4c^3n-3\,a^2b^3c^2dn+3\,a^3b^2cd^2n-a^4bd^3n)x^n}\,dx$$
$$-\left(bcd^2(3\,n-1)-ad^3(n-1)\right)\int -\frac{1}{b^3c^5n-3\,ab^2c^4dn+3\,a^2bc^3d^2n-a^3c^2d^3n+(b^3c^4dn-3\,ab^2c^3d^2n+3\,a^2bc^2d^3n-a^3cd^4n)x^n}\,dx$$
$$+\frac{\left(b^2cd+abd^2\right)xx^n+\left(b^2c^2+a^2d^2\right)x}{a^2b^2c^4n-2\,a^3bc^3dn+a^4c^2d^2n+(ab^3c^3dn-2\,a^2b^2c^2d^2n+a^3bcd^3n)x^{2\,n}+(ab^3c^4n-a^2b^2c^3dn-a^3bc^2d^2n+a^4cd^3n)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)^2*(d*x^n + c)^2),x, algorithm="maxima")
```

```
[Out] (a*b^2*d*(3*n - 1) - b^3*c*(n - 1))*integrate(-1/(a^2*b^3*c^3*n -
 3*a^3*b^2*c^2*d*n + 3*a^4*b*c*d^2*n - a^5*d^3*n + (a*b^4*c^3*n -
 3*a^2*b^3*c^2*d*n + 3*a^3*b^2*c*d^2*n - a^4*b*d^3*n)*x^n), x) -
(b*c*d^2*(3*n - 1) - a*d^3*(n - 1))*integrate(-1/(b^3*c^5*n - 3*a
*b^2*c^4*d*n + 3*a^2*b*c^3*d^2*n - a^3*c^2*d^3*n + (b^3*c^4*d*n -
 3*a*b^2*c^3*d^2*n + 3*a^2*b*c^2*d^3*n - a^3*c*d^4*n)*x^n), x) +
((b^2*c*d + a*b*d^2)*x*x^n + (b^2*c^2 + a^2*d^2)*x)/(a^2*b^2*c^4*
n - 2*a^3*b*c^3*d*n + a^4*c^2*d^2*n + (a*b^3*c^3*d*n - 2*a^2*b^2*
c^2*d^2*n + a^3*b*c*d^3*n)*x^(2*n) + (a*b^3*c^4*n - a^2*b^2*c^3*d
*n - a^3*b*c^2*d^2*n + a^4*c*d^3*n)*x^n)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2d^2x^{4\,n}+a^2c^2+2\,(b^2cd+abd^2)x^{3\,n}+(b^2c^2+4\,abcd+a^2d^2)x^{2\,n}+2\,(abc^2+a^2cd)x^n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)^2*(d*x^n + c)^2),x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d^2*x^(4*n) + a^2*c^2 + 2*(b^2*c*d + a*b*d^2)*x^(
3*n) + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(2*n) + 2*(a*b*c^2 + a^2
*c*d)*x^n), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*x**n)**2/(c+d*x**n)**2,x)

[Out] Timed out
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)^2 (dx^n + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)^2*(d*x^n + c)^2),x, algorithm="giac")

[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)^2), x)
```

## 3.212 $\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx$

**Optimal**. Leaf size=299

$$\frac{b^3x(ad(1-4n)-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^4}-\frac{dx\left(-a^2d^2(1-2n)+abcd(1-6n)-2b^2c^2n\right)}{2ac^2n^2(bc-ad)^3\,(c+dx^n)}+\frac{d^2x\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(4n^2-5n+1\right)+b^2c^2\left(12n^2-7n+1\right)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3n^2(bc-ad)^4}+\frac{bx}{an(bc-ad)\,(a+bx^n)\,(c+dx^n)^2}+\frac{dx(ad+2bc)}{2acn(bc-ad)^2\,(c+dx^n)^2}$$

```
[Out] (d*(2*b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*n*(c + d*x^n)^2) + (b*x)
/(a*(b*c - a*d)*n*(a + b*x^n)*(c + d*x^n)^2) - (d*(a*b*c*d*(1 - 6
*n) - a^2*d^2*(1 - 2*n) - 2*b^2*c^2*n)*x)/(2*a*c^2*(b*c - a*d)^3*
n^2*(c + d*x^n)) + (b^3*(a*d*(1 - 4*n) - b*c*(1 - n))*x*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*(b*c - a*d)^4
*n) + (d^2*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d*(1 - 5*n + 4*n^
2) + b^2*c^2*(1 - 7*n + 12*n^2))*x*Hypergeometric2F1[1, n^(-1), 1
 + n^(-1), -((d*x^n)/c)])/(2*c^3*(b*c - a*d)^4*n^2)
```

---

**Rubi [A]** time = 1.30478, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{b^3x(ad(1-4n)-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^4}-\frac{dx\left(-a^2d^2(1-2n)+abcd(1-6n)-2b^2c^2n\right)}{2ac^2n^2(bc-ad)^3\,(c+dx^n)}+\frac{d^2x\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(4n^2-5n+1\right)+b^2c^2\left(12n^2-7n+1\right)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3n^2(bc-ad)^4}+\frac{bx}{an(bc-ad)\,(a+bx^n)\,(c+dx^n)^2}+\frac{dx(ad+2bc)}{2acn(bc-ad)^2\,(c+dx^n)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^n)^2*(c + d*x^n)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*n*(c + d*x^n)^2) + (b*x)
/(a*(b*c - a*d)*n*(a + b*x^n)*(c + d*x^n)^2) - (d*(a*b*c*d*(1 - 6
*n) - a^2*d^2*(1 - 2*n) - 2*b^2*c^2*n)*x)/(2*a*c^2*(b*c - a*d)^3*
n^2*(c + d*x^n)) + (b^3*(a*d*(1 - 4*n) - b*c*(1 - n))*x*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*(b*c - a*d)^4
*n) + (d^2*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d*(1 - 5*n + 4*n^
2) + b^2*c^2*(1 - 7*n + 12*n^2))*x*Hypergeometric2F1[1, n^(-1), 1
 + n^(-1), -((d*x^n)/c)])/(2*c^3*(b*c - a*d)^4*n^2)
```

---

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*x**n)**2/(c+d*x**n)**3,x)
```

```
[Out] Timed out
```

---

**Mathematica [A]** time = 0.535841, size = 233, normalized size = 0.78

$$\frac{x\left(\frac{2b^3n(ad(1-4n)+bc(n-1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}+\frac{d^2\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(4n^2-5n+1\right)+b^2c^2\left(12n^2-7n+1\right)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^3}+\frac{2b^3n(bc-ad)}{a(a+bx^n)}\right.}{2n^2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^n)^2*(c + d*x^n)^3),x]
```

```
[Out] (x*((2*b^3*(b*c - a*d)*n)/(a*(a + b*x^n)) + (d^2*(b*c - a*d)^2*n)
/(c*(c + d*x^n)^2) + (d^2*(-(b*c) + a*d)*(a*d*(-1 + 2*n) + b*(c -
 6*c*n)))/(c^2*(c + d*x^n)) + (2*b^3*(a*d*(1 - 4*n) + b*c*(-1 + n
))*n*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a^2
+ (d^2*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d*(1 - 5*n + 4*n^2) +
 b^2*c^2*(1 - 7*n + 12*n^2))*Hypergeometric2F1[1, n^(-1), 1 + n^(
-1), -((d*x^n)/c)])/c^3))/(2*(b*c - a*d)^4*n^2)
```

---

**Maple [F]** time = 0.187, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)^2\,(c+dx^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*x^n)^2/(c+d*x^n)^3,x)
```

```
[Out] int(1/(a+b*x^n)^2/(c+d*x^n)^3,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)^2*(d*x^n + c)^3),x, algorithm="maxima")
```

```
[Out] ((12*n^2 - 7*n + 1)*b^2*c^2*d^2 - 2*(4*n^2 - 5*n + 1)*a*b*c*d^3 +
 (2*n^2 - 3*n + 1)*a^2*d^4)*integrate(1/2/(b^4*c^7*n^2 - 4*a*b^3*
c^6*d*n^2 + 6*a^2*b^2*c^5*d^2*n^2 - 4*a^3*b*c^4*d^3*n^2 + a^4*c^3
*d^4*n^2 + (b^4*c^6*d*n^2 - 4*a*b^3*c^5*d^2*n^2 + 6*a^2*b^2*c^4*d
^3*n^2 - 4*a^3*b*c^3*d^4*n^2 + a^4*c^2*d^5*n^2)*x^n), x) - (a*b^3
*d*(4*n - 1) - b^4*c*(n - 1))*integrate(1/(a^2*b^4*c^4*n - 4*a^3*
b^3*c^3*d*n + 6*a^4*b^2*c^2*d^2*n - 4*a^5*b*c*d^3*n + a^6*d^4*n +
 (a*b^5*c^4*n - 4*a^2*b^4*c^3*d*n + 6*a^3*b^3*c^2*d^2*n - 4*a^4*b
^2*c*d^3*n + a^5*b*d^4*n)*x^n), x) + 1/2*((a*b^2*c*d^3*(6*n - 1)
- a^2*b*d^4*(2*n - 1) + 2*b^3*c^2*d^2*n)*x*x^(2*n) + (a*b^2*c^2*d
^2*(7*n - 1) - a^3*d^4*(2*n - 1) + 4*b^3*c^3*d*n + 3*a^2*b*c*d^3*
n)*x*x^n + (a^2*b*c^2*d^2*(7*n - 1) - a^3*c*d^3*(3*n - 1) + 2*b^3
*c^4*n)*x)/(a^2*b^3*c^7*n^2 - 3*a^3*b^2*c^6*d*n^2 + 3*a^4*b*c^5*d
^2*n^2 - a^5*c^4*d^3*n^2 + (a*b^4*c^5*d^2*n^2 - 3*a^2*b^3*c^4*d^3
*n^2 + 3*a^3*b^2*c^3*d^4*n^2 - a^4*b*c^2*d^5*n^2)*x^(3*n) + (2*a*
b^4*c^6*d*n^2 - 5*a^2*b^3*c^5*d^2*n^2 + 3*a^3*b^2*c^4*d^3*n^2 + a
^4*b*c^3*d^4*n^2 - a^5*c^2*d^5*n^2)*x^(2*n) + (a*b^4*c^7*n^2 - a^
2*b^3*c^6*d*n^2 - 3*a^3*b^2*c^5*d^2*n^2 + 5*a^4*b*c^4*d^3*n^2 - 2
*a^5*c^3*d^4*n^2)*x^n)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{b^2d^3x^{5\,n} + a^2c^3 + (3\,b^2cd^2 + 2\,abd^3)x^{4\,n} + (3\,b^2c^2d + 6\,abcd^2 + a^2d^3)x^{3\,n} + (b^2c^3 + 6\,abc^2d + 3\,a^2cd^2)x^{2\,n} + (2\,abc^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)^2*(d*x^n + c)^3),x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d^3*x^(5*n) + a^2*c^3 + (3*b^2*c*d^2 + 2*a*b*d^3)
*x^(4*n) + (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(3*n) + (b^2*c
^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(2*n) + (2*a*b*c^3 + 3*a^2*c^2*
d)*x^n), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*x**n)**2/(c+d*x**n)**3,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)^2 (dx^n + c)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)^2*(d*x^n + c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)^3), x)
```

# 3.213 $\int (a + bx^n)^p (c + dx^n)^q \, dx$

**Optimal**. Leaf size=81

$$x (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} (c + dx^n)^q \left(\frac{dx^n}{c} + 1\right)^{-q} F_1\left(\frac{1}{n}; -p, -q; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)$$

```
[Out] (x*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[n^(-1), -p, -q, 1 + n^(-1
), -((b*x^n)/a), -((d*x^n)/c)])/((1 + (b*x^n)/a)^p*(1 + (d*x^n)/c
)^q)
```

---

**Rubi [A]** time = 0.128784, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$x (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} (c + dx^n)^q \left(\frac{dx^n}{c} + 1\right)^{-q} F_1\left(\frac{1}{n}; -p, -q; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)^p*(c + d*x^n)^q,x]
```

```
[Out] (x*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[n^(-1), -p, -q, 1 + n^(-1
), -((b*x^n)/a), -((d*x^n)/c)])/((1 + (b*x^n)/a)^p*(1 + (d*x^n)/c
)^q)
```

---

**Rubi in Sympy [A]** time = 21.6673, size = 63, normalized size = 0.78

$$x \left(1 + \frac{bx^n}{a}\right)^{-p} \left(1 + \frac{dx^n}{c}\right)^{-q} (a + bx^n)^p (c + dx^n)^q \operatorname{appellf_1}\left(\frac{1}{n}, -p, -q, 1 + \frac{1}{n}, -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)**p*(c+d*x**n)**q,x)
```

```
[Out] x*(1 + b*x**n/a)**(-p)*(1 + d*x**n/c)**(-q)*(a + b*x**n)**p*(c +
d*x**n)**q*appellf1(1/n, -p, -q, 1 + 1/n, -b*x**n/a, -d*x**n/c)
```

---

**Mathematica [B]** time = 0.661915, size = 190, normalized size = 2.35

$$\frac{ac(n+1)x (a + bx^n)^p (c + dx^n)^q F_1\left(\frac{1}{n}; -p, -q; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{bcnpx^n F_1\left(1 + \frac{1}{n}; 1 - p, -q; 2 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) + adnqx^n F_1\left(1 + \frac{1}{n}; -p, 1 - q; 2 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) + ac(n+1)F_1\left(\frac{1}{n}; -p, -\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^n)^p*(c + d*x^n)^q,x]
```

```
[Out] (a*c*(1 + n)*x*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[n^(-1), -p, -
q, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)])/(b*c*n*p*x^n*AppellF1
[1 + n^(-1), 1 - p, -q, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] +
 a*d*n*q*x^n*AppellF1[1 + n^(-1), -p, 1 - q, 2 + n^(-1), -((b*x^n
)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF1[n^(-1), -p, -q, 1 + n^
(-1), -((b*x^n)/a), -((d*x^n)/c)])
```

---

**Maple [F]** time = 0.207, size = 0, normalized size = 0.

$$\int (a+bx^n)^p (c+dx^n)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)^p*(c+d*x^n)^q,x)
```

```
[Out] int((a+b*x^n)^p*(c+d*x^n)^q,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n+a)^p (dx^n+c)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p*(d*x^n + c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^q, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^n+a)^p (dx^n+c)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p*(d*x^n + c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*(d*x^n + c)^q, x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)**p*(c+d*x**n)**q,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p (dx^n + c)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p*(d*x^n + c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^q, x)
```

## 3.214 $\int (a+bx^n)^p (c+dx^n)^3 \, dx$

**Optimal**. Leaf size=402

$$\frac{dx\,(a+bx^n)^{p+1}\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(n^2(p+7)+n(2p+9)+2\right)+b^2c^2\left(n^2\left(p^2+6p+11\right)+2n(p+3)+1\right)\right)}{b^3(np+n+1)(n(p+2)+1)(n(p+3)+1)}$$

$$-\frac{x\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(a^3d^3\left(2n^2+3n+1\right)-3a^2bcd^2(n+1)(n(p+3)+1)+3ab^2c^2d\left(n^2\left(p^2+5p+6\right)+n(2p+5)+1\right)\right.}{b^3(np+n+1)(n(p+2)+1)(n(p}$$

$$-\frac{dx\,(c+dx^n)\,(a+bx^n)^{p+1}\,(ad(2n+1)-bc(n(p+5)+1))}{b^2(n(p+2)+1)(n(p+3)+1)}+\frac{dx\,(c+dx^n)^2\,(a+bx^n)^{p+1}}{b(np+3n+1)}$$

```
[Out] (d*(a^2*d^2*(1 + 3*n + 2*n^2) - a*b*c*d*(2 + n^2*(7 + p) + n*(9 +
 2*p)) + b^2*c^2*(1 + 2*n*(3 + p) + n^2*(11 + 6*p + p^2)))*x*(a +
 b*x^n)^(1 + p))/(b^3*(1 + n + n*p)*(1 + n*(2 + p))*(1 + n*(3 + p
))) - (d*(a*d*(1 + 2*n) - b*c*(1 + n*(5 + p)))*x*(a + b*x^n)^(1 +
 p)*(c + d*x^n))/(b^2*(1 + n*(2 + p))*(1 + n*(3 + p))) + (d*x*(a
+ b*x^n)^(1 + p)*(c + d*x^n)^2)/(b*(1 + 3*n + n*p)) - ((a^3*d^3*(
1 + 3*n + 2*n^2) - 3*a^2*b*c*d^2*(1 + n)*(1 + n*(3 + p)) + 3*a*b^
2*c^2*d*(1 + n*(5 + 2*p) + n^2*(6 + 5*p + p^2)) - b^3*c^3*(1 + 3*
n*(2 + p) + n^2*(11 + 12*p + 3*p^2) + n^3*(6 + 11*p + 6*p^2 + p^3
)))*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((
b*x^n)/a)])/(b^3*(1 + n + n*p)*(1 + n*(2 + p))*(1 + n*(3 + p))*(1
 + (b*x^n)/a)^p)
```

---

**Rubi [A]** time = 1.28482, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{dx\,(a+bx^n)^{p+1}\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(n^2(p+7)+n(2p+9)+2\right)+b^2c^2\left(n^2\left(p^2+6p+11\right)+2n(p+3)+1\right)\right)}{b^3(np+n+1)(n(p+2)+1)(n(p+3)+1)}$$

$$-\frac{x\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(a^3d^3\left(2n^2+3n+1\right)-3a^2bcd^2(n+1)(n(p+3)+1)+3ab^2c^2d\left(n^2\left(p^2+5p+6\right)+n(2p+5)+1\right)\right.}{b^3(np+n+1)(n(p+2)+1)(n(p}$$

$$-\frac{dx\,(c+dx^n)\,(a+bx^n)^{p+1}\,(ad(2n+1)-b(cn(p+5)+c))}{b^2(n(p+2)+1)(n(p+3)+1)}+\frac{dx\,(c+dx^n)^2\,(a+bx^n)^{p+1}}{b(n(p+3)+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)^p*(c + d*x^n)^3,x]
```

```
[Out] (d*(a^2*d^2*(1 + 3*n + 2*n^2) - a*b*c*d*(2 + n^2*(7 + p) + n*(9 +
 2*p)) + b^2*c^2*(1 + 2*n*(3 + p) + n^2*(11 + 6*p + p^2)))*x*(a +
 b*x^n)^(1 + p))/(b^3*(1 + n + n*p)*(1 + n*(2 + p))*(1 + n*(3 + p
))) - (d*(a*d*(1 + 2*n) - b*(c + c*n*(5 + p)))*x*(a + b*x^n)^(1 +
 p)*(c + d*x^n))/(b^2*(1 + n*(2 + p))*(1 + n*(3 + p))) + (d*x*(a
+ b*x^n)^(1 + p)*(c + d*x^n)^2)/(b*(1 + n*(3 + p))) - ((a^3*d^3*(
1 + 3*n + 2*n^2) - 3*a^2*b*c*d^2*(1 + n)*(1 + n*(3 + p)) + 3*a*b^
2*c^2*d*(1 + n*(5 + 2*p) + n^2*(6 + 5*p + p^2)) - b^3*c^3*(1 + 3*
```

```
n*(2 + p) + n^2*(11 + 12*p + 3*p^2) + n^3*(6 + 11*p + 6*p^2 + p^3
)))*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((
b*x^n)/a)])/(b^3*(1 + n + n*p)*(1 + n*(2 + p))*(1 + n*(3 + p))*(1
 + (b*x^n)/a)^p)
```

---

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)**p*(c+d*x**n)**3,x)
```

```
[Out] Timed out
```

---

**Mathematica [A]** time = 0.444014, size = 168, normalized size = 0.42

$$x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c^3\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)+\frac{3c^2dx^n\,{}_2F_1\left(1+\frac{1}{n},-p;2+\frac{1}{n};-\frac{bx^n}{a}\right)}{n+1}+\frac{3cd^2x^{2n}\,{}_2F_1\left(2+\frac{1}{n},-p;3+\frac{1}{n};-\frac{bx^n}{a}\right)}{2n+1}+\frac{d^3x^{3n}\,{}_2F_1\left(3+\frac{1}{n},-p;4+\frac{1}{n};-\frac{bx^n}{a}\right)}{3n+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)^p*(c + d*x^n)^3,x]
```

```
[Out] (x*(a + b*x^n)^p*((3*c^2*d*x^n*Hypergeometric2F1[1 + n^(-1), -p,
2 + n^(-1), -((b*x^n)/a)])/(1 + n) + (3*c*d^2*x^(2*n)*Hypergeomet
ric2F1[2 + n^(-1), -p, 3 + n^(-1), -((b*x^n)/a)])/(1 + 2*n) + (d^
3*x^(3*n)*Hypergeometric2F1[3 + n^(-1), -p, 4 + n^(-1), -((b*x^n)
/a)])/(1 + 3*n) + c^3*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -
((b*x^n)/a)]))/(1 + (b*x^n)/a)^p
```

---

**Maple [F]** time = 0.159, size = 0, normalized size = 0.

$$\int (a+bx^n)^p\,(c+dx^n)^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)^p*(c+d*x^n)^3,x)
```

```
[Out] int((a+b*x^n)^p*(c+d*x^n)^3,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^n + c)^3 (bx^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^3*(b*x^n + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^3*(b*x^n + a)^p, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^3 x^{3n} + 3\,cd^2 x^{2n} + 3\,c^2 dx^n + c^3\right)(bx^n + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^3*(b*x^n + a)^p,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)*(b*x
^n + a)^p, x)
```

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)**p*(c+d*x**n)**3,x)
```

```
[Out] Exception raised: TypeError
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^3*(b*x^n + a)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.215 $\int (a+bx^n)^p (c+dx^n)^2 \, dx$

**Optimal**. Leaf size=202

$$-\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(bc(np+n+1)(ad-bc(n(p+2)+1))-ad(ad(n+1)-bc(n(p+3)+1)))\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)}{b^2(np+n+1)(n(p+2)+1)}$$
$$-\frac{dx(a+bx^n)^{p+1}(ad(n+1)-bc(n(p+3)+1))}{b^2(np+n+1)(n(p+2)+1)}+\frac{dx(c+dx^n)(a+bx^n)^{p+1}}{b(np+2n+1)}$$

```
[Out] -((d*(a*d*(1 + n) - b*c*(1 + n*(3 + p)))*x*(a + b*x^n)^(1 + p))/(
b^2*(1 + n + n*p)*(1 + n*(2 + p)))) + (d*x*(a + b*x^n)^(1 + p)*(c
 + d*x^n))/(b*(1 + 2*n + n*p)) - ((b*c*(1 + n + n*p)*(a*d - b*c*(
1 + n*(2 + p))) - a*d*(a*d*(1 + n) - b*c*(1 + n*(3 + p))))*x*(a +
 b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]
)/(b^2*(1 + n + n*p)*(1 + n*(2 + p))*(1 + (b*x^n)/a)^p)
```

---

**Rubi [A]** time = 0.624457, antiderivative size = 197, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$-\frac{dx(a+bx^n)^{p+1}(ad(n+1)-b(cn(p+3)+c))}{b^2(np+n+1)(n(p+2)+1)}$$
$$-\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c(ad-b(cn(p+2)+c))-\frac{ad(ad(n+1)-b(cn(p+3)+c))}{b(np+n+1)}\right)\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)}{b(n(p+2)+1)}$$
$$+\frac{dx(c+dx^n)(a+bx^n)^{p+1}}{b(n(p+2)+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)^p*(c + d*x^n)^2,x]
```

```
[Out] -((d*(a*d*(1 + n) - b*(c + c*n*(3 + p)))*x*(a + b*x^n)^(1 + p))/(
b^2*(1 + n + n*p)*(1 + n*(2 + p)))) + (d*x*(a + b*x^n)^(1 + p)*(c
 + d*x^n))/(b*(1 + n*(2 + p))) - ((c*(a*d - b*(c + c*n*(2 + p)))
- (a*d*(a*d*(1 + n) - b*(c + c*n*(3 + p))))/(b*(1 + n + n*p)))*x*
(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)
/a)])/(b*(1 + n*(2 + p))*(1 + (b*x^n)/a)^p)
```

---

**Rubi in Sympy [A]** time = 36.2797, size = 173, normalized size = 0.86

$$\frac{dx\,(a+bx^n)^{p+1}\,(c+dx^n)}{b\,(n\,(p+2)+1)} - \frac{dx\,(a+bx^n)^{p+1}\,(ad\,(n+1)-bc\,(n\,(p+3)+1))}{b^2\,(n\,(p+1)+1)\,(n\,(p+2)+1)} + \frac{x\left(1+\frac{bx^n}{a}\right)^{-p}(a+bx^n)^p\,(ad\,(ad\,(n+1)-bc\,(n\,(p+3)+1))-bc\,(ad-bc\,(n\,(p+2)+1))\,(n\,(p+1)+1))\,{}_2F_1\left(\begin{matrix}-p,\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|-\frac{bx^n}{a}\right)}{b^2\,(n\,(p+1)+1)\,(n\,(p+2)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)**p*(c+d*x**n)**2,x)
```

```
[Out] d*x*(a + b*x**n)**(p + 1)*(c + d*x**n)/(b*(n*(p + 2) + 1)) - d*x*
(a + b*x**n)**(p + 1)*(a*d*(n + 1) - b*c*(n*(p + 3) + 1))/(b**2*(
n*(p + 1) + 1)*(n*(p + 2) + 1)) + x*(1 + b*x**n/a)**(-p)*(a + b*x
**n)**p*(a*d*(a*d*(n + 1) - b*c*(n*(p + 3) + 1)) - b*c*(a*d - b*c
*(n*(p + 2) + 1))*(n*(p + 1) + 1))*hyper((-p, 1/n), (1 + 1/n,), -
b*x**n/a)/(b**2*(n*(p + 1) + 1)*(n*(p + 2) + 1))
```

---

**Mathematica [A]** time = 0.24786, size = 140, normalized size = 0.69

$$\frac{x\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left((n+1)\left(c^2(2n+1)\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)+d^2x^{2n}\,{}_2F_1\left(2+\frac{1}{n},-p;3+\frac{1}{n};-\frac{bx^n}{a}\right)\right)+2cd(2n+1)\right.}{(n+1)(2n+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)^p*(c + d*x^n)^2,x]
```

```
[Out] (x*(a + b*x^n)^p*(2*c*d*(1 + 2*n)*x^n*Hypergeometric2F1[1 + n^(-1
), -p, 2 + n^(-1), -((b*x^n)/a)] + (1 + n)*(d^2*x^(2*n)*Hypergeom
etric2F1[2 + n^(-1), -p, 3 + n^(-1), -((b*x^n)/a)] + c^2*(1 + 2*n
)*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])))/((1
+ n)*(1 + 2*n)*(1 + (b*x^n)/a)^p)
```

---

**Maple [F]** time = 0.137, size = 0, normalized size = 0.

$$\int (a+bx^n)^p\,(c+dx^n)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)^p*(c+d*x^n)^2,x)
```

```
[Out] int((a+b*x^n)^p*(c+d*x^n)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^n + c)^2 (bx^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^n + c)^2*(b*x^n + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^2*(b*x^n + a)^p, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^2 x^{2\,n} + 2\,c d x^n + c^2\right) (bx^n + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^n + c)^2*(b*x^n + a)^p,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)*(b*x^n + a)^p, x)
```

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**2,x)
```

```
[Out] Exception raised: TypeError
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^2*(b*x^n + a)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.216 $\int (a+bx^n)^p (c+dx^n) \, dx$

**Optimal**. Leaf size=98

$$\frac{dx(a+bx^n)^{p+1}}{b(np+n+1)} - \frac{x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (ad-bc(np+n+1))\, {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)}{b(np+n+1)}$$

```
[Out] (d*x*(a + b*x^n)^(1 + p))/(b*(1 + n + n*p)) - ((a*d - b*c*(1 + n
+ n*p))*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1),
 -((b*x^n)/a)])/(b*(1 + n + n*p)*(1 + (b*x^n)/a)^p)
```

---

**Rubi [A]** time = 0.119854, antiderivative size = 89, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \left(c-\frac{ad}{bnp+bn+b}\right) {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right) + \frac{dx(a+bx^n)^{p+1}}{b(np+n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)^p*(c + d*x^n),x]
```

```
[Out] (d*x*(a + b*x^n)^(1 + p))/(b*(1 + n + n*p)) + ((c - (a*d)/(b + b*
n + b*n*p))*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(
-1), -((b*x^n)/a)])/(1 + (b*x^n)/a)^p
```

---

**Rubi in Sympy [A]** time = 11.0367, size = 80, normalized size = 0.82

$$\frac{dx(a+bx^n)^{p+1}}{b(np+n+1)} - \frac{x\left(1+\frac{bx^n}{a}\right)^{-p} (a+bx^n)^p (ad-bc(n(p+1)+1))\, {}_2F_1\left(\begin{matrix} -p, \frac{1}{n} \\ 1+\frac{1}{n} \end{matrix}\middle| -\frac{bx^n}{a}\right)}{b(n(p+1)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)**p*(c+d*x**n),x)
```

```
[Out] d*x*(a + b*x**n)**(p + 1)/(b*(n*p + n + 1)) - x*(1 + b*x**n/a)**(
-p)*(a + b*x**n)**p*(a*d - b*c*(n*(p + 1) + 1))*hyper((-p, 1/n),
(1 + 1/n,), -b*x**n/a)/(b*(n*(p + 1) + 1))
```

---

**Mathematica [A]** time = 0.102702, size = 85, normalized size = 0.87

$$\frac{x\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c(n+1)\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)+dx^n\,{}_2F_1\left(1+\frac{1}{n},-p;2+\frac{1}{n};-\frac{bx^n}{a}\right)\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)^p*(c + d*x^n),x]
```

```
[Out] (x*(a + b*x^n)^p*(d*x^n*Hypergeometric2F1[1 + n^(-1), -p, 2 + n^(
-1), -((b*x^n)/a)] + c*(1 + n)*Hypergeometric2F1[n^(-1), -p, 1 +
n^(-1), -((b*x^n)/a)]))/((1 + n)*(1 + (b*x^n)/a)^p)
```

---

**Maple [F]** time = 0.135, size = 0, normalized size = 0.

$$\int (a+bx^n)^p\,(c+dx^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)^p*(c+d*x^n),x)
```

```
[Out] int((a+b*x^n)^p*(c+d*x^n),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^n+c)(bx^n+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)*(b*x^n + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)*(b*x^n + a)^p, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx^n+c)(bx^n+a)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)*(b*x^n + a)^p,x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)*(b*x^n + a)^p, x)
```

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)**p*(c+d*x**n),x)
```

```
[Out] Exception raised: TypeError
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)*(b*x^n + a)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.217 $\int (a+bx^n)^p \, dx$

**Optimal**. Leaf size=46

$$x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p} {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

```
[Out] (x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x
^n)/a)])/(1 + (b*x^n)/a)^p
```

---

**Rubi [A]** time = 0.0316681, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p} {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)^p,x]
```

```
[Out] (x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x
^n)/a)])/(1 + (b*x^n)/a)^p
```

---

**Rubi in Sympy [A]** time = 3.70442, size = 36, normalized size = 0.78

$$x\left(1+\frac{bx^n}{a}\right)^{-p}(a+bx^n)^p {}_2F_1\left(\begin{matrix} -p, \frac{1}{n} \\ 1+\frac{1}{n} \end{matrix}\middle| -\frac{bx^n}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)**p,x)
```

```
[Out] x*(1 + b*x**n/a)**(-p)*(a + b*x**n)**p*hyper((-p, 1/n), (1 + 1/n,
), -b*x**n/a)
```

---

**Mathematica [A]** time = 0.0267131, size = 46, normalized size = 1.

$$x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p} {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)^p,x]
```

```
[Out] (x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x
^n)/a)])/(1 + (b*x^n)/a)^p
```

---

**Maple [F]** time = 0.002, size = 0, normalized size = 0.

$$\int (a + bx^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)^p,x)
```

```
[Out] int((a+b*x^n)^p,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^n + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p, x)
```

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)**p,x)
```

```
[Out] Exception raised: TypeError
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p, x)
```

## 3.218 $\int \frac{(a+bx^n)^p}{c+dx^n}\,dx$

**Optimal**. Leaf size=59

$$\frac{x\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p} F_1\left(\frac{1}{n};-p,1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c}$$

```
[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -((b*x^n)/a)
, -((d*x^n)/c)])/(c*(1 + (b*x^n)/a)^p)
```

---

**Rubi [A]** time = 0.084008, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{x\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p} F_1\left(\frac{1}{n};-p,1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)^p/(c + d*x^n),x]
```

```
[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -((b*x^n)/a)
, -((d*x^n)/c)])/(c*(1 + (b*x^n)/a)^p)
```

---

**Rubi in Sympy [A]** time = 21.4334, size = 44, normalized size = 0.75

$$\frac{x\left(1+\frac{bx^n}{a}\right)^{-p}(a+bx^n)^p \operatorname{appellf}_1\left(\frac{1}{n},1,-p,1+\frac{1}{n},-\frac{dx^n}{c},-\frac{bx^n}{a}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)**p/(c+d*x**n),x)
```

```
[Out] x*(1 + b*x**n/a)**(-p)*(a + b*x**n)**p*appellf1(1/n, 1, -p, 1 + 1
/n, -d*x**n/c, -b*x**n/a)/c
```

---

**Mathematica [B]** time = 0.418044, size = 180, normalized size = 3.05

$$\frac{ac(n+1)x\,(a+bx^n)^p F_1\left(\frac{1}{n};-p,1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(c+dx^n)\left(bcnpx^nF_1\left(1+\frac{1}{n};1-p,1;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-adnx^nF_1\left(1+\frac{1}{n};-p,2;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+ac(n+1)F_1\left(\frac{1}{n};-\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^n)^p/(c + d*x^n),x]
```

```
[Out] (a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 1, 1 + n^(-1),
-((b*x^n)/a), -((d*x^n)/c)])/((c + d*x^n)*(b*c*n*p*x^n*AppellF1[1
 + n^(-1), 1 - p, 1, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - a*
d*n*x^n*AppellF1[1 + n^(-1), -p, 2, 2 + n^(-1), -((b*x^n)/a), -((
d*x^n)/c)] + a*c*(1 + n)*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -((b
*x^n)/a), -((d*x^n)/c)]))
```

---

**Maple [F]** time = 0.122, size = 0, normalized size = 0.

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)^p/(c+d*x^n),x)
```

```
[Out] int((a+b*x^n)^p/(c+d*x^n),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n+a)^p}{dx^n+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p/(d*x^n + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^n+a)^p}{dx^n+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p/(d*x^n + c),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d*x^n + c), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)**p/(c+d*x**n),x)
```

```
[Out] Integral((a + b*x**n)**p/(c + d*x**n), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n+a)^p}{dx^n+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p/(d*x^n + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c), x)
```

## 3.219 $\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx$

**Optimal**. Leaf size=59

$$\frac{x\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}F_1\left(\frac{1}{n};-p,2;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2}$$

```
[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)/a)
, -((d*x^n)/c)])/(c^2*(1 + (b*x^n)/a)^p)
```

---

**Rubi [A]** time = 0.0831748, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{x\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}F_1\left(\frac{1}{n};-p,2;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)^p/(c + d*x^n)^2,x]
```

```
[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)/a)
, -((d*x^n)/c)])/(c^2*(1 + (b*x^n)/a)^p)
```

---

**Rubi in Sympy [A]** time = 19.9456, size = 46, normalized size = 0.78

$$\frac{x\left(1+\frac{bx^n}{a}\right)^{-p}(a+bx^n)^p\,\text{appellf}_1\left(\frac{1}{n},2,-p,1+\frac{1}{n},-\frac{dx^n}{c},-\frac{bx^n}{a}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)**p/(c+d*x**n)**2,x)
```

```
[Out] x*(1 + b*x**n/a)**(-p)*(a + b*x**n)**p*appellf1(1/n, 2, -p, 1 + 1
/n, -d*x**n/c, -b*x**n/a)/c**2
```

---

**Mathematica [B]** time = 0.473617, size = 180, normalized size = 3.05

$$\frac{ac(n+1)x\,(a+bx^n)^p\,F_1\left(\frac{1}{n};-p,2;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(c+dx^n)^2\left(bcnpx^nF_1\left(1+\frac{1}{n};1-p,2;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-2adnx^nF_1\left(1+\frac{1}{n};-p,3;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+ac(n+1)F_1\left(\frac{1}{n};\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^n)^p/(c + d*x^n)^2,x]
```

```
[Out] (a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 2, 1 + n^(-1),
-((b*x^n)/a), -((d*x^n)/c)])/((c + d*x^n)^2*(b*c*n*p*x^n*AppellF1
[1 + n^(-1), 1 - p, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] -
2*a*d*n*x^n*AppellF1[1 + n^(-1), -p, 3, 2 + n^(-1), -((b*x^n)/a),
 -((d*x^n)/c)] + a*c*(1 + n)*AppellF1[n^(-1), -p, 2, 1 + n^(-1),
-((b*x^n)/a), -((d*x^n)/c)]))
```

---

**Maple [F]** time = 0.103, size = 0, normalized size = 0.

$$\int \frac{(a + bx^n)^p}{(c + dx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)^p/(c+d*x^n)^2,x)
```

```
[Out] int((a+b*x^n)^p/(c+d*x^n)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^p}{(dx^n + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p/(d*x^n + c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c)^2, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^n + a)^p}{d^2x^{2n} + 2\,cdx^n + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p/(d*x^n + c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^n)^p}{(c + dx^n)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)**p/(c+d*x**n)**2,x)
```

```
[Out] Integral((a + b*x**n)**p/(c + d*x**n)**2, x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^p}{(dx^n + c)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p/(d*x^n + c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c)^2, x)
```

## 3.220 $\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx$

**Optimal**. Leaf size=59

$$\frac{x\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}F_1\left(\frac{1}{n};-p,3;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^3}$$

```
[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)/a)
, -((d*x^n)/c)])/(c^3*(1 + (b*x^n)/a)^p)
```

---

**Rubi [A]** time = 0.084313, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{x\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}F_1\left(\frac{1}{n};-p,3;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)^p/(c + d*x^n)^3,x]
```

```
[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)/a)
, -((d*x^n)/c)])/(c^3*(1 + (b*x^n)/a)^p)
```

---

**Rubi in Sympy [A]** time = 20.1158, size = 46, normalized size = 0.78

$$\frac{x\left(1+\frac{bx^n}{a}\right)^{-p}(a+bx^n)^p\,\text{appellf}_1\left(\frac{1}{n},3,-p,1+\frac{1}{n},-\frac{dx^n}{c},-\frac{bx^n}{a}\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)**p/(c+d*x**n)**3,x)
```

```
[Out] x*(1 + b*x**n/a)**(-p)*(a + b*x**n)**p*appellf1(1/n, 3, -p, 1 + 1
/n, -d*x**n/c, -b*x**n/a)/c**3
```

---

**Mathematica [B]** time = 0.692187, size = 180, normalized size = 3.05

$$\frac{ac(n+1)x\,(a+bx^n)^p\,F_1\left(\frac{1}{n};-p,3;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(c+dx^n)^3\left(bcnpx^nF_1\left(1+\frac{1}{n};1-p,3;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-3adnx^nF_1\left(1+\frac{1}{n};-p,4;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+ac(n+1)F_1\left(\frac{1}{n};\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^n)^p/(c + d*x^n)^3,x]
```

```
[Out] (a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 3, 1 + n^(-1),
-((b*x^n)/a), -((d*x^n)/c)])/((c + d*x^n)^3*(b*c*n*p*x^n*AppellF1
[1 + n^(-1), 1 - p, 3, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] -
3*a*d*n*x^n*AppellF1[1 + n^(-1), -p, 4, 2 + n^(-1), -((b*x^n)/a),
 -((d*x^n)/c)] + a*c*(1 + n)*AppellF1[n^(-1), -p, 3, 1 + n^(-1),
-((b*x^n)/a), -((d*x^n)/c)]))
```

---

**Maple [F]** time = 0.112, size = 0, normalized size = 0.

$$\int \frac{(a + bx^n)^p}{(c + dx^n)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)^p/(c+d*x^n)^3,x)
```

```
[Out] int((a+b*x^n)^p/(c+d*x^n)^3,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^p}{(dx^n + c)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p/(d*x^n + c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c)^3, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^n + a)^p}{d^3x^{3n} + 3cd^2x^{2n} + 3c^2dx^n + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p/(d*x^n + c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x
^n + c^3), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)**p/(c+d*x**n)**3,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^p}{(dx^n + c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p/(d*x^n + c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c)^3, x)
```

## 3.221 $\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p} \, dx$

**Optimal**. Leaf size=93

$$\frac{x(a+bx^n)^p (c+dx^n)^{-\frac{1}{n}-p} \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p} {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{c}$$

```
[Out] (x*(a + b*x^n)^p*(c + d*x^n)^(-n^(-1) - p)*Hypergeometric2F1[n^(-
1), -p, 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(c*((c
*(a + b*x^n))/(a*(c + d*x^n)))^p)
```

---

**Rubi [A]** time = 0.0726765, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$\frac{x(a+bx^n)^p (c+dx^n)^{-\frac{1}{n}-p} \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p} {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)^p*(c + d*x^n)^(-1 - n^(-1) - p),x]
```

```
[Out] (x*(a + b*x^n)^p*(c + d*x^n)^(-n^(-1) - p)*Hypergeometric2F1[n^(-
1), -p, 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(c*((c
*(a + b*x^n))/(a*(c + d*x^n)))^p)
```

---

**Rubi in Sympy [A]** time = 8.19064, size = 83, normalized size = 0.89

$$\frac{x\left(\frac{a(c+dx^n)}{c(a+bx^n)}\right)^{p+1+\frac{1}{n}} (a+bx^n)^{p+1} (c+dx^n)^{-p-1-\frac{1}{n}} {}_2F_1\left(\begin{matrix}\frac{1}{n},p+1+\frac{1}{n}\\ 1+\frac{1}{n}\end{matrix}\middle| -\frac{x^n(ad-bc)}{c(a+bx^n)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)**p*(c+d*x**n)**(-1-1/n-p),x)
```

```
[Out] x*(a*(c + d*x**n)/(c*(a + b*x**n)))**(p + 1 + 1/n)*(a + b*x**n)**
(p + 1)*(c + d*x**n)**(-p - 1 - 1/n)*hyper((1/n, p + 1 + 1/n), (1
 + 1/n,), -x**n*(a*d - b*c)/(c*(a + b*x**n)))/a
```

---

**Mathematica [A]** time = 0.558127, size = 93, normalized size = 1.

$$\frac{x\left(a+bx^n\right)^p\left(c+dx^n\right)^{-\frac{np+1}{n}}\left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};\frac{(ad-bc)x^n}{a(dx^n+c)}\right)}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)^p*(c + d*x^n)^(-1 - n^(-1) - p),x]
```

```
[Out] (x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), ((-(b*
c) + a*d)*x^n)/(a*(c + d*x^n))])/(c*((c*(a + b*x^n))/(a*(c + d*x^
n)))^p*(c + d*x^n)^((1 + n*p)/n))
```

---

**Maple [F]** time = 0.249, size = 0, normalized size = 0.

$$\int \left(a+bx^n\right)^p\left(c+dx^n\right)^{-1-n^{-1}-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x)
```

```
[Out] int((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n+a)^p(dx^n+c)^{-p-\frac{1}{n}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 1),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 1), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^n+a)^p}{(dx^n+c)^{\frac{np+n+1}{n}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 1),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d*x^n + c)^((n*p + n + 1)/n), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)**p*(c+d*x**n)**(-1-1/n-p),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p (dx^n + c)^{-p - \frac{1}{n} - 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 1),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 1), x)
```

## 3.222 $\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\, dx$

**Optimal**. Leaf size=178

$$\frac{6a^3n^3x(c+dx^n)^{-1/n}}{c^4(n+1)(2n+1)(3n+1)}+\frac{6a^2n^2x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^3(n+1)(2n+1)(3n+1)}+\frac{3anx(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c^2(6n^2+5n+1)}+\frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}$$

```
[Out] (x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(c*(1 + 3*n)) + (3*a*
n*x*(a + b*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c^2*(1 + 5*n + 6*n^
2)) + (6*a^2*n^2*x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^3*(1
 + n)*(1 + 2*n)*(1 + 3*n)) + (6*a^3*n^3*x)/(c^4*(1 + n)*(1 + 2*n)
*(1 + 3*n)*(c + d*x^n)^n^(-1))
```

---

**Rubi [A]** time = 0.240627, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{6a^3n^3x(c+dx^n)^{-1/n}}{c^4(n+1)(2n+1)(3n+1)}+\frac{6a^2n^2x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^3(n+1)(2n+1)(3n+1)}+\frac{3anx(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c^2(6n^2+5n+1)}+\frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)^3*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(c*(1 + 3*n)) + (3*a*
n*x*(a + b*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c^2*(1 + 5*n + 6*n^
2)) + (6*a^2*n^2*x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^3*(1
 + n)*(1 + 2*n)*(1 + 3*n)) + (6*a^3*n^3*x)/(c^4*(1 + n)*(1 + 2*n)
*(1 + 3*n)*(c + d*x^n)^n^(-1))
```

---

**Rubi in Sympy [A]** time = 35.5955, size = 156, normalized size = 0.88

$$\frac{6a^3n^3x(c+dx^n)^{-\frac{1}{n}}}{c^4(n+1)(2n+1)(3n+1)}+\frac{6a^2n^2x(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c^3(n+1)(2n+1)(3n+1)}+\frac{3anx(a+bx^n)^2(c+dx^n)^{-2-\frac{1}{n}}}{c^2(2n+1)(3n+1)}+\frac{x(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{c(3n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)**3*(c+d*x**n)**(-4-1/n),x)
```

```
[Out] 6*a**3*n**3*x*(c + d*x**n)**(-1/n)/(c**4*(n + 1)*(2*n + 1)*(3*n +
 1)) + 6*a**2*n**2*x*(a + b*x**n)*(c + d*x**n)**(-1 - 1/n)/(c**3*
(n + 1)*(2*n + 1)*(3*n + 1)) + 3*a*n*x*(a + b*x**n)**2*(c + d*x**
n)**(-2 - 1/n)/(c**2*(2*n + 1)*(3*n + 1)) + x*(a + b*x**n)**3*(c
+ d*x**n)**(-3 - 1/n)/(c*(3*n + 1))
```

---

**Mathematica [C]** time = 0.607878, size = 198, normalized size = 1.11

$$\frac{x\left(c+dx^n\right)^{-1/n}\left(a^3\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(4+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+\frac{3a^2bx^n\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(1+\frac{1}{n},4+\frac{1}{n};2+\frac{1}{n};-\frac{dx^n}{c}\right)}{n+1}+\frac{3ab^2x^{2n}\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(\right.}{2n+}\right.}{c^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)^3*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] (x*((b^3*c^3*x^(3*n))/((1 + 3*n)*(c + d*x^n)^3) + (3*a^2*b*x^n*(1
 + (d*x^n)/c)^n^(-1)*Hypergeometric2F1[1 + n^(-1), 4 + n^(-1), 2
+ n^(-1), -((d*x^n)/c)])/(1 + n) + (3*a*b^2*x^(2*n)*(1 + (d*x^n)/
c)^n^(-1)*Hypergeometric2F1[2 + n^(-1), 4 + n^(-1), 3 + n^(-1), -
((d*x^n)/c)])/(1 + 2*n) + a^3*(1 + (d*x^n)/c)^n^(-1)*Hypergeometr
ic2F1[4 + n^(-1), n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(c^4*(c + d
*x^n)^n^(-1))
```

---

**Maple [F]** time = 0.203, size = 0, normalized size = 0.

$$\int \left(a+bx^n\right)^3\left(c+dx^n\right)^{-4-n^{-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x)
```

```
[Out] int((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^n+a\right)^3\left(dx^n+c\right)^{-\frac{1}{n}-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n + a)^3*(d*x^n + c)^(-1/n - 4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^3*(d*x^n + c)^(-1/n - 4), x)
```

---

**Fricas [A]** time = 0.255308, size = 645, normalized size = 3.62

$$\left(6\,a^3d^4n^3 + b^3c^3d + \left(2\,b^3c^3d + 3\,ab^2c^2d^2 + 6\,a^2bcd^3\right)n^2 + 3\left(b^3c^3d + ab^2c^2d^2\right)n\right)xx^{4\,n} + \left(24\,a^3cd^3n^3 + b^3c^4 + 3\,ab^2c^3d + 2\,(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n + a)^3*(d*x^n + c)^(-1/n - 4),x, algorithm="fricas")
```

```
[Out] ((6*a^3*d^4*n^3 + b^3*c^3*d + (2*b^3*c^3*d + 3*a*b^2*c^2*d^2 + 6*
a^2*b*c*d^3)*n^2 + 3*(b^3*c^3*d + a*b^2*c^2*d^2)*n)*x*x^(4*n) + (
24*a^3*c*d^3*n^3 + b^3*c^4 + 3*a*b^2*c^3*d + 2*(b^3*c^4 + 6*a*b^2
*c^3*d + 12*a^2*b*c^2*d^2 + 3*a^3*c*d^3)*n^2 + 3*(b^3*c^4 + 5*a*b
^2*c^3*d + 2*a^2*b*c^2*d^2)*n)*x*x^(3*n) + 3*(12*a^3*c^2*d^2*n^3
+ a*b^2*c^4 + a^2*b*c^3*d + (3*a*b^2*c^4 + 12*a^2*b*c^3*d + 7*a^3
*c^2*d^2)*n^2 + (4*a*b^2*c^4 + 7*a^2*b*c^3*d + a^3*c^2*d^2)*n)*x*
x^(2*n) + (24*a^3*c^3*d*n^3 + 3*a^2*b*c^4 + a^3*c^3*d + 2*(9*a^2*
b*c^4 + 13*a^3*c^3*d)*n^2 + 3*(5*a^2*b*c^4 + 3*a^3*c^3*d)*n)*x*x^
n + (6*a^3*c^4*n^3 + 11*a^3*c^4*n^2 + 6*a^3*c^4*n + a^3*c^4)*x)/(
(6*c^4*n^3 + 11*c^4*n^2 + 6*c^4*n + c^4)*(d*x^n + c)^((4*n + 1)/n
))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**3*(c+d*x**n)**(-4-1/n),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^3*(d*x^n + c)^(-1/n - 4),x, algorithm="giac")

[Out] Exception raised: TypeError
```

# 3.223 $\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\,dx$

**Optimal**. Leaf size=116

$$\frac{2a^2n^2x(c+dx^n)^{-1/n}}{c^3(n+1)(2n+1)}+\frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^2(n+1)(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}$$

```
[Out] (x*(a + b*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c*(1 + 2*n)) + (2*a*
n*x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^2*(1 + n)*(1 + 2*n)
) + (2*a^2*n^2*x)/(c^3*(1 + n)*(1 + 2*n)*(c + d*x^n)^n^(-1))
```

---

**Rubi [A]** time = 0.112751, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08

$$\frac{2a^2n^2x(c+dx^n)^{-1/n}}{c^3(n+1)(2n+1)}+\frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^2(n+1)(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)^2*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c*(1 + 2*n)) + (2*a*
n*x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^2*(1 + n)*(1 + 2*n)
) + (2*a^2*n^2*x)/(c^3*(1 + n)*(1 + 2*n)*(c + d*x^n)^n^(-1))
```

---

**Rubi in Sympy [A]** time = 18.1007, size = 100, normalized size = 0.86

$$\frac{2a^2n^2x(c+dx^n)^{-\frac{1}{n}}}{c^3(n+1)(2n+1)}+\frac{2anx(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c^2(n+1)(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-2-\frac{1}{n}}}{c(2n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)**2*(c+d*x**n)**(-3-1/n),x)
```

```
[Out] 2*a**2*n**2*x*(c + d*x**n)**(-1/n)/(c**3*(n + 1)*(2*n + 1)) + 2*a
*n*x*(a + b*x**n)*(c + d*x**n)**(-1 - 1/n)/(c**2*(n + 1)*(2*n + 1
)) + x*(a + b*x**n)**2*(c + d*x**n)**(-2 - 1/n)/(c*(2*n + 1))
```

---

**Mathematica [C]** time = 0.313694, size = 139, normalized size = 1.2

$$\frac{x\left(c+dx^n\right)^{-1/n}\left(a^2\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}{}_2F_1\left(3+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+\frac{2abx^n\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}{}_2F_1\left(1+\frac{1}{n},3+\frac{1}{n};2+\frac{1}{n};-\frac{dx^n}{c}\right)}{n+1}+\frac{b^2c^2x^{2n}}{(2n+1)(c+dx^n)^2}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)^2*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] (x*((b^2*c^2*x^(2*n))/((1 + 2*n)*(c + d*x^n)^2) + (2*a*b*x^n*(1 +
 (d*x^n)/c)^n^(-1)*Hypergeometric2F1[1 + n^(-1), 3 + n^(-1), 2 +
n^(-1), -((d*x^n)/c)])/(1 + n) + a^2*(1 + (d*x^n)/c)^n^(-1)*Hyper
geometric2F1[3 + n^(-1), n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(c^3
*(c + d*x^n)^n^(-1))
```

---

**Maple [F]** time = 0.177, size = 0, normalized size = 0.

$$\int \left(a+bx^n\right)^2\left(c+dx^n\right)^{-3-n^{-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x)
```

```
[Out] int((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n+a)^2(dx^n+c)^{-\frac{1}{n}-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 3), x)
```

---

**Fricas [A]** time = 0.256654, size = 312, normalized size = 2.69

$$\frac{\left(2\,a^2d^3n^2+b^2c^2d+\left(b^2c^2d+2\,abcd^2\right)n\right)xx^{3\,n}+\left(6\,a^2cd^2n^2+b^2c^3+2\,abc^2d+\left(b^2c^3+6\,abc^2d+2\,a^2cd^2\right)n\right)xx^{2\,n}+\left(6\,a^2c^2d\right.}{\left(2\,c^3n^2+3\,c^3n+c^3\right)\left(dx^n+c\right)^{\frac{3\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 3),x, algorithm="fricas")
```

```
[Out] ((2*a^2*d^3*n^2 + b^2*c^2*d + (b^2*c^2*d + 2*a*b*c*d^2)*n)*x*x^(3
*n) + (6*a^2*c*d^2*n^2 + b^2*c^3 + 2*a*b*c^2*d + (b^2*c^3 + 6*a*b
*c^2*d + 2*a^2*c*d^2)*n)*x*x^(2*n) + (6*a^2*c^2*d*n^2 + 2*a*b*c^3
 + a^2*c^2*d + (4*a*b*c^3 + 5*a^2*c^2*d)*n)*x*x^n + (2*a^2*c^3*n^
2 + 3*a^2*c^3*n + a^2*c^3)*x)/((2*c^3*n^2 + 3*c^3*n + c^3)*(d*x^n
 + c)^((3*n + 1)/n))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)**2*(c+d*x**n)**(-3-1/n),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.224 $\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}}\,dx$

**Optimal**. Leaf size=58

$$\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(n+1)}$$

```
[Out] (x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (a*n*x)/(
c^2*(1 + n)*(c + d*x^n)^n^(-1))
```

---

**Rubi [A]** time = 0.0483245, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)*(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (a*n*x)/(
c^2*(1 + n)*(c + d*x^n)^n^(-1))
```

---

**Rubi in Sympy [A]** time = 6.74866, size = 48, normalized size = 0.83

$$\frac{anx(c+dx^n)^{-\frac{1}{n}}}{c^2(n+1)}+\frac{x(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)*(c+d*x**n)**(-2-1/n),x)
```

```
[Out] a*n*x*(c + d*x**n)**(-1/n)/(c**2*(n + 1)) + x*(a + b*x**n)*(c + d
*x**n)**(-1 - 1/n)/(c*(n + 1))
```

---

**Mathematica [C]** time = 0.169288, size = 82, normalized size = 1.41

$$\frac{x(c+dx^n)^{-\frac{n+1}{n}}\left(a(n+1)(c+dx^n)\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+bcx^n\right)}{c^2(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)*(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(b*c*x^n + a*(1 + n)*(c + d*x^n)*(1 + (d*x^n)/c)^n^(-1)*Hyperg
eometric2F1[2 + n^(-1), n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(c^2*
(1 + n)*(c + d*x^n)^((1 + n)/n))
```

---

**Maple [F]** time = 0.165, size = 0, normalized size = 0.

$$\int (a + bx^n)(c + dx^n)^{-2-n^{-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)*(c+d*x^n)^(-2-1/n),x)
```

```
[Out] int((a+b*x^n)*(c+d*x^n)^(-2-1/n),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)(dx^n + c)^{-\frac{1}{n}-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 2), x)
```

---

**Fricas [A]** time = 0.267403, size = 115, normalized size = 1.98

$$\frac{\left(ad^2n + bcd\right)xx^{2\,n} + \left(2\,acdn + bc^2 + acd\right)xx^n + \left(ac^2n + ac^2\right)x}{(c^2n + c^2)(dx^n + c)^{\frac{2\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 2),x, algorithm="fricas")
```

```
[Out] ((a*d^2*n + b*c*d)*x*x^(2*n) + (2*a*c*d*n + b*c^2 + a*c*d)*x*x^n
+ (a*c^2*n + a*c^2)*x)/((c^2*n + c^2)*(d*x^n + c)^((2*n + 1)/n))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)*(c+d*x**n)**(-2-1/n),x)

[Out] Timed out
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 2),x, algorithm="giac")

[Out] Exception raised: TypeError
```

# 3.225 $\int (c + dx^n)^{-1-\frac{1}{n}} \, dx$

**Optimal**. Leaf size=18

$$\frac{x (c + dx^n)^{-1/n}}{c}$$

```
[Out] x/(c*(c + d*x^n)^n^(-1))
```

---

**Rubi [A]** time = 0.0116419, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{x (c + dx^n)^{-1/n}}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^n)^(-1 - n^(-1)),x]

[Out] x/(c*(c + d*x^n)^n^(-1))
```

---

**Rubi in Sympy [A]** time = 1.39143, size = 12, normalized size = 0.67

$$\frac{x (c + dx^n)^{-\frac{1}{n}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**n)**(-1-1/n),x)

[Out] x*(c + d*x**n)**(-1/n)/c
```

---

**Mathematica [A]** time = 0.0399012, size = 18, normalized size = 1.

$$\frac{x (c + dx^n)^{-1/n}}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^n)^(-1 - n^(-1)),x]
```

```
[Out] x/(c*(c + d*x^n)^n^(-1))
```

---

**Maple [B]** time = 0.036, size = 53, normalized size = 2.9

$$xe^{(-1-n^{-1})\ln\left(c+de^{n\ln(x)}\right)} + \frac{dxe^{n\ln(x)}}{c}e^{(-1-n^{-1})\ln\left(c+de^{n\ln(x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^n)^(-1-1/n),x)
```

```
[Out] x*exp((-1-1/n)*ln(c+d*exp(n*ln(x))))+d/c*x*exp(n*ln(x))*exp((-1-1
/n)*ln(c+d*exp(n*ln(x))))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^n + c)^{-\frac{1}{n}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^(-1/n - 1),x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 1), x)
```

---

**Fricas [A]** time = 0.248533, size = 42, normalized size = 2.33

$$\frac{dxx^n + cx}{(dx^n + c)^{\frac{n+1}{n}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^(-1/n - 1),x, algorithm="fricas")
```

```
[Out] (d*x*x^n + c*x)/((d*x^n + c)^((n + 1)/n)*c)
```

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**n)**(-1-1/n),x)
```

```
[Out] Exception raised: RecursionError
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^n + c)^{-\frac{1}{n}-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^(-1/n - 1),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 1), x)
```

# 3.226 $\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx$

**Optimal**. Leaf size=53

$$\frac{x\,(c+dx^n)^{-1/n}\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a}$$

```
[Out] (x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(
a*(c + d*x^n)))])/(a*(c + d*x^n)^n^(-1))
```

---

**Rubi [A]** time = 0.0478586, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{x\,(c+dx^n)^{-1/n}\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^n)*(c + d*x^n)^n^(-1)),x]
```

```
[Out] (x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(
a*(c + d*x^n)))])/(a*(c + d*x^n)^n^(-1))
```

---

**Rubi in Sympy [A]** time = 6.25931, size = 39, normalized size = 0.74

$$\frac{x\,(c+dx^n)^{-\frac{1}{n}}\ {}_2F_1\left(\begin{matrix}\frac{1}{n},1\\1+\frac{1}{n}\end{matrix}\middle|-\frac{x^n(-ad+bc)}{a(c+dx^n)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*x**n)/((c+d*x**n)**(1/n)),x)
```

```
[Out] x*(c + d*x**n)**(-1/n)*hyper((1/n, 1), (1 + 1/n,), -x**n*(-a*d +
b*c)/(a*(c + d*x**n)))/a
```

---

**Mathematica [A]** time = 0.300655, size = 79, normalized size = 1.49

$$\frac{x\,(c+dx^n)^{-1/n}\left(\frac{a(c+dx^n)}{c(a+bx^n)}\right)^{\frac{1}{n}}\ {}_2F_1\left(\frac{1}{n},\frac{1}{n};1+\frac{1}{n};\frac{(bc-ad)x^n}{c(bx^n+a)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^n)*(c + d*x^n)^n^(-1)),x]
```

```
[Out] (x*((a*(c + d*x^n))/(c*(a + b*x^n)))^n^(-1)*Hypergeometric2F1[n^(
-1), n^(-1), 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])/(a*(
c + d*x^n)^n^(-1))
```

---

**Maple [F]** time = 0.12, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)\sqrt[n]{c+dx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x)
```

```
[Out] int(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n+c)^{-\frac{1}{n}}}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)*(d*x^n + c)^(1/n)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n)/(b*x^n + a), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^n+a)(dx^n+c)^{\left(\frac{1}{n}\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)*(d*x^n + c)^(1/n)),x, algorithm="fricas")
```

```
[Out] integral(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx^n)^{-\frac{1}{n}}}{a+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*x**n)/((c+d*x**n)**(1/n)),x)

[Out] Integral((c + d*x**n)**(-1/n)/(a + b*x**n), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n+a)(dx^n+c)^{\left(\frac{1}{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)*(d*x^n + c)^(1/n)),x, algorithm="giac")

[Out] integrate(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)
```

## 3.227 $\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx$

**Optimal**. Leaf size=54

$$\frac{cx\left(c+dx^n\right)^{-1/n}\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2}$$

```
[Out] (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)
/(a*(c + d*x^n)))])/(a^2*(c + d*x^n)^n^(-1))
```

---

**Rubi [A]** time = 0.0502847, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04

$$\frac{cx\left(c+dx^n\right)^{-1/n}\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^2,x]
```

```
[Out] (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)
/(a*(c + d*x^n)))])/(a^2*(c + d*x^n)^n^(-1))
```

---

**Rubi in Sympy [A]** time = 6.1553, size = 42, normalized size = 0.78

$$\frac{cx\left(c+dx^n\right)^{-\frac{1}{n}}\,{}_2F_1\left(\begin{matrix}\frac{1}{n},2\\1+\frac{1}{n}\end{matrix}\middle|-\frac{x^n(-ad+bc)}{a(c+dx^n)}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**n)**(1-1/n)/(a+b*x**n)**2,x)
```

```
[Out] c*x*(c + d*x**n)**(-1/n)*hyper((1/n, 2), (1 + 1/n,), -x**n*(-a*d
+ b*c)/(a*(c + d*x**n)))/a**2
```

---

**Mathematica [A]** time = 0.134758, size = 53, normalized size = 0.98

$$\frac{cx\left(c+dx^n\right)^{-1/n}\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};\frac{(ad-bc)x^n}{a(dx^n+c)}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^2,x]
```

```
[Out] (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), ((-(b*c) + a*d)*x^n
)/(a*(c + d*x^n))])/(a^2*(c + d*x^n)^n^(-1))
```

---

**Maple [F]** time = 0.112, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b x^{n}\right)^{2}}\left(c+d x^{n}\right)^{1-n^{-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x)
```

```
[Out] int((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^{n}+c\right)^{-\frac{1}{n}+1}}{\left(bx^{n}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(dx^{n}+c\right)^{\frac{n-1}{n}}}{b^{2}x^{2\,n}+2\,abx^{n}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)^((n - 1)/n)/(b^2*x^(2*n) + 2*a*b*x^n + a^2),
 x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**n)**(1-1/n)/(a+b*x**n)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+1}}{(bx^n + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2, x)
```

## 3.228 $\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx$

**Optimal**. Leaf size=56

$$\frac{c^2x\,(c+dx^n)^{-1/n}\;{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^3}$$

```
[Out] (c^2*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^
n)/(a*(c + d*x^n)))])/(a^3*(c + d*x^n)^n^(-1))
```

---

**Rubi [A]** time = 0.0518859, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{c^2x\,(c+dx^n)^{-1/n}\;{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^3,x]
```

```
[Out] (c^2*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^
n)/(a*(c + d*x^n)))])/(a^3*(c + d*x^n)^n^(-1))
```

---

**Rubi in Sympy [A]** time = 6.46959, size = 44, normalized size = 0.79

$$\frac{c^2x\,(c+dx^n)^{-\frac{1}{n}}\;{}_2F_1\left(\begin{matrix}\frac{1}{n},3\\1+\frac{1}{n}\end{matrix}\middle|-\frac{x^n(-ad+bc)}{a(c+dx^n)}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**n)**(2-1/n)/(a+b*x**n)**3,x)
```

```
[Out] c**2*x*(c + d*x**n)**(-1/n)*hyper((1/n, 3), (1 + 1/n,), -x**n*(-a
*d + b*c)/(a*(c + d*x**n)))/a**3
```

---

**Mathematica [F]** time = 179.999, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In]  Integrate[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^3,x]
```

```
[Out] $Aborted
```

---

**Maple [F]** time = 0.115, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)^3}(c+dx^n)^{2-n^{-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x)
```

```
[Out] int((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^n+c)^{\frac{2n-1}{n}}}{b^3x^{3n}+3ab^2x^{2n}+3a^2bx^n+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)^((2*n - 1)/n)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n)
 + 3*a^2*b*x^n + a^3), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**n)**(2-1/n)/(a+b*x**n)**3,x)
```

```
[Out] Timed out
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+2}}{(bx^n + a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3, x)
```

## 3.229 $\int (a+bx^n)^p \left(c+dx^n\right)^{-2-\frac{1}{n}-p} \, dx$

**Optimal**. Leaf size=193

$$\frac{x\left(a+bx^n\right)^{p+1}\left(c+dx^n\right)^{-\frac{1}{n}-p-1}\left(n(p+1)(bc-ad)+bc\right)\left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p-1}{}_2F_1\left(\frac{1}{n},-p-1;1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{acn(p+1)(bc-ad)} - \frac{bx\left(a+bx^n\right)^{p+1}\left(c+dx^n\right)^{-\frac{1}{n}-p-1}}{an(p+1)(bc-ad)}$$

```
[Out] -((b*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(-1 - n^(-1) - p))/(a*(b*c
 - a*d)*n*(1 + p))) + ((b*c + (b*c - a*d)*n*(1 + p))*x*(a + b*x^n
)^(1 + p)*((c*(a + b*x^n))/(a*(c + d*x^n)))^(-1 - p)*(c + d*x^n)^
(-1 - n^(-1) - p)*Hypergeometric2F1[n^(-1), -1 - p, 1 + n^(-1), -
(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(a*c*(b*c - a*d)*n*(1 + p))
```

---

**Rubi [A]** time = 0.206221, antiderivative size = 179, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{x\left(a+bx^n\right)^{p+1}\left(c+dx^n\right)^{-\frac{1}{n}-p-1}\left(\frac{b}{n(p+1)(bc-ad)}+\frac{1}{c}\right)\left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p-1}{}_2F_1\left(\frac{1}{n},-p-1;1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a} - \frac{bx\left(a+bx^n\right)^{p+1}\left(c+dx^n\right)^{-\frac{1}{n}-p-1}}{an(p+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)^p*(c + d*x^n)^(-2 - n^(-1) - p),x]
```

```
[Out] -((b*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(-1 - n^(-1) - p))/(a*(b*c
 - a*d)*n*(1 + p))) + ((c^(-1) + b/((b*c - a*d)*n*(1 + p)))*x*(a
+ b*x^n)^(1 + p)*((c*(a + b*x^n))/(a*(c + d*x^n)))^(-1 - p)*(c +
d*x^n)^(-1 - n^(-1) - p)*Hypergeometric2F1[n^(-1), -1 - p, 1 + n^
(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/a
```

---

**Rubi in Sympy [A]** time = 25.2427, size = 155, normalized size = 0.8

$$\frac{bx\left(a+bx^n\right)^{p+1}\left(c+dx^n\right)^{-p-1-\frac{1}{n}}}{an\left(p+1\right)\left(ad-bc\right)} - \frac{x\left(\frac{a(c+dx^n)}{c(a+bx^n)}\right)^{p+2+\frac{1}{n}}\left(a+bx^n\right)^{p+2}\left(c+dx^n\right)^{-p-2-\frac{1}{n}}\left(bc-n\left(p+1\right)\left(ad-bc\right)\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{n},p+2+\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\right|-\frac{x^n(ad-bc)}{c(a+bx^n)}\right)}{a^2n\left(p+1\right)\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)**p*(c+d*x**n)**(-2-1/n-p),x)
```

```
[Out] b*x*(a + b*x**n)**(p + 1)*(c + d*x**n)**(-p - 1 - 1/n)/(a*n*(p +
1)*(a*d - b*c)) - x*(a*(c + d*x**n)/(c*(a + b*x**n)))**(p + 2 + 1
/n)*(a + b*x**n)**(p + 2)*(c + d*x**n)**(-p - 2 - 1/n)*(b*c - n*(
p + 1)*(a*d - b*c))*hyper((1/n, p + 2 + 1/n), (1 + 1/n,), -x**n*(
a*d - b*c)/(c*(a + b*x**n)))/(a**2*n*(p + 1)*(a*d - b*c))
```

---

**Mathematica [B]** time = 53.2575, size = 1414, normalized size = 7.33

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^n)^p*(c + d*x^n)^(-2 - n^(-1) - p),x]
```

```
[Out] (c^4*(1 + n)*(1 + 2*n)*(1 + 3*n)*x*(a + b*x^n)^(3 + p)*(c + d*x^n
)^(-2 - n^(-1) - p)*(1 + (d*x^n)/c)*Gamma[2 + n^(-1)]*Gamma[-p]*(
Hypergeometric2F1[1, -p, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*
x^n))] + (d*n*x^n*((c*Hypergeometric2F1[1, -p, 2 + n^(-1), ((b*c
- a*d)*x^n)/(c*(a + b*x^n))])/(1 + n) + ((b*c - a*d)*x^n*Gamma[1
+ n^(-1)]*Gamma[1 - p]*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((
b*c - a*d)*x^n)/(c*(a + b*x^n))])/((1 + 2*n)*(a + b*x^n)*Gamma[2
+ n^(-1)]*Gamma[-p])))/c^2))/(-(c*d*(1 + 3*n)*(1 + n + n*p)*x^n*(
a + b*x^n)^2*(c^2*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]
*Gamma[-p]*Hypergeometric2F1[1, -p, 1 + n^(-1), ((b*c - a*d)*x^n)
/(c*(a + b*x^n))] + d*n*x^n*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^
(-1)]*Gamma[-p]*Hypergeometric2F1[1, -p, 2 + n^(-1), ((b*c - a*d)
*x^n)/(c*(a + b*x^n))] + (b*c - a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)
]*Gamma[1 - p]*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b*c - a*
d)*x^n)/(c*(a + b*x^n))]))) + b*c*n*(1 + 3*n)*p*x^n*(a + b*x^n)*(
c + d*x^n)*(c^2*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*G
amma[-p]*Hypergeometric2F1[1, -p, 1 + n^(-1), ((b*c - a*d)*x^n)/(
c*(a + b*x^n))] + d*n*x^n*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-
1)]*Gamma[-p]*Hypergeometric2F1[1, -p, 2 + n^(-1), ((b*c - a*d)*x
^n)/(c*(a + b*x^n))] + (b*c - a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*
Gamma[1 - p]*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b*c - a*d)
*x^n)/(c*(a + b*x^n))])) + c*(1 + 3*n)*(a + b*x^n)^2*(c + d*x^n)*
(c^2*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hy
pergeometric2F1[1, -p, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^
n))] + d*n*x^n*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-
p]*Hypergeometric2F1[1, -p, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a +
 b*x^n))] + (b*c - a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*Gamma[1 - p
]*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a
 + b*x^n))])) + n^2*x^n*(c + d*x^n)*(a*c^2*(-(b*c) + a*d)*(1 + 2*
n)*(1 + 3*n)*p*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hypergeome
tric2F1[2, 1 - p, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]
+ c*d*(1 + 3*n)*(a + b*x^n)^2*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 +
n^(-1)]*Gamma[-p]*Hypergeometric2F1[1, -p, 2 + n^(-1), ((b*c - a*
d)*x^n)/(c*(a + b*x^n))] + (b*c - a*d)*(1 + n)*x^n*Gamma[1 + n^(-
1)]*Gamma[1 - p]*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b*c -
```

```
a*d)*x^n)/(c*(a + b*x^n))]) - d*(b*c - a*d)*x^n*(b*c*(1 + n)*(1 +
 3*n)*x^n*(a + b*x^n)*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometr
ic2F1[2, 1 - p, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] -
c*(1 + n)*(1 + 3*n)*(a + b*x^n)^2*Gamma[1 + n^(-1)]*Gamma[1 - p]*
Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a +
 b*x^n))] + a*c*n*(1 + 3*n)*p*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma
[-p]*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b*c - a*d)*x^n)/(c
*(a + b*x^n))] - 2*a*(-(b*c) + a*d)*n*(1 + n)*(-1 + p)*x^n*Gamma[
1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2F1[3, 2 - p, 4 + n^(-1),
((b*c - a*d)*x^n)/(c*(a + b*x^n))]))
```

---

**Maple [F]** time = 0.264, size = 0, normalized size = 0.

$$\int (a+bx^n)^p (c+dx^n)^{-2-n^{-1}-p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x)
```

```
[Out] int((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p (dx^n + c)^{-p-\frac{1}{n}-2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^n + a)^p}{(dx^n + c)^{\frac{np+2\,n+1}{n}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d*x^n + c)^((n*p + 2*n + 1)/n), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)**p*(c+d*x**n)**(-2-1/n-p),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p (dx^n + c)^{-p-\frac{1}{n}-2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2), x)
```

## 3.230 $\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} \, dx$

**Optimal**. Leaf size=57

$$\frac{x(a+bx^n)^{-\frac{bc}{n(bc-ad)}}(c+dx^n)^{\frac{ad}{n(bc-ad)}}}{ac}$$

```
[Out] (x*(c + d*x^n)^((a*d)/((b*c - a*d)*n)))/(a*c*(a + b*x^n)^((b*c)/(
(b*c - a*d)*n)))
```

---

**Rubi [A]** time = 0.0845283, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 69, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.014

$$\frac{x(a+bx^n)^{-\frac{bc}{n(bc-ad)}}(c+dx^n)^{\frac{ad}{n(bc-ad)}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^((a*d*n - b*c*(1 + n))/((b*c - a*d)*n))*(c + d*x^n)^((a*d - b*c*n
```

```
[Out] (x*(c + d*x^n)^((a*d)/((b*c - a*d)*n)))/(a*c*(a + b*x^n)^((b*c)/(
(b*c - a*d)*n)))
```

---

**Rubi in Sympy [A]** time = 13.5218, size = 41, normalized size = 0.72

$$\frac{x(a+bx^n)^{\frac{bc}{n(ad-bc)}}(c+dx^n)^{-\frac{ad}{n(ad-bc)}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a+b*x**n)**((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x**n)**((a*d*
b*c*n+a*d)/(-a*d*n+b*c*n)),x)
```

```
[Out] x*(a + b*x**n)**(b*c/(n*(a*d - b*c)))*(c + d*x**n)**(-a*d/(n*(a*d
 - b*c)))/(a*c)
```

---

**Mathematica [C]** time = 1.93871, size = 461, normalized size = 8.09

$$\frac{ac(n+1)x(ad-bc)(a+bx^n)^{\frac{adn-bc(n+1)}{n(bc-ad)}}(c+dx^n)^{\frac{adn+ad-bcn}{bcn-adn}} F}{bcx^n(bc(n+1)-adn)F_1\left(1+\frac{1}{n};\frac{bc+2bnc-2adn}{bcn-adn},\frac{bcn-ad(n+1)}{(bc-ad)n};2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-a\left(dx^n(ad(n+1)-bcn)F_1\left(1+\frac{1}{n};\frac{bc+bnc-}{bcn-a}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^n)^((a*d*n - b*c*(1 + n))/((b*c - a*d)*n))*(c + d*x^n)^((a*d -
```

```
[Out] (a*c*(-(b*c) + a*d)*(1 + n)*x*(a + b*x^n)^((a*d*n - b*c*(1 + n))/
((b*c - a*d)*n))*(c + d*x^n)^((a*d - b*c*n + a*d*n)/(b*c*n - a*d*
n))*AppellF1[n^(-1), (b*c + b*c*n - a*d*n)/(b*c*n - a*d*n), (b*c*
n - a*d*(1 + n))/((b*c - a*d)*n), 1 + n^(-1), -((b*x^n)/a), -((d*
x^n)/c)])/(b*c*(-(a*d*n) + b*c*(1 + n))*x^n*AppellF1[1 + n^(-1),
(b*c + 2*b*c*n - 2*a*d*n)/(b*c*n - a*d*n), (b*c*n - a*d*(1 + n))/
((b*c - a*d)*n), 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - a*(d*(
-(b*c*n) + a*d*(1 + n))*x^n*AppellF1[1 + n^(-1), (b*c + b*c*n - a
*d*n)/(b*c*n - a*d*n), -((a*d - 2*b*c*n + 2*a*d*n)/(b*c*n - a*d*n
)), 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + c*(b*c - a*d)*(1 +
n)*AppellF1[n^(-1), (b*c + b*c*n - a*d*n)/(b*c*n - a*d*n), (b*c*n
 - a*d*(1 + n))/((b*c - a*d)*n), 1 + n^(-1), -((b*x^n)/a), -((d*x
^n)/c)]))
```

---

**Maple [F]** time = 0.306, size = 0, normalized size = 0.

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(-ad+bc)n}}(c+dx^n)^{\frac{adn-bcn+ad}{-adn+bcn}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-
b*c*n+a*d)/(-a*d*n+b*c*n)),x)
```

```
[Out] int((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-
b*c*n+a*d)/(-a*d*n+b*c*n)),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n+a)^{-\frac{bc(n+1)-adn}{(bc-ad)n}}(dx^n+c)^{-\frac{bcn-adn-ad}{bcn-adn}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^(-(b*c*(n + 1) - a*d*n)/((b*c - a*d)*n))*(d*x^n + c)^(-
(b*c*n - a*d*n - a*d)/(b*c*n - a*d*n)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(-(b*c*(n + 1) - a*d*n)/((b*c - a*d)*n))*(d
*x^n + c)^(-(b*c*n - a*d*n - a*d)/(b*c*n - a*d*n)), x)
```

---

**Fricas [A]** time = 0.264862, size = 146, normalized size = 2.56

$$\frac{\left(bdxx^{2n} + acx + (bc + ad)xx^{n}\right)(dx^{n} + c)^{\frac{ad-(bc-ad)n}{(bc-ad)n}}}{(bx^{n} + a)^{\frac{bc+(bc-ad)n}{(bc-ad)n}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^n + a)^((b*c*(n + 1) - a*d*n)/((b*c - a*d)*n))*(d*x^n + c)^((b*
```

```
[Out] (b*d*x*x^(2*n) + a*c*x + (b*c + a*d)*x*x^n)*(d*x^n + c)^((a*d - (
b*c - a*d)*n)/((b*c - a*d)*n))/((b*x^n + a)^((b*c + (b*c - a*d)*n
)/((b*c - a*d)*n))*a*c)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x**n)**((a*d*n-
b*c*n+a*d)/(-a*d*n+b*c*n)),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^{n} + a)^{\frac{bc(n+1)-adn}{(bc-ad)n}}(dx^{n} + c)^{\frac{bcn-adn-ad}{bcn-adn}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^n + a)^((b*c*(n + 1) - a*d*n)/((b*c - a*d)*n))*(d*x^n + c)^((b*
```

```
[Out] integrate(1/((b*x^n + a)^((b*c*(n + 1) - a*d*n)/((b*c - a*d)*n))*
(d*x^n + c)^((b*c*n - a*d*n - a*d)/(b*c*n - a*d*n))), x)
```

# 3.231 $\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}}\,dx$

**Optimal**. Leaf size=327

$$-\frac{2a^2n^2x(c+dx^n)^{-1/n}(3adn-b(3cn+c))}{c^4(n+1)(2n+1)(3n+1)(bc-ad)}-\frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}(3adn-b(3cn+c))}{c^3(n+1)(2n+1)(3n+1)(bc-ad)}-\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}(3adn-b(3cn+c))}{c^2(6n^2+5n+1)(bc-ad)}-\frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}(3adn-b(3cn+c))}{3acn(3n+1)(bc-ad)}-\frac{bx(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{3an(bc-ad)}$$

```
[Out] -(b*x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(3*a*(b*c - a*d)*n
) - ((3*a*d*n - b*(c + 3*c*n))*x*(a + b*x^n)^3*(c + d*x^n)^(-3 -
n^(-1)))/(3*a*c*(b*c - a*d)*n*(1 + 3*n)) - ((3*a*d*n - b*(c + 3*c
*n))*x*(a + b*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c^2*(b*c - a*d)*
(1 + 5*n + 6*n^2)) - (2*a*n*(3*a*d*n - b*(c + 3*c*n))*x*(a + b*x^
n)*(c + d*x^n)^(-1 - n^(-1)))/(c^3*(b*c - a*d)*(1 + n)*(1 + 2*n)*
(1 + 3*n)) - (2*a^2*n^2*(3*a*d*n - b*(c + 3*c*n))*x)/(c^4*(b*c -
a*d)*(1 + n)*(1 + 2*n)*(1 + 3*n)*(c + d*x^n)^n^(-1))
```

---

**Rubi [A]** time = 0.548162, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$-\frac{2a^2n^2x(c+dx^n)^{-1/n}(3adn-b(3cn+c))}{c^4(n+1)(2n+1)(3n+1)(bc-ad)}-\frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}(3adn-b(3cn+c))}{c^3(n+1)(2n+1)(3n+1)(bc-ad)}-\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}(3adn-b(3cn+c))}{c^2(6n^2+5n+1)(bc-ad)}-\frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}(3adn-b(3cn+c))}{3acn(3n+1)(bc-ad)}-\frac{bx(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{3an(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)^2*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] -(b*x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(3*a*(b*c - a*d)*n
) - ((3*a*d*n - b*(c + 3*c*n))*x*(a + b*x^n)^3*(c + d*x^n)^(-3 -
n^(-1)))/(3*a*c*(b*c - a*d)*n*(1 + 3*n)) - ((3*a*d*n - b*(c + 3*c
*n))*x*(a + b*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c^2*(b*c - a*d)*
(1 + 5*n + 6*n^2)) - (2*a*n*(3*a*d*n - b*(c + 3*c*n))*x*(a + b*x^
n)*(c + d*x^n)^(-1 - n^(-1)))/(c^3*(b*c - a*d)*(1 + n)*(1 + 2*n)*
(1 + 3*n)) - (2*a^2*n^2*(3*a*d*n - b*(c + 3*c*n))*x)/(c^4*(b*c -
a*d)*(1 + n)*(1 + 2*n)*(1 + 3*n)*(c + d*x^n)^n^(-1))
```

---

**Rubi in Sympy [A]** time = 81.8544, size = 286, normalized size = 0.87

$$-\frac{2a^2n^2x\left(c+dx^n\right)^{-\frac{1}{n}}\left(-3adn+3bcn+bc\right)}{c^4\left(n+1\right)\left(2n+1\right)\left(3n+1\right)\left(ad-bc\right)}-\frac{2anx\left(a+bx^n\right)\left(c+dx^n\right)^{-1-\frac{1}{n}}\left(-3adn+3bcn+bc\right)}{c^3\left(n+1\right)\left(2n+1\right)\left(3n+1\right)\left(ad-bc\right)}-\frac{x\left(a+bx^n\right)^2\left(c+dx^n\right)^{-2-\frac{1}{n}}\left(-3adn+3bcn+bc\right)}{c^2\left(2n+1\right)\left(3n+1\right)\left(ad-bc\right)}+\frac{bx\left(a+bx^n\right)^3\left(c+dx^n\right)^{-3-\frac{1}{n}}}{3an\left(ad-bc\right)}-\frac{x\left(a+bx^n\right)^3\left(c+dx^n\right)^{-3-\frac{1}{n}}\left(-3adn+3bcn+bc\right)}{3acn\left(3n+1\right)\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)**2*(c+d*x**n)**(-4-1/n),x)
```

```
[Out] -2*a**2*n**2*x*(c + d*x**n)**(-1/n)*(-3*a*d*n + 3*b*c*n + b*c)/(c
**4*(n + 1)*(2*n + 1)*(3*n + 1)*(a*d - b*c)) - 2*a*n*x*(a + b*x**
n)*(c + d*x**n)**(-1 - 1/n)*(-3*a*d*n + 3*b*c*n + b*c)/(c**3*(n +
 1)*(2*n + 1)*(3*n + 1)*(a*d - b*c)) - x*(a + b*x**n)**2*(c + d*x
**n)**(-2 - 1/n)*(-3*a*d*n + 3*b*c*n + b*c)/(c**2*(2*n + 1)*(3*n
+ 1)*(a*d - b*c)) + b*x*(a + b*x**n)**3*(c + d*x**n)**(-3 - 1/n)/
(3*a*n*(a*d - b*c)) - x*(a + b*x**n)**3*(c + d*x**n)**(-3 - 1/n)*
(-3*a*d*n + 3*b*c*n + b*c)/(3*a*c*n*(3*n + 1)*(a*d - b*c))
```

---

**Mathematica [C]** time = 0.304528, size = 153, normalized size = 0.47

$$\frac{x\left(c+dx^n\right)^{-1/n}\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}\left((n+1)\left(a^2(2n+1)\,{}_2F_1\left(4+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+b^2x^{2n}\,{}_2F_1\left(2+\frac{1}{n},4+\frac{1}{n};3+\frac{1}{n};-\frac{dx^n}{c}\right)\right)+2ab\cdots}{c^4(n+1)(2n+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)^2*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] (x*(1 + (d*x^n)/c)^n^(-1)*(2*a*b*(1 + 2*n)*x^n*Hypergeometric2F1[
1 + n^(-1), 4 + n^(-1), 2 + n^(-1), -((d*x^n)/c)] + (1 + n)*(b^2*
x^(2*n)*Hypergeometric2F1[2 + n^(-1), 4 + n^(-1), 3 + n^(-1), -((
d*x^n)/c)] + a^2*(1 + 2*n)*Hypergeometric2F1[4 + n^(-1), n^(-1),
1 + n^(-1), -((d*x^n)/c)])))/(c^4*(1 + n)*(1 + 2*n)*(c + d*x^n)^n
^(-1))
```

---

**Maple [F]** time = 0.21, size = 0, normalized size = 0.

$$\int \left(a+bx^n\right)^2\left(c+dx^n\right)^{-4-n^{-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x)
```

```
[Out] int((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^2 (dx^n + c)^{-\frac{1}{n}-4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 4), x)
```

---

**Fricas [A]** time = 0.260355, size = 540, normalized size = 1.65

$$\frac{\left(6\,a^2 d^4 n^3 + b^2 c^2 d^2 n + \left(b^2 c^2 d^2 + 4\,abcd^3\right) n^2\right) x x^{4\,n} + \left(24\,a^2 c d^3 n^3 + b^2 c^3 d + 2\left(2\,b^2 c^3 d + 8\,abc^2 d^2 + 3\,a^2 c d^3\right) n^2 + \left(5\,b^2 c^3 d + \cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 4),x, algorithm="fricas")
```

```
[Out] ((6*a^2*d^4*n^3 + b^2*c^2*d^2*n + (b^2*c^2*d^2 + 4*a*b*c*d^3)*n^2
)*x*x^(4*n) + (24*a^2*c*d^3*n^3 + b^2*c^3*d + 2*(2*b^2*c^3*d + 8*
a*b*c^2*d^2 + 3*a^2*c*d^3)*n^2 + (5*b^2*c^3*d + 4*a*b*c^2*d^2)*n)
*x*x^(3*n) + (36*a^2*c^2*d^2*n^3 + b^2*c^4 + 2*a*b*c^3*d + 3*(b^2
*c^4 + 8*a*b*c^3*d + 7*a^2*c^2*d^2)*n^2 + (4*b^2*c^4 + 14*a*b*c^3
*d + 3*a^2*c^2*d^2)*n)*x*x^(2*n) + (24*a^2*c^3*d*n^3 + 2*a*b*c^4
+ a^2*c^3*d + 2*(6*a*b*c^4 + 13*a^2*c^3*d)*n^2 + (10*a*b*c^4 + 9*
a^2*c^3*d)*n)*x*x^n + (6*a^2*c^4*n^3 + 11*a^2*c^4*n^2 + 6*a^2*c^4
*n + a^2*c^4)*x)/((6*c^4*n^3 + 11*c^4*n^2 + 6*c^4*n + c^4)*(d*x^n
 + c)^((4*n + 1)/n))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)**2*(c+d*x**n)**(-4-1/n),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.232 $\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx$

**Optimal**. Leaf size=127

$$\frac{nx\,(c+dx^n)^{-1/n}\,(2adn+bc)}{c^3d(n+1)(2n+1)}+\frac{x\,(c+dx^n)^{-\frac{1}{n}-1}\,(2adn+bc)}{c^2d(n+1)(2n+1)}-\frac{x(bc-ad)\,(c+dx^n)^{-\frac{1}{n}-2}}{cd(2n+1)}$$

```
[Out] -(((b*c - a*d)*x*(c + d*x^n)^(-2 - n^(-1)))/(c*d*(1 + 2*n))) + ((
b*c + 2*a*d*n)*x*(c + d*x^n)^(-1 - n^(-1)))/(c^2*d*(1 + n)*(1 + 2
*n)) + (n*(b*c + 2*a*d*n)*x)/(c^3*d*(1 + n)*(1 + 2*n)*(c + d*x^n)
^n^(-1))
```

---

**Rubi [A]** time = 0.172598, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$

$$\frac{nx\,(c+dx^n)^{-1/n}\,(2adn+bc)}{c^3d(n+1)(2n+1)}+\frac{x\,(c+dx^n)^{-\frac{1}{n}-1}\,(2adn+bc)}{c^2d(n+1)(2n+1)}-\frac{x(bc-ad)\,(c+dx^n)^{-\frac{1}{n}-2}}{cd(2n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^n)*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] -(((b*c - a*d)*x*(c + d*x^n)^(-2 - n^(-1)))/(c*d*(1 + 2*n))) + ((
b*c + 2*a*d*n)*x*(c + d*x^n)^(-1 - n^(-1)))/(c^2*d*(1 + n)*(1 + 2
*n)) + (n*(b*c + 2*a*d*n)*x)/(c^3*d*(1 + n)*(1 + 2*n)*(c + d*x^n)
^n^(-1))
```

---

**Rubi in Sympy [A]** time = 17.8297, size = 105, normalized size = 0.83

$$\frac{x\,(c+dx^n)^{-2-\frac{1}{n}}\,(ad-bc)}{cd\,(2n+1)}+\frac{x\,(c+dx^n)^{-1-\frac{1}{n}}\,(2adn+bc)}{c^2d\,(n+1)\,(2n+1)}+\frac{nx\,(c+dx^n)^{-\frac{1}{n}}\,(2adn+bc)}{c^3d\,(n+1)\,(2n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+b*x**n)*(c+d*x**n)**(-3-1/n),x)
```

```
[Out] x*(c + d*x**n)**(-2 - 1/n)*(a*d - b*c)/(c*d*(2*n + 1)) + x*(c + d
*x**n)**(-1 - 1/n)*(2*a*d*n + b*c)/(c**2*d*(n + 1)*(2*n + 1)) + n
*x*(c + d*x**n)**(-1/n)*(2*a*d*n + b*c)/(c**3*d*(n + 1)*(2*n + 1)
)
```

---

**Mathematica [C]** time = 0.139311, size = 96, normalized size = 0.76

$$\frac{x\left(c+dx^n\right)^{-1/n}\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}\left(a(n+1)\,{}_2F_1\left(3+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+bx^n\,{}_2F_1\left(1+\frac{1}{n},3+\frac{1}{n};2+\frac{1}{n};-\frac{dx^n}{c}\right)\right)}{c^3(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^n)*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] (x*(1 + (d*x^n)/c)^n^(-1)*(b*x^n*Hypergeometric2F1[1 + n^(-1), 3
+ n^(-1), 2 + n^(-1), -((d*x^n)/c)] + a*(1 + n)*Hypergeometric2F1
[3 + n^(-1), n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(c^3*(1 + n)*(c
+ d*x^n)^n^(-1))
```

---

**Maple [F]** time = 0.18, size = 0, normalized size = 0.

$$\int (a+bx^n)\,(c+dx^n)^{-3-n^{-1}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+b*x^n)*(c+d*x^n)^(-3-1/n),x)
```

```
[Out] int((a+b*x^n)*(c+d*x^n)^(-3-1/n),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n+a)(dx^n+c)^{-\frac{1}{n}-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 3), x)
```

---

**Fricas [A]** time = 0.260778, size = 234, normalized size = 1.84

$$\frac{\left(2\,ad^3n^2+bcd^2n\right)xx^{3\,n}+\left(6\,acd^2n^2+bc^2d+\left(3\,bc^2d+2\,acd^2\right)n\right)xx^{2\,n}+\left(6\,ac^2dn^2+bc^3+ac^2d+\left(2\,bc^3+5\,ac^2d\right)n\right)xx^n+}{(2\,c^3n^2+3\,c^3n+c^3)(dx^n+c)^{\frac{3\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 3),x, algorithm="fricas")
```

```
[Out] ((2*a*d^3*n^2 + b*c*d^2*n)*x*x^(3*n) + (6*a*c*d^2*n^2 + b*c^2*d +
 (3*b*c^2*d + 2*a*c*d^2)*n)*x*x^(2*n) + (6*a*c^2*d*n^2 + b*c^3 +
a*c^2*d + (2*b*c^3 + 5*a*c^2*d)*n)*x*x^n + (2*a*c^3*n^2 + 3*a*c^3
*n + a*c^3)*x)/((2*c^3*n^2 + 3*c^3*n + c^3)*(d*x^n + c)^((3*n + 1
)/n))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+b*x**n)*(c+d*x**n)**(-3-1/n),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.233 $\int (c+dx^n)^{-2-\frac{1}{n}}\,dx$

**Optimal**. Leaf size=50

$$\frac{nx(c+dx^n)^{-1/n}}{c^2(n+1)}+\frac{x(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}$$

```
[Out] (x*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (n*x)/(c^2*(1 + n)*(c
 + d*x^n)^n^(-1))
```

---

**Rubi [A]** time = 0.0396865, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{nx(c+dx^n)^{-1/n}}{c^2(n+1)}+\frac{x(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (n*x)/(c^2*(1 + n)*(c
 + d*x^n)^n^(-1))
```

---

**Rubi in Sympy [A]** time = 3.68181, size = 39, normalized size = 0.78

$$\frac{x(c+dx^n)^{-1-\frac{1}{n}}}{c(n+1)}+\frac{nx(c+dx^n)^{-\frac{1}{n}}}{c^2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**n)**(-2-1/n),x)
```

```
[Out] x*(c + d*x**n)**(-1 - 1/n)/(c*(n + 1)) + n*x*(c + d*x**n)**(-1/n)
/(c**2*(n + 1))
```

---

**Mathematica [C]** time = 0.0370937, size = 55, normalized size = 1.1

$$\frac{x(c+dx^n)^{-1/n}\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(1 + (d*x^n)/c)^n^(-1)*Hypergeometric2F1[2 + n^(-1), n^(-1), 1
 + n^(-1), -((d*x^n)/c)])/(c^2*(c + d*x^n)^n^(-1))
```

---

**Maple [F]** time = 0.131, size = 0, normalized size = 0.

$$\int (c+dx^n)^{-2-n^{-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^n)^(-2-1/n),x)
```

```
[Out] int((c+d*x^n)^(-2-1/n),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^n+c)^{-\frac{1}{n}-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^(-1/n - 2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 2), x)
```

---

**Fricas [A]** time = 0.256259, size = 92, normalized size = 1.84

$$\frac{d^2nxx^{2\,n}+(2\,cdn+cd)xx^n+\left(c^2n+c^2\right)x}{(c^2n+c^2)(dx^n+c)^{\frac{2\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^(-1/n - 2),x, algorithm="fricas")
```

```
[Out] (d^2*n*x*x^(2*n) + (2*c*d*n + c*d)*x*x^n + (c^2*n + c^2)*x)/((c^2
*n + c^2)*(d*x^n + c)^((2*n + 1)/n))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**n)**(-2-1/n),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^n + c)^{-\frac{1}{n}-2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^(-1/n - 2),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 2), x)
```

## 3.234 $\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n} \, dx$

**Optimal**. Leaf size=95

$$\frac{bx(c+dx^n)^{-1/n} \, {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a(bc-ad)} - \frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)}$$

```
[Out] -((d*x)/(c*(b*c - a*d)*(c + d*x^n)^n^(-1))) + (b*x*Hypergeometric
2F1[1, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])
/(a*(b*c - a*d)*(c + d*x^n)^n^(-1))
```

---

**Rubi [A]** time = 0.105637, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{bx(c+dx^n)^{-1/n} \, {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a(bc-ad)} - \frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^n)^(-1 - n^(-1))/(a + b*x^n),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*(c + d*x^n)^n^(-1))) + (b*x*Hypergeometric
2F1[1, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])
/(a*(b*c - a*d)*(c + d*x^n)^n^(-1))
```

---

**Rubi in Sympy [A]** time = 13.6199, size = 70, normalized size = 0.74

$$\frac{dx(c+dx^n)^{-\frac{1}{n}}}{c(ad-bc)} - \frac{bx(c+dx^n)^{-\frac{1}{n}} \, {}_2F_1\left(\begin{matrix} \frac{1}{n}, 1 \\ 1+\frac{1}{n} \end{matrix}\middle| -\frac{x^n(-ad+bc)}{a(c+dx^n)}\right)}{a(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**n)**(-1-1/n)/(a+b*x**n),x)
```

```
[Out] d*x*(c + d*x**n)**(-1/n)/(c*(a*d - b*c)) - b*x*(c + d*x**n)**(-1/
n)*hyper((1/n, 1), (1 + 1/n,), -x**n*(-a*d + b*c)/(a*(c + d*x**n)
))/(a*(a*d - b*c))
```

---

**Mathematica [C]** time = 2.23558, size = 153, normalized size = 1.61

$$\frac{x\left(c+dx^n\right)^{-\frac{n+1}{n}}\left(\frac{bnx^{2n}(ad-bc)\,{}_2F_1\left(2,2+\frac{1}{n};3+\frac{1}{n};\frac{(ad-bc)x^n}{a(dx^n+c)}\right)}{a^2(2n+1)(c+dx^n)}+\frac{bx^n\Phi\left(\frac{(ad-bc)x^n}{a(dx^n+c)},1,1+\frac{1}{n}\right)}{a}+\frac{a(c+dx^n)}{c(a+bx^n)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^n)^(-1 - n^(-1))/(a + b*x^n),x]
```

```
[Out] (x*((a*(c + d*x^n))/(c*(a + b*x^n)) + (b*x^n*HurwitzLerchPhi[((-(
b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, 1 + n^(-1)])/a + (b*(-(b*c)
+ a*d)*n*x^(2*n)*Hypergeometric2F1[2, 2 + n^(-1), 3 + n^(-1), ((-
(b*c) + a*d)*x^n)/(a*(c + d*x^n))])/(a^2*(1 + 2*n)*(c + d*x^n))))
/(a*(c + d*x^n)^((1 + n)/n))
```

---

**Maple [F]** time = 0.126, size = 0, normalized size = 0.

$$\int \frac{1}{a+bx^n}\left(c+dx^n\right)^{-1-n^{-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^n)^(-1-1/n)/(a+b*x^n),x)
```

```
[Out] int((c+d*x^n)^(-1-1/n)/(a+b*x^n),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^n+c\right)^{-\frac{1}{n}-1}}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(bx^n+a\right)\left(dx^n+c\right)^{\frac{n+1}{n}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a),x, algorithm="fricas")
```

```
[Out] integral(1/((b*x^n + a)*(d*x^n + c)^((n + 1)/n)), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**n)**(-1-1/n)/(a+b*x**n),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}-1}}{bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a), x)
```

## 3.235 $\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx$

**Optimal**. Leaf size=127

$$\frac{bx\,(c+dx^n)^{-\frac{1-n}{n}}}{an(bc-ad)\,(a+bx^n)}-\frac{x\,(c+dx^n)^{-1/n}\,(adn+bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2n(bc-ad)}$$

```
[Out] (b*x)/(a*(b*c - a*d)*n*(a + b*x^n)*(c + d*x^n)^((1 - n)/n)) - ((b
*c*(1 - n) + a*d*n)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(
((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(a^2*(b*c - a*d)*n*(c + d*x^
n)^n^(-1))
```

---

**Rubi [A]** time = 0.151078, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{bx\,(c+dx^n)^{-\frac{1-n}{n}}}{an(bc-ad)\,(a+bx^n)}-\frac{x\,(c+dx^n)^{-1/n}\,(adn+b(c-cn))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^n)^2*(c + d*x^n)^n^(-1)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*n*(a + b*x^n)*(c + d*x^n)^((1 - n)/n)) - ((a
*d*n + b*(c - c*n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(
((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(a^2*(b*c - a*d)*n*(c + d*x^
n)^n^(-1))
```

---

**Rubi in Sympy [A]** time = 19.2463, size = 95, normalized size = 0.75

$$-\frac{bx\,(c+dx^n)^{\frac{n-1}{n}}}{an\,(a+bx^n)\,(ad-bc)}+\frac{x\,(c+dx^n)^{-\frac{1}{n}}\,(adn-bcn+bc)\,{}_2F_1\left(\begin{matrix}\frac{1}{n},1\\1+\frac{1}{n}\end{matrix}\middle|-\frac{x^n(-ad+bc)}{a(c+dx^n)}\right)}{a^2n\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+b*x**n)**2/((c+d*x**n)**(1/n)),x)
```

```
[Out] -b*x*(c + d*x**n)**((n - 1)/n)/(a*n*(a + b*x**n)*(a*d - b*c)) + x
*(c + d*x**n)**(-1/n)*(a*d*n - b*c*n + b*c)*hyper((1/n, 1), (1 +
1/n,), -x**n*(-a*d + b*c)/(a*(c + d*x**n)))/(a**2*n*(a*d - b*c))
```

**Mathematica [B]** time = 62.1905, size = 1070, normalized size = 8.43

$$-cd(1-n)(2n+1)(3n+1)\left(bx^n+a\right)^2\left(2(bc-ad)n\left(dx^n+c\right)\left(2+\frac{1}{n}\right)\ {}_2F_1\left(2,3;3+\frac{1}{n};\frac{(bc-ad)x^n}{c(bx^n+a)}\right)x^n+c\left(bx^n+a\right)\left(dnx^n+c\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^n)^2*(c + d*x^n)^n^(-1)),x]
```

```
[Out] (c^2*(1 + 2*n)*(1 + 3*n)*x*(a + b*x^n)*(1 + (d*x^n)/c)*Gamma[2 +
n^(-1)]*Gamma[3 + n^(-1)]*((c*(c + c*n + d*n*x^n)*Hypergeometric2
F1[1, 2, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])/Gamma[2
+ n^(-1)] + (2*(b*c - a*d)*n*x^n*(c + d*x^n)*Hypergeometric2F1[2,
 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])/((a + b*x^n)*
Gamma[3 + n^(-1)])))/((c + d*x^n)^n^(-1)*(-(c*d*(1 - n)*(1 + 2*n)
*(1 + 3*n)*x^n*(a + b*x^n)^2*(c*(a + b*x^n)*(c + c*n + d*n*x^n)*G
amma[3 + n^(-1)]*Hypergeometric2F1[1, 2, 2 + n^(-1), ((b*c - a*d)
*x^n)/(c*(a + b*x^n))] + 2*(b*c - a*d)*n*x^n*(c + d*x^n)*Gamma[2
+ n^(-1)]*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(
c*(a + b*x^n))])) - 2*b*c*n*(1 + 2*n)*(1 + 3*n)*x^n*(a + b*x^n)*(
c + d*x^n)*(c*(a + b*x^n)*(c + c*n + d*n*x^n)*Gamma[3 + n^(-1)]*H
ypergeometric2F1[1, 2, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^
n))] + 2*(b*c - a*d)*n*x^n*(c + d*x^n)*Gamma[2 + n^(-1)]*Hypergeo
metric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]) +
 c*(1 + 2*n)*(1 + 3*n)*(a + b*x^n)^2*(c + d*x^n)*(c*(a + b*x^n)*(
c + c*n + d*n*x^n)*Gamma[3 + n^(-1)]*Hypergeometric2F1[1, 2, 2 +
n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + 2*(b*c - a*d)*n*x^n*
(c + d*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F1[2, 3, 3 + n^(-1),
 ((b*c - a*d)*x^n)/(c*(a + b*x^n))]) + n^2*x^n*(c + d*x^n)*(c^2*d
*(1 + 2*n)*(1 + 3*n)*(a + b*x^n)^3*Gamma[3 + n^(-1)]*Hypergeometr
ic2F1[1, 2, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + 2*c*
d*(b*c - a*d)*(1 + 2*n)*(1 + 3*n)*x^n*(a + b*x^n)^2*Gamma[2 + n^(
-1)]*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a
+ b*x^n))] - 2*b*c*(b*c - a*d)*(1 + 2*n)*(1 + 3*n)*x^n*(a + b*x^n
)*(c + d*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F1[2, 3, 3 + n^(-1
), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + 2*c*(b*c - a*d)*(1 + 2*n)
*(1 + 3*n)*(a + b*x^n)^2*(c + d*x^n)*Gamma[2 + n^(-1)]*Hypergeome
tric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + 2*
a*c*(b*c - a*d)*(1 + 3*n)*(a + b*x^n)*(c + c*n + d*n*x^n)*Gamma[3
 + n^(-1)]*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/
(c*(a + b*x^n))] + 12*a*(b*c - a*d)^2*n*(1 + 2*n)*x^n*(c + d*x^n)
*Gamma[2 + n^(-1)]*Hypergeometric2F1[3, 4, 4 + n^(-1), ((b*c - a*
d)*x^n)/(c*(a + b*x^n))])))
```

**Maple [F]** time = 0.116, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^n\right)^2\sqrt[n]{c+dx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x)
```

```
[Out] int(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}}}{(bx^n + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)^2*(d*x^n + c)^(1/n)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n)/(b*x^n + a)^2, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(b^2x^{2\,n} + 2\,abx^n + a^2)(dx^n + c)^{\left(\frac{1}{n}\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)^2*(d*x^n + c)^(1/n)),x, algorithm="fricas")
```

```
[Out] integral(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)*(d*x^n + c)^(1/n)), x
)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+b*x**n)**2/((c+d*x**n)**(1/n)),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)^2 (dx^n + c)^{\left(\frac{1}{n}\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^n + a)^2*(d*x^n + c)^(1/n)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)^(1/n)), x)
```

## 3.236 $\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx$

**Optimal**. Leaf size=131

$$\frac{bx\left(c+dx^n\right)^{2-\frac{1}{n}}}{2an(bc-ad)\left(a+bx^n\right)^2}-\frac{cx\left(c+dx^n\right)^{-1/n}\left(2adn+bc(1-2n)\right){}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{2a^3n(bc-ad)}$$

```
[Out] (b*x*(c + d*x^n)^(2 - n^(-1)))/(2*a*(b*c - a*d)*n*(a + b*x^n)^2)
- (c*(b*c*(1 - 2*n) + 2*a*d*n)*x*Hypergeometric2F1[2, n^(-1), 1 +
 n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(2*a^3*(b*c - a*d
)*n*(c + d*x^n)^n^(-1))
```

---

**Rubi [A]** time = 0.16193, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08

$$\frac{bx\left(c+dx^n\right)^{2-\frac{1}{n}}}{2an(bc-ad)\left(a+bx^n\right)^2}-\frac{cx\left(c+dx^n\right)^{-1/n}\left(2adn+b(c-2cn)\right){}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{2a^3n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^3,x]
```

```
[Out] (b*x*(c + d*x^n)^(2 - n^(-1)))/(2*a*(b*c - a*d)*n*(a + b*x^n)^2)
- (c*(2*a*d*n + b*(c - 2*c*n))*x*Hypergeometric2F1[2, n^(-1), 1 +
 n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(2*a^3*(b*c - a*d
)*n*(c + d*x^n)^n^(-1))
```

---

**Rubi in Sympy [A]** time = 19.6972, size = 105, normalized size = 0.8

$$-\frac{bx\left(c+dx^n\right)^{2-\frac{1}{n}}}{2an\left(a+bx^n\right)^2(ad-bc)}+\frac{cx\left(c+dx^n\right)^{-\frac{1}{n}}\left(2adn-2bcn+bc\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{n},2\\1+\frac{1}{n}\end{matrix}\right|-\frac{x^n(-ad+bc)}{a(c+dx^n)}\right)}{2a^3n(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**n)**(1-1/n)/(a+b*x**n)**3,x)
```

```
[Out] -b*x*(c + d*x**n)**(2 - 1/n)/(2*a*n*(a + b*x**n)**2*(a*d - b*c))
+ c*x*(c + d*x**n)**(-1/n)*(2*a*d*n - 2*b*c*n + b*c)*hyper((1/n,
2), (1 + 1/n,), -x**n*(-a*d + b*c)/(a*(c + d*x**n)))/(2*a**3*n*(a
```

```
*d - b*c))
```

---

**Mathematica [B]** time = 43.7676, size = 1241, normalized size = 9.47

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^3,x]
```

```
[Out] -((c^3*(1 + n)*(1 + 2*n)*(1 + 3*n)*x*(c + d*x^n)^(2 - n^(-1))*Gam
ma[2 + n^(-1)]*(Hypergeometric2F1[1, 3, 1 + n^(-1), ((b*c - a*d)*
x^n)/(c*(a + b*x^n))] + (d*n*x^n*((c*Hypergeometric2F1[1, 3, 2 +
n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])/(1 + n) + (3*(b*c - a
*d)*x^n*Gamma[1 + n^(-1)]*Hypergeometric2F1[2, 4, 3 + n^(-1), ((b
*c - a*d)*x^n)/(c*(a + b*x^n))])/((1 + 2*n)*(a + b*x^n)*Gamma[2 +
 n^(-1)])))/c^2))/(c*d*(1 - 2*n)*(1 + 3*n)*x^n*(a + b*x^n)^2*(c^2
*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F
1[1, 3, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + d*n*x^n*
(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F1[1, 3
, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + 3*(b*c - a*d)*
(1 + n)*x^n*Gamma[1 + n^(-1)]*Hypergeometric2F1[2, 4, 3 + n^(-1),
 ((b*c - a*d)*x^n)/(c*(a + b*x^n))])) + 3*b*c*n*(1 + 3*n)*x^n*(a
+ b*x^n)*(c + d*x^n)*(c^2*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 +
 n^(-1)]*Hypergeometric2F1[1, 3, 1 + n^(-1), ((b*c - a*d)*x^n)/(c
*(a + b*x^n))] + d*n*x^n*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1
)]*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a +
b*x^n))] + 3*(b*c - a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hypergeome
tric2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])) -
c*(1 + 3*n)*(a + b*x^n)^2*(c + d*x^n)*(c^2*(1 + n)*(1 + 2*n)*(a +
 b*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F1[1, 3, 1 + n^(-1), ((b
*c - a*d)*x^n)/(c*(a + b*x^n))] + d*n*x^n*(c*(1 + 2*n)*(a + b*x^n
)*Gamma[2 + n^(-1)]*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b*c - a
*d)*x^n)/(c*(a + b*x^n))] + 3*(b*c - a*d)*(1 + n)*x^n*Gamma[1 + n
^(-1)]*Hypergeometric2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(
a + b*x^n))])) + n^2*x^n*(c + d*x^n)*(3*a*c^2*(-(b*c) + a*d)*(1 +
 2*n)*(1 + 3*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F1[2
, 4, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] - c*d*(1 + 3*
n)*(a + b*x^n)^2*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hyper
geometric2F1[1, 3, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]
 + 3*(b*c - a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hypergeometric2F1[
2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]) + 3*d*(b*c
- a*d)*x^n*(b*c*(1 + n)*(1 + 3*n)*x^n*(a + b*x^n)*Gamma[1 + n^(-1
)]*Hypergeometric2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a +
b*x^n))] - c*(1 + n)*(1 + 3*n)*(a + b*x^n)^2*Gamma[1 + n^(-1)]*Hy
pergeometric2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n
))] - a*c*n*(1 + 3*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hypergeometri
c2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + 8*a*(
-(b*c) + a*d)*n*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hypergeometric2F1[3
, 5, 4 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]))))
```

---

**Maple [F]** time = 0.113, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)^3}(c+dx^n)^{1-n^{-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x)
```

```
[Out] int((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n+c)^{-\frac{1}{n}+1}}{(bx^n+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^3, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx^n+c)^{\frac{n-1}{n}}}{b^3x^{3n}+3\,ab^2x^{2n}+3\,a^2bx^n+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)^((n - 1)/n)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) +
 3*a^2*b*x^n + a^3), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**n)**(1-1/n)/(a+b*x**n)**3,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+1}}{(bx^n + a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^3, x)
```

## 3.237 $\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx$

**Optimal**. Leaf size=133

$$\frac{bx\,(c+dx^n)^{3-\frac{1}{n}}}{3an(bc-ad)\,(a+bx^n)^3}-\frac{c^2x\,(c+dx^n)^{-1/n}\,(3adn+bc(1-3n))\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{3a^4n(bc-ad)}$$

```
[Out] (b*x*(c + d*x^n)^(3 - n^(-1)))/(3*a*(b*c - a*d)*n*(a + b*x^n)^3)
- (c^2*(b*c*(1 - 3*n) + 3*a*d*n)*x*Hypergeometric2F1[3, n^(-1), 1
 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(3*a^4*(b*c - a
*d)*n*(c + d*x^n)^n^(-1))
```

---

**Rubi [A]** time = 0.171492, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08

$$\frac{bx\,(c+dx^n)^{3-\frac{1}{n}}}{3an(bc-ad)\,(a+bx^n)^3}-\frac{c^2x\,(c+dx^n)^{-1/n}\,(3adn+b(c-3cn))\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{3a^4n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^4,x]
```

```
[Out] (b*x*(c + d*x^n)^(3 - n^(-1)))/(3*a*(b*c - a*d)*n*(a + b*x^n)^3)
- (c^2*(3*a*d*n + b*(c - 3*c*n))*x*Hypergeometric2F1[3, n^(-1), 1
 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(3*a^4*(b*c - a
*d)*n*(c + d*x^n)^n^(-1))
```

---

**Rubi in Sympy [A]** time = 20.6569, size = 107, normalized size = 0.8

$$-\frac{bx\,(c+dx^n)^{3-\frac{1}{n}}}{3an\,(a+bx^n)^3\,(ad-bc)}+\frac{c^2x\,(c+dx^n)^{-\frac{1}{n}}\,(3adn-3bcn+bc)\,{}_2F_1\left(\begin{matrix}\frac{1}{n},3\\1+\frac{1}{n}\end{matrix}\middle|-\frac{x^n(-ad+bc)}{a(c+dx^n)}\right)}{3a^4n\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**n)**(2-1/n)/(a+b*x**n)**4,x)
```

```
[Out] -b*x*(c + d*x**n)**(3 - 1/n)/(3*a*n*(a + b*x**n)**3*(a*d - b*c))
+ c**2*x*(c + d*x**n)**(-1/n)*(3*a*d*n - 3*b*c*n + b*c)*hyper((1/
n, 3), (1 + 1/n,), -x**n*(-a*d + b*c)/(a*(c + d*x**n)))/(3*a**4*n
```

```
*(a*d - b*c))
```

---

**Mathematica [F]** time = 179.999, size = 0, normalized size = 0.

$$\$Aborted$$

Verification is Not applicable to the result.

```
[In]  Integrate[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^4,x]
```

```
[Out] $Aborted
```

---

**Maple [F]** time = 0.137, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)^4}(c+dx^n)^{2-n^{-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x)
```

```
[Out] int((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^4, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^n+c)^{\frac{2n-1}{n}}}{b^4x^{4n}+4ab^3x^{3n}+6a^2b^2x^{2n}+4a^3bx^n+a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^4,x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)^((2*n - 1)/n)/(b^4*x^(4*n) + 4*a*b^3*x^(3*n)
 + 6*a^2*b^2*x^(2*n) + 4*a^3*b*x^n + a^4), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c+d*x**n)**(2-1/n)/(a+b*x**n)**4,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+2}}{(bx^n + a)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^4,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^4, x)
```

# 3.238 $\int x^5\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$

**Optimal**. Leaf size=152

$$\frac{(dx-c)^{7/2}(c+dx)^{7/2}\left(ad^2+3bc^2\right)}{7d^8}+\frac{c^2(dx-c)^{5/2}(c+dx)^{5/2}\left(2ad^2+3bc^2\right)}{5d^8}+\frac{c^4(dx-c)^{3/2}(c+dx)^{3/2}\left(ad^2+bc^2\right)}{3d^8}+\frac{b(dx-c)^{9/2}(c+dx)^{9/2}}{9d^8}$$

```
[Out] (c^4*(b*c^2 + a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(3*d^8) +
(c^2*(3*b*c^2 + 2*a*d^2)*(-c + d*x)^(5/2)*(c + d*x)^(5/2))/(5*d^8
) + ((3*b*c^2 + a*d^2)*(-c + d*x)^(7/2)*(c + d*x)^(7/2))/(7*d^8)
+ (b*(-c + d*x)^(9/2)*(c + d*x)^(9/2))/(9*d^8)
```

---

**Rubi [A]** time = 0.406873, antiderivative size = 164, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129

$$\frac{4c^2x^2(dx-c)^{3/2}(c+dx)^{3/2}\left(3ad^2+2bc^2\right)}{105d^6}+\frac{x^4(dx-c)^{3/2}(c+dx)^{3/2}\left(3ad^2+2bc^2\right)}{21d^4}+\frac{8c^4(dx-c)^{3/2}(c+dx)^{3/2}\left(3ad^2+2bc^2\right)}{315d^8}+\frac{bx^6(dx-c)^{3/2}(c+dx)^{3/2}}{9d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (8*c^4*(2*b*c^2 + 3*a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(315
*d^8) + (4*c^2*(2*b*c^2 + 3*a*d^2)*x^2*(-c + d*x)^(3/2)*(c + d*x)
^(3/2))/(105*d^6) + ((2*b*c^2 + 3*a*d^2)*x^4*(-c + d*x)^(3/2)*(c
+ d*x)^(3/2))/(21*d^4) + (b*x^6*(-c + d*x)^(3/2)*(c + d*x)^(3/2))
/(9*d^2)
```

---

**Rubi in Sympy [A]** time = 25.8529, size = 150, normalized size = 0.99

$$\frac{bx^6\left(-c+dx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{3}{2}}}{9d^2}+\frac{8c^4\left(-c+dx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{3}{2}}\left(3ad^2+2bc^2\right)}{315d^8}+\frac{4c^2x^2\left(-c+dx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{3}{2}}\left(3ad^2+2bc^2\right)}{105d^6}+\frac{x^4\left(-c+dx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{3}{2}}\left(3ad^2+2bc^2\right)}{21d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] b*x**6*(-c + d*x)**(3/2)*(c + d*x)**(3/2)/(9*d**2) + 8*c**4*(-c +
 d*x)**(3/2)*(c + d*x)**(3/2)*(3*a*d**2 + 2*b*c**2)/(315*d**8) +
4*c**2*x**2*(-c + d*x)**(3/2)*(c + d*x)**(3/2)*(3*a*d**2 + 2*b*c*
*2)/(105*d**6) + x**4*(-c + d*x)**(3/2)*(c + d*x)**(3/2)*(3*a*d**
2 + 2*b*c**2)/(21*d**4)
```

---

**Mathematica [A]** time = 0.094186, size = 109, normalized size = 0.72

$$-\frac{\sqrt{dx-c}\sqrt{c+dx}\left(c^2-d^2x^2\right)\left(3ad^2\left(8c^4+12c^2d^2x^2+15d^4x^4\right)+b\left(16c^6+24c^4d^2x^2+30c^2d^4x^4+35d^6x^6\right)\right)}{315d^8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] -(Sqrt[-c + d*x]*Sqrt[c + d*x]*(c^2 - d^2*x^2)*(3*a*d^2*(8*c^4 +
12*c^2*d^2*x^2 + 15*d^4*x^4) + b*(16*c^6 + 24*c^4*d^2*x^2 + 30*c^
2*d^4*x^4 + 35*d^6*x^6)))/(315*d^8)
```

---

**Maple [A]** time = 0.01, size = 92, normalized size = 0.6

$$\frac{35\,bx^6d^6+45\,ad^6x^4+30\,bc^2d^4x^4+36\,ac^2d^4x^2+24\,bc^4d^2x^2+24\,ac^4d^2+16\,bc^6}{315\,d^8}\left(dx+c\right)^{\frac{3}{2}}\left(dx-c\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/315*(d*x+c)^(3/2)*(35*b*d^6*x^6+45*a*d^6*x^4+30*b*c^2*d^4*x^4+3
6*a*c^2*d^4*x^2+24*b*c^4*d^2*x^2+24*a*c^4*d^2+16*b*c^6)*(d*x-c)^(
3/2)/d^8
```

---

**Maxima [A]** time = 1.39231, size = 240, normalized size = 1.58

$$\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^6}{9\,d^2}+\frac{2\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2x^4}{21\,d^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ax^4}{7\,d^2}+\frac{8\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^4x^2}{105\,d^6}+\frac{4\left(d^2x^2-c^2\right)^{\frac{3}{2}}ac^2x^2}{35\,d^4}+\frac{16\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^6}{315\,d^8}+\frac{8\left(d^2x^2-c^2\right)^{\frac{3}{2}}ac^4}{105\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x^5,x, algorithm="maxima")
```

```
[Out] 1/9*(d^2*x^2 - c^2)^(3/2)*b*x^6/d^2 + 2/21*(d^2*x^2 - c^2)^(3/2)*
b*c^2*x^4/d^4 + 1/7*(d^2*x^2 - c^2)^(3/2)*a*x^4/d^2 + 8/105*(d^2*
x^2 - c^2)^(3/2)*b*c^4*x^2/d^6 + 4/35*(d^2*x^2 - c^2)^(3/2)*a*c^2
*x^2/d^4 + 16/315*(d^2*x^2 - c^2)^(3/2)*b*c^6/d^8 + 8/105*(d^2*x^
2 - c^2)^(3/2)*a*c^4/d^6
```

---

**Fricas [A]** time = 0.743305, size = 741, normalized size = 4.88

$$-\frac{8960\,bd^{18}x^{18} + 16\,bc^{18} + 24\,ac^{16}d^{2} - 2880\left(9\,bc^{2}d^{16} - 4\,ad^{18}\right)x^{16} + 144\left(181\,bc^{4}d^{14} - 236\,ac^{2}d^{16}\right)x^{14} - 24\left(461\,bc^{6}d^{12} - 1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x^5,x, algorithm="fricas")
```

```
[Out] -1/315*(8960*b*d^18*x^18 + 16*b*c^18 + 24*a*c^16*d^2 - 2880*(9*b*
c^2*d^16 - 4*a*d^18)*x^16 + 144*(181*b*c^4*d^14 - 236*a*c^2*d^16)
*x^14 - 24*(461*b*c^6*d^12 - 1426*a*c^4*d^14)*x^12 + 9*(27*b*c^8*
d^10 - 1832*a*c^6*d^12)*x^10 + 45*(160*b*c^10*d^8 + 289*a*c^8*d^1
0)*x^8 - 21*(429*b*c^12*d^6 + 646*a*c^10*d^8)*x^6 + 2079*(2*b*c^1
4*d^4 + 3*a*c^12*d^6)*x^4 - 324*(2*b*c^16*d^2 + 3*a*c^14*d^4)*x^2
 - (8960*b*d^17*x^17 - 320*(67*b*c^2*d^15 - 36*a*d^17)*x^15 + 235
2*(7*b*c^4*d^13 - 12*a*c^2*d^15)*x^13 - 8*(619*b*c^6*d^11 - 2694*
a*c^4*d^13)*x^11 - 5*(233*b*c^8*d^9 + 1704*a*c^6*d^11)*x^9 + 45*(
143*b*c^10*d^7 + 225*a*c^8*d^9)*x^7 - 3003*(2*b*c^12*d^5 + 3*a*c^
10*d^7)*x^5 + 924*(2*b*c^14*d^3 + 3*a*c^12*d^5)*x^3 - 72*(2*b*c^1
6*d + 3*a*c^14*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c))/(256*d^17*x^9
 - 576*c^2*d^15*x^7 + 432*c^4*d^13*x^5 - 120*c^6*d^11*x^3 + 9*c^8
*d^9*x - (256*d^16*x^8 - 448*c^2*d^14*x^6 + 240*c^4*d^12*x^4 - 40
*c^6*d^10*x^2 + c^8*d^8)*sqrt(d*x + c)*sqrt(d*x - c))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{5}\left(a + bx^{2}\right)\sqrt{-c + dx}\sqrt{c + dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

---

**GIAC/XCAS [A]** time = 0.257139, size = 309, normalized size = 2.03

$$\frac{3\left(\left(3\left((dx+c)\left(5\,(dx+c)\left(\frac{dx+c}{d^5}-\frac{6\,c}{d^5}\right)+\frac{74\,c^2}{d^5}\right)-\frac{96\,c^3}{d^5}\right)(dx+c)+\frac{203\,c^4}{d^5}\right)(dx+c)-\frac{70\,c^5}{d^5}\right)(dx+c)^{\frac{3}{2}}\sqrt{dx-c}a+\left(\left(\left(\left(5\left((dx+\right.\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x^5,x, algorithm="giac")
```

```
[Out] 1/315*(3*((3*((d*x + c)*(5*(d*x + c)*((d*x + c)/d^5 - 6*c/d^5) +
74*c^2/d^5) - 96*c^3/d^5)*(d*x + c) + 203*c^4/d^5)*(d*x + c) - 70
*c^5/d^5)*(d*x + c)^(3/2)*sqrt(d*x - c)*a + ((((5*((d*x + c)*(7*(
d*x + c)*((d*x + c)/d^7 - 8*c/d^7) + 195*c^2/d^7) - 386*c^3/d^7)*
(d*x + c) + 2369*c^4/d^7)*(d*x + c) - 1836*c^5/d^7)*(d*x + c) + 8
61*c^6/d^7)*(d*x + c) - 210*c^7/d^7)*(d*x + c)^(3/2)*sqrt(d*x - c
)*b)/d
```

## 3.239 $\int x^3\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$

**Optimal**. Leaf size=109

$$\frac{(dx-c)^{5/2}(c+dx)^{5/2}\left(ad^2+2bc^2\right)}{5d^6}+\frac{c^2(dx-c)^{3/2}(c+dx)^{3/2}\left(ad^2+bc^2\right)}{3d^6}+\frac{b(dx-c)^{7/2}(c+dx)^{7/2}}{7d^6}$$

```
[Out] (c^2*(b*c^2 + a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(3*d^6) +
((2*b*c^2 + a*d^2)*(-c + d*x)^(5/2)*(c + d*x)^(5/2))/(5*d^6) + (b
*(-c + d*x)^(7/2)*(c + d*x)^(7/2))/(7*d^6)
```

---

**Rubi [A]** time = 0.291181, antiderivative size = 118, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129

$$\frac{2c^2(dx-c)^{3/2}(c+dx)^{3/2}\left(7ad^2+4bc^2\right)}{105d^6}+\frac{x^2(dx-c)^{3/2}(c+dx)^{3/2}\left(7ad^2+4bc^2\right)}{35d^4}+\frac{bx^4(dx-c)^{3/2}(c+dx)^{3/2}}{7d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (2*c^2*(4*b*c^2 + 7*a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(105
*d^6) + ((4*b*c^2 + 7*a*d^2)*x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2)
)/(35*d^4) + (b*x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(7*d^2)
```

---

**Rubi in Sympy [A]** time = 17.5402, size = 105, normalized size = 0.96

$$\frac{bx^4\left(-c+dx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{3}{2}}}{7d^2}+\frac{2c^2\left(-c+dx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{3}{2}}\left(7ad^2+4bc^2\right)}{105d^6}+\frac{x^2\left(-c+dx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{3}{2}}\left(7ad^2+4bc^2\right)}{35d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] b*x**4*(-c + d*x)**(3/2)*(c + d*x)**(3/2)/(7*d**2) + 2*c**2*(-c +
 d*x)**(3/2)*(c + d*x)**(3/2)*(7*a*d**2 + 4*b*c**2)/(105*d**6) +
x**2*(-c + d*x)**(3/2)*(c + d*x)**(3/2)*(7*a*d**2 + 4*b*c**2)/(35
*d**4)
```

---

**Mathematica [A]** time = 0.0750872, size = 88, normalized size = 0.81

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(d^2x^2-c^2\right)\left(7ad^2\left(2c^2+3d^2x^2\right)+b\left(8c^4+12c^2d^2x^2+15d^4x^4\right)\right)}{105d^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-c^2 + d^2*x^2)*(7*a*d^2*(2*c^2 +
3*d^2*x^2) + b*(8*c^4 + 12*c^2*d^2*x^2 + 15*d^4*x^4)))/(105*d^6)
```

---

**Maple [A]**   time = 0.009, size = 68, normalized size = 0.6

$$\frac{15\,bd^{4}x^{4}+21\,ad^{4}x^{2}+12\,bc^{2}d^{2}x^{2}+14\,ac^{2}d^{2}+8\,bc^{4}}{105\,d^{6}}\,(dx+c)^{\frac{3}{2}}\,(dx-c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/105*(d*x+c)^(3/2)*(15*b*d^4*x^4+21*a*d^4*x^2+12*b*c^2*d^2*x^2+1
4*a*c^2*d^2+8*b*c^4)*(d*x-c)^(3/2)/d^6
```

---

**Maxima [A]**   time = 1.38088, size = 167, normalized size = 1.53

$$\frac{\left(d^{2}x^{2}-c^{2}\right)^{\frac{3}{2}}bx^{4}}{7\,d^{2}}+\frac{4\left(d^{2}x^{2}-c^{2}\right)^{\frac{3}{2}}bc^{2}x^{2}}{35\,d^{4}}+\frac{\left(d^{2}x^{2}-c^{2}\right)^{\frac{3}{2}}ax^{2}}{5\,d^{2}}+\frac{8\left(d^{2}x^{2}-c^{2}\right)^{\frac{3}{2}}bc^{4}}{105\,d^{6}}+\frac{2\left(d^{2}x^{2}-c^{2}\right)^{\frac{3}{2}}ac^{2}}{15\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x^3,x, algorithm="maxima")
```

```
[Out] 1/7*(d^2*x^2 - c^2)^(3/2)*b*x^4/d^2 + 4/35*(d^2*x^2 - c^2)^(3/2)*
b*c^2*x^2/d^4 + 1/5*(d^2*x^2 - c^2)^(3/2)*a*x^2/d^2 + 8/105*(d^2*
x^2 - c^2)^(3/2)*b*c^4/d^6 + 2/15*(d^2*x^2 - c^2)^(3/2)*a*c^2/d^4
```

---

**Fricas [A]**   time = 0.413169, size = 582, normalized size = 5.34

$$-\frac{960\,bd^{14}x^{14}-8\,bc^{14}-14\,ac^{12}d^{2}-336\left(7\,bc^{2}d^{12}-4\,ad^{14}\right)x^{12}+1736\left(bc^{4}d^{10}-2\,ac^{2}d^{12}\right)x^{10}-7\left(89\,bc^{6}d^{8}-328\,ac^{4}d^{10}\right)x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x^3,x, algorithm="fricas")
```

```
[Out] -1/105*(960*b*d^14*x^14 - 8*b*c^14 - 14*a*c^12*d^2 - 336*(7*b*c^2
*d^12 - 4*a*d^14)*x^12 + 1736*(b*c^4*d^10 - 2*a*c^2*d^12)*x^10 -
7*(89*b*c^6*d^8 - 328*a*c^4*d^10)*x^8 + 7*(118*b*c^8*d^6 + 109*a*
c^6*d^8)*x^6 - 105*(7*b*c^10*d^4 + 12*a*c^8*d^6)*x^4 + 49*(4*b*c^
12*d^2 + 7*a*c^10*d^4)*x^2 - (960*b*d^13*x^13 - 48*(39*b*c^2*d^11
 - 28*a*d^13)*x^11 + 40*(23*b*c^4*d^9 - 70*a*c^2*d^11)*x^9 - (337
*b*c^6*d^7 - 1064*a*c^4*d^9)*x^7 + 21*(33*b*c^8*d^5 + 49*a*c^6*d^
7)*x^5 - 105*(4*b*c^10*d^3 + 7*a*c^8*d^5)*x^3 + 14*(4*b*c^12*d +
7*a*c^10*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c))/(64*d^13*x^7 - 112*
c^2*d^11*x^5 + 56*c^4*d^9*x^3 - 7*c^6*d^7*x - (64*d^12*x^6 - 80*c
^2*d^10*x^4 + 24*c^4*d^8*x^2 - c^6*d^6)*sqrt(d*x + c)*sqrt(d*x -
c))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(a + bx^2\right) \sqrt{-c + dx}\sqrt{c + dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

---

**GIAC/XCAS [A]** time = 0.230614, size = 227, normalized size = 2.08

$$\frac{7\Big(\big((dx+c)\big(3(dx+c)\big(\frac{dx+c}{d^3}-\frac{4c}{d^3}\big)+\frac{17c^2}{d^3}\big)-\frac{10c^3}{d^3}\big)(dx+c)^{\frac{3}{2}}\sqrt{dx-c}\,a+\Big(\Big(3\Big((dx+c)\Big(5(dx+c)\Big(\frac{dx+c}{d^5}-\frac{6c}{d^5}\Big)+\frac{74c^2}{d^5}\Big)-\frac{96c}{d^5}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x^3,x, algorithm="giac")
```

```
[Out] 1/105*(7*((d*x + c)*(3*(d*x + c)*((d*x + c)/d^3 - 4*c/d^3) + 17*c
^2/d^3) - 10*c^3/d^3)*(d*x + c)^(3/2)*sqrt(d*x - c)*a + ((3*((d*x
 + c)*(5*(d*x + c)*((d*x + c)/d^5 - 6*c/d^5) + 74*c^2/d^5) - 96*c
^3/d^5)*(d*x + c) + 203*c^4/d^5)*(d*x + c) - 70*c^5/d^5)*(d*x + c
)^(3/2)*sqrt(d*x - c)*b)/d
```

# 3.240 $\int x\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$

**Optimal**. Leaf size=67

$$\frac{(dx-c)^{3/2}(c+dx)^{3/2}\left(ad^2+bc^2\right)}{3d^4}+\frac{b(dx-c)^{5/2}(c+dx)^{5/2}}{5d^4}$$

```
[Out] ((b*c^2 + a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(3*d^4) + (b*(
-c + d*x)^(5/2)*(c + d*x)^(5/2))/(5*d^4)
```

---

**Rubi [A]** time = 0.151357, antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{(dx-c)^{3/2}(c+dx)^{3/2}\left(5ad^2+2bc^2\right)}{15d^4}+\frac{bx^2(dx-c)^{3/2}(c+dx)^{3/2}}{5d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] ((2*b*c^2 + 5*a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(15*d^4) +
 (b*x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(5*d^2)
```

---

**Rubi in Sympy [A]** time = 9.89443, size = 61, normalized size = 0.91

$$\frac{bx^2\left(-c+dx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{3}{2}}}{5d^2}+\frac{\left(-c+dx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{3}{2}}\left(5ad^2+2bc^2\right)}{15d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] b*x**2*(-c + d*x)**(3/2)*(c + d*x)**(3/2)/(5*d**2) + (-c + d*x)**
(3/2)*(c + d*x)**(3/2)*(5*a*d**2 + 2*b*c**2)/(15*d**4)
```

---

**Mathematica [A]** time = 0.0582542, size = 62, normalized size = 0.93

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(d^2x^2-c^2\right)\left(5ad^2+2bc^2+3bd^2x^2\right)}{15d^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-c^2 + d^2*x^2)*(2*b*c^2 + 5*a*d^2
 + 3*b*d^2*x^2))/(15*d^4)
```

---

**Maple [A]** time = 0.006, size = 44, normalized size = 0.7

$$\frac{3\,bd^2x^2 + 5\,ad^2 + 2\,bc^2}{15\,d^4}\,(dx+c)^{\frac{3}{2}}\,(dx-c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/15*(d*x+c)^(3/2)*(3*b*d^2*x^2+5*a*d^2+2*b*c^2)*(d*x-c)^(3/2)/d^
4
```

---

**Maxima [A]** time = 1.37831, size = 95, normalized size = 1.42

$$\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^2}{5\,d^2} + \frac{2\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2}{15\,d^4} + \frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}a}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x,x, algorithm="maxima")
```

```
[Out] 1/5*(d^2*x^2 - c^2)^(3/2)*b*x^2/d^2 + 2/15*(d^2*x^2 - c^2)^(3/2)*
b*c^2/d^4 + 1/3*(d^2*x^2 - c^2)^(3/2)*a/d^2
```

---

**Fricas [A]** time = 0.279759, size = 421, normalized size = 6.28

$$-\frac{48\,bd^{10}x^{10} + 2\,bc^{10} + 5\,ac^8d^2 - 20\left(5\,bc^2d^8 - 4\,ad^{10}\right)x^8 + 5\left(7\,bc^4d^6 - 44\,ac^2d^8\right)x^6 + 5\left(8\,bc^6d^4 + 41\,ac^4d^6\right)x^4 - 5\left(5\,bc^8d^2\right.}{15\left(16\,d^9x^5 - 20\,c^2d^7x^3 + 5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x,x, algorithm="fricas")
```

```
[Out] -1/15*(48*b*d^10*x^10 + 2*b*c^10 + 5*a*c^8*d^2 - 20*(5*b*c^2*d^8
- 4*a*d^10)*x^8 + 5*(7*b*c^4*d^6 - 44*a*c^2*d^8)*x^6 + 5*(8*b*c^6
```

```
*d^4 + 41*a*c^4*d^6)*x^4 - 5*(5*b*c^8*d^2 + 14*a*c^6*d^4)*x^2 - (
48*b*d^9*x^9 - 4*(19*b*c^2*d^7 - 20*a*d^9)*x^7 + 3*(b*c^4*d^5 - 6
0*a*c^2*d^7)*x^5 + 5*(7*b*c^6*d^3 + 25*a*c^4*d^5)*x^3 - 5*(2*b*c^
8*d + 5*a*c^6*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c))/(16*d^9*x^5 -
20*c^2*d^7*x^3 + 5*c^4*d^5*x - (16*d^8*x^4 - 12*c^2*d^6*x^2 + c^4
*d^4)*sqrt(d*x + c)*sqrt(d*x - c))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x\left(a+bx^2\right)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

---

**GIAC/XCAS [A]** time = 0.217173, size = 126, normalized size = 1.88

$$\frac{\left((dx+c)\left(3\,(dx+c)\left(\frac{dx+c}{d^3}-\frac{4\,c}{d^3}\right)+\frac{17\,c^2}{d^3}\right)-\frac{10\,c^3}{d^3}\right)(dx+c)^{\frac{3}{2}}\sqrt{dx-c}\,b+\frac{5\,(dx+c)^{\frac{3}{2}}(dx-c)^{\frac{3}{2}}\,a}{d}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x,x, algorithm="giac")
```

```
[Out] 1/15*((((d*x + c)*(3*(d*x + c)*((d*x + c)/d^3 - 4*c/d^3) + 17*c^2/
d^3) - 10*c^3/d^3)*(d*x + c)^(3/2)*sqrt(d*x - c)*b + 5*(d*x + c)^
(3/2)*(d*x - c)^(3/2)*a/d)/d
```

## 3.241 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x}\,dx$

**Optimal**. Leaf size=80

$$a\sqrt{dx-c}\sqrt{c+dx}-ac\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)+\frac{b(dx-c)^{3/2}(c+dx)^{3/2}}{3d^2}$$

```
[Out] a*Sqrt[-c + d*x]*Sqrt[c + d*x] + (b*(-c + d*x)^(3/2)*(c + d*x)^(3
/2))/(3*d^2) - a*c*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c]
```

---

**Rubi [A]** time = 0.286411, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161

$$a\sqrt{dx-c}\sqrt{c+dx}-ac\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)+\frac{b(dx-c)^{3/2}(c+dx)^{3/2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x,x]
```

```
[Out] a*Sqrt[-c + d*x]*Sqrt[c + d*x] + (b*(-c + d*x)^(3/2)*(c + d*x)^(3
/2))/(3*d^2) - a*c*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c]
```

---

**Rubi in Sympy [A]** time = 17.3838, size = 65, normalized size = 0.81

$$-ac\,\mathrm{atan}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)+a\sqrt{-c+dx}\sqrt{c+dx}+\frac{b\left(-c+dx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{3}{2}}}{3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x,x)
```

```
[Out] -a*c*atan(sqrt(-c + d*x)*sqrt(c + d*x)/c) + a*sqrt(-c + d*x)*sqrt
(c + d*x) + b*(-c + d*x)**(3/2)*(c + d*x)**(3/2)/(3*d**2)
```

---

**Mathematica [A]** time = 0.142888, size = 75, normalized size = 0.94

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(3ad^2-bc^2+bd^2x^2\right)}{3d^2}+ac\tan^{-1}\left(\frac{c}{\sqrt{dx-c}\sqrt{c+dx}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x,x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-(b*c^2) + 3*a*d^2 + b*d^2*x^2))/(
3*d^2) + a*c*ArcTan[c/(Sqrt[-c + d*x]*Sqrt[c + d*x])]
```

---

**Maple [B]** time = 0.02, size = 174, normalized size = 2.2

$$\frac{1}{3\,d^2}\sqrt{dx-c}\sqrt{dx+c}\left(x^2bd^2\sqrt{-c^2}\sqrt{d^2x^2-c^2}+3\,ac^2\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)d^2+3\,a\sqrt{d^2x^2-c^2}d^2\sqrt{-c^2}-bc^2\sqrt{-c^2}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x)
```

```
[Out] 1/3*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(x^2*b*d^2*(-c^2)^(1/2)*(d^2*x^2-
c^2)^(1/2)+3*a*c^2*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x
)*d^2+3*a*(d^2*x^2-c^2)^(1/2)*d^2*(-c^2)^(1/2)-b*c^2*(-c^2)^(1/2)
*(d^2*x^2-c^2)^(1/2))/(d^2*x^2-c^2)^(1/2)/d^2/(-c^2)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.250421, size = 383, normalized size = 4.79

$$-\frac{4\,bd^6x^6-bc^6+3\,ac^4d^2-3\left(3\,bc^2d^4-4\,ad^6\right)x^4+3\left(2\,bc^4d^2-5\,ac^2d^4\right)x^2-\left(4\,bd^5x^5-\left(7\,bc^2d^3-12\,ad^5\right)x^3+3\left(bc^4d-\right.\right.}{3\left(4\,d^5x^3-3\,c^2d^3x-(4\,d^4x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x,x, algorithm="fricas")
```

```
[Out] -1/3*(4*b*d^6*x^6 - b*c^6 + 3*a*c^4*d^2 - 3*(3*b*c^2*d^4 - 4*a*d^
6)*x^4 + 3*(2*b*c^4*d^2 - 5*a*c^2*d^4)*x^2 - (4*b*d^5*x^5 - (7*b*
c^2*d^3 - 12*a*d^5)*x^3 + 3*(b*c^4*d - 3*a*c^2*d^3)*x)*sqrt(d*x +
 c)*sqrt(d*x - c) + 6*(4*a*c*d^5*x^3 - 3*a*c^3*d^3*x - (4*a*c*d^4
*x^2 - a*c^3*d^2)*sqrt(d*x + c)*sqrt(d*x - c))*arctan(-(d*x - sqr
t(d*x + c)*sqrt(d*x - c))/c))/(4*d^5*x^3 - 3*c^2*d^3*x - (4*d^4*x
^2 - c^2*d^2)*sqrt(d*x + c)*sqrt(d*x - c))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)\sqrt{-c+dx}\sqrt{c+dx}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x,x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x, x)
```

---

**GIAC/XCAS [A]** time = 0.224712, size = 109, normalized size = 1.36

$$2\,ac\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)+\frac{1}{1920}\left(3\,ad^6+\left((dx+c)bd^4-2\,bcd^4\right)(dx+c)\right)\sqrt{dx+c}\sqrt{dx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x,x, algorithm="giac")
```

```
[Out] 2*a*c*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c) + 1/1920*(3
*a*d^6 + ((d*x + c)*b*d^4 - 2*b*c*d^4)*(d*x + c))*sqrt(d*x + c)*s
qrt(d*x - c)
```

## 3.242 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3}\,dx$

**Optimal**. Leaf size=96

$$-\frac{(2bc^2-ad^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c}-\frac{a\sqrt{dx-c}\sqrt{c+dx}}{2x^2}+b\sqrt{dx-c}\sqrt{c+dx}$$

```
[Out] b*Sqrt[-c + d*x]*Sqrt[c + d*x] - (a*Sqrt[-c + d*x]*Sqrt[c + d*x])
/(2*x^2) - ((2*b*c^2 - a*d^2)*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x
])/c])/(2*c)
```

---

**Rubi [A]** time = 0.319887, antiderivative size = 114, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$

$$\frac{1}{2}\sqrt{dx-c}\sqrt{c+dx}\left(2b-\frac{ad^2}{c^2}\right)-\frac{(2bc^2-ad^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^3,x]
```

```
[Out] ((2*b - (a*d^2)/c^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/2 + (a*(-c + d
*x)^(3/2)*(c + d*x)^(3/2))/(2*c^2*x^2) - ((2*b*c^2 - a*d^2)*ArcTa
n[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c])/(2*c)
```

---

**Rubi in Sympy [A]** time = 18.789, size = 95, normalized size = 0.99

$$\frac{a(-c+dx)^{\frac{3}{2}}(c+dx)^{\frac{3}{2}}}{2c^2x^2}+\frac{(ad^2-2bc^2)\,\mathrm{atan}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{2c}-\frac{\sqrt{-c+dx}\sqrt{c+dx}\,(ad^2-2bc^2)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**3,x)
```

```
[Out] a*(-c + d*x)**(3/2)*(c + d*x)**(3/2)/(2*c**2*x**2) + (a*d**2 - 2*
b*c**2)*atan(sqrt(-c + d*x)*sqrt(c + d*x)/c)/(2*c) - sqrt(-c + d*
x)*sqrt(c + d*x)*(a*d**2 - 2*b*c**2)/(2*c**2)
```

---

**Mathematica [C]** time = 0.19657, size = 105, normalized size = 1.09

$$\frac{1}{2}\left(\frac{(2bx^2-a)\sqrt{dx-c}\sqrt{c+dx}}{x^2}+\left(2ibc-\frac{iad^2}{c}\right)\log\left(\frac{-4\sqrt{dx-c}\sqrt{c+dx}+4ic}{2bc^2x-ad^2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^3,x]
```

```
[Out] ((Sqrt[-c + d*x]*Sqrt[c + d*x]*(-a + 2*b*x^2))/x^2 + ((2*I)*b*c -
 (I*a*d^2)/c)*Log[((4*I)*c - 4*Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*b
*c^2*x - a*d^2*x)])/2
```

---

**Maple [B]** time = 0.02, size = 182, normalized size = 1.9

$$-\frac{1}{2x^2}\sqrt{dx-c}\sqrt{dx+c}\left(\ln\left(-2\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^2ad^2-2\ln\left(-2\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^2bc^2-2x^2b\sqrt{-c^2}\sqrt{d^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x)
```

```
[Out] -1/2*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^
2-c^2)^(1/2))/x)*x^2*a*d^2-2*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2
)^(1/2))/x)*x^2*b*c^2-2*x^2*b*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)+a*
(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2))/(d^2*x^2-c^2)^(1/2)/x^2/(-c^2)^
(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.25108, size = 419, normalized size = 4.36

$$-\frac{8bcd^4x^6-ac^5-2(5bc^3d^2+2acd^4)x^4+(2bc^5+5ac^3d^2)x^2-(8bcd^3x^5+3ac^3dx-2(3bc^3d+2acd^3)x^3)\sqrt{dx+c}\sqrt{dx}}{2(4cd^3x^5-3c^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(8*b*c*d^4*x^6 - a*c^5 - 2*(5*b*c^3*d^2 + 2*a*c*d^4)*x^4 + (
2*b*c^5 + 5*a*c^3*d^2)*x^2 - (8*b*c*d^3*x^5 + 3*a*c^3*d*x - 2*(3*
b*c^3*d + 2*a*c*d^3)*x^3)*sqrt(d*x + c)*sqrt(d*x - c) + 2*(4*(2*b
*c^2*d^3 - a*d^5)*x^5 - 3*(2*b*c^4*d - a*c^2*d^3)*x^3 - (4*(2*b*c
^2*d^2 - a*d^4)*x^4 - (2*b*c^4 - a*c^2*d^2)*x^2)*sqrt(d*x + c)*sq
rt(d*x - c))*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c))/(4*c
*d^3*x^5 - 3*c^3*d*x^3 - (4*c*d^2*x^4 - c^3*x^2)*sqrt(d*x + c)*sq
rt(d*x - c))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)\sqrt{-c + dx}\sqrt{c + dx}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**3,x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x**3, x)
```

---

**GIAC/XCAS [A]** time = 0.239757, size = 212, normalized size = 2.21

$$\frac{\sqrt{dx + c}\sqrt{dx - c}bd + \frac{(2\,bc^2d - ad^3)\arctan\left(\frac{\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^2}{2\,c}\right)}{c} + \frac{2\left(ad^3\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^6 - 4\,ac^2d^3\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^2\right)}{\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^4 + 4\,c^2\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x^3,x, algorithm="giac")
```

```
[Out] (sqrt(d*x + c)*sqrt(d*x - c)*b*d + (2*b*c^2*d - a*d^3)*arctan(1/2
*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c + 2*(a*d^3*(sqrt(d*x + c)
 - sqrt(d*x - c))^6 - 4*a*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))
^2)/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^2)/d
```

## 3.243 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5} dx$

**Optimal**. Leaf size=121

$$-\frac{\sqrt{dx-c}\sqrt{c+dx}\left(ad^2+4bc^2\right)}{8c^2x^2}+\frac{d^2\left(ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^3}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{4c^2x^4}$$

```
[Out] -((4*b*c^2 + a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*c^2*x^2) + (
a*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(4*c^2*x^4) + (d^2*(4*b*c^2 +
 a*d^2)*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c])/(8*c^3)
```

---

**Rubi [A]** time = 0.398807, antiderivative size = 164, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129

$$-\frac{\sqrt{dx-c}(c+dx)^{3/2}\left(ad^2+4bc^2\right)}{8c^3x^2}+\frac{d\sqrt{dx-c}\sqrt{c+dx}\left(ad^2+4bc^2\right)}{8c^3x}+\frac{d^2\left(ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^3}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{4c^2x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^5,x]
```

```
[Out] (d*(4*b*c^2 + a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*c^3*x) - ((
4*b*c^2 + a*d^2)*Sqrt[-c + d*x]*(c + d*x)^(3/2))/(8*c^3*x^2) + (a
*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(4*c^2*x^4) + (d^2*(4*b*c^2 +
a*d^2)*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c])/(8*c^3)
```

---

**Rubi in Sympy [A]** time = 22.6634, size = 141, normalized size = 1.17

$$\frac{a\left(-c+dx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{3}{2}}}{4c^2x^4}+\frac{d^2\left(ad^2+4bc^2\right)\operatorname{atan}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{8c^3}+\frac{d\sqrt{-c+dx}\sqrt{c+dx}\left(ad^2+4bc^2\right)}{8c^3x}-\frac{\sqrt{-c+dx}\left(c+dx\right)^{\frac{3}{2}}\left(ad^2+4bc^2\right)}{8c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**5,x)
```

```
[Out] a*(-c + d*x)**(3/2)*(c + d*x)**(3/2)/(4*c**2*x**4) + d**2*(a*d**2
 + 4*b*c**2)*atan(sqrt(-c + d*x)*sqrt(c + d*x)/c)/(8*c**3) + d*sq
```

```
rt(-c + d*x)*sqrt(c + d*x)*(a*d**2 + 4*b*c**2)/(8*c**3*x) - sqrt(
-c + d*x)*(c + d*x)**(3/2)*(a*d**2 + 4*b*c**2)/(8*c**3*x**2)
```

---

**Mathematica [C]** time = 0.179346, size = 132, normalized size = 1.09

$$\frac{c\sqrt{dx-c}\sqrt{c+dx}\left(-2ac^2+ad^2x^2-4bc^2x^2\right)-id^2x^4\left(ad^2+4bc^2\right)\log\left(\frac{16c^2\left(\sqrt{dx-c}\sqrt{c+dx}-ic\right)}{d^2x(ad^2+4bc^2)}\right)}{8c^3x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^5,x]
```

```
[Out] (c*Sqrt[-c + d*x]*Sqrt[c + d*x]*(-2*a*c^2 - 4*b*c^2*x^2 + a*d^2*x
^2) - I*d^2*(4*b*c^2 + a*d^2)*x^4*Log[(16*c^2*((-I)*c + Sqrt[-c +
 d*x]*Sqrt[c + d*x]))/(d^2*(4*b*c^2 + a*d^2)*x)])/(8*c^3*x^4)
```

---

**Maple [B]** time = 0.023, size = 226, normalized size = 1.9

$$-\frac{1}{8\,c^2x^4}\sqrt{dx-c}\sqrt{dx+c}\left(\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^4ad^4+4\,\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^4bc^2d^2-ad^2\sqrt{d^2x^2-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x)
```

```
[Out] -1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2*(ln(-2*(c^2-(-c^2)^(1/2)*(d^
2*x^2-c^2)^(1/2))/x)*x^4*a*d^4+4*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2
-c^2)^(1/2))/x)*x^4*b*c^2*d^2-a*d^2*(d^2*x^2-c^2)^(1/2)*x^2*(-c^2
)^(1/2)+4*b*(d^2*x^2-c^2)^(1/2)*c^2*x^2*(-c^2)^(1/2)+2*a*(d^2*x^2
-c^2)^(1/2)*c^2*(-c^2)^(1/2))/(d^2*x^2-c^2)^(1/2)/x^4/(-c^2)^(1/2
)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.263067, size = 539, normalized size = 4.45

$$\frac{8\,ac^7dx + 8\left(4\,bc^3d^5 - acd^7\right)x^7 - 4\left(12\,bc^5d^3 - 7\,ac^3d^5\right)x^5 + 4\left(4\,bc^7d - 7\,ac^5d^3\right)x^3 - \left(2\,ac^7 + 8\left(4\,bc^3d^4 - acd^6\right)x^6 - 8\,(4\,b\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a*c^7*d*x + 8*(4*b*c^3*d^5 - a*c*d^7)*x^7 - 4*(12*b*c^5*d^
3 - 7*a*c^3*d^5)*x^5 + 4*(4*b*c^7*d - 7*a*c^5*d^3)*x^3 - (2*a*c^7
 + 8*(4*b*c^3*d^4 - a*c*d^6)*x^6 - 8*(4*b*c^5*d^2 - 3*a*c^3*d^4)*
x^4 + (4*b*c^7 - 17*a*c^5*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c) +
 2*(8*(4*b*c^2*d^6 + a*d^8)*x^8 - 8*(4*b*c^4*d^4 + a*c^2*d^6)*x^6
 + (4*b*c^6*d^2 + a*c^4*d^4)*x^4 - 4*(2*(4*b*c^2*d^5 + a*d^7)*x^7
 - (4*b*c^4*d^3 + a*c^2*d^5)*x^5)*sqrt(d*x + c)*sqrt(d*x - c))*ar
ctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c))/(8*c^3*d^4*x^8 - 8*
c^5*d^2*x^6 + c^7*x^4 - 4*(2*c^3*d^3*x^7 - c^5*d*x^5)*sqrt(d*x +
c)*sqrt(d*x - c))
```

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: MellinTransformStripError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**5,x)
```

```
[Out] Exception raised: MellinTransformStripError
```

---

**GIAC/XCAS [A]** time = 0.244135, size = 437, normalized size = 3.61

$$-\frac{\frac{\left(4\,bc^2d^3+ad^5\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^3} - \frac{2\left(4\,bc^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14} - ad^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14} + 16\,bc^4d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10} + 28\,ac^2d^5\left(\sqrt{dx+c}-\sqrt{d\cdots}\right.\right.}{\left(\left(\sqrt{d\cdots}\right.\right.}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*((4*b*c^2*d^3 + a*d^5)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x
- c))^2/c)/c^3 - 2*(4*b*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^1
4 - a*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^14 + 16*b*c^4*d^3*(sqrt
(d*x + c) - sqrt(d*x - c))^10 + 28*a*c^2*d^5*(sqrt(d*x + c) - sqr
t(d*x - c))^10 - 64*b*c^6*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^6 -
 112*a*c^4*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^6 - 256*b*c^8*d^3*
(sqrt(d*x + c) - sqrt(d*x - c))^2 + 64*a*c^6*d^5*(sqrt(d*x + c) -
 sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^4
*c^2))/d
```

## 3.244 $\int x^4\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$

**Optimal**. Leaf size=208

$$\frac{c^2x(dx-c)^{3/2}(c+dx)^{3/2}\left(8ad^2+5bc^2\right)}{64d^6}+\frac{x^3(dx-c)^{3/2}(c+dx)^{3/2}\left(8ad^2+5bc^2\right)}{48d^4}-\frac{c^6\left(8ad^2+5bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{64d^7}+\frac{c^4x\sqrt{dx-c}\sqrt{c+dx}\left(8ad^2+5bc^2\right)}{128d^6}+\frac{bx^5(dx-c)^{3/2}(c+dx)^{3/2}}{8d^2}$$

```
[Out] (c^4*(5*b*c^2 + 8*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(128*d^6
) + (c^2*(5*b*c^2 + 8*a*d^2)*x*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/
(64*d^6) + ((5*b*c^2 + 8*a*d^2)*x^3*(-c + d*x)^(3/2)*(c + d*x)^(3
/2))/(48*d^4) + (b*x^5*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(8*d^2)
- (c^6*(5*b*c^2 + 8*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])
/(64*d^7)
```

---

**Rubi [A]** time = 0.476242, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226

$$\frac{c^2x(dx-c)^{3/2}(c+dx)^{3/2}\left(8ad^2+5bc^2\right)}{64d^6}+\frac{x^3(dx-c)^{3/2}(c+dx)^{3/2}\left(8ad^2+5bc^2\right)}{48d^4}-\frac{c^6\left(8ad^2+5bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{64d^7}+\frac{c^4x\sqrt{dx-c}\sqrt{c+dx}\left(8ad^2+5bc^2\right)}{128d^6}+\frac{bx^5(dx-c)^{3/2}(c+dx)^{3/2}}{8d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (c^4*(5*b*c^2 + 8*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(128*d^6
) + (c^2*(5*b*c^2 + 8*a*d^2)*x*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/
(64*d^6) + ((5*b*c^2 + 8*a*d^2)*x^3*(-c + d*x)^(3/2)*(c + d*x)^(3
/2))/(48*d^4) + (b*x^5*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(8*d^2)
- (c^6*(5*b*c^2 + 8*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])
/(64*d^7)
```

---

**Rubi in Sympy [A]** time = 34.3333, size = 187, normalized size = 0.9

$$\frac{bx^5\left(-c+dx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{3}{2}}}{8d^2}-\frac{c^6\left(8ad^2+5bc^2\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{64d^7}+\frac{c^4x\sqrt{-c+dx}\sqrt{c+dx}\left(8ad^2+5bc^2\right)}{128d^6}+\frac{c^2x\left(-c+dx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{3}{2}}\left(8ad^2+5bc^2\right)}{64d^6}+\frac{x^3\left(-c+dx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{3}{2}}\left(8ad^2+5bc^2\right)}{48d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] b*x**5*(-c + d*x)**(3/2)*(c + d*x)**(3/2)/(8*d**2) - c**6*(8*a*d*
*2 + 5*b*c**2)*atanh(sqrt(c + d*x)/sqrt(-c + d*x))/(64*d**7) + c*
*4*x*sqrt(-c + d*x)*sqrt(c + d*x)*(8*a*d**2 + 5*b*c**2)/(128*d**6
) + c**2*x*(-c + d*x)**(3/2)*(c + d*x)**(3/2)*(8*a*d**2 + 5*b*c**
2)/(64*d**6) + x**3*(-c + d*x)**(3/2)*(c + d*x)**(3/2)*(8*a*d**2
+ 5*b*c**2)/(48*d**4)
```

---

**Mathematica [A]** time = 0.177939, size = 146, normalized size = 0.7

$$\frac{dx\sqrt{dx-c}\sqrt{c+dx}\left(8ad^2\left(-3c^4-2c^2d^2x^2+8d^4x^4\right)-b\left(15c^6+10c^4d^2x^2+8c^2d^4x^4-48d^6x^6\right)\right)-3\left(8ac^6d^2+5bc^8\right)\log\left(\sqrt{\right.}$$
$$384d^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(8*a*d^2*(-3*c^4 - 2*c^2*d^2*x^
2 + 8*d^4*x^4) - b*(15*c^6 + 10*c^4*d^2*x^2 + 8*c^2*d^4*x^4 - 48*
d^6*x^6)) - 3*(5*b*c^8 + 8*a*c^6*d^2)*Log[d*x + Sqrt[-c + d*x]*Sq
rt[c + d*x]])/(384*d^7)
```

---

**Maple [C]** time = 0.032, size = 298, normalized size = 1.4

$$\frac{csgn(d)}{384\,d^7}\sqrt{dx-c}\sqrt{dx+c}\left(48\,csgn(d)\,x^7bd^7\sqrt{d^2x^2-c^2}+64\,csgn(d)\,x^5ad^7\sqrt{d^2x^2-c^2}-8\,csgn(d)\,x^5bc^2d^5\sqrt{d^2x^2-c^2}-16\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/384*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(48*csgn(d)*x^7*b*d^7*(d^2*x^2-
c^2)^(1/2)+64*csgn(d)*x^5*a*d^7*(d^2*x^2-c^2)^(1/2)-8*csgn(d)*x^5
*b*c^2*d^5*(d^2*x^2-c^2)^(1/2)-16*csgn(d)*x^3*a*c^2*d^5*(d^2*x^2-
c^2)^(1/2)-10*csgn(d)*x^3*b*c^4*d^3*(d^2*x^2-c^2)^(1/2)-24*a*c^4*
x*(d^2*x^2-c^2)^(1/2)*d^3*csgn(d)-15*b*c^6*x*(d^2*x^2-c^2)^(1/2)*
csgn(d)*d-24*a*c^6*ln((csgn(d)*(d^2*x^2-c^2)^(1/2)+d*x)*csgn(d))*
d^2-15*b*c^8*ln((csgn(d)*(d^2*x^2-c^2)^(1/2)+d*x)*csgn(d)))*csgn(
d)/(d^2*x^2-c^2)^(1/2)/d^7
```

---

**Maxima [A]** time = 1.41082, size = 356, normalized size = 1.71

$$\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^5}{8\,d^2}+\frac{5\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2x^3}{48\,d^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ax^3}{6\,d^2}-\frac{5\,bc^8\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{128\,\sqrt{d^2}d^6}-\frac{ac^6\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{16\,\sqrt{d^2}d^4}+\frac{5\,\sqrt{d^2x^2-c^2}bc^6x}{128\,d^6}+\frac{\sqrt{d^2x^2-c^2}ac^4x}{16\,d^4}+\frac{5\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^4x}{64\,d^6}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ac^2x}{8\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x^4,x, algorithm="maxima")
```

```
[Out] 1/8*(d^2*x^2 - c^2)^(3/2)*b*x^5/d^2 + 5/48*(d^2*x^2 - c^2)^(3/2)*
b*c^2*x^3/d^4 + 1/6*(d^2*x^2 - c^2)^(3/2)*a*x^3/d^2 - 5/128*b*c^8
*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^6) -
 1/16*a*c^6*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/(sqrt(
d^2)*d^4) + 5/128*sqrt(d^2*x^2 - c^2)*b*c^6*x/d^6 + 1/16*sqrt(d^2
*x^2 - c^2)*a*c^4*x/d^4 + 5/64*(d^2*x^2 - c^2)^(3/2)*b*c^4*x/d^6
+ 1/8*(d^2*x^2 - c^2)^(3/2)*a*c^2*x/d^4
```

---

**Fricas [A]** time = 0.651568, size = 976, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x^4,x, algorithm="fricas")
```

```
[Out] -1/384*(6144*b*d^16*x^16 - 8192*(2*b*c^2*d^14 - a*d^16)*x^14 + 20
48*(7*b*c^4*d^12 - 11*a*c^2*d^14)*x^12 - 1024*(5*b*c^6*d^10 - 19*
a*c^4*d^12)*x^10 + 288*(11*b*c^8*d^8 - 8*a*c^6*d^10)*x^8 - 192*(1
7*b*c^10*d^6 + 24*a*c^8*d^8)*x^6 + 248*(5*b*c^12*d^4 + 8*a*c^10*d
^6)*x^4 - 24*(5*b*c^14*d^2 + 8*a*c^12*d^4)*x^2 - (6144*b*d^15*x^1
5 - 1024*(13*b*c^2*d^13 - 8*a*d^15)*x^13 + 768*(11*b*c^4*d^11 - 2
4*a*c^2*d^13)*x^11 - 128*(17*b*c^6*d^9 - 88*a*c^4*d^11)*x^9 + 48*
(53*b*c^8*d^7 + 32*a*c^6*d^9)*x^7 - 24*(87*b*c^10*d^5 + 136*a*c^8
*d^7)*x^5 + 94*(5*b*c^12*d^3 + 8*a*c^10*d^5)*x^3 - 3*(5*b*c^14*d
+ 8*a*c^12*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) - 3*(5*b*c^16 + 8*
a*c^14*d^2 + 128*(5*b*c^8*d^8 + 8*a*c^6*d^10)*x^8 - 256*(5*b*c^10
*d^6 + 8*a*c^8*d^8)*x^6 + 160*(5*b*c^12*d^4 + 8*a*c^10*d^6)*x^4 -
 32*(5*b*c^14*d^2 + 8*a*c^12*d^4)*x^2 - 8*(16*(5*b*c^8*d^7 + 8*a*
c^6*d^9)*x^7 - 24*(5*b*c^10*d^5 + 8*a*c^8*d^7)*x^5 + 10*(5*b*c^12
*d^3 + 8*a*c^10*d^5)*x^3 - (5*b*c^14*d + 8*a*c^12*d^3)*x)*sqrt(d*
x + c)*sqrt(d*x - c))*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/(1
28*d^15*x^8 - 256*c^2*d^13*x^6 + 160*c^4*d^11*x^4 - 32*c^6*d^9*x^
2 + c^8*d^7 - 8*(16*d^14*x^7 - 24*c^2*d^12*x^5 + 10*c^4*d^10*x^3
```

```
- c^6*d^8*x)*sqrt(d*x + c)*sqrt(d*x - c))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 \left(a + bx^2\right) \sqrt{-c + dx}\sqrt{c + dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

---

**GIAC/XCAS [A]** time = 0.306101, size = 394, normalized size = 1.89

$$8\left(\frac{6\,c^6\ln\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^4} + \left(\left(2\left((dx+c)\left(4\,(dx+c)\left(\frac{dx+c}{d^4} - \frac{5\,c}{d^4}\right) + \frac{39\,c^2}{d^4}\right) - \frac{37\,c^3}{d^4}\right)(dx+c) + \frac{31\,c^4}{d^4}\right)(dx+c) - \frac{3\,c^5}{d^4}\right)\sqrt{dx+c}\sqrt{d\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x^4,x, algorithm="giac")
```

```
[Out] 1/384*(8*(6*c^6*ln(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^4 + ((2
*((d*x + c)*(4*(d*x + c)*((d*x + c)/d^4 - 5*c/d^4) + 39*c^2/d^4)
- 37*c^3/d^4)*(d*x + c) + 31*c^4/d^4)*(d*x + c) - 3*c^5/d^4)*sqrt
(d*x + c)*sqrt(d*x - c))*a + (30*c^8*ln(abs(-sqrt(d*x + c) + sqrt
(d*x - c)))/d^6 + ((2*((4*((d*x + c)*(6*(d*x + c)*((d*x + c)/d^6
- 7*c/d^6) + 125*c^2/d^6) - 205*c^3/d^6)*(d*x + c) + 795*c^4/d^6)
*(d*x + c) - 449*c^5/d^6)*(d*x + c) + 251*c^6/d^6)*(d*x + c) - 15
*c^7/d^6)*sqrt(d*x + c)*sqrt(d*x - c))*b)/d
```

## 3.245 $\int x^2\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$

**Optimal**. Leaf size=159

$$\frac{c^2x\sqrt{dx-c}\sqrt{c+dx}\left(2ad^2+bc^2\right)}{16d^4}+\frac{x(dx-c)^{3/2}(c+dx)^{3/2}\left(2ad^2+bc^2\right)}{8d^4}-\frac{c^4\left(2ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^5}+\frac{bx^3(dx-c)^{3/2}(c+dx)^{3/2}}{6d^2}$$

```
[Out] (c^2*(b*c^2 + 2*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(16*d^4) +
 ((b*c^2 + 2*a*d^2)*x*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(8*d^4) +
 (b*x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(6*d^2) - (c^4*(b*c^2 +
 2*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(8*d^5)
```

---

**Rubi [A]** time = 0.377282, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194

$$\frac{c^2x\sqrt{dx-c}\sqrt{c+dx}\left(2ad^2+bc^2\right)}{16d^4}+\frac{x(dx-c)^{3/2}(c+dx)^{3/2}\left(2ad^2+bc^2\right)}{8d^4}-\frac{c^4\left(2ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^5}+\frac{bx^3(dx-c)^{3/2}(c+dx)^{3/2}}{6d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (c^2*(b*c^2 + 2*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(16*d^4) +
 ((b*c^2 + 2*a*d^2)*x*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(8*d^4) +
 (b*x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(6*d^2) - (c^4*(b*c^2 +
 2*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(8*d^5)
```

---

**Rubi in Sympy [A]** time = 23.9385, size = 139, normalized size = 0.87

$$\frac{bx^3\left(-c+dx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{3}{2}}}{6d^2}-\frac{c^4\left(2ad^2+bc^2\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{8d^5}+\frac{c^2x\sqrt{-c+dx}\sqrt{c+dx}\left(2ad^2+bc^2\right)}{16d^4}+\frac{x\left(-c+dx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{3}{2}}\left(2ad^2+bc^2\right)}{8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] b*x**3*(-c + d*x)**(3/2)*(c + d*x)**(3/2)/(6*d**2) - c**4*(2*a*d*
*2 + b*c**2)*atanh(sqrt(c + d*x)/sqrt(-c + d*x))/(8*d**5) + c**2*
x*sqrt(-c + d*x)*sqrt(c + d*x)*(2*a*d**2 + b*c**2)/(16*d**4) + x*
(-c + d*x)**(3/2)*(c + d*x)**(3/2)*(2*a*d**2 + b*c**2)/(8*d**4)
```

---

**Mathematica [A]** time = 0.122253, size = 120, normalized size = 0.75

$$\frac{dx\sqrt{dx-c}\sqrt{c+dx}\left(b\left(-3c^4-2c^2d^2x^2+8d^4x^4\right)-6ad^2\left(c^2-2d^2x^2\right)\right)-3\left(2ac^4d^2+bc^6\right)\log\left(\sqrt{dx-c}\sqrt{c+dx}+dx\right)}{48d^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(-6*a*d^2*(c^2 - 2*d^2*x^2) + b
*(-3*c^4 - 2*c^2*d^2*x^2 + 8*d^4*x^4)) - 3*(b*c^6 + 2*a*c^4*d^2)*
Log[d*x + Sqrt[-c + d*x]*Sqrt[c + d*x]])/(48*d^5)
```

---

**Maple [C]** time = 0.018, size = 240, normalized size = 1.5

$$\frac{csgn(d)}{48\,d^5}\sqrt{dx-c}\sqrt{dx+c}\left(8\,csgn(d)\,x^5bd^5\sqrt{d^2x^2-c^2}+12\,csgn(d)\,x^3ad^5\sqrt{d^2x^2-c^2}-2\,csgn(d)\,x^3bc^2d^3\sqrt{d^2x^2-c^2}-6\,ac^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/48*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(8*csgn(d)*x^5*b*d^5*(d^2*x^2-c^
2)^(1/2)+12*csgn(d)*x^3*a*d^5*(d^2*x^2-c^2)^(1/2)-2*csgn(d)*x^3*b
*c^2*d^3*(d^2*x^2-c^2)^(1/2)-6*a*c^2*x*(d^2*x^2-c^2)^(1/2)*d^3*cs
gn(d)-3*b*c^4*x*(d^2*x^2-c^2)^(1/2)*csgn(d)*d-6*a*c^4*ln((csgn(d)
*(d^2*x^2-c^2)^(1/2)+d*x)*csgn(d))*d^2-3*b*c^6*ln((csgn(d)*(d^2*x
^2-c^2)^(1/2)+d*x)*csgn(d)))*csgn(d)/(d^2*x^2-c^2)^(1/2)/d^5
```

---

**Maxima [A]** time = 1.3891, size = 284, normalized size = 1.79

$$\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^3}{6\,d^2}-\frac{bc^6\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{16\,\sqrt{d^2}d^4}-\frac{ac^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{8\,\sqrt{d^2}d^2}+\frac{\sqrt{d^2x^2-c^2}bc^4x}{16\,d^4}+\frac{\sqrt{d^2x^2-c^2}ac^2x}{8\,d^2}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2x}{8\,d^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ax}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x^2,x, algorithm="maxima")
```

```
[Out] 1/6*(d^2*x^2 - c^2)^(3/2)*b*x^3/d^2 - 1/16*b*c^6*log(2*d^2*x + 2*
sqrt(d^2*x^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^4) - 1/8*a*c^4*log(2*
d^2*x + 2*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^2) + 1/16*s
qrt(d^2*x^2 - c^2)*b*c^4*x/d^4 + 1/8*sqrt(d^2*x^2 - c^2)*a*c^2*x/
d^2 + 1/8*(d^2*x^2 - c^2)^(3/2)*b*c^2*x/d^4 + 1/4*(d^2*x^2 - c^2)
^(3/2)*a*x/d^2
```

---

**Fricas [A]** time = 0.345833, size = 741, normalized size = 4.66

$$-\frac{256\,bd^{12}x^{12} - 192\left(3\,bc^{2}d^{10} - 2\,ad^{12}\right)x^{10} + 48\left(7\,bc^{4}d^{8} - 20\,ac^{2}d^{10}\right)x^{8} + 4\left(17\,bc^{6}d^{6} + 210\,ac^{4}d^{8}\right)x^{6} - 6\left(17\,bc^{8}d^{4} + 50\,ac^{6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x^2,x, algorithm="fricas")
```

```
[Out] -1/48*(256*b*d^12*x^12 - 192*(3*b*c^2*d^10 - 2*a*d^12)*x^10 + 48*
(7*b*c^4*d^8 - 20*a*c^2*d^10)*x^8 + 4*(17*b*c^6*d^6 + 210*a*c^4*d
^8)*x^6 - 6*(17*b*c^8*d^4 + 50*a*c^6*d^6)*x^4 + 18*(b*c^10*d^2 +
2*a*c^8*d^4)*x^2 - (256*b*d^11*x^11 - 64*(7*b*c^2*d^9 - 6*a*d^11)
*x^9 + 48*(3*b*c^4*d^7 - 16*a*c^2*d^9)*x^7 + 4*(25*b*c^6*d^5 + 12
6*a*c^4*d^7)*x^5 - 4*(13*b*c^8*d^3 + 30*a*c^6*d^5)*x^3 + 3*(b*c^1
0*d + 2*a*c^8*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) + 3*(b*c^12 + 2
*a*c^10*d^2 - 32*(b*c^6*d^6 + 2*a*c^4*d^8)*x^6 + 48*(b*c^8*d^4 +
2*a*c^6*d^6)*x^4 - 18*(b*c^10*d^2 + 2*a*c^8*d^4)*x^2 + 2*(16*(b*c
^6*d^5 + 2*a*c^4*d^7)*x^5 - 16*(b*c^8*d^3 + 2*a*c^6*d^5)*x^3 + 3*
(b*c^10*d + 2*a*c^8*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c))*log(-d*x
 + sqrt(d*x + c)*sqrt(d*x - c)))/(32*d^11*x^6 - 48*c^2*d^9*x^4 +
18*c^4*d^7*x^2 - c^6*d^5 - 2*(16*d^10*x^5 - 16*c^2*d^8*x^3 + 3*c^
4*d^6*x)*sqrt(d*x + c)*sqrt(d*x - c))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{2}\left(a + bx^{2}\right)\sqrt{-c + dx}\sqrt{c + dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

---

**GIAC/XCAS [A]** time = 0.28994, size = 311, normalized size = 1.96

$$\frac{6\left(\frac{2\,c^4\ln\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^2}+\left((dx+c)\left(2\,(dx+c)\left(\frac{dx+c}{d^2}-\frac{3\,c}{d^2}\right)+\frac{5\,c^2}{d^2}\right)-\frac{c^3}{d^2}\right)\sqrt{dx+c}\sqrt{dx-c}\right)a+\left(\frac{6\,c^6\ln\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^4}+\left(\left(2\right.\right.\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)*x^2,x, algorithm="giac")
```

```
[Out] 1/48*(6*(2*c^4*ln(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^2 + ((d*
x + c)*(2*(d*x + c)*((d*x + c)/d^2 - 3*c/d^2) + 5*c^2/d^2) - c^3/
d^2)*sqrt(d*x + c)*sqrt(d*x - c))*a + (6*c^6*ln(abs(-sqrt(d*x + c
) + sqrt(d*x - c)))/d^4 + ((2*((d*x + c)*(4*(d*x + c)*((d*x + c)/
d^4 - 5*c/d^4) + 39*c^2/d^4) - 37*c^3/d^4)*(d*x + c) + 31*c^4/d^4
)*(d*x + c) - 3*c^5/d^4)*sqrt(d*x + c)*sqrt(d*x - c))*b)/d
```

## 3.246 $\int \sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$

**Optimal**. Leaf size=114

$$\frac{x\sqrt{dx-c}\sqrt{c+dx}\left(4ad^2+bc^2\right)}{8d^2}-\frac{c^2\left(4ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^3}+\frac{bx(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}$$

```
[Out] ((b*c^2 + 4*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*d^2) + (b*x
*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(4*d^2) - (c^2*(b*c^2 + 4*a*d^
2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(4*d^3)
```

---

**Rubi [A]** time = 0.161227, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{1}{8}x\sqrt{dx-c}\sqrt{c+dx}\left(4a+\frac{bc^2}{d^2}\right)-\frac{c^2\left(4ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^3}+\frac{bx(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] ((4*a + (b*c^2)/d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/8 + (b*x*(-c
 + d*x)^(3/2)*(c + d*x)^(3/2))/(4*d^2) - (c^2*(b*c^2 + 4*a*d^2)*A
rcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(4*d^3)
```

---

**Rubi in Sympy [A]** time = 19.3635, size = 129, normalized size = 1.13

$$-\frac{ac^2\operatorname{atanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}+\frac{ax\sqrt{-c+dx}\sqrt{c+dx}}{2}-\frac{bc^4\operatorname{atanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{4d^3}+\frac{bc^2x\sqrt{-c+dx}\sqrt{c+dx}}{8d^2}+\frac{bx\left(-c+dx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{3}{2}}}{4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] -a*c**2*atanh(sqrt(-c + d*x)/sqrt(c + d*x))/d + a*x*sqrt(-c + d*x
)*sqrt(c + d*x)/2 - b*c**4*atanh(sqrt(-c + d*x)/sqrt(c + d*x))/(4
*d**3) + b*c**2*x*sqrt(-c + d*x)*sqrt(c + d*x)/(8*d**2) + b*x*(-c
 + d*x)**(3/2)*(c + d*x)**(3/2)/(4*d**2)
```

---

**Mathematica [A]** time = 0.0825454, size = 96, normalized size = 0.84

$$\frac{dx\sqrt{dx-c}\sqrt{c+dx}\left(4ad^2-bc^2+2bd^2x^2\right)-\left(4ac^2d^2+bc^4\right)\log\left(\sqrt{dx-c}\sqrt{c+dx}+dx\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(-(b*c^2) + 4*a*d^2 + 2*b*d^2*x
^2) - (b*c^4 + 4*a*c^2*d^2)*Log[d*x + Sqrt[-c + d*x]*Sqrt[c + d*x
]])/(8*d^3)
```

---

**Maple [C]** time = 0.016, size = 182, normalized size = 1.6

$$\frac{csgn(d)}{8\,d^3}\sqrt{dx-c}\sqrt{dx+c}\left(2\,csgn(d)\,x^3bd^3\sqrt{d^2x^2-c^2}+4\,ax\sqrt{d^2x^2-c^2}d^3\,csgn(d)-bc^2x\sqrt{d^2x^2-c^2}\,csgn(d)\,d-4\,ac^2\ln\left(\left(c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(2*csgn(d)*x^3*b*d^3*(d^2*x^2-c^2
)^(1/2)+4*a*x*(d^2*x^2-c^2)^(1/2)*d^3*csgn(d)-b*c^2*x*(d^2*x^2-c^
2)^(1/2)*csgn(d)*d-4*a*c^2*ln((csgn(d)*(d^2*x^2-c^2)^(1/2)+d*x)*c
sgn(d))*d^2-b*c^4*ln((csgn(d)*(d^2*x^2-c^2)^(1/2)+d*x)*csgn(d)))*
csgn(d)/(d^2*x^2-c^2)^(1/2)/d^3
```

---

**Maxima [A]** time = 1.43454, size = 205, normalized size = 1.8

$$-\frac{ac^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{2\,\sqrt{d^2}}-\frac{bc^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{8\,\sqrt{d^2}d^2}+\frac{1}{2}\,\sqrt{d^2x^2-c^2}ax+\frac{\sqrt{d^2x^2-c^2}bc^2x}{8\,d^2}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c),x, algorithm="maxima")
```

```
[Out] -1/2*a*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/sqrt(d^
2) - 1/8*b*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/(sq
rt(d^2)*d^2) + 1/2*sqrt(d^2*x^2 - c^2)*a*x + 1/8*sqrt(d^2*x^2 - c
```

```
^2)*b*c^2*x/d^2 + 1/4*(d^2*x^2 - c^2)^(3/2)*b*x/d^2
```

---

**Fricas [A]** time = 0.259909, size = 517, normalized size = 4.54

$$-\frac{16\,bd^8x^8 - 32\left(bc^2d^6 - ad^8\right)x^6 + 4\left(5\,bc^4d^4 - 12\,ac^2d^6\right)x^4 - 4\left(bc^6d^2 - 4\,ac^4d^4\right)x^2 - \left(16\,bd^7x^7 - 8\left(3\,bc^2d^5 - 4\,ad^7\right)x^5 + \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c),x, algorithm="fricas")
```

```
[Out] -1/8*(16*b*d^8*x^8 - 32*(b*c^2*d^6 - a*d^8)*x^6 + 4*(5*b*c^4*d^4
- 12*a*c^2*d^6)*x^4 - 4*(b*c^6*d^2 - 4*a*c^4*d^4)*x^2 - (16*b*d^7
*x^7 - 8*(3*b*c^2*d^5 - 4*a*d^7)*x^5 + 2*(5*b*c^4*d^3 - 16*a*c^2*
d^5)*x^3 - (b*c^6*d - 4*a*c^4*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c)
 - (b*c^8 + 4*a*c^6*d^2 + 8*(b*c^4*d^4 + 4*a*c^2*d^6)*x^4 - 8*(b*
c^6*d^2 + 4*a*c^4*d^4)*x^2 - 4*(2*(b*c^4*d^3 + 4*a*c^2*d^5)*x^3 -
 (b*c^6*d + 4*a*c^4*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c))*log(-d*x
 + sqrt(d*x + c)*sqrt(d*x - c)))/(8*d^7*x^4 - 8*c^2*d^5*x^2 + c^4
*d^3 - 4*(2*d^6*x^3 - c^2*d^4*x)*sqrt(d*x + c)*sqrt(d*x - c))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(a + bx^2\right)\sqrt{-c + dx}\sqrt{c + dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

---

**GIAC/XCAS [A]** time = 0.279106, size = 204, normalized size = 1.79

$$\frac{4\left(\sqrt{dx+c}\sqrt{dx-c}dx + 2\,c^2\ln\left(\left|-\sqrt{dx+c} + \sqrt{dx-c}\right|\right)\right)a + \left(\frac{2\,c^4\ln\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^2} + \left((dx+c)\left(2\,(dx+c)\left(\frac{dx+c}{d^2} - \frac{3\,c}{d^2}\right) + \frac{5\,c^{\cdots}}{d^2}\cdots\right.\right.\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c),x, algorithm="giac")
```

```
[Out] 1/8*(4*(sqrt(d*x + c)*sqrt(d*x - c)*d*x + 2*c^2*ln(abs(-sqrt(d*x
+ c) + sqrt(d*x - c))))*a + (2*c^4*ln(abs(-sqrt(d*x + c) + sqrt(d
*x - c)))/d^2 + ((d*x + c)*(2*(d*x + c)*((d*x + c)/d^2 - 3*c/d^2)
 + 5*c^2/d^2) - c^3/d^2)*sqrt(d*x + c)*sqrt(d*x - c))*b)/d
```

# 3.247 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)}{x^2}\,dx$

**Optimal**. Leaf size=104

$$\frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}\left(b-\frac{2ad^2}{c^2}\right)-\frac{\left(bc^2-2ad^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{c^2x}$$

```
[Out] ((b - (2*a*d^2)/c^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/2 + (a*(-c +
 d*x)^(3/2)*(c + d*x)^(3/2))/(c^2*x) - ((b*c^2 - 2*a*d^2)*ArcTanh
[Sqrt[-c + d*x]/Sqrt[c + d*x]])/d
```

---

**Rubi [A]** time = 0.255858, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129

$$\frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}\left(b-\frac{2ad^2}{c^2}\right)-\frac{\left(bc^2-2ad^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{c^2x}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^2,x]
```

```
[Out] ((b - (2*a*d^2)/c^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/2 + (a*(-c +
 d*x)^(3/2)*(c + d*x)^(3/2))/(c^2*x) - ((b*c^2 - 2*a*d^2)*ArcTanh
[Sqrt[-c + d*x]/Sqrt[c + d*x]])/d
```

---

**Rubi in Sympy [A]** time = 16.7444, size = 90, normalized size = 0.87

$$\frac{a\left(-c+dx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{3}{2}}}{c^2x}+\frac{\left(2ad^2-bc^2\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{d}-\frac{x\sqrt{-c+dx}\sqrt{c+dx}\left(2ad^2-bc^2\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**2,x)
```

```
[Out] a*(-c + d*x)**(3/2)*(c + d*x)**(3/2)/(c**2*x) + (2*a*d**2 - b*c**
2)*atanh(sqrt(c + d*x)/sqrt(-c + d*x))/d - x*sqrt(-c + d*x)*sqrt(
c + d*x)*(2*a*d**2 - b*c**2)/(2*c**2)
```

---

**Mathematica [A]** time = 0.107133, size = 81, normalized size = 0.78

$$\frac{(2ad^2-bc^2)\log\left(\sqrt{dx-c}\sqrt{c+dx}+dx\right)}{2d}+\left(\frac{bx}{2}-\frac{a}{x}\right)\sqrt{dx-c}\sqrt{c+dx}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^2,x]
```

```
[Out] (-(a/x) + (b*x)/2)*Sqrt[-c + d*x]*Sqrt[c + d*x] + ((-(b*c^2) + 2*
a*d^2)*Log[d*x + Sqrt[-c + d*x]*Sqrt[c + d*x]])/(2*d)
```

---

**Maple [C]** time = 0.021, size = 153, normalized size = 1.5

$$\frac{csgn(d)}{2\,dx}\sqrt{dx-c}\sqrt{dx+c}\left(csgn(d)\,x^2bd\sqrt{d^2x^2-c^2}+2\,\ln\left(\left(csgn(d)\,\sqrt{d^2x^2-c^2}+dx\right)csgn(d)\right)xad^2-\ln\left(\left(csgn(d)\,\sqrt{d^2x^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x)
```

```
[Out] 1/2*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(csgn(d)*x^2*b*d*(d^2*x^2-c^2)^(1
/2)+2*ln((csgn(d)*(d^2*x^2-c^2)^(1/2)+d*x)*csgn(d))*x*a*d^2-ln((c
sgn(d)*(d^2*x^2-c^2)^(1/2)+d*x)*csgn(d))*x*b*c^2-2*a*(d^2*x^2-c^2
)^(1/2)*csgn(d)*d)*csgn(d)/(d^2*x^2-c^2)^(1/2)/d/x
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.241695, size = 379, normalized size = 3.64

$$-\frac{4\,bd^5x^6-5\,bc^2d^3x^4-2\,ac^4d+\left(bc^4d+4\,ac^2d^3\right)x^2-\left(4\,bd^4x^5-3\,bc^2d^2x^3+4\,ac^2d^2x\right)\sqrt{dx+c}\sqrt{dx-c}-\left(4\left(bc^2d^3-2\,ac\right.\right.}{2\left(4\,d^4x^4-3\,c^2d^2x^2-(4\,d^3x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*b*d^5*x^6 - 5*b*c^2*d^3*x^4 - 2*a*c^4*d + (b*c^4*d + 4*a*
c^2*d^3)*x^2 - (4*b*d^4*x^5 - 3*b*c^2*d^2*x^3 + 4*a*c^2*d^2*x)*sq
rt(d*x + c)*sqrt(d*x - c) - (4*(b*c^2*d^3 - 2*a*d^5)*x^4 - 3*(b*c
^4*d - 2*a*c^2*d^3)*x^2 - (4*(b*c^2*d^2 - 2*a*d^4)*x^3 - (b*c^4 -
 2*a*c^2*d^2)*x)*sqrt(d*x + c)*sqrt(d*x - c))*log(-d*x + sqrt(d*x
 + c)*sqrt(d*x - c)))/(4*d^4*x^4 - 3*c^2*d^2*x^2 - (4*d^3*x^3 - c
^2*d*x)*sqrt(d*x + c)*sqrt(d*x - c))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)\sqrt{-c+dx}\sqrt{c+dx}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**2,x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x**2, x)
```

---

**GIAC/XCAS [A]** time = 0.230448, size = 149, normalized size = 1.43

$$-\frac{\frac{6144\,ac^2d^2}{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2}-2\left((dx+c)b-bc\right)\sqrt{dx+c}\sqrt{dx-c}-\left(bc^2-2\,ad^2\right)\ln\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right)}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x^2,x, algorithm="giac")
```

```
[Out] -1/768*(6144*a*c^2*d^2/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2
) - 2*((d*x + c)*b - b*c)*sqrt(d*x + c)*sqrt(d*x - c) - (b*c^2 -
2*a*d^2)*ln((sqrt(d*x + c) - sqrt(d*x - c))^4))/d
```

# 3.248 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4}\,dx$

**Optimal**. Leaf size=84

$$\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{3c^2x^3}-\frac{b\sqrt{dx-c}\sqrt{c+dx}}{x}+2bd\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)$$

```
[Out] -((b*Sqrt[-c + d*x]*Sqrt[c + d*x])/x) + (a*(-c + d*x)^(3/2)*(c +
d*x)^(3/2))/(3*c^2*x^3) + 2*b*d*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d
*x]]
```

---

**Rubi [A]** time = 0.259594, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161

$$\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{3c^2x^3}-\frac{b\sqrt{dx-c}\sqrt{c+dx}}{x}+2bd\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^4,x]
```

```
[Out] -((b*Sqrt[-c + d*x]*Sqrt[c + d*x])/x) + (a*(-c + d*x)^(3/2)*(c +
d*x)^(3/2))/(3*c^2*x^3) + 2*b*d*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d
*x]]
```

---

**Rubi in Sympy [A]** time = 15.5248, size = 70, normalized size = 0.83

$$\frac{a\left(-c+dx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{3}{2}}}{3c^2x^3}+2bd\,\operatorname{atanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)-\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**4,x)
```

```
[Out] a*(-c + d*x)**(3/2)*(c + d*x)**(3/2)/(3*c**2*x**3) + 2*b*d*atanh(
sqrt(-c + d*x)/sqrt(c + d*x)) - b*sqrt(-c + d*x)*sqrt(c + d*x)/x
```

---

**Mathematica [A]** time = 0.162109, size = 78, normalized size = 0.93

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(a\left(\frac{d^2x^2}{c^2}-1\right)-3bx^2\right)}{3x^3}+bd\log\left(\sqrt{dx-c}\sqrt{c+dx}+dx\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^4,x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-3*b*x^2 + a*(-1 + (d^2*x^2)/c^2))
)/(3*x^3) + b*d*Log[d*x + Sqrt[-c + d*x]*Sqrt[c + d*x]]
```

---

**Maple [C]** time = 0.022, size = 153, normalized size = 1.8

$$\frac{csgn(d)}{3\,c^2x^3}\sqrt{dx-c}\sqrt{dx+c}\left(3\,\ln\left(\left(csgn(d)\sqrt{d^2x^2-c^2}+dx\right)csgn(d)\right)x^3bc^2d+csgn(d)\,x^2ad^2\sqrt{d^2x^2-c^2}-3\,csgn(d)\,x^2bc^2\sqrt{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x)
```

```
[Out] 1/3*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(3*ln((csgn(d)*(d^2*x^2-c^2)^(1/2
)+d*x)*csgn(d))*x^3*b*c^2*d+csgn(d)*x^2*a*d^2*(d^2*x^2-c^2)^(1/2)
-3*csgn(d)*x^2*b*c^2*(d^2*x^2-c^2)^(1/2)-csgn(d)*a*c^2*(d^2*x^2-c
^2)^(1/2))*csgn(d)/(d^2*x^2-c^2)^(1/2)/c^2/x^3
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.237663, size = 313, normalized size = 3.73

$$\frac{ac^4-6\left(bc^2d^2-ad^4\right)x^4+3\left(bc^4-2\,ac^2d^2\right)x^2+3\left(ac^2dx+2\left(bc^2d-ad^3\right)x^3\right)\sqrt{dx+c}\sqrt{dx-c}-3\left(4\,bd^4x^6-3\,bc^2d^2x^4-\ldots\right.}{3\left(4\,d^3x^6-3\,c^2dx^4-\left(4\,d^2x^5-c^2x^3\right)\sqrt{dx+c}\sqrt{dx-c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(a*c^4 - 6*(b*c^2*d^2 - a*d^4)*x^4 + 3*(b*c^4 - 2*a*c^2*d^2)*
x^2 + 3*(a*c^2*d*x + 2*(b*c^2*d - a*d^3)*x^3)*sqrt(d*x + c)*sqrt(
d*x - c) - 3*(4*b*d^4*x^6 - 3*b*c^2*d^2*x^4 - (4*b*d^3*x^5 - b*c^
2*d*x^3)*sqrt(d*x + c)*sqrt(d*x - c))*log(-d*x + sqrt(d*x + c)*sq
rt(d*x - c)))/(4*d^3*x^6 - 3*c^2*d*x^4 - (4*d^2*x^5 - c^2*x^3)*sq
rt(d*x + c)*sqrt(d*x - c))
```

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: MellinTransformStripError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**4,x)
```

```
[Out] Exception raised: MellinTransformStripError
```

---

**GIAC/XCAS [A]** time = 0.23387, size = 231, normalized size = 2.75

$$-\frac{3\,bd^2\ln\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right)+\frac{16\left(3\,bc^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8-3\,ad^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+24\,bc^4d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+48\,bc^6d^2-16\,ac^4d^4\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x + c)*sqrt(d*x - c)/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*d^2*ln((sqrt(d*x + c) - sqrt(d*x - c))^4) + 16*(3*b*c^2
*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^8 - 3*a*d^4*(sqrt(d*x + c) -
 sqrt(d*x - c))^8 + 24*b*c^4*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^
4 + 48*b*c^6*d^2 - 16*a*c^4*d^4)/((sqrt(d*x + c) - sqrt(d*x - c))
^4 + 4*c^2)^3)/d
```

## 3.249 $\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

**Optimal**. Leaf size=125

$$\frac{(6ac^2+5b)\cosh^{-1}(cx)}{16c^7}+\frac{x\sqrt{cx-1}\sqrt{cx+1}\,(6ac^2+5b)}{16c^6}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}\,(6ac^2+5b)}{24c^4}+\frac{bx^5\sqrt{cx-1}\sqrt{cx+1}}{6c^2}$$

```
[Out] ((5*b + 6*a*c^2)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c^6) + ((5*b
 + 6*a*c^2)*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(24*c^4) + (b*x^5*S
qrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c^2) + ((5*b + 6*a*c^2)*ArcCosh[c
*x])/(16*c^7)
```

---

**Rubi [A]** time = 0.33463, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{(6ac^2+5b)\cosh^{-1}(cx)}{16c^7}+\frac{x\sqrt{cx-1}\sqrt{cx+1}\,(6ac^2+5b)}{16c^6}+\frac{x^3\sqrt{cx-1}\sqrt{cx+1}\,(6ac^2+5b)}{24c^4}+\frac{bx^5\sqrt{cx-1}\sqrt{cx+1}}{6c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((5*b + 6*a*c^2)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c^6) + ((5*b
 + 6*a*c^2)*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(24*c^4) + (b*x^5*S
qrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c^2) + ((5*b + 6*a*c^2)*ArcCosh[c
*x])/(16*c^7)
```

---

**Rubi in Sympy [A]** time = 17.229, size = 116, normalized size = 0.93

$$\frac{bx^5\sqrt{cx-1}\sqrt{cx+1}}{6c^2}+\frac{x^3\,(6ac^2+5b)\,\sqrt{cx-1}\sqrt{cx+1}}{24c^4}+\frac{x\,(6ac^2+5b)\,\sqrt{cx-1}\sqrt{cx+1}}{16c^6}+\frac{(6ac^2+5b)\operatorname{acosh}(cx)}{16c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] b*x**5*sqrt(c*x - 1)*sqrt(c*x + 1)/(6*c**2) + x**3*(6*a*c**2 + 5*
b)*sqrt(c*x - 1)*sqrt(c*x + 1)/(24*c**4) + x*(6*a*c**2 + 5*b)*sqr
t(c*x - 1)*sqrt(c*x + 1)/(16*c**6) + (6*a*c**2 + 5*b)*acosh(c*x)/
(16*c**7)
```

---

**Mathematica [A]** time = 0.13668, size = 102, normalized size = 0.82

$$\frac{3\left(6ac^2+5b\right)\log\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)+cx\sqrt{cx-1}\sqrt{cx+1}\left(6ac^2\left(2c^2x^2+3\right)+b\left(8c^4x^4+10c^2x^2+15\right)\right)}{48c^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(6*a*c^2*(3 + 2*c^2*x^2) + b*(1
5 + 10*c^2*x^2 + 8*c^4*x^4)) + 3*(5*b + 6*a*c^2)*Log[c*x + Sqrt[-
1 + c*x]*Sqrt[1 + c*x]])/(48*c^7)
```

---

**Maple [C]** time = 0.05, size = 191, normalized size = 1.5

$$\frac{csgn(c)}{48\,c^7}\sqrt{cx-1}\sqrt{cx+1}\left(8\,csgn(c)\,x^5bc^5\sqrt{c^2x^2-1}+12\,csgn(c)\,x^3ac^5\sqrt{c^2x^2-1}+10\,bx^3\sqrt{c^2x^2-1}c^3csgn(c)+18\,ax\sqrt{c^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/48*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(8*csgn(c)*x^5*b*c^5*(c^2*x^2-1)
^(1/2)+12*csgn(c)*x^3*a*c^5*(c^2*x^2-1)^(1/2)+10*b*x^3*(c^2*x^2-1
)^(1/2)*c^3*csgn(c)+18*a*x*(c^2*x^2-1)^(1/2)*c^3*csgn(c)+15*b*x*(
c^2*x^2-1)^(1/2)*csgn(c)*c+18*a*ln((csgn(c)*(c^2*x^2-1)^(1/2)+c*x
)*csgn(c))*c^2+15*b*ln((csgn(c)*(c^2*x^2-1)^(1/2)+c*x)*csgn(c)))*
csgn(c)/(c^2*x^2-1)^(1/2)/c^7
```

---

**Maxima [A]** time = 1.48597, size = 231, normalized size = 1.85

$$\frac{\sqrt{c^2x^2-1}bx^5}{6\,c^2}+\frac{\sqrt{c^2x^2-1}ax^3}{4\,c^2}+\frac{5\sqrt{c^2x^2-1}bx^3}{24\,c^4}+\frac{3\sqrt{c^2x^2-1}ax}{8\,c^4}+\frac{3\,a\log\left(2\,c^2x+2\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{8\sqrt{c^2}c^4}+\frac{5\sqrt{c^2x^2-1}bx}{16\,c^6}+\frac{5\,b\log\left(2\,c^2x+2\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{16\sqrt{c^2}c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^4/(sqrt(c*x + 1)*sqrt(c*x - 1)),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(c^2*x^2 - 1)*b*x^5/c^2 + 1/4*sqrt(c^2*x^2 - 1)*a*x^3/c^2
 + 5/24*sqrt(c^2*x^2 - 1)*b*x^3/c^4 + 3/8*sqrt(c^2*x^2 - 1)*a*x/c
^4 + 3/8*a*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2
)*c^4) + 5/16*sqrt(c^2*x^2 - 1)*b*x/c^6 + 5/16*b*log(2*c^2*x + 2*
sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^6)
```

---

**Fricas [A]** time = 0.270148, size = 581, normalized size = 4.65

$$-\frac{256\,bc^{12}x^{12} + 192\left(2\,ac^{12} - bc^{10}\right)x^{10} - 48\left(4\,ac^{10} - 3\,bc^{8}\right)x^{8} - 4\left(174\,ac^{8} + 157\,bc^{6}\right)x^{6} + 102\left(6\,ac^{6} + 5\,bc^{4}\right)x^{4} - 18\left(6\,ac^{4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^4/(sqrt(c*x + 1)*sqrt(c*x - 1)),x, algorithm="fricas")
```

```
[Out] -1/48*(256*b*c^12*x^12 + 192*(2*a*c^12 - b*c^10)*x^10 - 48*(4*a*c
^10 - 3*b*c^8)*x^8 - 4*(174*a*c^8 + 157*b*c^6)*x^6 + 102*(6*a*c^6
 + 5*b*c^4)*x^4 - 18*(6*a*c^4 + 5*b*c^2)*x^2 - (256*b*c^11*x^11 +
 144*b*c^7*x^7 + 64*(6*a*c^11 - b*c^9)*x^9 - 4*(162*a*c^7 + 137*b
*c^5)*x^5 + 52*(6*a*c^5 + 5*b*c^3)*x^3 - 3*(6*a*c^3 + 5*b*c)*x)*s
qrt(c*x + 1)*sqrt(c*x - 1) + 3*(32*(6*a*c^8 + 5*b*c^6)*x^6 - 48*(
6*a*c^6 + 5*b*c^4)*x^4 - 6*a*c^2 + 18*(6*a*c^4 + 5*b*c^2)*x^2 - 2
*(16*(6*a*c^7 + 5*b*c^5)*x^5 - 16*(6*a*c^5 + 5*b*c^3)*x^3 + 3*(6*
a*c^3 + 5*b*c)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) - 5*b)*log(-c*x + s
qrt(c*x + 1)*sqrt(c*x - 1)))/(32*c^13*x^6 - 48*c^11*x^4 + 18*c^9*
x^2 - c^7 - 2*(16*c^12*x^5 - 16*c^10*x^3 + 3*c^8*x)*sqrt(c*x + 1)
*sqrt(c*x - 1))
```

---

**Sympy [A]** time = 134.074, size = 216, normalized size = 1.73

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix} -\frac{7}{4}, -\frac{5}{4} & -\frac{3}{2}, -\frac{3}{2}, -1, 1 \\ -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, 0 & \end{matrix} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^5} - \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -\frac{5}{2}, -\frac{9}{4}, -2, -\frac{7}{4}, -\frac{3}{2}, 1 & \\ -\frac{9}{4}, -\frac{7}{4} & -\frac{5}{2}, -2, -2, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^5} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{11}{4}, -\frac{9}{4} & -\frac{5}{2}, -\frac{5}{2}, -2, 1 \\ -3, -\frac{11}{4}, -\frac{5}{2}, -\frac{9}{4}, -2, 0 & \end{matrix} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^7} - \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -\frac{7}{2}, -\frac{13}{4}, -3, -\frac{11}{4}, -\frac{5}{2}, 1 & \\ -\frac{13}{4}, -\frac{11}{4} & -\frac{7}{2}, -3, -3, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((-7/4, -5/4), (-3/2, -3/2, -1, 1)), ((-2, -7/4, -3/2,
-5/4, -1, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**5) - I*a*meijer
g(((-5/2, -9/4, -2, -7/4, -3/2, 1), ()), ((-9/4, -7/4), (-5/2, -2
, -2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**5) + b*
meijerg(((-11/4, -9/4), (-5/2, -5/2, -2, 1)), ((-3, -11/4, -5/2,
-9/4, -2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**7) - I*b*meijer
g(((-7/2, -13/4, -3, -11/4, -5/2, 1), ()), ((-13/4, -11/4), (-7/2
, -3, -3, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**7)
```

---

**GIAC/XCAS [A]** time = 0.246501, size = 205, normalized size = 1.64

$$-\frac{\left(30\,ac^{38} + 33\,bc^{36} - \left(54\,ac^{38} + 85\,bc^{36} - 2\left(18\,ac^{38} + 55\,bc^{36} - \left(6\,ac^{38} + 45\,bc^{36} + 4\left((cx+1)bc^{36} - 5\,bc^{36}\right)(cx+1)\right)(cx+1\right.\right.\right.}{34603008\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^4/(sqrt(c*x + 1)*sqrt(c*x - 1)),x, algorithm="giac")
```

```
[Out] -1/34603008*((30*a*c^38 + 33*b*c^36 - (54*a*c^38 + 85*b*c^36 - 2*
(18*a*c^38 + 55*b*c^36 - (6*a*c^38 + 45*b*c^36 + 4*((c*x + 1)*b*c
^36 - 5*b*c^36)*(c*x + 1))*(c*x + 1))*(c*x + 1))*(c*x + 1))*sqrt(
c*x + 1)*sqrt(c*x - 1) + 6*(6*a*c^38 + 5*b*c^36)*ln(abs(-sqrt(c*x
 + 1) + sqrt(c*x - 1))))/c
```

## 3.250 $\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

**Optimal**. Leaf size=103

$$\frac{2\sqrt{cx-1}\sqrt{cx+1}\,(5ac^2+4b)}{15c^6}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\,(5ac^2+4b)}{15c^4}+\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}$$

```
[Out] (2*(4*b + 5*a*c^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^6) + ((4*b
 + 5*a*c^2)*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^4) + (b*x^4*S
qrt[-1 + c*x]*Sqrt[1 + c*x])/(5*c^2)
```

---

**Rubi [A]** time = 0.26647, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138

$$\frac{2\sqrt{cx-1}\sqrt{cx+1}\,(5ac^2+4b)}{15c^6}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\,(5ac^2+4b)}{15c^4}+\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (2*(4*b + 5*a*c^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^6) + ((4*b
 + 5*a*c^2)*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^4) + (b*x^4*S
qrt[-1 + c*x]*Sqrt[1 + c*x])/(5*c^2)
```

---

**Rubi in Sympy [A]** time = 13.3902, size = 95, normalized size = 0.92

$$\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}+\frac{x^2\,(5ac^2+4b)\,\sqrt{cx-1}\sqrt{cx+1}}{15c^4}+\frac{2\,(5ac^2+4b)\,\sqrt{cx-1}\sqrt{cx+1}}{15c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] b*x**4*sqrt(c*x - 1)*sqrt(c*x + 1)/(5*c**2) + x**2*(5*a*c**2 + 4*
b)*sqrt(c*x - 1)*sqrt(c*x + 1)/(15*c**4) + 2*(5*a*c**2 + 4*b)*sqr
t(c*x - 1)*sqrt(c*x + 1)/(15*c**6)
```

---

**Mathematica [A]** time = 0.0631832, size = 61, normalized size = 0.59

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\,\left(5ac^2\,(c^2x^2+2)+b\,(3c^4x^4+4c^2x^2+8)\right)}{15c^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(5*a*c^2*(2 + c^2*x^2) + b*(8 + 4*c
^2*x^2 + 3*c^4*x^4)))/(15*c^6)
```

---

**Maple [A]** time = 0.009, size = 57, normalized size = 0.6

$$\frac{3\,b x^4 c^4 + 5\,a c^4 x^2 + 4\,b c^2 x^2 + 10\,a c^2 + 8\,b}{15\,c^6}\sqrt{cx-1}\sqrt{cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/15*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(3*b*c^4*x^4+5*a*c^4*x^2+4*b*c^2
*x^2+10*a*c^2+8*b)/c^6
```

---

**Maxima [A]** time = 1.39394, size = 128, normalized size = 1.24

$$\frac{\sqrt{c^2x^2-1}bx^4}{5\,c^2} + \frac{\sqrt{c^2x^2-1}ax^2}{3\,c^2} + \frac{4\,\sqrt{c^2x^2-1}bx^2}{15\,c^4} + \frac{2\,\sqrt{c^2x^2-1}a}{3\,c^4} + \frac{8\,\sqrt{c^2x^2-1}b}{15\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^3/(sqrt(c*x + 1)*sqrt(c*x - 1)),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(c^2*x^2 - 1)*b*x^4/c^2 + 1/3*sqrt(c^2*x^2 - 1)*a*x^2/c^2
 + 4/15*sqrt(c^2*x^2 - 1)*b*x^2/c^4 + 2/3*sqrt(c^2*x^2 - 1)*a/c^4
 + 8/15*sqrt(c^2*x^2 - 1)*b/c^6
```

---

**Fricas [A]** time = 0.253568, size = 333, normalized size = 3.23

$$-\frac{48\,bc^{10}x^{10} + 20\left(4\,ac^{10} - bc^8\right)x^8 + 5\left(4\,ac^8 + 11\,bc^6\right)x^6 - 5\left(43\,ac^6 + 35\,bc^4\right)x^4 - 10\,ac^2 + 25\left(5\,ac^4 + 4\,bc^2\right)x^2 - \left(48\,bc^9x^9\right.}{15\left(16\,c^{11}x^5 - 20\,c^9x^3 + 5\,c^7x - (16\,c^{10}x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^3/(sqrt(c*x + 1)*sqrt(c*x - 1)),x, algorithm="fricas")
```

```
[Out] -1/15*(48*b*c^10*x^10 + 20*(4*a*c^10 - b*c^8)*x^8 + 5*(4*a*c^8 +
11*b*c^6)*x^6 - 5*(43*a*c^6 + 35*b*c^4)*x^4 - 10*a*c^2 + 25*(5*a*
c^4 + 4*b*c^2)*x^2 - (48*b*c^9*x^9 + 4*(20*a*c^9 + b*c^7)*x^7 + 3
*(20*a*c^7 + 21*b*c^5)*x^5 - 35*(5*a*c^5 + 4*b*c^3)*x^3 + 10*(5*a
*c^3 + 4*b*c)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) - 8*b)/(16*c^11*x^5
- 20*c^9*x^3 + 5*c^7*x - (16*c^10*x^4 - 12*c^8*x^2 + c^6)*sqrt(c*
x + 1)*sqrt(c*x - 1))
```

---

**Sympy [A]** time = 89.8548, size = 216, normalized size = 2.1

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4}, -\frac{3}{4} & -1, -1, -\frac{1}{2}, 1 \\ -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 0 & \end{matrix} \middle| \frac{1}{c^{2}x^{2}} \right)}{4\pi^{\frac{3}{2}}c^{4}} + \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, 1 & \\ -\frac{7}{4}, -\frac{5}{4} & -2, -\frac{3}{2}, -\frac{3}{2}, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^{2}x^{2}} \right)}{4\pi^{\frac{3}{2}}c^{4}} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{9}{4}, -\frac{7}{4} & -2, -2, -\frac{3}{2}, 1 \\ -\frac{5}{2}, -\frac{9}{4}, -2, -\frac{7}{4}, -\frac{3}{2}, 0 & \end{matrix} \middle| \frac{1}{c^{2}x^{2}} \right)}{4\pi^{\frac{3}{2}}c^{6}} + \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -3, -\frac{11}{4}, -\frac{5}{2}, -\frac{9}{4}, -2, 1 & \\ -\frac{11}{4}, -\frac{9}{4} & -3, -\frac{5}{2}, -\frac{5}{2}, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^{2}x^{2}} \right)}{4\pi^{\frac{3}{2}}c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3
/4, -1/2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**4) + I*a*meijer
g(((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2,
-3/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**4) + b*
meijerg(((-9/4, -7/4), (-2, -2, -3/2, 1)), ((-5/2, -9/4, -2, -7/4
, -3/2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**6) + I*b*meijerg(
((-3, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4), (-3, -5/2,
-5/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**6)
```

---

**GIAC/XCAS [A]** time = 0.21793, size = 130, normalized size = 1.26

$$\frac{\left(15\,ac^{27} + 15\,bc^{25} - \left(10\,ac^{27} + 20\,bc^{25} - \left(5\,ac^{27} + 22\,bc^{25} + 3\,\left((cx+1)bc^{25} - 4\,bc^{25}\right)(cx+1)\right)(cx+1)\right)(cx+1)\right)\sqrt{cx+1}\sqrt{cx}}{276480\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^3/(sqrt(c*x + 1)*sqrt(c*x - 1)),x, algorithm="giac")
```

```
[Out] 1/276480*(15*a*c^27 + 15*b*c^25 - (10*a*c^27 + 20*b*c^25 - (5*a*c
^27 + 22*b*c^25 + 3*((c*x + 1)*b*c^25 - 4*b*c^25)*(c*x + 1))*(c*x
 + 1))*(c*x + 1))*sqrt(c*x + 1)*sqrt(c*x - 1)/c
```

## 3.251 $\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

**Optimal**. Leaf size=87

$$\frac{(4ac^2+3b)\cosh^{-1}(cx)}{8c^5}+\frac{x\sqrt{cx-1}\sqrt{cx+1}\,(4ac^2+3b)}{8c^4}+\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}$$

```
[Out] ((3*b + 4*a*c^2)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(8*c^4) + (b*x^3
*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c^2) + ((3*b + 4*a*c^2)*ArcCosh
[c*x])/(8*c^5)
```

---

**Rubi [A]** time = 0.261869, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$\frac{(4ac^2+3b)\cosh^{-1}(cx)}{8c^5}+\frac{x\sqrt{cx-1}\sqrt{cx+1}\,(4ac^2+3b)}{8c^4}+\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((3*b + 4*a*c^2)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(8*c^4) + (b*x^3
*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c^2) + ((3*b + 4*a*c^2)*ArcCosh
[c*x])/(8*c^5)
```

---

**Rubi in Sympy [A]** time = 12.7528, size = 80, normalized size = 0.92

$$\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}+\frac{x\,(4ac^2+3b)\,\sqrt{cx-1}\sqrt{cx+1}}{8c^4}+\frac{(4ac^2+3b)\operatorname{acosh}(cx)}{8c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] b*x**3*sqrt(c*x - 1)*sqrt(c*x + 1)/(4*c**2) + x*(4*a*c**2 + 3*b)*
sqrt(c*x - 1)*sqrt(c*x + 1)/(8*c**4) + (4*a*c**2 + 3*b)*acosh(c*x
)/(8*c**5)
```

---

**Mathematica [A]** time = 0.0936331, size = 83, normalized size = 0.95

$$\frac{cx\sqrt{cx-1}\sqrt{cx+1}\,\left(4ac^2+b\left(2c^2x^2+3\right)\right)+(4ac^2+3b)\log\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)}{8c^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*a*c^2 + b*(3 + 2*c^2*x^2)) +
 (3*b + 4*a*c^2)*Log[c*x + Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/(8*c^5)
```

---

**Maple [C]** time = 0.028, size = 147, normalized size = 1.7

$$\frac{csgn(c)}{8\,c^5}\sqrt{cx-1}\sqrt{cx+1}\left(2\,bx^3\sqrt{c^2x^2-1}c^3\,csgn(c)+4\,ax\sqrt{c^2x^2-1}c^3\,csgn(c)+3\,bx\sqrt{c^2x^2-1}csgn(c)\,c+4\,a\ln\left(\left(csgn(c)\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(2*b*x^3*(c^2*x^2-1)^(1/2)*c^3*cs
gn(c)+4*a*x*(c^2*x^2-1)^(1/2)*c^3*csgn(c)+3*b*x*(c^2*x^2-1)^(1/2)
*csgn(c)*c+4*a*ln((csgn(c)*(c^2*x^2-1)^(1/2)+c*x)*csgn(c))*c^2+3*
b*ln((csgn(c)*(c^2*x^2-1)^(1/2)+c*x)*csgn(c)))*csgn(c)/(c^2*x^2-1
)^(1/2)/c^5
```

---

**Maxima [A]** time = 1.38053, size = 177, normalized size = 2.03

$$\frac{\sqrt{c^2x^2-1}bx^3}{4\,c^2}+\frac{\sqrt{c^2x^2-1}ax}{2\,c^2}+\frac{a\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{2\,\sqrt{c^2}c^2}+\frac{3\,\sqrt{c^2x^2-1}bx}{8\,c^4}+\frac{3\,b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{8\,\sqrt{c^2}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^2/(sqrt(c*x + 1)*sqrt(c*x - 1)),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c^2*x^2 - 1)*b*x^3/c^2 + 1/2*sqrt(c^2*x^2 - 1)*a*x/c^2 +
 1/2*a*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^
2) + 3/8*sqrt(c^2*x^2 - 1)*b*x/c^4 + 3/8*b*log(2*c^2*x + 2*sqrt(c
^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^4)
```

---

**Fricas [A]** time = 0.243741, size = 406, normalized size = 4.67

$$-\frac{16\,bc^{8}x^{8} + 32\,ac^{8}x^{6} - 4\left(12\,ac^{6} + 7\,bc^{4}\right)x^{4} + 4\left(4\,ac^{4} + 3\,bc^{2}\right)x^{2} - \left(16\,bc^{7}x^{7} + 8\left(4\,ac^{7} + bc^{5}\right)x^{5} - 2\left(16\,ac^{5} + 11\,bc^{3}\right)x^{3} + \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^2/(sqrt(c*x + 1)*sqrt(c*x - 1)),x, algorithm="fricas")
```

```
[Out] -1/8*(16*b*c^8*x^8 + 32*a*c^8*x^6 - 4*(12*a*c^6 + 7*b*c^4)*x^4 +
4*(4*a*c^4 + 3*b*c^2)*x^2 - (16*b*c^7*x^7 + 8*(4*a*c^7 + b*c^5)*x
^5 - 2*(16*a*c^5 + 11*b*c^3)*x^3 + (4*a*c^3 + 3*b*c)*x)*sqrt(c*x
+ 1)*sqrt(c*x - 1) + (8*(4*a*c^6 + 3*b*c^4)*x^4 + 4*a*c^2 - 8*(4*
a*c^4 + 3*b*c^2)*x^2 - 4*(2*(4*a*c^5 + 3*b*c^3)*x^3 - (4*a*c^3 +
3*b*c)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + 3*b)*log(-c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1)))/(8*c^9*x^4 - 8*c^7*x^2 + c^5 - 4*(2*c^8*x^3
- c^6*x)*sqrt(c*x + 1)*sqrt(c*x - 1))
```

---

**Sympy [A]** time = 86.6118, size = 212, normalized size = 2.44

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{4} & -\frac{1}{2}, -\frac{1}{2}, 0, 1 \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 & \end{matrix} \middle| {\frac{1}{c^{2}x^{2}}} \right)}{4\pi^{\frac{3}{2}}c^{3}} - \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 & \\ -\frac{5}{4}, -\frac{3}{4} & -\frac{3}{2}, -1, -1, 0 \end{matrix} \middle| {\frac{e^{2i\pi}}{c^{2}x^{2}}} \right)}{4\pi^{\frac{3}{2}}c^{3}} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{7}{4}, -\frac{5}{4} & -\frac{3}{2}, -\frac{3}{2}, -1, 1 \\ -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, 0 & \end{matrix} \middle| {\frac{1}{c^{2}x^{2}}} \right)}{4\pi^{\frac{3}{2}}c^{5}} - \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -\frac{5}{2}, -\frac{9}{4}, -2, -\frac{7}{4}, -\frac{3}{2}, 1 & \\ -\frac{9}{4}, -\frac{7}{4} & -\frac{5}{2}, -2, -2, 0 \end{matrix} \middle| {\frac{e^{2i\pi}}{c^{2}x^{2}}} \right)}{4\pi^{\frac{3}{2}}c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -
1/4, 0, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**3) - I*a*meijerg(
((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1,
-1, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**3) + b*me
ijerg(((-7/4, -5/4), (-3/2, -3/2, -1, 1)), ((-2, -7/4, -3/2, -5/4
, -1, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**5) - I*b*meijerg(((
```

```
-5/2, -9/4, -2, -7/4, -3/2, 1), ()), ((-9/4, -7/4), (-5/2, -2, -2
, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**5)
```

---

**GIAC/XCAS [A]** time = 0.249062, size = 151, normalized size = 1.74

$$-\frac{\left(4\,ac^{18}+5\,bc^{16}-\left(4\,ac^{18}+9\,bc^{16}+2\left((cx+1)bc^{16}-3\,bc^{16}\right)(cx+1)\right)(cx+1)\right)\sqrt{cx+1}\sqrt{cx-1}+2\left(4\,ac^{18}+3\,bc^{16}\right)\ln\left(\left|-\sqrt{\vphantom{1}}\right.\right.}{114688\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^2/(sqrt(c*x + 1)*sqrt(c*x - 1)),x, algorithm="giac")
```

```
[Out] -1/114688*((4*a*c^18 + 5*b*c^16 - (4*a*c^18 + 9*b*c^16 + 2*((c*x
+ 1)*b*c^16 - 3*b*c^16)*(c*x + 1))*(c*x + 1))*sqrt(c*x + 1)*sqrt(
c*x - 1) + 2*(4*a*c^18 + 3*b*c^16)*ln(abs(-sqrt(c*x + 1) + sqrt(c
*x - 1))))/c
```

# 3.252 $\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

**Optimal**. Leaf size=65

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(3ac^2+2b\right)}{3c^4}+\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}$$

```
[Out] ((2*b + 3*a*c^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^4) + (b*x^2*S
qrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^2)
```

---

**Rubi [A]** time = 0.142211, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(3ac^2+2b\right)}{3c^4}+\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((2*b + 3*a*c^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^4) + (b*x^2*S
qrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^2)
```

---

**Rubi in Sympy [A]** time = 8.28265, size = 58, normalized size = 0.89

$$\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}+\frac{\left(3ac^2+2b\right)\sqrt{cx-1}\sqrt{cx+1}}{3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] b*x**2*sqrt(c*x - 1)*sqrt(c*x + 1)/(3*c**2) + (3*a*c**2 + 2*b)*sq
rt(c*x - 1)*sqrt(c*x + 1)/(3*c**4)
```

---

**Mathematica [A]** time = 0.0491001, size = 43, normalized size = 0.66

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(3ac^2+b\left(c^2x^2+2\right)\right)}{3c^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3*a*c^2 + b*(2 + c^2*x^2)))/(3*c^4
)
```

---

**Maple [A]** time = 0.006, size = 38, normalized size = 0.6

$$\frac{bx^2c^2 + 3\,ac^2 + 2\,b}{3\,c^4}\sqrt{cx-1}\sqrt{cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(b*c^2*x^2+3*a*c^2+2*b)/c^4
```

---

**Maxima [A]** time = 1.41744, size = 73, normalized size = 1.12

$$\frac{\sqrt{c^2x^2-1}bx^2}{3\,c^2} + \frac{\sqrt{c^2x^2-1}a}{c^2} + \frac{2\,\sqrt{c^2x^2-1}b}{3\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*x/(sqrt(c*x + 1)*sqrt(c*x - 1)),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c^2*x^2 - 1)*b*x^2/c^2 + sqrt(c^2*x^2 - 1)*a/c^2 + 2/3*s
qrt(c^2*x^2 - 1)*b/c^4
```

---

**Fricas [A]** time = 0.242697, size = 216, normalized size = 3.32

$$-\frac{4\,bc^6x^6 + 3\left(4\,ac^6 + bc^4\right)x^4 + 3\,ac^2 - 3\left(5\,ac^4 + 3\,bc^2\right)x^2 - \left(4\,bc^5x^5 + \left(12\,ac^5 + 5\,bc^3\right)x^3 - 3\left(3\,ac^3 + 2\,bc\right)x\right)\sqrt{cx+1}\sqrt{cx-}}{3\left(4\,c^7x^3 - 3\,c^5x - \left(4\,c^6x^2 - c^4\right)\sqrt{cx+1}\sqrt{cx-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*x/(sqrt(c*x + 1)*sqrt(c*x - 1)),x, algorithm="fricas")
```

```
[Out] -1/3*(4*b*c^6*x^6 + 3*(4*a*c^6 + b*c^4)*x^4 + 3*a*c^2 - 3*(5*a*c^
4 + 3*b*c^2)*x^2 - (4*b*c^5*x^5 + (12*a*c^5 + 5*b*c^3)*x^3 - 3*(3
*a*c^3 + 2*b*c)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + 2*b)/(4*c^7*x^3
```

```
- 3*c^5*x - (4*c^6*x^2 - c^4)*sqrt(c*x + 1)*sqrt(c*x - 1))
```

---

**Sympy [A]** time = 57.3893, size = 202, normalized size = 3.11

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}$$
$$+\frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4}$$
$$+\frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/
2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**2) + I*a*meijerg(((-1,
 -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)
), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**2) + b*meijerg(
((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2,
0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**4) + I*b*meijerg(((-2, -7
/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)),
 exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**4)
```

---

**GIAC/XCAS [A]** time = 0.220132, size = 74, normalized size = 1.14

$$\frac{\left(3\,ac^{11}+3\,bc^9+\left((cx+1)bc^9-2\,bc^9\right)(cx+1)\right)\sqrt{cx+1}\sqrt{cx-1}}{1920\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*x/(sqrt(c*x + 1)*sqrt(c*x - 1)),x, algorithm="giac")
```

```
[Out] 1/1920*(3*a*c^11 + 3*b*c^9 + ((c*x + 1)*b*c^9 - 2*b*c^9)*(c*x + 1
))*sqrt(c*x + 1)*sqrt(c*x - 1)/c
```

## 3.253 $\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

**Optimal**. Leaf size=47

$$\frac{(2ac^2+b)\cosh^{-1}(cx)}{2c^3}+\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{2c^2}$$

```
[Out] (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ((b + 2*a*c^2)*ArcCo
sh[c*x])/(2*c^3)
```

---

**Rubi [A]** time = 0.0904182, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{(2ac^2+b)\cosh^{-1}(cx)}{2c^3}+\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ((b + 2*a*c^2)*ArcCo
sh[c*x])/(2*c^3)
```

---

**Rubi in Sympy [A]** time = 8.93401, size = 44, normalized size = 0.94

$$\frac{a\,\mathrm{acosh}\,(cx)}{c}+\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{2c^2}+\frac{b\,\mathrm{acosh}\,(cx)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*acosh(c*x)/c + b*x*sqrt(c*x - 1)*sqrt(c*x + 1)/(2*c**2) + b*aco
sh(c*x)/(2*c**3)
```

---

**Mathematica [A]** time = 0.0543974, size = 63, normalized size = 1.34

$$\frac{(2ac^2+b)\log\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)+bcx\sqrt{cx-1}\sqrt{cx+1}}{2c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + (b + 2*a*c^2)*Log[c*x + Sqr
t[-1 + c*x]*Sqrt[1 + c*x]])/(2*c^3)
```

---

**Maple [C]** time = 0.023, size = 103, normalized size = 2.2

$$\frac{csgn\left(c\right)}{2\,c^{3}}\sqrt{cx-1}\sqrt{cx+1}\left(bx\sqrt{c^{2}x^{2}-1}csgn\left(c\right)c+2\,a\ln\left(\left(csgn\left(c\right)\sqrt{c^{2}x^{2}-1}+cx\right)csgn\left(c\right)\right)c^{2}+b\ln\left(\left(csgn\left(c\right)\sqrt{c^{2}x^{2}-1}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(b*x*(c^2*x^2-1)^(1/2)*csgn(c)*c+
2*a*ln((csgn(c)*(c^2*x^2-1)^(1/2)+c*x)*csgn(c))*c^2+b*ln((csgn(c)
*(c^2*x^2-1)^(1/2)+c*x)*csgn(c)))/(c^2*x^2-1)^(1/2)/c^3*csgn(c)
```

---

**Maxima [A]** time = 1.4254, size = 120, normalized size = 2.55

$$\frac{a\log\left(2\,c^{2}x+2\,\sqrt{c^{2}x^{2}-1}\sqrt{c^{2}}\right)}{\sqrt{c^{2}}}+\frac{\sqrt{c^{2}x^{2}-1}bx}{2\,c^{2}}+\frac{b\log\left(2\,c^{2}x+2\,\sqrt{c^{2}x^{2}-1}\sqrt{c^{2}}\right)}{2\,\sqrt{c^{2}}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)),x, algorithm="maxima")
```

```
[Out] a*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/sqrt(c^2) + 1/2*sq
rt(c^2*x^2 - 1)*b*x/c^2 + 1/2*b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)
*sqrt(c^2))/(sqrt(c^2)*c^2)
```

---

**Fricas [A]** time = 0.242437, size = 220, normalized size = 4.68

$$-\frac{2\,bc^{4}x^{4}-2\,bc^{2}x^{2}-\left(2\,bc^{3}x^{3}-bcx\right)\sqrt{cx+1}\sqrt{cx-1}-\left(2\,ac^{2}+2\left(2\,ac^{3}+bc\right)\sqrt{cx+1}\sqrt{cx-1}x-2\left(2\,ac^{4}+bc^{2}\right)x^{2}+b\right)\mathrm{lo}}{2\left(2\,c^{5}x^{2}-2\,\sqrt{cx+1}\sqrt{cx-1}c^{4}x-c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*c^4*x^4 - 2*b*c^2*x^2 - (2*b*c^3*x^3 - b*c*x)*sqrt(c*x
+ 1)*sqrt(c*x - 1) - (2*a*c^2 + 2*(2*a*c^3 + b*c)*sqrt(c*x + 1)*s
qrt(c*x - 1)*x - 2*(2*a*c^4 + b*c^2)*x^2 + b)*log(-c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)))/(2*c^5*x^2 - 2*sqrt(c*x + 1)*sqrt(c*x - 1)*
c^4*x - c^3)
```

---

**Sympy [A]** time = 37.0999, size = 182, normalized size = 3.87

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c}-\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1\\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^3}-\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1,
0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c) - I*a*meijerg(((-1/2, -1/
4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_pola
r(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c) + b*meijerg(((-3/4, -1/4),
 (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(c**2
*x**2))/(4*pi**(3/2)*c**3) - I*b*meijerg(((-3/2, -5/4, -1, -3/4,
-1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0)), exp_polar(2*I*p
i)/(c**2*x**2))/(4*pi**(3/2)*c**3)
```

---

**GIAC/XCAS [A]** time = 0.246122, size = 96, normalized size = 2.04

$$\frac{\left((cx+1)bc^4-bc^4\right)\sqrt{cx+1}\sqrt{cx-1}-2\left(2\,ac^6+bc^4\right)\ln\left(\left|-\sqrt{cx+1}+\sqrt{cx-1}\right|\right)}{384\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)),x, algorithm="giac")
```

```
[Out] 1/384*(((c*x + 1)*b*c^4 - b*c^4)*sqrt(c*x + 1)*sqrt(c*x - 1) - 2*
(2*a*c^6 + b*c^4)*ln(abs(-sqrt(c*x + 1) + sqrt(c*x - 1))))/c
```

## 3.254 $\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

**Optimal**. Leaf size=46

$$a\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^2}$$

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c^2 + a*ArcTan[Sqrt[-1 + c*x]*Sq
rt[1 + c*x]]
```

---

**Rubi [A]** time = 0.211028, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$a\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c^2 + a*ArcTan[Sqrt[-1 + c*x]*Sq
rt[1 + c*x]]
```

---

**Rubi in Sympy [A]** time = 9.71313, size = 41, normalized size = 0.89

$$a\,\mathrm{atan}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/x/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*atan(sqrt(c*x - 1)*sqrt(c*x + 1)) + b*sqrt(c*x - 1)*sqrt(c*x +
1)/c**2
```

---

**Mathematica [A]** time = 0.0651869, size = 47, normalized size = 1.02

$$\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^2}-a\tan^{-1}\left(\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c^2 - a*ArcTan[1/(Sqrt[-1 + c*x]
*Sqrt[1 + c*x])]
```

---

**Maple [A]** time = 0.026, size = 62, normalized size = 1.4

$$\frac{1}{c^2}\left(-a\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^2+b\sqrt{c^2x^2-1}\right)\sqrt{cx-1}\sqrt{cx+1}\frac{1}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] (-a*arctan(1/(c^2*x^2-1)^(1/2))*c^2+b*(c^2*x^2-1)^(1/2))*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)/c^2
```

---

**Maxima [A]** time = 1.70736, size = 42, normalized size = 0.91

$$-a\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)+\frac{\sqrt{c^2x^2-1}b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x),x, algorithm="maxima")
```

```
[Out] -a*arcsin(1/(sqrt(c^2)*abs(x))) + sqrt(c^2*x^2 - 1)*b/c^2
```

---

**Fricas [A]** time = 0.242114, size = 149, normalized size = 3.24

$$-\frac{bc^2x^2-\sqrt{cx+1}\sqrt{cx-1}bcx-2\left(ac^3x-\sqrt{cx+1}\sqrt{cx-1}ac^2\right)\arctan\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)-b}{c^3x-\sqrt{cx+1}\sqrt{cx-1}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x),x, algorithm="fricas")
```

```
[Out] -(b*c^2*x^2 - sqrt(c*x + 1)*sqrt(c*x - 1)*b*c*x - 2*(a*c^3*x - sq
rt(c*x + 1)*sqrt(c*x - 1)*a*c^2)*arctan(-c*x + sqrt(c*x + 1)*sqrt
```

```
(c*x - 1)) - b)/(c^3*x - sqrt(c*x + 1)*sqrt(c*x - 1)*c^2)
```

---

**Sympy [A]** time = 36.4893, size = 162, normalized size = 3.52

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}+\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/x/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2)
, (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2
, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*
pi)/(c**2*x**2))/(4*pi**(3/2)) + b*meijerg(((-1/4, 1/4), (0, 0, 1
/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()), 1/(c**2*x**2))/(4*pi
**(3/2)*c**2) + I*b*meijerg(((-1, -3/4, -1/2, -1/4, 0, 1), ()), (
(-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2)
)/(4*pi**(3/2)*c**2)
```

---

**GIAC/XCAS [A]** time = 0.218052, size = 61, normalized size = 1.33

$$-2\,a\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)+\frac{\sqrt{cx+1}\sqrt{cx-1}b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) + sqrt(c*x + 1
)*sqrt(c*x - 1)*b/c^2
```

## 3.255 $\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

**Optimal**. Leaf size=33

$$\frac{a\sqrt{cx-1}\sqrt{cx+1}}{x}+\frac{b\cosh^{-1}(cx)}{c}$$

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/x + (b*ArcCosh[c*x])/c
```

---

**Rubi [A]** time = 0.183341, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069

$$\frac{a\sqrt{cx-1}\sqrt{cx+1}}{x}+\frac{b\cosh^{-1}(cx)}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]

[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/x + (b*ArcCosh[c*x])/c
```

---

**Rubi in Sympy [A]** time = 8.5764, size = 27, normalized size = 0.82

$$\frac{a\sqrt{cx-1}\sqrt{cx+1}}{x}+\frac{b\,\mathrm{acosh}\,(cx)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/x**2/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)

[Out] a*sqrt(c*x - 1)*sqrt(c*x + 1)/x + b*acosh(c*x)/c
```

---

**Mathematica [A]** time = 0.0546701, size = 53, normalized size = 1.61

$$\frac{a\sqrt{cx-1}\sqrt{cx+1}}{x}+\frac{b\log\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/x + (b*Log[c*x + Sqrt[-1 + c*x]*
Sqrt[1 + c*x]])/c
```

---

**Maple [C]** time = 0.029, size = 77, normalized size = 2.3

$$\frac{csgn(c)}{cx}\sqrt{cx-1}\sqrt{cx+1}\left(a\sqrt{c^2x^2-1}csgn(c)\,c+b\ln\left(\left(\left(csgn(c)\,\sqrt{c^2x^2-1}+cx\right)\,csgn(c)\right)x\right)\frac{1}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] (c*x-1)^(1/2)*(c*x+1)^(1/2)*(a*(c^2*x^2-1)^(1/2)*csgn(c)*c+b*ln((
csgn(c)*(c^2*x^2-1)^(1/2)+c*x)*csgn(c))*x)*csgn(c)/(c^2*x^2-1)^(1
/2)/x/c
```

---

**Maxima [A]** time = 1.69408, size = 68, normalized size = 2.06

$$\frac{b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{\sqrt{c^2}}+\frac{\sqrt{c^2x^2-1}a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x^2),x, algorithm="maxima")
```

```
[Out] b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/sqrt(c^2) + sqrt(c
^2*x^2 - 1)*a/x
```

---

**Fricas [A]** time = 0.239542, size = 109, normalized size = 3.3

$$\frac{ac-\left(bcx^2-\sqrt{cx+1}\sqrt{cx-1}bx\right)\log\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{c^2x^2-\sqrt{cx+1}\sqrt{cx-1}cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x^2),x, algorithm="fricas")
```

```
[Out] (a*c - (b*c*x^2 - sqrt(c*x + 1)*sqrt(c*x - 1)*b*x)*log(-c*x + sqr
t(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^2 - sqrt(c*x + 1)*sqrt(c*x - 1)
*c*x)
```

---

**Sympy [A]** time = 42.4737, size = 148, normalized size = 4.48

$$-\frac{acG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{iacG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c}-\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/x**2/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*c*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4,
2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) - I*a*c*meijerg(((1/2, 3/
4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(
2*I*pi)/(c**2*x**2))/(4*pi**(3/2)) + b*meijerg(((1/4, 3/4), (1/2,
 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 1/(c**2*x**2))/(4*p
i**(3/2)*c) - I*b*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-
1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi
**(3/2)*c)
```

---

**GIAC/XCAS [A]** time = 0.223461, size = 78, normalized size = 2.36

$$\frac{\frac{16\,ac^2}{\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+4}-b\ln\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(16*a*c^2/((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4) - b*ln((sqr
t(c*x + 1) - sqrt(c*x - 1))^4))/c
```

## 3.256 $\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

**Optimal**. Leaf size=60

$$\frac{1}{2}\left(ac^2+2b\right)\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{2x^2}$$

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x^2) + ((2*b + a*c^2)*ArcTan[
Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/2
```

---

**Rubi [A]** time = 0.215672, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{1}{2}\left(ac^2+2b\right)\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x^2) + ((2*b + a*c^2)*ArcTan[
Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/2
```

---

**Rubi in Sympy [A]** time = 9.67229, size = 49, normalized size = 0.82

$$\frac{a\sqrt{cx-1}\sqrt{cx+1}}{2x^2}+\left(\frac{ac^2}{2}+b\right)\operatorname{atan}\left(\sqrt{cx-1}\sqrt{cx+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/x**3/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*sqrt(c*x - 1)*sqrt(c*x + 1)/(2*x**2) + (a*c**2/2 + b)*atan(sqrt
(c*x - 1)*sqrt(c*x + 1))
```

---

**Mathematica [A]** time = 0.0891357, size = 61, normalized size = 1.02

$$\frac{1}{2}\left(-ac^2-2b\right)\tan^{-1}\left(\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x^2) + ((-2*b - a*c^2)*ArcTan
[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])])/2
```

---

**Maple [A]** time = 0.028, size = 84, normalized size = 1.4

$$-\frac{1}{2x^2}\sqrt{cx-1}\sqrt{cx+1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)ac^2x^2+2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)bx^2-a\sqrt{c^2x^2-1}\right)\frac{1}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] -1/2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(arctan(1/(c^2*x^2-1)^(1/2))*a*c
^2*x^2+2*arctan(1/(c^2*x^2-1)^(1/2))*b*x^2-a*(c^2*x^2-1)^(1/2))/(
c^2*x^2-1)^(1/2)/x^2
```

---

**Maxima [A]** time = 1.54408, size = 66, normalized size = 1.1

$$-\frac{1}{2}ac^2\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)-b\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)+\frac{\sqrt{c^2x^2-1}a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x^3),x, algorithm="maxima")
```

```
[Out] -1/2*a*c^2*arcsin(1/(sqrt(c^2)*abs(x))) - b*arcsin(1/(sqrt(c^2)*a
bs(x))) + 1/2*sqrt(c^2*x^2 - 1)*a/x^2
```

---

**Fricas [A]** time = 0.242027, size = 223, normalized size = 3.72

$$-\frac{2ac^3x^3-2acx-\left(2ac^2x^2-a\right)\sqrt{cx+1}\sqrt{cx-1}+2\left(2\left(ac^3+2bc\right)\sqrt{cx+1}\sqrt{cx-1}x^3-2\left(ac^4+2bc^2\right)x^4+\left(ac^2+2b\right)x^2\right.}{2\left(2c^2x^4-2\sqrt{cx+1}\sqrt{cx-1}cx^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x^3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*c^3*x^3 - 2*a*c*x - (2*a*c^2*x^2 - a)*sqrt(c*x + 1)*sqr
t(c*x - 1) + 2*(2*(a*c^3 + 2*b*c)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^3
 - 2*(a*c^4 + 2*b*c^2)*x^4 + (a*c^2 + 2*b)*x^2)*arctan(-c*x + sqr
t(c*x + 1)*sqrt(c*x - 1)))/(2*c^2*x^4 - 2*sqrt(c*x + 1)*sqrt(c*x
- 1)*c*x^3 - x^2)
```

---

**Sympy [A]** time = 56.1081, size = 141, normalized size = 2.35

$$-\frac{ac^2G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iac^2G_{6,6}^{2,6}\left(\begin{matrix}1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2,1 & \\ \frac{5}{4},\frac{7}{4} & 1,\frac{3}{2},\frac{3}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/x**3/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*c**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4,
 5/2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*a*c**2*meijerg(((1
, 5/4, 3/2, 7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_
polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)) - b*meijerg(((3/4, 5/4,
1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1/(c**2*x**2))
/(4*pi**(3/2)) + I*b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/
4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi*
*(3/2))
```

---

**GIAC/XCAS [A]** time = 0.22855, size = 154, normalized size = 2.57

$$-\frac{\left(ac^3+2\,bc\right)\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)+\frac{2\left(ac^3\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^6-4\,ac^3\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)}{\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+4\right)^2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x^3),x, algorithm="giac")
```

```
[Out] -((a*c^3 + 2*b*c)*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) +
 2*(a*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1))^6 - 4*a*c^3*(sqrt(c*x +
 1) - sqrt(c*x - 1))^2)/((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4)^2
)/c
```

## 3.257 $\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

**Optimal**. Leaf size=62

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2ac^2+3b\right)}{3x}+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{3x^3}$$

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*x^3) + ((3*b + 2*a*c^2)*Sqrt[
-1 + c*x]*Sqrt[1 + c*x])/(3*x)
```

---

**Rubi [A]** time = 0.217287, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2ac^2+3b\right)}{3x}+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*x^3) + ((3*b + 2*a*c^2)*Sqrt[
-1 + c*x]*Sqrt[1 + c*x])/(3*x)
```

---

**Rubi in Sympy [A]** time = 9.31846, size = 51, normalized size = 0.82

$$\frac{a\sqrt{cx-1}\sqrt{cx+1}}{3x^3}+\frac{\left(\frac{2ac^2}{3}+b\right)\sqrt{cx-1}\sqrt{cx+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/x**4/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*sqrt(c*x - 1)*sqrt(c*x + 1)/(3*x**3) + (2*a*c**2/3 + b)*sqrt(c*
x - 1)*sqrt(c*x + 1)/x
```

---

**Mathematica [A]** time = 0.0517534, size = 42, normalized size = 0.68

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2ac^2x^2+a+3bx^2\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + 3*b*x^2 + 2*a*c^2*x^2))/(3*x^3
)
```

---

**Maple [A]** time = 0.008, size = 37, normalized size = 0.6

$$\frac{2\,ac^2x^2+3\,bx^2+a}{3\,x^3}\sqrt{cx-1}\sqrt{cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(2*a*c^2*x^2+3*b*x^2+a)/x^3
```

---

**Maxima [A]** time = 1.61251, size = 73, normalized size = 1.18

$$\frac{2\sqrt{c^2x^2-1}ac^2}{3\,x}+\frac{\sqrt{c^2x^2-1}b}{x}+\frac{\sqrt{c^2x^2-1}a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x^4),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(c^2*x^2 - 1)*a*c^2/x + sqrt(c^2*x^2 - 1)*b/x + 1/3*sqrt(
c^2*x^2 - 1)*a/x^3
```

---

**Fricas [A]** time = 0.23031, size = 140, normalized size = 2.26

$$\frac{6\,bc^2x^4+3\left(ac^2-b\right)x^2-3\left(2\,bcx^3+acx\right)\sqrt{cx+1}\sqrt{cx-1}-a}{3\left(4\,c^3x^6-3\,cx^4-(4\,c^2x^5-x^3)\sqrt{cx+1}\sqrt{cx-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x^4),x, algorithm="fricas")
```

```
[Out] 1/3*(6*b*c^2*x^4 + 3*(a*c^2 - b)*x^2 - 3*(2*b*c*x^3 + a*c*x)*sqrt
(c*x + 1)*sqrt(c*x - 1) - a)/(4*c^3*x^6 - 3*c*x^4 - (4*c^2*x^5 -
x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))
```

**Sympy [A]** time = 84.0966, size = 146, normalized size = 2.35

$$-\frac{ac^{3}G_{6,6}^{5,3}\left(\begin{matrix}\frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3\\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0\end{matrix}\middle|\frac{1}{c^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}-\frac{iac^{3}G_{6,6}^{2,6}\left(\begin{matrix}\frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0\end{matrix}\middle|\frac{e^{2i\pi}}{c^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}-\frac{bcG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{c^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}-\frac{ibcG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{2i\pi}}{c^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/x**4/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*c**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 1
1/4, 3), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) - I*a*c**3*meijerg((
(3/2, 7/4, 2, 9/4, 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), ex
p_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)) - b*c*meijerg(((5/4, 7
/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), 1/(c**2*x*
*2))/(4*pi**(3/2)) - I*b*c*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), (
)), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(c**2*x**2))/
(4*pi**(3/2))
```

**GIAC/XCAS [A]** time = 0.224913, size = 157, normalized size = 2.53

$$\frac{8\left(3\,bc^{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{8}+24\,ac^{4}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{4}+24\,bc^{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{4}+32\,ac^{4}+48\,bc^{2}\right)}{3\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{4}+4\right)^{3}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x^4),x, algorithm="giac")
```

```
[Out] 8/3*(3*b*c^2*(sqrt(c*x + 1) - sqrt(c*x - 1))^8 + 24*a*c^4*(sqrt(c
*x + 1) - sqrt(c*x - 1))^4 + 24*b*c^2*(sqrt(c*x + 1) - sqrt(c*x -
 1))^4 + 32*a*c^4 + 48*b*c^2)/(((sqrt(c*x + 1) - sqrt(c*x - 1))^4
 + 4)^3*c)
```

## 3.258 $\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

**Optimal**. Leaf size=99

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(3ac^2+4b\right)}{8x^2}+\frac{1}{8}c^2\left(3ac^2+4b\right)\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{4x^4}$$

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*x^4) + ((4*b + 3*a*c^2)*Sqrt[
-1 + c*x]*Sqrt[1 + c*x])/(8*x^2) + (c^2*(4*b + 3*a*c^2)*ArcTan[Sq
rt[-1 + c*x]*Sqrt[1 + c*x]])/8
```

---

**Rubi [A]** time = 0.285726, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(3ac^2+4b\right)}{8x^2}+\frac{1}{8}c^2\left(3ac^2+4b\right)\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{4x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*x^4) + ((4*b + 3*a*c^2)*Sqrt[
-1 + c*x]*Sqrt[1 + c*x])/(8*x^2) + (c^2*(4*b + 3*a*c^2)*ArcTan[Sq
rt[-1 + c*x]*Sqrt[1 + c*x]])/8
```

---

**Rubi in Sympy [A]** time = 13.7745, size = 88, normalized size = 0.89

$$\frac{a\sqrt{cx-1}\sqrt{cx+1}}{4x^4}+\frac{c^2\left(\frac{3ac^2}{4}+b\right)\operatorname{atan}\left(\sqrt{cx-1}\sqrt{cx+1}\right)}{2}+\frac{\left(\frac{3ac^2}{8}+\frac{b}{2}\right)\sqrt{cx-1}\sqrt{cx+1}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/x**5/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*sqrt(c*x - 1)*sqrt(c*x + 1)/(4*x**4) + c**2*(3*a*c**2/4 + b)*at
an(sqrt(c*x - 1)*sqrt(c*x + 1))/2 + (3*a*c**2/8 + b/2)*sqrt(c*x -
 1)*sqrt(c*x + 1)/x**2
```

---

**Mathematica [A]** time = 0.132844, size = 80, normalized size = 0.81

$$\frac{1}{8}\left(\frac{\sqrt{cx-1}\sqrt{cx+1}\left(x^2\left(3ac^2+4b\right)+2a\right)}{x^4}-c^2\left(3ac^2+4b\right)\tan^{-1}\left(\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2*a + (4*b + 3*a*c^2)*x^2))/x^4 -
 c^2*(4*b + 3*a*c^2)*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])])/8
```

---

**Maple [A]** time = 0.029, size = 125, normalized size = 1.3

$$-\frac{1}{8\,x^4}\sqrt{cx-1}\sqrt{cx+1}\left(3\,c^4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)ax^4+4\,c^2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)bx^4-3\,\sqrt{c^2x^2-1}ac^2x^2-4\,\sqrt{c^2x^2-1}bx^2-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] -1/8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(3*c^4*arctan(1/(c^2*x^2-1)^(1/2
))*a*x^4+4*c^2*arctan(1/(c^2*x^2-1)^(1/2))*b*x^4-3*(c^2*x^2-1)^(1
/2)*a*c^2*x^2-4*(c^2*x^2-1)^(1/2)*b*x^2-2*a*(c^2*x^2-1)^(1/2))/(c
^2*x^2-1)^(1/2)/x^4
```

---

**Maxima [A]** time = 1.58707, size = 120, normalized size = 1.21

$$-\frac{3}{8}\,ac^4\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)-\frac{1}{2}\,bc^2\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)+\frac{3\,\sqrt{c^2x^2-1}ac^2}{8\,x^2}+\frac{\sqrt{c^2x^2-1}b}{2\,x^2}+\frac{\sqrt{c^2x^2-1}a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x^5),x, algorithm="maxima")
```

```
[Out] -3/8*a*c^4*arcsin(1/(sqrt(c^2)*abs(x))) - 1/2*b*c^2*arcsin(1/(sqr
t(c^2)*abs(x))) + 3/8*sqrt(c^2*x^2 - 1)*a*c^2/x^2 + 1/2*sqrt(c^2*
x^2 - 1)*b/x^2 + 1/4*sqrt(c^2*x^2 - 1)*a/x^4
```

---

**Fricas [A]** time = 0.244998, size = 423, normalized size = 4.27

$$-\frac{8\left(3\,ac^7+4\,bc^5\right)x^7-4\left(5\,ac^5+12\,bc^3\right)x^5-4\left(3\,ac^3-4\,bc\right)x^3+8\,acx-\left(8\left(3\,ac^6+4\,bc^4\right)x^6-8\left(ac^4+4\,bc^2\right)x^4-\left(13\,ac^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x^5),x, algorithm="fricas")
```

```
[Out] -1/8*(8*(3*a*c^7 + 4*b*c^5)*x^7 - 4*(5*a*c^5 + 12*b*c^3)*x^5 - 4*
(3*a*c^3 - 4*b*c)*x^3 + 8*a*c*x - (8*(3*a*c^6 + 4*b*c^4)*x^6 - 8*
(a*c^4 + 4*b*c^2)*x^4 - (13*a*c^2 - 4*b)*x^2 + 2*a)*sqrt(c*x + 1)
*sqrt(c*x - 1) - 2*(8*(3*a*c^8 + 4*b*c^6)*x^8 - 8*(3*a*c^6 + 4*b*
c^4)*x^6 + (3*a*c^4 + 4*b*c^2)*x^4 - 4*(2*(3*a*c^7 + 4*b*c^5)*x^7
 - (3*a*c^5 + 4*b*c^3)*x^5)*sqrt(c*x + 1)*sqrt(c*x - 1))*arctan(-
c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(8*c^4*x^8 - 8*c^2*x^6 + x^4
- 4*(2*c^3*x^7 - c*x^5)*sqrt(c*x + 1)*sqrt(c*x - 1))
```

---

**Sympy [A]** time = 124.583, size = 148, normalized size = 1.49

$$-\frac{ac^4 G^{5,3}_{6,6}\left(\begin{matrix} \frac{11}{4}, \frac{13}{4}, 1 & 3, 3, \frac{7}{2} \\ \frac{5}{2}, \frac{11}{4}, 3, \frac{13}{4}, \frac{7}{2} & 0 \end{matrix} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{iac^4 G^{2,6}_{6,6}\left(\begin{matrix} 2, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}, 3, 1 & \\ \frac{9}{4}, \frac{11}{4} & 2, \frac{5}{2}, \frac{5}{2}, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{bc^2 G^{5,3}_{6,6}\left(\begin{matrix} \frac{7}{4}, \frac{9}{4}, 1 & 2, 2, \frac{5}{2} \\ \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2} & 0 \end{matrix} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{ibc^2 G^{2,6}_{6,6}\left(\begin{matrix} 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 1 & \\ \frac{5}{4}, \frac{7}{4} & 1, \frac{3}{2}, \frac{3}{2}, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/x**5/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*c**4*meijerg(((11/4, 13/4, 1), (3, 3, 7/2)), ((5/2, 11/4, 3, 1
3/4, 7/2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*a*c**4*meijerg
(((2, 9/4, 5/2, 11/4, 3, 1), ()), ((9/4, 11/4), (2, 5/2, 5/2, 0))
, exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)) - b*c**2*meijerg((
(7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,)), 1/(
c**2*x**2))/(4*pi**(3/2)) + I*b*c**2*meijerg(((1, 5/4, 3/2, 7/4,
2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(2*I*pi)/(c*
*2*x**2))/(4*pi**(3/2))
```

---

**GIAC/XCAS [A]** time = 0.229307, size = 362, normalized size = 3.66

$$-\frac{\left(3\,ac^5 + 4\,bc^3\right)\arctan\left(\frac{1}{2}\left(\sqrt{cx+1} - \sqrt{cx-1}\right)^2\right) + \frac{2\left(3\,ac^5\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{14} + 4\,bc^3\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{14} + 44\,ac^5\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{10} + 16\,bc^3\left(\right.\right.}{}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x^5),x, algorithm="giac")
```

```
[Out] -1/4*((3*a*c^5 + 4*b*c^3)*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x -
1))^2) + 2*(3*a*c^5*(sqrt(c*x + 1) - sqrt(c*x - 1))^14 + 4*b*c^3*
(sqrt(c*x + 1) - sqrt(c*x - 1))^14 + 44*a*c^5*(sqrt(c*x + 1) - sq
rt(c*x - 1))^10 + 16*b*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1))^10 - 1
76*a*c^5*(sqrt(c*x + 1) - sqrt(c*x - 1))^6 - 64*b*c^3*(sqrt(c*x +
 1) - sqrt(c*x - 1))^6 - 192*a*c^5*(sqrt(c*x + 1) - sqrt(c*x - 1)
)^2 - 256*b*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1))^2)/((sqrt(c*x + 1
) - sqrt(c*x - 1))^4 + 4)^4)/c
```

## 3.259 $\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=164

$$\frac{c^2x\sqrt{dx-c}\sqrt{c+dx}\left(6ad^2+5bc^2\right)}{16d^6}+\frac{x^3\sqrt{dx-c}\sqrt{c+dx}\left(6ad^2+5bc^2\right)}{24d^4}+\frac{c^4\left(6ad^2+5bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^7}+\frac{bx^5\sqrt{dx-c}\sqrt{c+dx}}{6d^2}$$

```
[Out] (c^2*(5*b*c^2 + 6*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(16*d^6)
 + ((5*b*c^2 + 6*a*d^2)*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x])/(24*d^4
) + (b*x^5*Sqrt[-c + d*x]*Sqrt[c + d*x])/(6*d^2) + (c^4*(5*b*c^2
+ 6*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(8*d^7)
```

---

**Rubi [A]** time = 0.422232, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194

$$\frac{c^2x\sqrt{dx-c}\sqrt{c+dx}\left(6ad^2+5bc^2\right)}{16d^6}+\frac{x^3\sqrt{dx-c}\sqrt{c+dx}\left(6ad^2+5bc^2\right)}{24d^4}+\frac{c^4\left(6ad^2+5bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^7}+\frac{bx^5\sqrt{dx-c}\sqrt{c+dx}}{6d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (c^2*(5*b*c^2 + 6*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(16*d^6)
 + ((5*b*c^2 + 6*a*d^2)*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x])/(24*d^4
) + (b*x^5*Sqrt[-c + d*x]*Sqrt[c + d*x])/(6*d^2) + (c^4*(5*b*c^2
+ 6*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(8*d^7)
```

---

**Rubi in Sympy [A]** time = 26.1904, size = 146, normalized size = 0.89

$$\frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2}+\frac{c^4\left(6ad^2+5bc^2\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{8d^7}+\frac{c^2x\sqrt{-c+dx}\sqrt{c+dx}\left(6ad^2+5bc^2\right)}{16d^6}+\frac{x^3\sqrt{-c+dx}\sqrt{c+dx}\left(6ad^2+5bc^2\right)}{24d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] b*x**5*sqrt(-c + d*x)*sqrt(c + d*x)/(6*d**2) + c**4*(6*a*d**2 + 5
*b*c**2)*atanh(sqrt(c + d*x)/sqrt(-c + d*x))/(8*d**7) + c**2*x*sq
rt(-c + d*x)*sqrt(c + d*x)*(6*a*d**2 + 5*b*c**2)/(16*d**6) + x**3
*sqrt(-c + d*x)*sqrt(c + d*x)*(6*a*d**2 + 5*b*c**2)/(24*d**4)
```

---

**Mathematica [A]** time = 0.170729, size = 123, normalized size = 0.75

$$\frac{3\left(6ac^4d^2+5bc^6\right)\log\left(\sqrt{dx-c}\sqrt{c+dx}+dx\right)+dx\sqrt{dx-c}\sqrt{c+dx}\left(6ad^2\left(3c^2+2d^2x^2\right)+b\left(15c^4+10c^2d^2x^2+8d^4x^4\right)\right)}{48d^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(6*a*d^2*(3*c^2 + 2*d^2*x^2) +
b*(15*c^4 + 10*c^2*d^2*x^2 + 8*d^4*x^4)) + 3*(5*b*c^6 + 6*a*c^4*d
^2)*Log[d*x + Sqrt[-c + d*x]*Sqrt[c + d*x]])/(48*d^7)
```

---

**Maple [C]** time = 0.04, size = 240, normalized size = 1.5

$$\frac{csgn(d)}{48\,d^7}\sqrt{dx-c}\sqrt{dx+c}\left(8\,csgn(d)\,x^5bd^5\sqrt{d^2x^2-c^2}+12\,csgn(d)\,x^3ad^5\sqrt{d^2x^2-c^2}+10\,csgn(d)\,x^3bc^2d^3\sqrt{d^2x^2-c^2}+18\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/48*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(8*csgn(d)*x^5*b*d^5*(d^2*x^2-c^
2)^(1/2)+12*csgn(d)*x^3*a*d^5*(d^2*x^2-c^2)^(1/2)+10*csgn(d)*x^3*
b*c^2*d^3*(d^2*x^2-c^2)^(1/2)+18*a*c^2*x*(d^2*x^2-c^2)^(1/2)*d^3*
csgn(d)+15*b*c^4*x*(d^2*x^2-c^2)^(1/2)*csgn(d)*d+18*a*c^4*ln((csg
n(d)*(d^2*x^2-c^2)^(1/2)+d*x)*csgn(d))*d^2+15*b*c^6*ln((csgn(d)*(
d^2*x^2-c^2)^(1/2)+d*x)*csgn(d)))*csgn(d)/(d^2*x^2-c^2)^(1/2)/d^7
```

---

**Maxima [A]** time = 1.39756, size = 289, normalized size = 1.76

$$\frac{\sqrt{d^2x^2-c^2}bx^5}{6\,d^2}+\frac{5\,\sqrt{d^2x^2-c^2}bc^2x^3}{24\,d^4}+\frac{\sqrt{d^2x^2-c^2}ax^3}{4\,d^2}+\frac{5\,bc^6\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{16\,\sqrt{d^2}d^6}$$
$$+\frac{3\,ac^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{8\,\sqrt{d^2}d^4}+\frac{5\,\sqrt{d^2x^2-c^2}bc^4x}{16\,d^6}+\frac{3\,\sqrt{d^2x^2-c^2}ac^2x}{8\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^4/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(d^2*x^2 - c^2)*b*x^5/d^2 + 5/24*sqrt(d^2*x^2 - c^2)*b*c^
2*x^3/d^4 + 1/4*sqrt(d^2*x^2 - c^2)*a*x^3/d^2 + 5/16*b*c^6*log(2*
d^2*x + 2*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^6) + 3/8*a*
c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^4
) + 5/16*sqrt(d^2*x^2 - c^2)*b*c^4*x/d^6 + 3/8*sqrt(d^2*x^2 - c^2
)*a*c^2*x/d^4
```

---

**Fricas [A]** time = 0.361266, size = 734, normalized size = 4.48

$$-\frac{256\,bd^{12}x^{12} - 192\left(bc^{2}d^{10} - 2\,ad^{12}\right)x^{10} + 48\left(3\,bc^{4}d^{8} - 4\,ac^{2}d^{10}\right)x^{8} - 4\left(157\,bc^{6}d^{6} + 174\,ac^{4}d^{8}\right)x^{6} + 102\left(5\,bc^{8}d^{4} + 6\,ac^{6}d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^4/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="fricas")
```

```
[Out] -1/48*(256*b*d^12*x^12 - 192*(b*c^2*d^10 - 2*a*d^12)*x^10 + 48*(3
*b*c^4*d^8 - 4*a*c^2*d^10)*x^8 - 4*(157*b*c^6*d^6 + 174*a*c^4*d^8
)*x^6 + 102*(5*b*c^8*d^4 + 6*a*c^6*d^6)*x^4 - 18*(5*b*c^10*d^2 +
6*a*c^8*d^4)*x^2 - (256*b*d^11*x^11 + 144*b*c^4*d^7*x^7 - 64*(b*c
^2*d^9 - 6*a*d^11)*x^9 - 4*(137*b*c^6*d^5 + 162*a*c^4*d^7)*x^5 +
52*(5*b*c^8*d^3 + 6*a*c^6*d^5)*x^3 - 3*(5*b*c^10*d + 6*a*c^8*d^3)
*x)*sqrt(d*x + c)*sqrt(d*x - c) - 3*(5*b*c^12 + 6*a*c^10*d^2 - 32
*(5*b*c^6*d^6 + 6*a*c^4*d^8)*x^6 + 48*(5*b*c^8*d^4 + 6*a*c^6*d^6)
*x^4 - 18*(5*b*c^10*d^2 + 6*a*c^8*d^4)*x^2 + 2*(16*(5*b*c^6*d^5 +
 6*a*c^4*d^7)*x^5 - 16*(5*b*c^8*d^3 + 6*a*c^6*d^5)*x^3 + 3*(5*b*c
^10*d + 6*a*c^8*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c))*log(-d*x + s
qrt(d*x + c)*sqrt(d*x - c)))/(32*d^13*x^6 - 48*c^2*d^11*x^4 + 18*
c^4*d^9*x^2 - c^6*d^7 - 2*(16*d^12*x^5 - 16*c^2*d^10*x^3 + 3*c^4*
d^8*x)*sqrt(d*x + c)*sqrt(d*x - c))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [A]** time = 0.258263, size = 246, normalized size = 1.5

$$-\frac{\left(33\,bc^{5}d^{36}+30\,ac^{3}d^{38}-\left(85\,bc^{4}d^{36}+54\,ac^{2}d^{38}-2\left(55\,bc^{3}d^{36}+18\,acd^{38}-\left(45\,bc^{2}d^{36}+6\,ad^{38}+4\left((dx+c)bd^{36}-5\,bcd^{36}\right.\right.\right.\right.\right.}{34603008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^4/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="giac")
```

```
[Out] -1/34603008*((33*b*c^5*d^36 + 30*a*c^3*d^38 - (85*b*c^4*d^36 + 54
*a*c^2*d^38 - 2*(55*b*c^3*d^36 + 18*a*c*d^38 - (45*b*c^2*d^36 + 6
*a*d^38 + 4*((d*x + c)*b*d^36 - 5*b*c*d^36)*(d*x + c))*(d*x + c))
*(d*x + c))*(d*x + c))*sqrt(d*x + c)*sqrt(d*x - c) + 6*(5*b*c^6*d
^36 + 6*a*c^4*d^38)*ln(abs(-sqrt(d*x + c) + sqrt(d*x - c))))/d
```

## 3.260 $\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=118

$$\frac{2c^2\sqrt{dx-c}\sqrt{c+dx}\,(5ad^2+4bc^2)}{15d^6}+\frac{x^2\sqrt{dx-c}\sqrt{c+dx}\,(5ad^2+4bc^2)}{15d^4}+\frac{bx^4\sqrt{dx-c}\sqrt{c+dx}}{5d^2}$$

```
[Out] (2*c^2*(4*b*c^2 + 5*a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(15*d^6)
 + ((4*b*c^2 + 5*a*d^2)*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x])/(15*d^4
) + (b*x^4*Sqrt[-c + d*x]*Sqrt[c + d*x])/(5*d^2)
```

---

**Rubi [A]** time = 0.305834, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129

$$\frac{2c^2\sqrt{dx-c}\sqrt{c+dx}\,(5ad^2+4bc^2)}{15d^6}+\frac{x^2\sqrt{dx-c}\sqrt{c+dx}\,(5ad^2+4bc^2)}{15d^4}+\frac{bx^4\sqrt{dx-c}\sqrt{c+dx}}{5d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*c^2*(4*b*c^2 + 5*a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(15*d^6)
 + ((4*b*c^2 + 5*a*d^2)*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x])/(15*d^4
) + (b*x^4*Sqrt[-c + d*x]*Sqrt[c + d*x])/(5*d^2)
```

---

**Rubi in Sympy [A]** time = 17.574, size = 105, normalized size = 0.89

$$\frac{bx^4\sqrt{-c+dx}\sqrt{c+dx}}{5d^2}+\frac{2c^2\sqrt{-c+dx}\sqrt{c+dx}\,(5ad^2+4bc^2)}{15d^6}+\frac{x^2\sqrt{-c+dx}\sqrt{c+dx}\,(5ad^2+4bc^2)}{15d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] b*x**4*sqrt(-c + d*x)*sqrt(c + d*x)/(5*d**2) + 2*c**2*sqrt(-c + d
*x)*sqrt(c + d*x)*(5*a*d**2 + 4*b*c**2)/(15*d**6) + x**2*sqrt(-c
+ d*x)*sqrt(c + d*x)*(5*a*d**2 + 4*b*c**2)/(15*d**4)
```

---

**Mathematica [A]** time = 0.079503, size = 74, normalized size = 0.63

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\,\left(5ad^2\left(2c^2+d^2x^2\right)+b\left(8c^4+4c^2d^2x^2+3d^4x^4\right)\right)}{15d^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(5*a*d^2*(2*c^2 + d^2*x^2) + b*(8*c
^4 + 4*c^2*d^2*x^2 + 3*d^4*x^4)))/(15*d^6)
```

---

**Maple [A]** time = 0.009, size = 68, normalized size = 0.6

$$\frac{3\,b{d}^{4}{x}^{4}+5\,a{d}^{4}{x}^{2}+4\,b{c}^{2}{d}^{2}{x}^{2}+10\,a{c}^{2}{d}^{2}+8\,b{c}^{4}}{15\,{d}^{6}}\sqrt{dx+c}\sqrt{dx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/15*(d*x+c)^(1/2)*(3*b*d^4*x^4+5*a*d^4*x^2+4*b*c^2*d^2*x^2+10*a*
c^2*d^2+8*b*c^4)/d^6*(d*x-c)^(1/2)
```

---

**Maxima [A]** time = 1.37497, size = 167, normalized size = 1.42

$$\frac{\sqrt{d^{2}x^{2}-c^{2}}bx^{4}}{5\,d^{2}}+\frac{4\,\sqrt{d^{2}x^{2}-c^{2}}bc^{2}x^{2}}{15\,d^{4}}+\frac{\sqrt{d^{2}x^{2}-c^{2}}ax^{2}}{3\,d^{2}}+\frac{8\,\sqrt{d^{2}x^{2}-c^{2}}bc^{4}}{15\,d^{6}}+\frac{2\,\sqrt{d^{2}x^{2}-c^{2}}ac^{2}}{3\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*x^3/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(d^2*x^2 - c^2)*b*x^4/d^2 + 4/15*sqrt(d^2*x^2 - c^2)*b*c^
2*x^2/d^4 + 1/3*sqrt(d^2*x^2 - c^2)*a*x^2/d^2 + 8/15*sqrt(d^2*x^2
 - c^2)*b*c^4/d^6 + 2/3*sqrt(d^2*x^2 - c^2)*a*c^2/d^4
```

---

**Fricas [A]** time = 0.2754, size = 420, normalized size = 3.56

$$-\frac{48\,bd^{10}x^{10}-8\,bc^{10}-10\,ac^{8}d^{2}-20\left(bc^{2}d^{8}-4\,ad^{10}\right)x^{8}+5\left(11\,bc^{4}d^{6}+4\,ac^{2}d^{8}\right)x^{6}-5\left(35\,bc^{6}d^{4}+43\,ac^{4}d^{6}\right)x^{4}+25\left(4\,bc^{8}\cdots\right.}{15\left(16\,d^{11}x^{5}-20\,c^{2}d^{9}x^{3}+5\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*x^3/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="fricas")
```

```
[Out] -1/15*(48*b*d^10*x^10 - 8*b*c^10 - 10*a*c^8*d^2 - 20*(b*c^2*d^8 -
 4*a*d^10)*x^8 + 5*(11*b*c^4*d^6 + 4*a*c^2*d^8)*x^6 - 5*(35*b*c^6
*d^4 + 43*a*c^4*d^6)*x^4 + 25*(4*b*c^8*d^2 + 5*a*c^6*d^4)*x^2 - (
48*b*d^9*x^9 + 4*(b*c^2*d^7 + 20*a*d^9)*x^7 + 3*(21*b*c^4*d^5 + 2
0*a*c^2*d^7)*x^5 - 35*(4*b*c^6*d^3 + 5*a*c^4*d^5)*x^3 + 10*(4*b*c
^8*d + 5*a*c^6*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c))/(16*d^11*x^5
- 20*c^2*d^9*x^3 + 5*c^4*d^7*x - (16*d^10*x^4 - 12*c^2*d^8*x^2 +
c^4*d^6)*sqrt(d*x + c)*sqrt(d*x - c))
```

---

**Sympy [A]** time = 136.219, size = 240, normalized size = 2.03

$$\frac{ac^{3}G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{c^{2}}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}d^{4}}+\frac{iac^{3}G_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0\end{matrix}\middle|\frac{c^{2}e^{2i\pi}}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}d^{4}}+\frac{bc^{5}G_{6,6}^{6,2}\left(\begin{matrix} -\frac{9}{4},-\frac{7}{4} & -2,-2,-\frac{3}{2},1 \\ -\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},0 & \end{matrix}\middle|\frac{c^{2}}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}d^{6}}+\frac{ibc^{5}G_{6,6}^{2,6}\left(\begin{matrix} -3,-\frac{11}{4},-\frac{5}{2},-\frac{9}{4},-2,1 & \\ -\frac{11}{4},-\frac{9}{4} & -3,-\frac{5}{2},-\frac{5}{2},0\end{matrix}\middle|\frac{c^{2}e^{2i\pi}}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*c**3*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -
1, -3/4, -1/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**4) + I*
a*c**3*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4)
, (-2, -3/2, -3/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi
**(3/2)*d**4) + b*c**5*meijerg(((-9/4, -7/4), (-2, -2, -3/2, 1)),
 ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), c**2/(d**2*x**2))/(4*pi**
(3/2)*d**6) + I*b*c**5*meijerg(((-3, -11/4, -5/2, -9/4, -2, 1), (
)), ((-11/4, -9/4), (-3, -5/2, -5/2, 0)), c**2*exp_polar(2*I*pi)/
(d**2*x**2))/(4*pi**(3/2)*d**6)
```

---

**GIAC/XCAS [A]** time = 0.221716, size = 151, normalized size = 1.28

$$\frac{\left(15\,bc^{4}d^{25}+15\,ac^{2}d^{27}-\left(20\,bc^{3}d^{25}+10\,acd^{27}-\left(22\,bc^{2}d^{25}+5\,ad^{27}+3\,\left((dx+c)bd^{25}-4\,bcd^{25}\right)(dx+c)\right)(dx+c)\right)(dx+c)\right)}{276480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*x^3/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="giac")
```

```
[Out] 1/276480*(15*b*c^4*d^25 + 15*a*c^2*d^27 - (20*b*c^3*d^25 + 10*a*c
*d^27 - (22*b*c^2*d^25 + 5*a*d^27 + 3*((d*x + c)*b*d^25 - 4*b*c*d
^25)*(d*x + c))*(d*x + c))*(d*x + c))*sqrt(d*x + c)*sqrt(d*x - c)
/d
```

## 3.261 $\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=118

$$\frac{c^2\left(4ad^2+3bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^5}+\frac{x\sqrt{dx-c}\sqrt{c+dx}\left(4ad^2+3bc^2\right)}{8d^4}+\frac{bx^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}$$

```
[Out] ((3*b*c^2 + 4*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*d^4) + (b
*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x])/(4*d^2) + (c^2*(3*b*c^2 + 4*a*
d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(4*d^5)
```

---

**Rubi [A]** time = 0.317815, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161

$$\frac{c^2\left(4ad^2+3bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^5}+\frac{x\sqrt{dx-c}\sqrt{c+dx}\left(4ad^2+3bc^2\right)}{8d^4}+\frac{bx^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```


```
[Out] ((3*b*c^2 + 4*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*d^4) + (b
*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x])/(4*d^2) + (c^2*(3*b*c^2 + 4*a*
d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(4*d^5)
```


---

**Rubi in Sympy [A]** time = 17.9971, size = 104, normalized size = 0.88

$$\frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2}+\frac{c^2\left(4ad^2+3bc^2\right)\operatorname{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{4d^5}+\frac{x\sqrt{-c+dx}\sqrt{c+dx}\left(4ad^2+3bc^2\right)}{8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] b*x**3*sqrt(-c + d*x)*sqrt(c + d*x)/(4*d**2) + c**2*(4*a*d**2 + 3
*b*c**2)*atanh(sqrt(c + d*x)/sqrt(-c + d*x))/(4*d**5) + x*sqrt(-c
 + d*x)*sqrt(c + d*x)*(4*a*d**2 + 3*b*c**2)/(8*d**4)
```

---

**Mathematica [A]** time = 0.106383, size = 96, normalized size = 0.81

$$\frac{dx\sqrt{dx-c}\sqrt{c+dx}\left(4ad^2+3bc^2+2bd^2x^2\right)+\left(4ac^2d^2+3bc^4\right)\log\left(\sqrt{dx-c}\sqrt{c+dx}+dx\right)}{8d^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(3*b*c^2 + 4*a*d^2 + 2*b*d^2*x^
2) + (3*b*c^4 + 4*a*c^2*d^2)*Log[d*x + Sqrt[-c + d*x]*Sqrt[c + d*
x]])/(8*d^5)
```

---

**Maple [C]** time = 0.03, size = 182, normalized size = 1.5

$$\frac{csgn(d)}{8\,d^5}\sqrt{dx-c}\sqrt{dx+c}\left(2\,csgn(d)\,x^3bd^3\sqrt{d^2x^2-c^2}+4\,ax\sqrt{d^2x^2-c^2}d^3csgn(d)+3\,bc^2x\sqrt{d^2x^2-c^2}csgn(d)\,d+4\,ac^2\ln\left(\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(2*csgn(d)*x^3*b*d^3*(d^2*x^2-c^2
)^(1/2)+4*a*x*(d^2*x^2-c^2)^(1/2)*d^3*csgn(d)+3*b*c^2*x*(d^2*x^2-
c^2)^(1/2)*csgn(d)*d+4*a*c^2*ln((csgn(d)*(d^2*x^2-c^2)^(1/2)+d*x)
*csgn(d))*d^2+3*b*c^4*ln((csgn(d)*(d^2*x^2-c^2)^(1/2)+d*x)*csgn(d
)))*csgn(d)/(d^2*x^2-c^2)^(1/2)/d^5
```

---

**Maxima [A]** time = 1.38641, size = 216, normalized size = 1.83

$$\frac{\sqrt{d^2x^2-c^2}bx^3}{4\,d^2}+\frac{3\,bc^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{8\,\sqrt{d^2}d^4}+\frac{ac^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{2\,\sqrt{d^2}d^2}+\frac{3\,\sqrt{d^2x^2-c^2}bc^2x}{8\,d^4}+\frac{\sqrt{d^2x^2-c^2}ax}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^2/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(d^2*x^2 - c^2)*b*x^3/d^2 + 3/8*b*c^4*log(2*d^2*x + 2*sqr
t(d^2*x^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^4) + 1/2*a*c^2*log(2*d^2
*x + 2*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^2) + 3/8*sqrt(
```

```
d^2*x^2 - c^2)*b*c^2*x/d^4 + 1/2*sqrt(d^2*x^2 - c^2)*a*x/d^2
```

---

**Fricas [A]** time = 0.252791, size = 508, normalized size = 4.31

$$-\frac{16\,bd^8x^8 + 32\,ad^8x^6 - 4\left(7\,bc^4d^4 + 12\,ac^2d^6\right)x^4 + 4\left(3\,bc^6d^2 + 4\,ac^4d^4\right)x^2 - \left(16\,bd^7x^7 + 8\left(bc^2d^5 + 4\,ad^7\right)x^5 - 2\left(11\,bc^4d^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^2/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="fricas")
```

```
[Out] -1/8*(16*b*d^8*x^8 + 32*a*d^8*x^6 - 4*(7*b*c^4*d^4 + 12*a*c^2*d^6
)*x^4 + 4*(3*b*c^6*d^2 + 4*a*c^4*d^4)*x^2 - (16*b*d^7*x^7 + 8*(b*
c^2*d^5 + 4*a*d^7)*x^5 - 2*(11*b*c^4*d^3 + 16*a*c^2*d^5)*x^3 + (3
*b*c^6*d + 4*a*c^4*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) + (3*b*c^8
 + 4*a*c^6*d^2 + 8*(3*b*c^4*d^4 + 4*a*c^2*d^6)*x^4 - 8*(3*b*c^6*d
^2 + 4*a*c^4*d^4)*x^2 - 4*(2*(3*b*c^4*d^3 + 4*a*c^2*d^5)*x^3 - (3
*b*c^6*d + 4*a*c^4*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c))*log(-d*x
+ sqrt(d*x + c)*sqrt(d*x - c)))/(8*d^9*x^4 - 8*c^2*d^7*x^2 + c^4*
d^5 - 4*(2*d^8*x^3 - c^2*d^6*x)*sqrt(d*x + c)*sqrt(d*x - c))
```

---

**Sympy [A]** time = 117.765, size = 236, normalized size = 2.

$$\frac{ac^2G_{6,6}^{6,2}\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{4} & -\frac{1}{2}, -\frac{1}{2}, 0, 1 \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 & \end{matrix} \middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3} - \frac{iac^2G_{6,6}^{2,6}\left(\begin{matrix} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 & \\ -\frac{5}{4}, -\frac{3}{4} & -\frac{3}{2}, -1, -1, 0 \end{matrix} \middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3} + \frac{bc^4G_{6,6}^{6,2}\left(\begin{matrix} -\frac{7}{4}, -\frac{5}{4} & -\frac{3}{2}, -\frac{3}{2}, -1, 1 \\ -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, 0 & \end{matrix} \middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^5} - \frac{ibc^4G_{6,6}^{2,6}\left(\begin{matrix} -\frac{5}{2}, -\frac{9}{4}, -2, -\frac{7}{4}, -\frac{3}{2}, 1 & \\ -\frac{9}{4}, -\frac{7}{4} & -\frac{5}{2}, -2, -2, 0 \end{matrix} \middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*c**2*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1
/2, -1/4, 0, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**3) - I*a*
c**2*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4)
, (-3/2, -1, -1, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**
(3/2)*d**3) + b*c**4*meijerg(((-7/4, -5/4), (-3/2, -3/2, -1, 1)),
 ((-2, -7/4, -3/2, -5/4, -1, 0), ()), c**2/(d**2*x**2))/(4*pi**(3
/2)*d**5) - I*b*c**4*meijerg(((-5/2, -9/4, -2, -7/4, -3/2, 1), ()
), ((-9/4, -7/4), (-5/2, -2, -2, 0)), c**2*exp_polar(2*I*pi)/(d**
2*x**2))/(4*pi**(3/2)*d**5)
```

---

**GIAC/XCAS [A]** time = 0.255064, size = 176, normalized size = 1.49

$$-\frac{\left(5\,bc^{3}d^{16}+4\,acd^{18}-\left(9\,bc^{2}d^{16}+4\,ad^{18}+2\,\left((dx+c)bd^{16}-3\,bcd^{16}\right)(dx+c)\right)(dx+c)\right)\sqrt{dx+c}\sqrt{dx-c}+2\,\left(3\,bc^{4}d^{16}+4\,ac\right.}{114688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^2/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="giac")
```

```
[Out] -1/114688*((5*b*c^3*d^16 + 4*a*c*d^18 - (9*b*c^2*d^16 + 4*a*d^18
+ 2*((d*x + c)*b*d^16 - 3*b*c*d^16)*(d*x + c))*(d*x + c))*sqrt(d*
x + c)*sqrt(d*x - c) + 2*(3*b*c^4*d^16 + 4*a*c^2*d^18)*ln(abs(-sq
rt(d*x + c) + sqrt(d*x - c))))/d
```

# 3.262 $\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=72

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(3ad^2+2bc^2\right)}{3d^4}+\frac{bx^2\sqrt{dx-c}\sqrt{c+dx}}{3d^2}$$

```
[Out] ((2*b*c^2 + 3*a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*d^4) + (b*x
^2*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*d^2)
```

---

**Rubi [A]** time = 0.158959, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(3ad^2+2bc^2\right)}{3d^4}+\frac{bx^2\sqrt{dx-c}\sqrt{c+dx}}{3d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] ((2*b*c^2 + 3*a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*d^4) + (b*x
^2*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*d^2)
```

---

**Rubi in Sympy [A]** time = 9.97023, size = 61, normalized size = 0.85

$$\frac{bx^2\sqrt{-c+dx}\sqrt{c+dx}}{3d^2}+\frac{\sqrt{-c+dx}\sqrt{c+dx}\left(3ad^2+2bc^2\right)}{3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] b*x**2*sqrt(-c + d*x)*sqrt(c + d*x)/(3*d**2) + sqrt(-c + d*x)*sqr
t(c + d*x)*(3*a*d**2 + 2*b*c**2)/(3*d**4)
```

---

**Mathematica [A]** time = 0.0556434, size = 48, normalized size = 0.67

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(3ad^2+2bc^2+bd^2x^2\right)}{3d^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(2*b*c^2 + 3*a*d^2 + b*d^2*x^2))/(3
*d^4)
```

---

**Maple [A]** time = 0.006, size = 43, normalized size = 0.6

$$\frac{bd^2x^2 + 3\,ad^2 + 2\,bc^2}{3\,d^4}\sqrt{dx+c}\sqrt{dx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/3*(d*x+c)^(1/2)*(b*d^2*x^2+3*a*d^2+2*b*c^2)/d^4*(d*x-c)^(1/2)
```

---

**Maxima [A]** time = 1.38412, size = 93, normalized size = 1.29

$$\frac{\sqrt{d^2x^2-c^2}bx^2}{3\,d^2} + \frac{2\,\sqrt{d^2x^2-c^2}bc^2}{3\,d^4} + \frac{\sqrt{d^2x^2-c^2}a}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(d^2*x^2 - c^2)*b*x^2/d^2 + 2/3*sqrt(d^2*x^2 - c^2)*b*c^2
/d^4 + sqrt(d^2*x^2 - c^2)*a/d^2
```

---

**Fricas [A]** time = 0.240768, size = 262, normalized size = 3.64

$$-\frac{4\,bd^6x^6 + 2\,bc^6 + 3\,ac^4d^2 + 3\left(bc^2d^4 + 4\,ad^6\right)x^4 - 3\left(3\,bc^4d^2 + 5\,ac^2d^4\right)x^2 - \left(4\,bd^5x^5 + \left(5\,bc^2d^3 + 12\,ad^5\right)x^3 - 3\left(2\,bc^4d + \right.\right.}{3\left(4\,d^7x^3 - 3\,c^2d^5x - (4\,d^6x^2 - c^2d^4)\sqrt{dx+c}\sqrt{dx-c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="fricas")
```

```
[Out] -1/3*(4*b*d^6*x^6 + 2*b*c^6 + 3*a*c^4*d^2 + 3*(b*c^2*d^4 + 4*a*d^
6)*x^4 - 3*(3*b*c^4*d^2 + 5*a*c^2*d^4)*x^2 - (4*b*d^5*x^5 + (5*b*
c^2*d^3 + 12*a*d^5)*x^3 - 3*(2*b*c^4*d + 3*a*c^2*d^3)*x)*sqrt(d*x
```

```
 + c)*sqrt(d*x - c))/(4*d^7*x^3 - 3*c^2*d^5*x - (4*d^6*x^2 - c^2*
d^4)*sqrt(d*x + c)*sqrt(d*x - c))
```

---

**Sympy [A]** time = 76.0226, size = 223, normalized size = 3.1

$$\frac{acG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{iacG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{bc^3G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}+\frac{ibc^3G_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4,
1/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*a*c*meijer
g(((-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -
1/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
+ b*c**3*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4,
 -1, -3/4, -1/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**4) +
I*b*c**3*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/
4), (-2, -3/2, -3/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)*d**4)
```

---

**GIAC/XCAS [A]** time = 0.217631, size = 82, normalized size = 1.14

$$\frac{\left(3\,bc^2d^9+3\,ad^{11}+\left((dx+c)bd^9-2\,bcd^9\right)(dx+c)\right)\sqrt{dx+c}\sqrt{dx-c}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*x/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="giac")
```

```
[Out] 1/1920*(3*b*c^2*d^9 + 3*a*d^11 + ((d*x + c)*b*d^9 - 2*b*c*d^9)*(d
*x + c))*sqrt(d*x + c)*sqrt(d*x - c)/d
```

## 3.263 $\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=68

$$\frac{(2ad^2+bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}+\frac{bx\sqrt{dx-c}\sqrt{c+dx}}{2d^2}$$

```
[Out] (b*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*d^2) + ((b*c^2 + 2*a*d^2)*A
rcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/d^3
```

---

**Rubi [A]** time = 0.115793, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107

$$\frac{(2ad^2+bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}+\frac{bx\sqrt{dx-c}\sqrt{c+dx}}{2d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (b*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*d^2) + ((b*c^2 + 2*a*d^2)*A
rcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/d^3
```

---

**Rubi in Sympy [A]** time = 13.259, size = 75, normalized size = 1.1

$$\frac{2a\,\text{atanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}+\frac{bc^2\,\text{atanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}+\frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] 2*a*atanh(sqrt(-c + d*x)/sqrt(c + d*x))/d + b*c**2*atanh(sqrt(-c
+ d*x)/sqrt(c + d*x))/d**3 + b*x*sqrt(-c + d*x)*sqrt(c + d*x)/(2*
d**2)
```

---

**Mathematica [A]** time = 0.0665225, size = 71, normalized size = 1.04

$$\frac{(2ad^2+bc^2)\log\left(\sqrt{dx-c}\sqrt{c+dx}+dx\right)+bdx\sqrt{dx-c}\sqrt{c+dx}}{2d^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (b*d*x*Sqrt[-c + d*x]*Sqrt[c + d*x] + (b*c^2 + 2*a*d^2)*Log[d*x +
 Sqrt[-c + d*x]*Sqrt[c + d*x]])/(2*d^3)
```

---

**Maple [C]** time = 0.025, size = 124, normalized size = 1.8

$$\frac{csgn(d)}{2\,d^3}\sqrt{dx-c}\sqrt{dx+c}\left(bx\sqrt{d^2x^2-c^2}\,csgn(d)\,d+bc^2\ln\left(\left(csgn(d)\sqrt{d^2x^2-c^2}+dx\right)csgn(d)\right)+2\ln\left(\left(csgn(d)\sqrt{d^2x^2-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/2*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(b*x*(d^2*x^2-c^2)^(1/2)*csgn(d)*
d+b*c^2*ln((csgn(d)*(d^2*x^2-c^2)^(1/2)+d*x)*csgn(d))+2*ln((csgn(
d)*(d^2*x^2-c^2)^(1/2)+d*x)*csgn(d))*a*d^2)/(d^2*x^2-c^2)^(1/2)/d
^3*csgn(d)
```

---

**Maxima [A]** time = 1.39622, size = 140, normalized size = 2.06

$$\frac{a\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{\sqrt{d^2}}+\frac{bc^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{2\,\sqrt{d^2}d^2}+\frac{\sqrt{d^2x^2-c^2}bx}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="maxima")
```

```
[Out] a*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/sqrt(d^2) + 1/2*
b*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d
^2) + 1/2*sqrt(d^2*x^2 - c^2)*b*x/d^2
```

---

**Fricas [A]** time = 0.237688, size = 261, normalized size = 3.84

$$-\frac{2\,bd^4x^4-2\,bc^2d^2x^2-\left(2\,bd^3x^3-bc^2dx\right)\sqrt{dx+c}\sqrt{dx-c}-\left(bc^4+2\,ac^2d^2+2\left(bc^2d+2\,ad^3\right)\sqrt{dx+c}\sqrt{dx-c}x-2\left(bc^2d^2\right.\right.}{2\left(2\,d^5x^2-2\,\sqrt{dx+c}\sqrt{dx-c}d^4x-c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d^4*x^4 - 2*b*c^2*d^2*x^2 - (2*b*d^3*x^3 - b*c^2*d*x)*s
qrt(d*x + c)*sqrt(d*x - c) - (b*c^4 + 2*a*c^2*d^2 + 2*(b*c^2*d +
2*a*d^3)*sqrt(d*x + c)*sqrt(d*x - c)*x - 2*(b*c^2*d^2 + 2*a*d^4)*
x^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/(2*d^5*x^2 - 2*sqrt
(d*x + c)*sqrt(d*x - c)*d^4*x - c^2*d^3)
```

---

**Sympy [A]** time = 43.6661, size = 199, normalized size = 2.93

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{bc^2G_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1\\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}-\frac{ibc^2G_{6,6}^{2,6}\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1,
0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d) - I*a*meijerg(((-1/2,
-1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), c**2*
exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d) + b*c**2*meijerg((
(-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0),
 ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**3) - I*b*c**2*meijerg(((-
3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1,
 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3)
```

---

**GIAC/XCAS [A]** time = 0.255644, size = 107, normalized size = 1.57

$$\frac{\left((dx+c)bd^4-bcd^4\right)\sqrt{dx+c}\sqrt{dx-c}-2\left(bc^2d^4+2\,ad^6\right)\ln\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)),x, algorithm="giac")
```

```
[Out] 1/384*(((d*x + c)*b*d^4 - b*c*d^4)*sqrt(d*x + c)*sqrt(d*x - c) -
2*(b*c^2*d^4 + 2*a*d^6)*ln(abs(-sqrt(d*x + c) + sqrt(d*x - c))))/
d
```

# 3.264 $\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=56

$$\frac{a\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c}+\frac{b\sqrt{dx-c}\sqrt{c+dx}}{d^2}$$

```
[Out] (b*Sqrt[-c + d*x]*Sqrt[c + d*x])/d^2 + (a*ArcTan[(Sqrt[-c + d*x]*
Sqrt[c + d*x])/c])/c
```

---

**Rubi [A]** time = 0.241805, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$\frac{a\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c}+\frac{b\sqrt{dx-c}\sqrt{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(x*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (b*Sqrt[-c + d*x]*Sqrt[c + d*x])/d^2 + (a*ArcTan[(Sqrt[-c + d*x]*
Sqrt[c + d*x])/c])/c
```

---

**Rubi in Sympy [A]** time = 12.8825, size = 44, normalized size = 0.79

$$\frac{a\,\mathrm{atan}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{c}+\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/x/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*atan(sqrt(-c + d*x)*sqrt(c + d*x)/c)/c + b*sqrt(-c + d*x)*sqrt(
c + d*x)/d**2
```

---

**Mathematica [A]** time = 0.0996939, size = 55, normalized size = 0.98

$$\frac{b\sqrt{dx-c}\sqrt{c+dx}}{d^2}-\frac{a\tan^{-1}\left(\frac{c}{\sqrt{dx-c}\sqrt{c+dx}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(x*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (b*Sqrt[-c + d*x]*Sqrt[c + d*x])/d^2 - (a*ArcTan[c/(Sqrt[-c + d*x
]*Sqrt[c + d*x])])/c
```

---

**Maple [B]** time = 0.026, size = 108, normalized size = 1.9

$$\frac{1}{d^2}\left(-\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)ad^2+b\sqrt{-c^2}\sqrt{d^2x^2-c^2}\right)\sqrt{dx-c}\sqrt{dx+c}\frac{1}{\sqrt{-c^2}}\frac{1}{\sqrt{d^2x^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] (-ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*d^2+b*(-c^2)^
(1/2)*(d^2*x^2-c^2)^(1/2))*(d*x-c)^(1/2)*(d*x+c)^(1/2)/(d^2*x^2-c
^2)^(1/2)/(-c^2)^(1/2)/d^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.240987, size = 176, normalized size = 3.14

$$-\frac{bcd^2x^2-\sqrt{dx+c}\sqrt{dx-c}bcdx-bc^3-2\left(ad^3x-\sqrt{dx+c}\sqrt{dx-c}ad^2\right)\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)}{cd^3x-\sqrt{dx+c}\sqrt{dx-c}cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)*x),x, algorithm="fricas")
```

```
[Out] -(b*c*d^2*x^2 - sqrt(d*x + c)*sqrt(d*x - c)*b*c*d*x - b*c^3 - 2*(
a*d^3*x - sqrt(d*x + c)*sqrt(d*x - c)*a*d^2)*arctan(-(d*x - sqrt(
```

```
d*x + c)*sqrt(d*x - c))/c))/(c*d^3*x - sqrt(d*x + c)*sqrt(d*x - c
)*c*d^2)
```

---

**Sympy [A]** time = 44.7243, size = 178, normalized size = 3.18

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c}+\frac{bcG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{ibcG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/x/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2)
, (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c) + I*a*meijerg(((0, 1/4
, 1/2, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), c**2*exp_
polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c) + b*c*meijerg(((-1/4,
1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()), c**2/(
d**2*x**2))/(4*pi**(3/2)*d**2) + I*b*c*meijerg(((-1, -3/4, -1/2,
-1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), c**2*exp_p
olar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

---

**GIAC/XCAS [A]** time = 0.220305, size = 74, normalized size = 1.32

$$-\frac{2\,a\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c}+\frac{\sqrt{dx+c}\sqrt{dx-c}b}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)*x),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c + sqrt(d*x
 + c)*sqrt(d*x - c)*b/d^2
```

## 3.265 $\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=57

$$\frac{a\sqrt{dx-c}\sqrt{c+dx}}{c^2x}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}$$

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*x) + (2*b*ArcTanh[Sqrt[-c +
 d*x]/Sqrt[c + d*x]])/d
```

---

**Rubi [A]** time = 0.227326, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$\frac{a\sqrt{dx-c}\sqrt{c+dx}}{c^2x}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*x) + (2*b*ArcTanh[Sqrt[-c +
 d*x]/Sqrt[c + d*x]])/d
```

---

**Rubi in Sympy [A]** time = 11.5265, size = 46, normalized size = 0.81

$$\frac{a\sqrt{-c+dx}\sqrt{c+dx}}{c^2x}+\frac{2b\,\mathrm{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{-c+dx}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/x**2/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*sqrt(-c + d*x)*sqrt(c + d*x)/(c**2*x) + 2*b*atanh(sqrt(c + d*x)
/sqrt(-c + d*x))/d
```

---

**Mathematica [A]** time = 0.0726016, size = 60, normalized size = 1.05

$$\frac{a\sqrt{dx-c}\sqrt{c+dx}}{c^2x}+\frac{b\log\left(\sqrt{dx-c}\sqrt{c+dx}+dx\right)}{d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*x) + (b*Log[d*x + Sqrt[-c +
 d*x]*Sqrt[c + d*x]])/d
```

---

**Maple [C]** time = 0.029, size = 97, normalized size = 1.7

$$\frac{csgn(d)}{c^2xd}\sqrt{dx-c}\sqrt{dx+c}\left(\ln\left(\left(csgn(d)\sqrt{d^2x^2-c^2}+dx\right)csgn(d)\right)xbc^2+a\sqrt{d^2x^2-c^2}csgn(d)\,d\right)\frac{1}{\sqrt{d^2x^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] (d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2*(ln((csgn(d)*(d^2*x^2-c^2)^(1/2)+
d*x)*csgn(d))*x*b*c^2+a*(d^2*x^2-c^2)^(1/2)*csgn(d)*d)*csgn(d)/(d
^2*x^2-c^2)^(1/2)/x/d
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.237346, size = 117, normalized size = 2.05

$$\frac{ad-\left(bdx^2-\sqrt{dx+c}\sqrt{dx-c}bx\right)\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)}{d^2x^2-\sqrt{dx+c}\sqrt{dx-c}dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)*x^2),x, algorithm="fricas")
```

```
[Out] (a*d - (b*d*x^2 - sqrt(d*x + c)*sqrt(d*x - c)*b*x)*log(-d*x + sqr
t(d*x + c)*sqrt(d*x - c)))/(d^2*x^2 - sqrt(d*x + c)*sqrt(d*x - c)
```

```
*d*x)
```

---

**Sympy [A]** time = 51.5059, size = 165, normalized size = 2.89

$$-\frac{adG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}-\frac{iadG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/x**2/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4,
2), (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**2) - I*a*d*meijerg((
(1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), c*
*2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c**2) + b*meijerg(
((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), c
**2/(d**2*x**2))/(4*pi**(3/2)*d) - I*b*meijerg(((-1/2, -1/4, 0, 1
/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), c**2*exp_polar(
2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d)
```

---

**GIAC/XCAS [A]** time = 0.224909, size = 89, normalized size = 1.56

$$\frac{\frac{16\,ad^2}{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2}-b\ln\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(16*a*d^2/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2) - b*ln(
(sqrt(d*x + c) - sqrt(d*x - c))^4))/d
```

# 3.266 $\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=76

$$\frac{(ad^2+2bc^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c^3}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{2c^2x^2}$$

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*c^2*x^2) + ((2*b*c^2 + a*d^2)
*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c])/(2*c^3)
```

---

**Rubi [A]** time = 0.260603, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$\frac{(ad^2+2bc^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c^3}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*c^2*x^2) + ((2*b*c^2 + a*d^2)
*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c])/(2*c^3)
```

---

**Rubi in Sympy [A]** time = 13.3616, size = 63, normalized size = 0.83

$$\frac{a\sqrt{-c+dx}\sqrt{c+dx}}{2c^2x^2}+\frac{(ad^2+2bc^2)\,\mathrm{atan}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/x**3/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*sqrt(-c + d*x)*sqrt(c + d*x)/(2*c**2*x**2) + (a*d**2 + 2*b*c**2
)*atan(sqrt(-c + d*x)*sqrt(c + d*x)/c)/(2*c**3)
```

---

**Mathematica [C]** time = 0.139132, size = 103, normalized size = 1.36

$$\frac{ac\sqrt{dx-c}\sqrt{c+dx}-ix^2(ad^2+2bc^2)\log\left(\frac{4c^2\left(\sqrt{dx-c}\sqrt{c+dx}-ic\right)}{x(ad^2+2bc^2)}\right)}{2c^3x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*c*Sqrt[-c + d*x]*Sqrt[c + d*x] - I*(2*b*c^2 + a*d^2)*x^2*Log[(
4*c^2*((-I)*c + Sqrt[-c + d*x]*Sqrt[c + d*x]))/((2*b*c^2 + a*d^2)
*x)])/(2*c^3*x^2)
```

---

**Maple [B]** time = 0.029, size = 158, normalized size = 2.1

$$-\frac{1}{2\,c^2x^2}\sqrt{dx-c}\sqrt{dx+c}\left(\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^2ad^2+2\,\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^2bc^2-a\sqrt{d^2x^2-c^2}\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] -1/2*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2*(ln(-2*(c^2-(-c^2)^(1/2)*(d^
2*x^2-c^2)^(1/2))/x)*x^2*a*d^2+2*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2
-c^2)^(1/2))/x)*x^2*b*c^2-a*(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2))/(d^
2*x^2-c^2)^(1/2)/(-c^2)^(1/2)/x^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.245142, size = 279, normalized size = 3.67

$$-\frac{2\,acd^3x^3-2\,ac^3dx-\left(2\,acd^2x^2-ac^3\right)\sqrt{dx+c}\sqrt{dx-c}+2\left(2\left(2\,bc^2d+ad^3\right)\sqrt{dx+c}\sqrt{dx-c}x^3-2\left(2\,bc^2d^2+ad^4\right)x^4+\left(\right.\right.}{2\left(2\,c^3d^2x^4-2\,\sqrt{dx+c}\sqrt{dx-c}c^3dx^3-c^5x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)*x^3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*c*d^3*x^3 - 2*a*c^3*d*x - (2*a*c*d^2*x^2 - a*c^3)*sqrt(
d*x + c)*sqrt(d*x - c) + 2*(2*(2*b*c^2*d + a*d^3)*sqrt(d*x + c)*s
qrt(d*x - c)*x^3 - 2*(2*b*c^2*d^2 + a*d^4)*x^4 + (2*b*c^4 + a*c^2
*d^2)*x^2)*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c))/(2*c^3
*d^2*x^4 - 2*sqrt(d*x + c)*sqrt(d*x - c)*c^3*d*x^3 - c^5*x^2)
```

---

**Sympy [A]** time = 68.5575, size = 162, normalized size = 2.13

$$-\frac{ad^2 G_{6,6}^{5,3}\left(\begin{matrix} \frac{7}{4}, \frac{9}{4}, 1 & 2, 2, \frac{5}{2} \\ \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2} & 0 \end{matrix} \middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^3} + \frac{iad^2 G_{6,6}^{2,6}\left(\begin{matrix} 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 1 & \\ \frac{5}{4}, \frac{7}{4} & 1, \frac{3}{2}, \frac{3}{2}, 0 \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^3} - \frac{bG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 & 1, 1, \frac{3}{2} \\ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2} & 0 \end{matrix} \middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c} + \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1 & \\ \frac{1}{4}, \frac{3}{4} & 0, \frac{1}{2}, \frac{1}{2}, 0 \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/x**3/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*d**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4,
 5/2), (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**3) + I*a*d**2*mei
jerg(((1, 5/4, 3/2, 7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0
)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c**3) - b*me
ijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,
)), c**2/(d**2*x**2))/(4*pi**(3/2)*c) + I*b*meijerg(((0, 1/4, 1/2
, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), c**2*exp_polar
(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c)
```

---

**GIAC/XCAS [A]** time = 0.229743, size = 190, normalized size = 2.5

$$-\frac{\frac{(2\,bc^2d+ad^3)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^3}+\frac{2\left(ad^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6-4\,ac^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^2c^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)*x^3),x, algorithm="giac")
```

```
[Out] -((2*b*c^2*d + a*d^3)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^
2/c)/c^3 + 2*(a*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^6 - 4*a*c^2*d
^3*(sqrt(d*x + c) - sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x
 - c))^4 + 4*c^2)^2*c^2))/d
```

## 3.267 $\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=75

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(2ad^2+3bc^2\right)}{3c^4x}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{3c^2x^3}$$

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*c^2*x^3) + ((3*b*c^2 + 2*a*d^
2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*c^4*x)
```

---

**Rubi [A]** time = 0.254355, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(2ad^2+3bc^2\right)}{3c^4x}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{3c^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*c^2*x^3) + ((3*b*c^2 + 2*a*d^
2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*c^4*x)
```

---

**Rubi in Sympy [A]** time = 11.9889, size = 63, normalized size = 0.84

$$\frac{a\sqrt{-c+dx}\sqrt{c+dx}}{3c^2x^3}+\frac{\sqrt{-c+dx}\sqrt{c+dx}\left(2ad^2+3bc^2\right)}{3c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/x**4/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*sqrt(-c + d*x)*sqrt(c + d*x)/(3*c**2*x**3) + sqrt(-c + d*x)*sqr
t(c + d*x)*(2*a*d**2 + 3*b*c**2)/(3*c**4*x)
```

---

**Mathematica [A]** time = 0.0741589, size = 56, normalized size = 0.75

$$\sqrt{dx-c}\sqrt{c+dx}\left(\frac{2ad^2+3bc^2}{3c^4x}+\frac{a}{3c^2x^3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a/(3*c^2*x^3) + (3*b*c^2 + 2*a*d^2)/(3*c^4*x))*Sqrt[-c + d*x]*Sq
rt[c + d*x]
```

---

**Maple [A]** time = 0.008, size = 49, normalized size = 0.7

$$\frac{2\,ad^2x^2+3\,bc^2x^2+ac^2}{3\,x^3c^4}\sqrt{dx+c}\sqrt{dx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/3*(d*x+c)^(1/2)*(2*a*d^2*x^2+3*b*c^2*x^2+a*c^2)/x^3/c^4*(d*x-c)
^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.237519, size = 162, normalized size = 2.16

$$\frac{6\,bd^2x^4-ac^2-3\left(bc^2-ad^2\right)x^2-3\left(2\,bdx^3+adx\right)\sqrt{dx+c}\sqrt{dx-c}}{3\left(4\,d^3x^6-3\,c^2dx^4-(4\,d^2x^5-c^2x^3)\sqrt{dx+c}\sqrt{dx-c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)*x^4),x, algorithm="fricas")
```

```
[Out] 1/3*(6*b*d^2*x^4 - a*c^2 - 3*(b*c^2 - a*d^2)*x^2 - 3*(2*b*d*x^3 +
 a*d*x)*sqrt(d*x + c)*sqrt(d*x - c))/(4*d^3*x^6 - 3*c^2*d*x^4 - (
4*d^2*x^5 - c^2*x^3)*sqrt(d*x + c)*sqrt(d*x - c))
```

**Sympy [A]** time = 108.996, size = 170, normalized size = 2.27

$$-\frac{ad^3G_{6,6}^{5,3}\left(\begin{matrix}\frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3\\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4}-\frac{iad^3G_{6,6}^{2,6}\left(\begin{matrix}\frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4}-\frac{bdG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}-\frac{ibdG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/x**4/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*d**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 1
1/4, 3), (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**4) - I*a*d**3*m
eijerg(((3/2, 7/4, 2, 9/4, 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2,
 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c**4) - b*
d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2),
 (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**2) - I*b*d*meijerg(((1/
2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), c**2*
exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c**2)
```

**GIAC/XCAS [A]** time = 0.234151, size = 185, normalized size = 2.47

$$\frac{8\left(3\,bd^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+24\,bc^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+24\,ad^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+48\,bc^4d^2+32\,ac^2d^4\right)}{3\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)*x^4),x, algorithm="giac")
```

```
[Out] 8/3*(3*b*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^8 + 24*b*c^2*d^2*(sq
rt(d*x + c) - sqrt(d*x - c))^4 + 24*a*d^4*(sqrt(d*x + c) - sqrt(d
*x - c))^4 + 48*b*c^4*d^2 + 32*a*c^2*d^4)/(((sqrt(d*x + c) - sqrt
(d*x - c))^4 + 4*c^2)^3*d)
```

## 3.268 $\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=123

$$\frac{d^2\left(3ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^5}+\frac{\sqrt{dx-c}\sqrt{c+dx}\left(3ad^2+4bc^2\right)}{8c^4x^2}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{4c^2x^4}$$

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(4*c^2*x^4) + ((4*b*c^2 + 3*a*d^
2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*c^4*x^2) + (d^2*(4*b*c^2 + 3*
a*d^2)*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c])/(8*c^5)
```

---

**Rubi [A]** time = 0.342501, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161

$$\frac{d^2\left(3ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^5}+\frac{\sqrt{dx-c}\sqrt{c+dx}\left(3ad^2+4bc^2\right)}{8c^4x^2}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{4c^2x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(4*c^2*x^4) + ((4*b*c^2 + 3*a*d^
2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*c^4*x^2) + (d^2*(4*b*c^2 + 3*
a*d^2)*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c])/(8*c^5)
```

---

**Rubi in Sympy [A]** time = 19.7361, size = 107, normalized size = 0.87

$$\frac{a\sqrt{-c+dx}\sqrt{c+dx}}{4c^2x^4}+\frac{\sqrt{-c+dx}\sqrt{c+dx}\left(3ad^2+4bc^2\right)}{8c^4x^2}+\frac{d^2\left(3ad^2+4bc^2\right)\operatorname{atan}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{8c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/x**5/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*sqrt(-c + d*x)*sqrt(c + d*x)/(4*c**2*x**4) + sqrt(-c + d*x)*sqr
t(c + d*x)*(3*a*d**2 + 4*b*c**2)/(8*c**4*x**2) + d**2*(3*a*d**2 +
 4*b*c**2)*atan(sqrt(-c + d*x)*sqrt(c + d*x)/c)/(8*c**5)
```

---

**Mathematica [C]** time = 0.19576, size = 135, normalized size = 1.1

$$\frac{c\sqrt{dx-c}\sqrt{c+dx}\left(2ac^2+3ad^2x^2+4bc^2x^2\right)-id^2x^4\left(3ad^2+4bc^2\right)\log\left(\frac{16c^4\left(\sqrt{dx-c}\sqrt{c+dx}-ic\right)}{d^2x(3ad^2+4bc^2)}\right)}{8c^5x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (c*Sqrt[-c + d*x]*Sqrt[c + d*x]*(2*a*c^2 + 4*b*c^2*x^2 + 3*a*d^2*
x^2) - I*d^2*(4*b*c^2 + 3*a*d^2)*x^4*Log[(16*c^4*((-I)*c + Sqrt[-
c + d*x]*Sqrt[c + d*x]))/(d^2*(4*b*c^2 + 3*a*d^2)*x)])/(8*c^5*x^4
)
```

---

**Maple [B]** time = 0.03, size = 227, normalized size = 1.9

$$-\frac{1}{8\,c^4x^4}\sqrt{dx-c}\sqrt{dx+c}\left(3\,\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^4ad^4+4\,\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^4bc^2d^2-3\,ad^2\sqrt{d^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] -1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^4*(3*ln(-2*(c^2-(-c^2)^(1/2)*(
d^2*x^2-c^2)^(1/2))/x)*x^4*a*d^4+4*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x
^2-c^2)^(1/2))/x)*x^4*b*c^2*d^2-3*a*d^2*(d^2*x^2-c^2)^(1/2)*x^2*(
-c^2)^(1/2)-4*b*(d^2*x^2-c^2)^(1/2)*c^2*x^2*(-c^2)^(1/2)-2*a*(d^2
*x^2-c^2)^(1/2)*c^2*(-c^2)^(1/2))/(d^2*x^2-c^2)^(1/2)/x^4/(-c^2)^
(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.272273, size = 544, normalized size = 4.42

$$-\frac{8\,ac^7dx + 8\left(4\,bc^3d^5 + 3\,acd^7\right)x^7 - 4\left(12\,bc^5d^3 + 5\,ac^3d^5\right)x^5 + 4\left(4\,bc^7d - 3\,ac^5d^3\right)x^3 - \left(2\,ac^7 + 8\left(4\,bc^3d^4 + 3\,acd^6\right)x^6 - 8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)*x^5),x, algorithm="fricas")
```

```
[Out] -1/8*(8*a*c^7*d*x + 8*(4*b*c^3*d^5 + 3*a*c*d^7)*x^7 - 4*(12*b*c^5
*d^3 + 5*a*c^3*d^5)*x^5 + 4*(4*b*c^7*d - 3*a*c^5*d^3)*x^3 - (2*a*
c^7 + 8*(4*b*c^3*d^4 + 3*a*c*d^6)*x^6 - 8*(4*b*c^5*d^2 + a*c^3*d^
4)*x^4 + (4*b*c^7 - 13*a*c^5*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c
) - 2*(8*(4*b*c^2*d^6 + 3*a*d^8)*x^8 - 8*(4*b*c^4*d^4 + 3*a*c^2*d
^6)*x^6 + (4*b*c^6*d^2 + 3*a*c^4*d^4)*x^4 - 4*(2*(4*b*c^2*d^5 + 3
*a*d^7)*x^7 - (4*b*c^4*d^3 + 3*a*c^2*d^5)*x^5)*sqrt(d*x + c)*sqrt
(d*x - c))*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c))/(8*c^5
*d^4*x^8 - 8*c^7*d^2*x^6 + c^9*x^4 - 4*(2*c^5*d^3*x^7 - c^7*d*x^5
)*sqrt(d*x + c)*sqrt(d*x - c))
```

---

**Sympy [A]** time = 157.614, size = 172, normalized size = 1.4

$$-\frac{ad^4G_{6,6}^{5,3}\left(\begin{matrix}\frac{11}{4},\frac{13}{4},1 & 3,3,\frac{7}{2}\\ \frac{5}{2},\frac{11}{4},3,\frac{13}{4},\frac{7}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^5}+\frac{iad^4G_{6,6}^{2,6}\left(\begin{matrix}2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3,1 & \\ \frac{9}{4},\frac{11}{4} & 2,\frac{5}{2},\frac{5}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^5}-\frac{bd^2G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^3}+\frac{ibd^2G_{6,6}^{2,6}\left(\begin{matrix}1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2,1 & \\ \frac{5}{4},\frac{7}{4} & 1,\frac{3}{2},\frac{3}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**5/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*d**4*meijerg(((11/4, 13/4, 1), (3, 3, 7/2)), ((5/2, 11/4, 3, 1
3/4, 7/2), (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**5) + I*a*d**4
*meijerg(((2, 9/4, 5/2, 11/4, 3, 1), ()), ((9/4, 11/4), (2, 5/2,
5/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c**5)
- b*d**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4
, 5/2), (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**3) + I*b*d**2*me
ijerg(((1, 5/4, 3/2, 7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2,
0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c**3)
```

---

**GIAC/XCAS [A]** time = 0.239979, size = 439, normalized size = 3.57

$$-\frac{\frac{\left(4\,bc^2d^3+3\,ad^5\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^5}+\frac{2\left(4\,bc^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+3\,ad^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+16\,bc^4d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}+44\,ac^2d^5\left(\sqrt{dx+c}-\cdots\right.\right.}{\left(\left(\cdots\right.\right.}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x + c)*sqrt(d*x - c)*x^5),x, algorithm="giac")
```

```
[Out] -1/4*((4*b*c^2*d^3 + 3*a*d^5)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*
x - c))^2/c)/c^5 + 2*(4*b*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))
^14 + 3*a*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^14 + 16*b*c^4*d^3*(
sqrt(d*x + c) - sqrt(d*x - c))^10 + 44*a*c^2*d^5*(sqrt(d*x + c) -
 sqrt(d*x - c))^10 - 64*b*c^6*d^3*(sqrt(d*x + c) - sqrt(d*x - c))
^6 - 176*a*c^4*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^6 - 256*b*c^8*
d^3*(sqrt(d*x + c) - sqrt(d*x - c))^2 - 192*a*c^6*d^5*(sqrt(d*x +
 c) - sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c
^2)^4*c^4))/d
```

# 3.269 $\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=161

$$\frac{3c^2\left(4ad^2+5bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^7}+\frac{3x\sqrt{dx-c}\sqrt{c+dx}\left(4ad^2+5bc^2\right)}{8d^6}-\frac{x^3\left(4ad^2+5bc^2\right)}{4d^4\sqrt{dx-c}\sqrt{c+dx}}+\frac{bx^5}{4d^2\sqrt{dx-c}\sqrt{c+dx}}$$

```
[Out] -((5*b*c^2 + 4*a*d^2)*x^3)/(4*d^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) +
 (b*x^5)/(4*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (3*(5*b*c^2 + 4*a
*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*d^6) + (3*c^2*(5*b*c^2 +
 4*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(4*d^7)
```

---

**Rubi [A]** time = 0.417564, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226

$$\frac{3c^2\left(4ad^2+5bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^7}+\frac{3x\sqrt{dx-c}\sqrt{c+dx}\left(4ad^2+5bc^2\right)}{8d^6}-\frac{x^3\left(4ad^2+5bc^2\right)}{4d^4\sqrt{dx-c}\sqrt{c+dx}}+\frac{bx^5}{4d^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -((5*b*c^2 + 4*a*d^2)*x^3)/(4*d^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) +
 (b*x^5)/(4*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (3*(5*b*c^2 + 4*a
*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*d^6) + (3*c^2*(5*b*c^2 +
 4*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(4*d^7)
```

---

**Rubi in Sympy [A]** time = 26.8087, size = 146, normalized size = 0.91

$$\frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}}+\frac{3c^2\left(4ad^2+5bc^2\right)\operatorname{atanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{4d^7}-\frac{x^3\left(4ad^2+5bc^2\right)}{4d^4\sqrt{-c+dx}\sqrt{c+dx}}+\frac{3x\sqrt{-c+dx}\sqrt{c+dx}\left(4ad^2+5bc^2\right)}{8d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] b*x**5/(4*d**2*sqrt(-c + d*x)*sqrt(c + d*x)) + 3*c**2*(4*a*d**2 +
 5*b*c**2)*atanh(sqrt(-c + d*x)/sqrt(c + d*x))/(4*d**7) - x**3*(4
*a*d**2 + 5*b*c**2)/(4*d**4*sqrt(-c + d*x)*sqrt(c + d*x)) + 3*x*s
qrt(-c + d*x)*sqrt(c + d*x)*(4*a*d**2 + 5*b*c**2)/(8*d**6)
```

---

**Mathematica [A]** time = 0.196303, size = 150, normalized size = 0.93

$$\frac{dx\sqrt{dx-c}\sqrt{c+dx}\left(4ad^2\left(d^2x^2-3c^2\right)+b\left(-15c^4+5c^2d^2x^2+2d^4x^4\right)\right)-3c^2\left(c^2-d^2x^2\right)\left(4ad^2+5bc^2\right)\log\left(\sqrt{dx-c}\sqrt{c+\right.}{8d^7(dx-c)(c+dx)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(4*a*d^2*(-3*c^2 + d^2*x^2) + b
*(-15*c^4 + 5*c^2*d^2*x^2 + 2*d^4*x^4)) - 3*c^2*(5*b*c^2 + 4*a*d^
2)*(c^2 - d^2*x^2)*Log[d*x + Sqrt[-c + d*x]*Sqrt[c + d*x]])/(8*d^
7*(-c + d*x)*(c + d*x))
```

---

**Maple [C]** time = 0.047, size = 316, normalized size = 2.

$$\frac{csgn(d)}{8d^7}\left(2\,csgn(d)\,x^5bd^5\sqrt{d^2x^2-c^2}+4\,csgn(d)\,x^3ad^5\sqrt{d^2x^2-c^2}+5\,csgn(d)\,x^3bc^2d^3\sqrt{d^2x^2-c^2}+12\,\ln\left(\left(csgn(d)\sqrt{d^2x^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/8*(2*csgn(d)*x^5*b*d^5*(d^2*x^2-c^2)^(1/2)+4*csgn(d)*x^3*a*d^5*
(d^2*x^2-c^2)^(1/2)+5*csgn(d)*x^3*b*c^2*d^3*(d^2*x^2-c^2)^(1/2)+1
2*ln((csgn(d)*(d^2*x^2-c^2)^(1/2)+d*x)*csgn(d))*x^2*a*c^2*d^4+15*
ln((csgn(d)*(d^2*x^2-c^2)^(1/2)+d*x)*csgn(d))*x^2*b*c^4*d^2-12*a*
c^2*x*(d^2*x^2-c^2)^(1/2)*d^3*csgn(d)-15*b*c^4*x*(d^2*x^2-c^2)^(1
/2)*csgn(d)*d-12*a*c^4*ln((csgn(d)*(d^2*x^2-c^2)^(1/2)+d*x)*csgn(
d))*d^2-15*b*c^6*ln((csgn(d)*(d^2*x^2-c^2)^(1/2)+d*x)*csgn(d)))*c
sgn(d)/(d^2*x^2-c^2)^(1/2)/d^7/(d*x+c)^(1/2)/(d*x-c)^(1/2)
```

---

**Maxima [A]** time = 1.43494, size = 289, normalized size = 1.8

$$\frac{bx^5}{4\sqrt{d^2x^2-c^2}d^2}+\frac{5\,bc^2x^3}{8\sqrt{d^2x^2-c^2}d^4}+\frac{ax^3}{2\sqrt{d^2x^2-c^2}d^2}-\frac{15\,bc^4x}{8\sqrt{d^2x^2-c^2}d^6}-\frac{3\,ac^2x}{2\sqrt{d^2x^2-c^2}d^4}+\frac{15\,bc^4\log\left(2\,d^2x+2\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{8\sqrt{d^2}d^6}+\frac{3\,ac^2\log\left(2\,d^2x+2\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{2\sqrt{d^2}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^4/((d*x + c)^(3/2)*(d*x - c)^(3/2)),x, algorithm="maxima
```

```
[Out] 1/4*b*x^5/(sqrt(d^2*x^2 - c^2)*d^2) + 5/8*b*c^2*x^3/(sqrt(d^2*x^2
 - c^2)*d^4) + 1/2*a*x^3/(sqrt(d^2*x^2 - c^2)*d^2) - 15/8*b*c^4*x
/(sqrt(d^2*x^2 - c^2)*d^6) - 3/2*a*c^2*x/(sqrt(d^2*x^2 - c^2)*d^4
) + 15/8*b*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/(sq
rt(d^2)*d^6) + 3/2*a*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*sqrt
(d^2))/(sqrt(d^2)*d^4)
```

---

**Fricas [A]** time = 0.244879, size = 702, normalized size = 4.36

$$-\frac{32\,bd^{10}x^{10} - 8\,bc^{10} - 8\,ac^{8}d^{2} + 8\left(5\,bc^{2}d^{8} + 8\,ad^{10}\right)x^{8} - 2\left(101\,bc^{4}d^{6} + 72\,ac^{2}d^{8}\right)x^{6} + \left(101\,bc^{6}d^{4} + 36\,ac^{4}d^{6}\right)x^{4} + \left(29\,bc^{8}d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^4/((d*x + c)^(3/2)*(d*x - c)^(3/2)),x, algorithm="fricas
```

```
[Out] -1/8*(32*b*d^10*x^10 - 8*b*c^10 - 8*a*c^8*d^2 + 8*(5*b*c^2*d^8 +
8*a*d^10)*x^8 - 2*(101*b*c^4*d^6 + 72*a*c^2*d^8)*x^6 + (101*b*c^6
*d^4 + 36*a*c^4*d^6)*x^4 + (29*b*c^8*d^2 + 44*a*c^6*d^4)*x^2 - (3
2*b*d^9*x^9 + 8*(7*b*c^2*d^7 + 8*a*d^9)*x^7 - 2*(85*b*c^4*d^5 + 5
6*a*c^2*d^7)*x^5 + (25*b*c^6*d^3 - 12*a*c^4*d^5)*x^3 + (25*b*c^8*
d + 28*a*c^6*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) - 3*(5*b*c^10 +
4*a*c^8*d^2 - 16*(5*b*c^4*d^6 + 4*a*c^2*d^8)*x^6 + 28*(5*b*c^6*d^
4 + 4*a*c^4*d^6)*x^4 - 13*(5*b*c^8*d^2 + 4*a*c^6*d^4)*x^2 + (16*(
5*b*c^4*d^5 + 4*a*c^2*d^7)*x^5 - 20*(5*b*c^6*d^3 + 4*a*c^4*d^5)*x
^3 + 5*(5*b*c^8*d + 4*a*c^6*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c))*
log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/(16*d^13*x^6 - 28*c^2*d^
11*x^4 + 13*c^4*d^9*x^2 - c^6*d^7 - (16*d^12*x^5 - 20*c^2*d^10*x^
3 + 5*c^4*d^8*x)*sqrt(d*x + c)*sqrt(d*x - c))
```

---

**Sympy [A]** time = 102.515, size = 233, normalized size = 1.45

$$a\left(\frac{c^{2}G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4}, -\frac{3}{4} & -\frac{3}{2}, -\frac{1}{2}, 0, 1 \\ -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 0, 0 & \end{matrix} \middle| \frac{c^{2}}{d^{2}x^{2}}\right)}{2\pi^{\frac{3}{2}}d^{5}} + \frac{ic^{2}G_{6,6}^{2,6}\left(\begin{matrix} -\frac{5}{2}, -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, 1 & \\ -\frac{7}{4}, -\frac{5}{4} & -\frac{5}{2}, -2, -1, 0 \end{matrix} \middle| \frac{c^{2}e^{2i\pi}}{d^{2}x^{2}}\right)}{2\pi^{\frac{3}{2}}d^{5}}\right) + b\left(\frac{c^{4}G_{6,6}^{6,2}\left(\begin{matrix} -\frac{9}{4}, -\frac{7}{4} & -\frac{5}{2}, -\frac{3}{2}, -1, 1 \\ -\frac{9}{4}, -2, -\frac{7}{4}, -\frac{3}{2}, -1, 0 & \end{matrix} \middle| \frac{c^{2}}{d^{2}x^{2}}\right)}{2\pi^{\frac{3}{2}}d^{7}} + \frac{ic^{4}G_{6,6}^{2,6}\left(\begin{matrix} -\frac{7}{2}, -3, -\frac{11}{4}, -\frac{5}{2}, -\frac{9}{4}, 1 & \\ -\frac{11}{4}, -\frac{9}{4} & -\frac{7}{2}, -3, -2, 0 \end{matrix} \middle| \frac{c^{2}e^{2i\pi}}{d^{2}x^{2}}\right)}{2\pi^{\frac{3}{2}}d^{7}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(c**2*meijerg(((-5/4, -3/4), (-3/2, -1/2, 0, 1)), ((-5/4, -1, -
3/4, -1/2, 0, 0), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**5) + I*c
**2*meijerg(((-5/2, -2, -7/4, -3/2, -5/4, 1), ()), ((-7/4, -5/4),
 (-5/2, -2, -1, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(
3/2)*d**5)) + b*(c**4*meijerg(((-9/4, -7/4), (-5/2, -3/2, -1, 1))
, ((-9/4, -2, -7/4, -3/2, -1, 0), ()), c**2/(d**2*x**2))/(2*pi**(
3/2)*d**7) + I*c**4*meijerg(((-7/2, -3, -11/4, -5/2, -9/4, 1), ()
), ((-11/4, -9/4), (-7/2, -3, -2, 0)), c**2*exp_polar(2*I*pi)/(d*
*2*x**2))/(2*pi**(3/2)*d**7))
```

---

**GIAC/XCAS [A]** time = 0.243284, size = 290, normalized size = 1.8

$$-\frac{1}{688128}\left(5\,bc^{3}d^{35} + 4\,acd^{37}\right)\ln\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^{2}\right) - \frac{\left(\left(\left(2\left(5\,bd^{35} - \frac{(dx+c)bd^{35}}{c}\right)(dx+c) - \frac{25\,bc^{2}d^{35}+4\,ad^{37}}{c}\right)(dx+c) + \frac{35\,bc^{3}d^{35}+12\,acd^{37}}{c}\right)(dx+c) - \frac{2\left(7\,bc^{4}d^{35}+2\,ac^{2}d^{37}\right)}{c}\right)\sqrt{dx+c}}{2064384\sqrt{dx-c}} - \frac{2\left(bc^{5} + ac^{3}d^{2}\right)}{\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^{2} + 2\,c\right)d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^4/((d*x + c)^(3/2)*(d*x - c)^(3/2)),x, algorithm="giac")
```

```
[Out] -1/688128*(5*b*c^3*d^35 + 4*a*c*d^37)*ln((sqrt(d*x + c) - sqrt(d*
x - c))^2) - 1/2064384*(((2*(5*b*d^35 - (d*x + c)*b*d^35/c)*(d*x
+ c) - (25*b*c^2*d^35 + 4*a*d^37)/c)*(d*x + c) + (35*b*c^3*d^35 +
 12*a*c*d^37)/c)*(d*x + c) - 2*(7*b*c^4*d^35 + 2*a*c^2*d^37)/c)*s
qrt(d*x + c)/sqrt(d*x - c) - 2*(b*c^5 + a*c^3*d^2)/(((sqrt(d*x +
c) - sqrt(d*x - c))^2 + 2*c)*d^7)
```

## 3.270 $\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=115

$$\frac{2\sqrt{dx-c}\sqrt{c+dx}\,(3ad^2+4bc^2)}{3d^6}-\frac{x^2\,(3ad^2+4bc^2)}{3d^4\sqrt{dx-c}\sqrt{c+dx}}+\frac{bx^4}{3d^2\sqrt{dx-c}\sqrt{c+dx}}$$

```
[Out] -((4*b*c^2 + 3*a*d^2)*x^2)/(3*d^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) +
 (b*x^4)/(3*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (2*(4*b*c^2 + 3*a
*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*d^6)
```

---

**Rubi [A]** time = 0.316393, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129

$$\frac{2\sqrt{dx-c}\sqrt{c+dx}\,(3ad^2+4bc^2)}{3d^6}-\frac{x^2\,(3ad^2+4bc^2)}{3d^4\sqrt{dx-c}\sqrt{c+dx}}+\frac{bx^4}{3d^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -((4*b*c^2 + 3*a*d^2)*x^2)/(3*d^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) +
 (b*x^4)/(3*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (2*(4*b*c^2 + 3*a
*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*d^6)
```

---

**Rubi in Sympy [A]** time = 18.6131, size = 102, normalized size = 0.89

$$\frac{bx^4}{3d^2\sqrt{-c+dx}\sqrt{c+dx}}-\frac{x^2\,(3ad^2+4bc^2)}{3d^4\sqrt{-c+dx}\sqrt{c+dx}}+\frac{2\sqrt{-c+dx}\sqrt{c+dx}\,(3ad^2+4bc^2)}{3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] b*x**4/(3*d**2*sqrt(-c + d*x)*sqrt(c + d*x)) - x**2*(3*a*d**2 + 4
*b*c**2)/(3*d**4*sqrt(-c + d*x)*sqrt(c + d*x)) + 2*sqrt(-c + d*x)
*sqrt(c + d*x)*(3*a*d**2 + 4*b*c**2)/(3*d**6)
```

---

**Mathematica [A]** time = 0.0896928, size = 72, normalized size = 0.63

$$\frac{-6ac^2d^2+3ad^4x^2-8bc^4+4bc^2d^2x^2+bd^4x^4}{3d^6\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-8*b*c^4 - 6*a*c^2*d^2 + 4*b*c^2*d^2*x^2 + 3*a*d^4*x^2 + b*d^4*x
^4)/(3*d^6*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

---

**Maple [A]** time = 0.008, size = 68, normalized size = 0.6

$$-\frac{-bd^4x^4 - 3\,ad^4x^2 - 4\,bc^2d^2x^2 + 6\,ac^2d^2 + 8\,bc^4}{3\,d^6}\frac{1}{\sqrt{dx+c}}\frac{1}{\sqrt{dx-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] -1/3*(-b*d^4*x^4-3*a*d^4*x^2-4*b*c^2*d^2*x^2+6*a*c^2*d^2+8*b*c^4)
/(d*x+c)^(1/2)/d^6/(d*x-c)^(1/2)
```

---

**Maxima [A]** time = 1.40378, size = 166, normalized size = 1.44

$$\frac{bx^4}{3\,\sqrt{d^2x^2 - c^2}d^2} + \frac{4\,bc^2x^2}{3\,\sqrt{d^2x^2 - c^2}d^4} + \frac{ax^2}{\sqrt{d^2x^2 - c^2}d^2} - \frac{8\,bc^4}{3\,\sqrt{d^2x^2 - c^2}d^6} - \frac{2\,ac^2}{\sqrt{d^2x^2 - c^2}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^3/((d*x + c)^(3/2)*(d*x - c)^(3/2)),x, algorithm="maxima
```

```
[Out] 1/3*b*x^4/(sqrt(d^2*x^2 - c^2)*d^2) + 4/3*b*c^2*x^2/(sqrt(d^2*x^2
 - c^2)*d^4) + a*x^2/(sqrt(d^2*x^2 - c^2)*d^2) - 8/3*b*c^4/(sqrt(
d^2*x^2 - c^2)*d^6) - 2*a*c^2/(sqrt(d^2*x^2 - c^2)*d^4)
```

---

**Fricas [A]** time = 0.254866, size = 354, normalized size = 3.08

$$-\frac{8\,bd^8x^8 - 8\,bc^8 - 6\,ac^6d^2 + 24\,(bc^2d^6 + ad^8)x^6 - (95\,bc^4d^4 + 72\,ac^2d^6)x^4 + 17\,(4\,bc^6d^2 + 3\,ac^4d^4)x^2 - 4\,(2\,bd^7x^7 + (7\,bc^2 \cdots}{3\,(8\,d^{11}x^5 - 12\,c^2d^9x^3 + 4\,c^4d^7x - (8\,d^{10}x^4 - 8\,c^2d^8x^2 + c^4d \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^3/((d*x + c)^(3/2)*(d*x - c)^(3/2)),x, algorithm="fricas
```

```
[Out] -1/3*(8*b*d^8*x^8 - 8*b*c^8 - 6*a*c^6*d^2 + 24*(b*c^2*d^6 + a*d^8
)*x^6 - (95*b*c^4*d^4 + 72*a*c^2*d^6)*x^4 + 17*(4*b*c^6*d^2 + 3*a
*c^4*d^4)*x^2 - 4*(2*b*d^7*x^7 + (7*b*c^2*d^5 + 6*a*d^7)*x^5 - 5*
(4*b*c^4*d^3 + 3*a*c^2*d^5)*x^3 + 2*(4*b*c^6*d + 3*a*c^4*d^3)*x)*
sqrt(d*x + c)*sqrt(d*x - c))/(8*d^11*x^5 - 12*c^2*d^9*x^3 + 4*c^4
*d^7*x - (8*d^10*x^4 - 8*c^2*d^8*x^2 + c^4*d^6)*sqrt(d*x + c)*sqr
t(d*x - c))
```

---

**Sympy [A]** time = 82.2935, size = 226, normalized size = 1.97

$$a\left(\frac{cG_{6,6}^{6,2}\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{4} & -1, 0, \frac{1}{2}, 1 \\ -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{2}, 0 & \end{matrix} \middle| \frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^4} - \frac{icG_{6,6}^{2,6}\left(\begin{matrix} -2, -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, 1 & \\ -\frac{5}{4}, -\frac{3}{4} & -2, -\frac{3}{2}, -\frac{1}{2}, 0 \end{matrix} \middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^4}\right) + b\left(\frac{c^3G_{6,6}^{6,2}\left(\begin{matrix} -\frac{7}{4}, -\frac{5}{4} & -2, -1, -\frac{1}{2}, 1 \\ -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{1}{2}, 0 & \end{matrix} \middle| \frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^6} - \frac{ic^3G_{6,6}^{2,6}\left(\begin{matrix} -3, -\frac{5}{2}, -\frac{9}{4}, -2, -\frac{7}{4}, 1 & \\ -\frac{9}{4}, -\frac{7}{4} & -3, -\frac{5}{2}, -\frac{3}{2}, 0 \end{matrix} \middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(c*meijerg(((-3/4, -1/4), (-1, 0, 1/2, 1)), ((-3/4, -1/2, -1/4,
 0, 1/2, 0), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**4) - I*c*meij
erg(((-2, -3/2, -5/4, -1, -3/4, 1), ()), ((-5/4, -3/4), (-2, -3/2
, -1/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**
4)) + b*(c**3*meijerg(((-7/4, -5/4), (-2, -1, -1/2, 1)), ((-7/4,
-3/2, -5/4, -1, -1/2, 0), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**
6) - I*c**3*meijerg(((-3, -5/2, -9/4, -2, -7/4, 1), ()), ((-9/4,
-7/4), (-3, -5/2, -3/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/
(2*pi**(3/2)*d**6))
```

---

**GIAC/XCAS [A]** time = 0.236065, size = 194, normalized size = 1.69

$$-\frac{\left(2\left(\left(4\,bd^{24}-\frac{(dx+c)bd^{24}}{c}\right)(dx+c)-\frac{10\,bc^2d^{24}+3\,ad^{26}}{c}\right)(dx+c)+\frac{3\left(9\,bc^3d^{24}+5\,acd^{26}\right)}{c}\right)\sqrt{dx+c}}{23040\,\sqrt{dx-c}}+\frac{2\left(bc^4+ac^2d^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^3/((d*x + c)^(3/2)*(d*x - c)^(3/2)),x, algorithm="giac")
```

```
[Out] -1/23040*(2*((4*b*d^24 - (d*x + c)*b*d^24/c)*(d*x + c) - (10*b*c^
2*d^24 + 3*a*d^26)/c)*(d*x + c) + 3*(9*b*c^3*d^24 + 5*a*c*d^26)/c
)*sqrt(d*x + c)/sqrt(d*x - c) + 2*(b*c^4 + a*c^2*d^2)/(((sqrt(d*x
 + c) - sqrt(d*x - c))^2 + 2*c)*d^6)
```

## 3.271 $\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=152

$$-\frac{\sqrt{dx-c}\left(2ad^2+3bc^2\right)}{2d^5\sqrt{c+dx}}-\frac{c\left(2ad^2+3bc^2\right)}{2d^5\sqrt{dx-c}\sqrt{c+dx}}+\frac{\left(2ad^2+3bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^5}+\frac{bx^3}{2d^2\sqrt{dx-c}\sqrt{c+dx}}$$

```
[Out] -(c*(3*b*c^2 + 2*a*d^2))/(2*d^5*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (
b*x^3)/(2*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((3*b*c^2 + 2*a*d^2
)*Sqrt[-c + d*x])/(2*d^5*Sqrt[c + d*x]) + ((3*b*c^2 + 2*a*d^2)*Ar
cTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/d^5
```

---

**Rubi [A]** time = 0.380678, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194

$$-\frac{\sqrt{dx-c}\left(2ad^2+3bc^2\right)}{2d^5\sqrt{c+dx}}-\frac{c\left(2ad^2+3bc^2\right)}{2d^5\sqrt{dx-c}\sqrt{c+dx}}+\frac{\left(2ad^2+3bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^5}+\frac{bx^3}{2d^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -(c*(3*b*c^2 + 2*a*d^2))/(2*d^5*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (
b*x^3)/(2*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((3*b*c^2 + 2*a*d^2
)*Sqrt[-c + d*x])/(2*d^5*Sqrt[c + d*x]) + ((3*b*c^2 + 2*a*d^2)*Ar
cTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/d^5
```

---

**Rubi in Sympy [A]** time = 24.6745, size = 134, normalized size = 0.88

$$\frac{bx^3}{2d^2\sqrt{-c+dx}\sqrt{c+dx}}-\frac{c\left(2ad^2+3bc^2\right)}{2d^5\sqrt{-c+dx}\sqrt{c+dx}}-\frac{\sqrt{-c+dx}\left(2ad^2+3bc^2\right)}{2d^5\sqrt{c+dx}}+\frac{\left(2ad^2+3bc^2\right)\operatorname{atanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] b*x**3/(2*d**2*sqrt(-c + d*x)*sqrt(c + d*x)) - c*(2*a*d**2 + 3*b*
c**2)/(2*d**5*sqrt(-c + d*x)*sqrt(c + d*x)) - sqrt(-c + d*x)*(2*a
*d**2 + 3*b*c**2)/(2*d**5*sqrt(c + d*x)) + (2*a*d**2 + 3*b*c**2)*
atanh(sqrt(-c + d*x)/sqrt(c + d*x))/d**5
```

**Mathematica [A]** time = 0.16081, size = 121, normalized size = 0.8

$$\frac{dx\sqrt{dx-c}\sqrt{c+dx}\left(-2ad^2-3bc^2+bd^2x^2\right)-\left(c^2-d^2x^2\right)\left(2ad^2+3bc^2\right)\log\left(\sqrt{dx-c}\sqrt{c+dx}+dx\right)}{2d^5(dx-c)(c+dx)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (d*x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(-3*b*c^2 - 2*a*d^2 + b*d^2*x^2
) - (3*b*c^2 + 2*a*d^2)*(c^2 - d^2*x^2)*Log[d*x + Sqrt[-c + d*x]*
Sqrt[c + d*x]])/(2*d^5*(-c + d*x)*(c + d*x))
```

**Maple [C]** time = 0.033, size = 254, normalized size = 1.7

$$\frac{csgn(d)}{2\,d^5}\left(csgn(d)\,x^3bd^3\sqrt{d^2x^2-c^2}+2\,\ln\left(\left(csgn(d)\,\sqrt{d^2x^2-c^2}+dx\right)csgn(d)\right)x^2ad^4+3\,\ln\left(\left(csgn(d)\,\sqrt{d^2x^2-c^2}+dx\right)c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/2*(csgn(d)*x^3*b*d^3*(d^2*x^2-c^2)^(1/2)+2*ln((csgn(d)*(d^2*x^2
-c^2)^(1/2)+d*x)*csgn(d))*x^2*a*d^4+3*ln((csgn(d)*(d^2*x^2-c^2)^(
1/2)+d*x)*csgn(d))*x^2*b*c^2*d^2-2*a*x*(d^2*x^2-c^2)^(1/2)*d^3*cs
gn(d)-3*b*c^2*x*(d^2*x^2-c^2)^(1/2)*csgn(d)*d-2*a*c^2*ln((csgn(d)
*(d^2*x^2-c^2)^(1/2)+d*x)*csgn(d))*d^2-3*b*c^4*ln((csgn(d)*(d^2*x
^2-c^2)^(1/2)+d*x)*csgn(d)))*csgn(d)/(d^2*x^2-c^2)^(1/2)/d^5/(d*x
+c)^(1/2)/(d*x-c)^(1/2)
```

**Maxima [A]** time = 1.41408, size = 211, normalized size = 1.39

$$\frac{bx^3}{2\sqrt{d^2x^2-c^2}d^2}-\frac{3\,bc^2x}{2\sqrt{d^2x^2-c^2}d^4}-\frac{ax}{\sqrt{d^2x^2-c^2}d^2}+\frac{3\,bc^2\log\left(2\,d^2x+2\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{2\sqrt{d^2}d^4}+\frac{a\log\left(2\,d^2x+2\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{\sqrt{d^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^2/((d*x + c)^(3/2)*(d*x - c)^(3/2)),x, algorithm="maxima
```

```
[Out] 1/2*b*x^3/(sqrt(d^2*x^2 - c^2)*d^2) - 3/2*b*c^2*x/(sqrt(d^2*x^2 -
 c^2)*d^4) - a*x/(sqrt(d^2*x^2 - c^2)*d^2) + 3/2*b*c^2*log(2*d^2*
x + 2*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^4) + a*log(2*d^
2*x + 2*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^2)
```

---

**Fricas [A]** time = 0.246244, size = 448, normalized size = 2.95

$$-\frac{4\,bd^6x^6 - 7\,bc^2d^4x^4 + 2\,bc^6 + 2\,ac^4d^2 - \left(bc^4d^2 + 4\,ac^2d^4\right)x^2 - \left(4\,bd^5x^5 - 5\,bc^2d^3x^3 - \left(3\,bc^4d + 4\,ac^2d^3\right)x\right)\sqrt{dx+c}\sqrt{dx-}}{2\left(4\,d^9x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^2/((d*x + c)^(3/2)*(d*x - c)^(3/2)),x, algorithm="fricas
```

```
[Out] -1/2*(4*b*d^6*x^6 - 7*b*c^2*d^4*x^4 + 2*b*c^6 + 2*a*c^4*d^2 - (b*
c^4*d^2 + 4*a*c^2*d^4)*x^2 - (4*b*d^5*x^5 - 5*b*c^2*d^3*x^3 - (3*
b*c^4*d + 4*a*c^2*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) + (3*b*c^6
+ 2*a*c^4*d^2 + 4*(3*b*c^2*d^4 + 2*a*d^6)*x^4 - 5*(3*b*c^4*d^2 +
2*a*c^2*d^4)*x^2 - (4*(3*b*c^2*d^3 + 2*a*d^5)*x^3 - 3*(3*b*c^4*d
+ 2*a*c^2*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c))*log(-d*x + sqrt(d*
x + c)*sqrt(d*x - c)))/(4*d^9*x^4 - 5*c^2*d^7*x^2 + c^4*d^5 - (4*
d^8*x^3 - 3*c^2*d^6*x)*sqrt(d*x + c)*sqrt(d*x - c))
```

---

**Sympy [A]** time = 80.9724, size = 212, normalized size = 1.39

$$a\left(\frac{G_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, \frac{1}{2}, 1, 1 \\ -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1, 0 & \end{matrix}\middle| \frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^3} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{3}{2}, -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 1 & \\ -\frac{3}{4}, -\frac{1}{4} & -\frac{3}{2}, -1, 0, 0 \end{matrix}\middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^3}\right) + b\left(\frac{c^2G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4}, -\frac{3}{4} & -\frac{3}{2}, -\frac{1}{2}, 0, 1 \\ -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 0, 0 & \end{matrix}\middle| \frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^5} + \frac{ic^2G_{6,6}^{2,6}\left(\begin{matrix} -\frac{5}{2}, -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, 1 & \\ -\frac{7}{4}, -\frac{5}{4} & -\frac{5}{2}, -2, -1, 0 \end{matrix}\middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(meijerg(((-1/4, 1/4), (-1/2, 1/2, 1, 1)), ((-1/4, 0, 1/4, 1/2,
 1, 0), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**3) + I*meijerg(((-
3/2, -1, -3/4, -1/2, -1/4, 1), ()), ((-3/4, -1/4), (-3/2, -1, 0,
0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**3)) + b*
(c**2*meijerg(((-5/4, -3/4), (-3/2, -1/2, 0, 1)), ((-5/4, -1, -3/
4, -1/2, 0, 0), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**5) + I*c**
2*meijerg(((-5/2, -2, -7/4, -3/2, -5/4, 1), ()), ((-7/4, -5/4), (
-5/2, -2, -1, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/
2)*d**5))
```

---

**GIAC/XCAS [A]** time = 0.241076, size = 208, normalized size = 1.37

$$-\frac{\left(\left(3\,bd^{15}-\frac{(dx+c)bd^{15}}{c}\right)(dx+c)-\frac{bc^2d^{15}-ad^{17}}{c}\right)\sqrt{dx+c}}{768\,\sqrt{dx-c}}-\frac{\left(3\,bc^2d^{15}+2\,ad^{17}\right)\ln\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{768\,c}-\frac{2\,\left(bc^3+acd^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x^2/((d*x + c)^(3/2)*(d*x - c)^(3/2)),x, algorithm="giac")
```

```
[Out] -1/768*((3*b*d^15 - (d*x + c)*b*d^15/c)*(d*x + c) - (b*c^2*d^15 -
 a*d^17)/c)*sqrt(d*x + c)/sqrt(d*x - c) - 1/768*(3*b*c^2*d^15 + 2
*a*d^17)*ln((sqrt(d*x + c) - sqrt(d*x - c))^2)/c - 2*(b*c^3 + a*c
*d^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*d^5)
```

## 3.272 $\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=76

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\,(ad^2+2bc^2)}{c^2d^4}-\frac{x^2\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

```
[Out] -(((a/c^2 + b/d^2)*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) + ((2*b*c
^2 + a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*d^4)
```

---

**Rubi [A]** time = 0.181657, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\,(ad^2+2bc^2)}{c^2d^4}-\frac{x^2\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -(((a/c^2 + b/d^2)*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) + ((2*b*c
^2 + a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*d^4)
```

---

**Rubi in Sympy [A]** time = 11.3348, size = 65, normalized size = 0.86

$$-\frac{x^2\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{-c+dx}\sqrt{c+dx}}+\frac{\sqrt{-c+dx}\sqrt{c+dx}\,(ad^2+2bc^2)}{c^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] -x**2*(a/c**2 + b/d**2)/(sqrt(-c + d*x)*sqrt(c + d*x)) + sqrt(-c
+ d*x)*sqrt(c + d*x)*(a*d**2 + 2*b*c**2)/(c**2*d**4)
```

---

**Mathematica [A]** time = 0.07601, size = 45, normalized size = 0.59

$$\frac{-ad^2-2bc^2+bd^2x^2}{d^4\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*b*c^2 - a*d^2 + b*d^2*x^2)/(d^4*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

---

**Maple [A]** time = 0.005, size = 43, normalized size = 0.6

$$-\frac{-bd^2x^2+ad^2+2\,bc^2}{d^4}\frac{1}{\sqrt{dx+c}}\frac{1}{\sqrt{dx-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] -(-b*d^2*x^2+a*d^2+2*b*c^2)/(d*x+c)^(1/2)/d^4/(d*x-c)^(1/2)
```

---

**Maxima [A]** time = 1.38135, size = 93, normalized size = 1.22

$$\frac{bx^2}{\sqrt{d^2x^2-c^2}d^2}-\frac{2\,bc^2}{\sqrt{d^2x^2-c^2}d^4}-\frac{a}{\sqrt{d^2x^2-c^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x/((d*x + c)^(3/2)*(d*x - c)^(3/2)),x, algorithm="maxima")
```

```
[Out] b*x^2/(sqrt(d^2*x^2 - c^2)*d^2) - 2*b*c^2/(sqrt(d^2*x^2 - c^2)*d^
4) - a/(sqrt(d^2*x^2 - c^2)*d^2)
```

---

**Fricas [A]** time = 0.247176, size = 196, normalized size = 2.58

$$-\frac{2\,bd^4x^4+2\,bc^4+ac^2d^2-\left(5\,bc^2d^2+2\,ad^4\right)x^2-2\left(bd^3x^3-\left(2\,bc^2d+ad^3\right)x\right)\sqrt{dx+c}\sqrt{dx-c}}{2\,d^7x^3-2\,c^2d^5x-(2\,d^6x^2-c^2d^4)\sqrt{dx+c}\sqrt{dx-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*x/((d*x + c)^(3/2)*(d*x - c)^(3/2)),x, algorithm="fricas")
```

```
[Out] -(2*b*d^4*x^4 + 2*b*c^4 + a*c^2*d^2 - (5*b*c^2*d^2 + 2*a*d^4)*x^2
 - 2*(b*d^3*x^3 - (2*b*c^2*d + a*d^3)*x)*sqrt(d*x + c)*sqrt(d*x -
```

```
c))/(2*d^7*x^3 - 2*c^2*d^5*x - (2*d^6*x^2 - c^2*d^4)*sqrt(d*x +
c)*sqrt(d*x - c))
```

---

**Sympy [A]** time = 71.8296, size = 201, normalized size = 2.64

$$a\left(-\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{1}{4},\frac{3}{4},1 & 0,1,\frac{3}{2}\\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{3}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}cd^2}-\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},1 & \\ -\frac{1}{4},\frac{1}{4} & -1,-\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}cd^2}\right)+b\left(\frac{cG_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & -1,0,\frac{1}{2},1\\ -\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,\frac{1}{2},0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^4}-\frac{icG_{6,6}^{2,6}\left(\begin{matrix}-2,-\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},1 & \\ -\frac{5}{4},-\frac{3}{4} & -2,-\frac{3}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(-meijerg(((1/4, 3/4, 1), (0, 1, 3/2)), ((1/4, 1/2, 3/4, 1, 3/2
), (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c*d**2) - I*meijerg(((-1
, -1/2, -1/4, 0, 1/4, 1), ()), ((-1/4, 1/4), (-1, -1/2, 1/2, 0)),
 c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c*d**2)) + b*(c
*meijerg(((-3/4, -1/4), (-1, 0, 1/2, 1)), ((-3/4, -1/2, -1/4, 0,
1/2, 0), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**4) - I*c*meijerg(
((-2, -3/2, -5/4, -1, -3/4, 1), ()), ((-5/4, -3/4), (-2, -3/2, -1
/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**4))
```

---

**GIAC/XCAS [A]** time = 0.22925, size = 127, normalized size = 1.67

$$\frac{\left(2\,(dx+c)bd^8-\frac{5\,bc^2d^8+ad^{10}}{c}\right)\sqrt{dx+c}}{32\,\sqrt{dx-c}}+\frac{2\,\left(bc^2+ad^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*x/((d*x + c)^(3/2)*(d*x - c)^(3/2)),x, algorithm="giac")
```

```
[Out] 1/32*(2*(d*x + c)*b*d^8 - (5*b*c^2*d^8 + a*d^10)/c)*sqrt(d*x + c)
/sqrt(d*x - c) + 2*(b*c^2 + a*d^2)/(((sqrt(d*x + c) - sqrt(d*x -
c))^2 + 2*c)*d^4)
```

## 3.273 $\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=63

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}-\frac{x\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

```
[Out] -(((a/c^2 + b/d^2)*x)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) + (2*b*ArcT
anh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/d^3
```

---

**Rubi [A]** time = 0.114977, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}-\frac{x\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -(((a/c^2 + b/d^2)*x)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) + (2*b*ArcT
anh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/d^3
```

---

**Rubi in Sympy [A]** time = 21.0818, size = 94, normalized size = 1.49

$$-\frac{ax}{c^2\sqrt{-c+dx}\sqrt{c+dx}}-\frac{bc}{d^3\sqrt{-c+dx}\sqrt{c+dx}}-\frac{b\sqrt{-c+dx}}{d^3\sqrt{c+dx}}+\frac{2b\,\mathrm{atanh}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] -a*x/(c**2*sqrt(-c + d*x)*sqrt(c + d*x)) - b*c/(d**3*sqrt(-c + d*
x)*sqrt(c + d*x)) - b*sqrt(-c + d*x)/(d**3*sqrt(c + d*x)) + 2*b*a
tanh(sqrt(-c + d*x)/sqrt(c + d*x))/d**3
```

---

**Mathematica [A]** time = 0.0934242, size = 100, normalized size = 1.59

$$\frac{dx\sqrt{dx-c}\sqrt{c+dx}\left(ad^2+bc^2\right)+bc^2\left(c^2-d^2x^2\right)\log\left(\sqrt{dx-c}\sqrt{c+dx}+dx\right)}{c^2d^3(c-dx)(c+dx)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (d*(b*c^2 + a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x] + b*c^2*(c^2 -
d^2*x^2)*Log[d*x + Sqrt[-c + d*x]*Sqrt[c + d*x]])/(c^2*d^3*(c - d
*x)*(c + d*x))
```

---

**Maple [C]** time = 0.027, size = 160, normalized size = 2.5

$$\frac{csgn(d)}{c^2d^3}\left(\ln\left(\left(csgn(d)\sqrt{d^2x^2-c^2}+dx\right)csgn(d)\right)x^2bc^2d^2-ax\sqrt{d^2x^2-c^2}d^3csgn(d)-bc^2x\sqrt{d^2x^2-c^2}csgn(d)d-bc^4\ln\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] (ln((csgn(d)*(d^2*x^2-c^2)^(1/2)+d*x)*csgn(d))*x^2*b*c^2*d^2-a*x*
(d^2*x^2-c^2)^(1/2)*d^3*csgn(d)-b*c^2*x*(d^2*x^2-c^2)^(1/2)*csgn(
d)*d-b*c^4*ln((csgn(d)*(d^2*x^2-c^2)^(1/2)+d*x)*csgn(d)))*csgn(d)
/(d^2*x^2-c^2)^(1/2)/c^2/d^3/(d*x+c)^(1/2)/(d*x-c)^(1/2)
```

---

**Maxima [A]** time = 1.41802, size = 115, normalized size = 1.83

$$-\frac{ax}{\sqrt{d^2x^2-c^2}c^2}-\frac{bx}{\sqrt{d^2x^2-c^2}d^2}+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{\sqrt{d^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)),x, algorithm="maxima")
```

```
[Out] -a*x/(sqrt(d^2*x^2 - c^2)*c^2) - b*x/(sqrt(d^2*x^2 - c^2)*d^2) +
b*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^2)
```

---

**Fricas [A]** time = 0.237107, size = 153, normalized size = 2.43

$$\frac{bc^2+ad^2-\left(bd^2x^2-\sqrt{dx+c}\sqrt{dx-c}bdx-bc^2\right)\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)}{d^5x^2-\sqrt{dx+c}\sqrt{dx-c}d^4x-c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)),x, algorithm="fricas")
```

```
[Out] (b*c^2 + a*d^2 - (b*d^2*x^2 - sqrt(d*x + c)*sqrt(d*x - c)*b*d*x -
 b*c^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/(d^5*x^2 - sqrt(
d*x + c)*sqrt(d*x - c)*d^4*x - c^2*d^3)
```

---

**Sympy [A]** time = 70.1161, size = 182, normalized size = 2.89

$$a\left(-\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2\\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^2d}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^2d}\right)+b\left(\frac{G_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & -\frac{1}{2},\frac{1}{2},1,1\\ -\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1,0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^3}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{3}{2},-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},1 & \\ -\frac{3}{4},-\frac{1}{4} & -\frac{3}{2},-1,0,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(-meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2
), (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c**2*d) + I*meijerg(((-1
/2, 0, 1/4, 1/2, 3/4, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), c**
2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c**2*d)) + b*(meije
rg(((-1/4, 1/4), (-1/2, 1/2, 1, 1)), ((-1/4, 0, 1/4, 1/2, 1, 0),
()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**3) + I*meijerg(((-3/2, -1,
 -3/4, -1/2, -1/4, 1), ()), ((-3/4, -1/4), (-3/2, -1, 0, 0)), c**
2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**3))
```

---

**GIAC/XCAS [A]** time = 0.231704, size = 153, normalized size = 2.43

$$-\frac{b\ln\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{d^3}-\frac{2\left(bc^2+ad^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)cd^3}-\frac{\left(bc^2d^3+ad^5\right)\sqrt{dx+c}}{2\sqrt{dx-c}c^2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)),x, algorithm="giac")
```

```
[Out] -b*ln((sqrt(d*x + c) - sqrt(d*x - c))^2)/d^3 - 2*(b*c^2 + a*d^2)/
(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c*d^3) - 1/2*(b*c^2*d^
3 + a*d^5)*sqrt(d*x + c)/(sqrt(d*x - c)*c^2*d^6)
```

## 3.274 $\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=65

$$-\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{dx-c}\sqrt{c+dx}}-\frac{a\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c^3}$$

```
[Out] -((a/c^2 + b/d^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) - (a*ArcTan[(Sq
rt[-c + d*x]*Sqrt[c + d*x])/c])/c^3
```

---

**Rubi [A]** time = 0.266444, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$-\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{dx-c}\sqrt{c+dx}}-\frac{a\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(x*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -((a/c^2 + b/d^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) - (a*ArcTan[(Sq
rt[-c + d*x]*Sqrt[c + d*x])/c])/c^3
```

---

**Rubi in Sympy [A]** time = 14.2202, size = 53, normalized size = 0.82

$$-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{c^3}-\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{-c+dx}\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/x/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] -a*atan(sqrt(-c + d*x)*sqrt(c + d*x)/c)/c**3 - (a/c**2 + b/d**2)/
(sqrt(-c + d*x)*sqrt(c + d*x))
```

---

**Mathematica [A]** time = 0.15873, size = 96, normalized size = 1.48

$$\frac{c\sqrt{dx-c}\sqrt{c+dx}\left(ad^2+bc^2\right)+ad^2\left(c^2-d^2x^2\right)\tan^{-1}\left(\frac{c}{\sqrt{dx-c}\sqrt{c+dx}}\right)}{c^3d^2(c-dx)(c+dx)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(x*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (c*(b*c^2 + a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x] + a*d^2*(c^2 - d^
2*x^2)*ArcTan[c/(Sqrt[-c + d*x]*Sqrt[c + d*x])])/(c^3*d^2*(c - d*
x)*(c + d*x))
```

---

**Maple [B]** time = 0.034, size = 188, normalized size = 2.9

$$\frac{1}{c^2d^2}\left(\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^2ad^4-ac^2\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)d^2-a\sqrt{d^2x^2-c^2}d^2\sqrt{-c^2}-bc^2\sqrt{-c^2}\sqrt{d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/c^2*(ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^2*a*d^4-
a*c^2*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*d^2-a*(d^2*
x^2-c^2)^(1/2)*d^2*(-c^2)^(1/2)-b*c^2*(-c^2)^(1/2)*(d^2*x^2-c^2)^
(1/2))/(-c^2)^(1/2)/(d^2*x^2-c^2)^(1/2)/d^2/(d*x+c)^(1/2)/(d*x-c)
^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.242387, size = 223, normalized size = 3.43

$$-\frac{\left(bc^3+acd^2\right)\sqrt{dx+c}\sqrt{dx-c}-\left(bc^3d+acd^3\right)x+2\left(ad^4x^2-\sqrt{dx+c}\sqrt{dx-c}\,cad^3x-ac^2d^2\right)\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)}{c^3d^4x^2-\sqrt{dx+c}\sqrt{dx-c}\,c^3d^3x-c^5d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] -((b*c^3 + a*c*d^2)*sqrt(d*x + c)*sqrt(d*x - c) - (b*c^3*d + a*c*
d^3)*x + 2*(a*d^4*x^2 - sqrt(d*x + c)*sqrt(d*x - c)*a*d^3*x - a*c
^2*d^2)*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c))/(c^3*d^4*
x^2 - sqrt(d*x + c)*sqrt(d*x - c)*c^3*d^3*x - c^5*d^2)
```

---

**Sympy [A]** time = 93.7092, size = 172, normalized size = 2.65

$$a\left(-\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & 1,2,\frac{5}{2}\\ \frac{5}{4},\frac{3}{2},\frac{7}{4},2,\frac{5}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^3}-\frac{iG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{2},\frac{3}{4},1,\frac{5}{4},1 & \\ \frac{3}{4},\frac{5}{4} & 0,\frac{1}{2},\frac{3}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^3}\right)+b\left(-\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{1}{4},\frac{3}{4},1 & 0,1,\frac{3}{2}\\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{3}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}cd^2}-\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},1 & \\ -\frac{1}{4},\frac{1}{4} & -1,-\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}cd^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/x/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(-meijerg(((5/4, 7/4, 1), (1, 2, 5/2)), ((5/4, 3/2, 7/4, 2, 5/2
), (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c**3) - I*meijerg(((0, 1
/2, 3/4, 1, 5/4, 1), ()), ((3/4, 5/4), (0, 1/2, 3/2, 0)), c**2*ex
p_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c**3)) + b*(-meijerg(((
1/4, 3/4, 1), (0, 1, 3/2)), ((1/4, 1/2, 3/4, 1, 3/2), (0,)), c**2
/(d**2*x**2))/(2*pi**(3/2)*c*d**2) - I*meijerg(((-1, -1/2, -1/4,
0, 1/4, 1), ()), ((-1/4, 1/4), (-1, -1/2, 1/2, 0)), c**2*exp_pola
r(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c*d**2))
```

---

**GIAC/XCAS [A]** time = 0.252711, size = 155, normalized size = 2.38

$$\frac{2\,a\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^3}-\frac{\left(bc^2+ad^2\right)\sqrt{dx+c}}{2\,\sqrt{dx-c}c^3d^2}+\frac{2\,\left(bc^2+ad^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)*x),x, algorithm="giac")
```

```
[Out] 2*a*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^3 - 1/2*(b*
c^2 + a*d^2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^3*d^2) + 2*(b*c^2 + a
```

```
*d^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^2*d^2)
```

## 3.275 $\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=67

$$\frac{a}{c^2x\sqrt{dx-c}\sqrt{c+dx}}-\frac{x\left(2ad^2+bc^2\right)}{c^4\sqrt{dx-c}\sqrt{c+dx}}$$

```
[Out] a/(c^2*x*Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((b*c^2 + 2*a*d^2)*x)/(c
^4*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

---

**Rubi [A]** time = 0.239412, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065

$$\frac{a}{c^2x\sqrt{dx-c}\sqrt{c+dx}}-\frac{x\left(2ad^2+bc^2\right)}{c^4\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] a/(c^2*x*Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((b*c^2 + 2*a*d^2)*x)/(c
^4*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

---

**Rubi in Sympy [A]** time = 11.3995, size = 56, normalized size = 0.84

$$\frac{a}{c^2x\sqrt{-c+dx}\sqrt{c+dx}}-\frac{x\left(2ad^2+bc^2\right)}{c^4\sqrt{-c+dx}\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/x**2/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a/(c**2*x*sqrt(-c + d*x)*sqrt(c + d*x)) - x*(2*a*d**2 + b*c**2)/(
c**4*sqrt(-c + d*x)*sqrt(c + d*x))
```

---

**Mathematica [A]** time = 0.0849212, size = 51, normalized size = 0.76

$$\frac{a\left(c^2-2d^2x^2\right)-bc^2x^2}{c^4x\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-(b*c^2*x^2) + a*(c^2 - 2*d^2*x^2))/(c^4*x*Sqrt[-c + d*x]*Sqrt[c
 + d*x])
```

---

**Maple [A]** time = 0.007, size = 48, normalized size = 0.7

$$\frac{-2\,a{d}^{2}{x}^{2}-b{c}^{2}{x}^{2}+a{c}^{2}}{x{c}^{4}}{\frac{1}{\sqrt{dx+c}}}{\frac{1}{\sqrt{dx-c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] (-2*a*d^2*x^2-b*c^2*x^2+a*c^2)/(d*x+c)^(1/2)/x/c^4/(d*x-c)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)*x^2),x, algorithm="maxima
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.237032, size = 117, normalized size = 1.75

$$\frac{b d x^{2} - \sqrt{d x + c} \sqrt{d x - c} b x + a d}{2 \, d^{4} x^{4} - 2 \, c^{2} d^{2} x^{2} - {\left(2 \, d^{3} x^{3} - c^{2} d x\right)} \sqrt{d x + c} \sqrt{d x - c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)*x^2),x, algorithm="fricas
```

```
[Out] (b*d*x^2 - sqrt(d*x + c)*sqrt(d*x - c)*b*x + a*d)/(2*d^4*x^4 - 2*
c^2*d^2*x^2 - (2*d^3*x^3 - c^2*d*x)*sqrt(d*x + c)*sqrt(d*x - c))
```

---

**Sympy [A]** time = 152.285, size = 165, normalized size = 2.46

$$a\left(-\frac{dG_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & \frac{3}{2},\frac{5}{2},3\\ \frac{7}{4},2,\frac{9}{4},\frac{5}{2},3 & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^4}+\frac{idG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},1,\frac{5}{4},\frac{3}{2},\frac{7}{4},1 & \\ \frac{5}{4},\frac{7}{4} & \frac{1}{2},1,2,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^4}\right)+b\left(-\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2\\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^2d}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^2d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/x**2/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(-d*meijerg(((7/4, 9/4, 1), (3/2, 5/2, 3)), ((7/4, 2, 9/4, 5/2,
 3), (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c**4) + I*d*meijerg(((
1/2, 1, 5/4, 3/2, 7/4, 1), ()), ((5/4, 7/4), (1/2, 1, 2, 0)), c**
2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c**4)) + b*(-meijer
g(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)),
c**2/(d**2*x**2))/(2*pi**(3/2)*c**2*d) + I*meijerg(((-1/2, 0, 1/4
, 1/2, 3/4, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), c**2*exp_pola
r(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c**2*d))
```

---

**GIAC/XCAS [A]** time = 0.277904, size = 296, normalized size = 4.42

$$-\frac{\left(bc^2+ad^2\right)\sqrt{dx+c}}{2\sqrt{dx-c}c^4d}-\frac{2\left(bc^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+ad^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,acd^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+4\,bc^4+12\,ac^2d^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6+2\,c\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+8\,c^3\right)c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)*x^2),x, algorithm="giac")
```

```
[Out] -1/2*(b*c^2 + a*d^2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^4*d) - 2*(b*c
^2*(sqrt(d*x + c) - sqrt(d*x - c))^4 + a*d^2*(sqrt(d*x + c) - sqr
t(d*x - c))^4 + 4*a*c*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 4*b
*c^4 + 12*a*c^2*d^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^6 + 2*c*(s
qrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2*(sqrt(d*x + c) - sqrt(d*x
 - c))^2 + 8*c^3)*c^3*d)
```

# 3.276 $\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=117

$$-\frac{\left(3ad^2+2bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c^5}-\frac{3ad^2+2bc^2}{2c^4\sqrt{dx-c}\sqrt{c+dx}}+\frac{a}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}$$

```
[Out] -(2*b*c^2 + 3*a*d^2)/(2*c^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + a/(2*
c^2*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((2*b*c^2 + 3*a*d^2)*ArcT
an[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c])/(2*c^5)
```

---

**Rubi [A]** time = 0.347629, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161

$$-\frac{\left(3ad^2+2bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c^5}-\frac{3ad^2+2bc^2}{2c^4\sqrt{dx-c}\sqrt{c+dx}}+\frac{a}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -(2*b*c^2 + 3*a*d^2)/(2*c^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + a/(2*
c^2*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((2*b*c^2 + 3*a*d^2)*ArcT
an[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c])/(2*c^5)
```

---

**Rubi in Sympy [A]** time = 21.4516, size = 100, normalized size = 0.85

$$\frac{a}{2c^2x^2\sqrt{-c+dx}\sqrt{c+dx}}-\frac{3ad^2+2bc^2}{2c^4\sqrt{-c+dx}\sqrt{c+dx}}-\frac{\left(3ad^2+2bc^2\right)\operatorname{atan}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/x**3/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a/(2*c**2*x**2*sqrt(-c + d*x)*sqrt(c + d*x)) - (3*a*d**2 + 2*b*c*
*2)/(2*c**4*sqrt(-c + d*x)*sqrt(c + d*x)) - (3*a*d**2 + 2*b*c**2)
*atan(sqrt(-c + d*x)*sqrt(c + d*x)/c)/(2*c**5)
```

---

**Mathematica [C]** time = 0.289895, size = 126, normalized size = 1.08

$$\frac{\frac{a(c^3-3cd^2x^2)-2bc^3x^2}{x^2\sqrt{dx-c}\sqrt{c+dx}}+i\left(3ad^2+2bc^2\right)\log\left(\frac{-4c^4\sqrt{dx-c}\sqrt{c+dx}+4ic^5}{3ad^2x+2bc^2x}\right)}{2c^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] ((-2*b*c^3*x^2 + a*(c^3 - 3*c*d^2*x^2))/(x^2*Sqrt[-c + d*x]*Sqrt[
c + d*x]) + I*(2*b*c^2 + 3*a*d^2)*Log[((4*I)*c^5 - 4*c^4*Sqrt[-c
+ d*x]*Sqrt[c + d*x])/(2*b*c^2*x + 3*a*d^2*x)])/(2*c^5)
```

---

**Maple [B]** time = 0.036, size = 315, normalized size = 2.7

$$\frac{1}{2\,c^4x^2}\left(3\,\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^4ad^4+2\,\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^4bc^2d^2-3\,\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c}}{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/2/c^4*(3*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^4*a*
d^4+2*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^4*b*c^2*d
^2-3*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^2*a*c^2*d^
2-2*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^2*b*c^4-3*a
*d^2*(d^2*x^2-c^2)^(1/2)*x^2*(-c^2)^(1/2)-2*b*(d^2*x^2-c^2)^(1/2)
*c^2*x^2*(-c^2)^(1/2)+a*(d^2*x^2-c^2)^(1/2)*c^2*(-c^2)^(1/2))/(-c
^2)^(1/2)/x^2/(d^2*x^2-c^2)^(1/2)/(d*x+c)^(1/2)/(d*x-c)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)*x^3),x, algorithm="maxima
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.254717, size = 474, normalized size = 4.05

$$\frac{3\,ac^{5}dx + 4\left(2\,bc^{3}d^{3} + 3\,acd^{5}\right)x^{5} - \left(6\,bc^{5}d + 13\,ac^{3}d^{3}\right)x^{3} - \left(ac^{5} + 4\left(2\,bc^{3}d^{2} + 3\,acd^{4}\right)x^{4} - \left(2\,bc^{5} + 7\,ac^{3}d^{2}\right)x^{2}\right)\sqrt{dx+c}\sqrt{d\cdots}}{2\left(4\,c^{5}d^{\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)*x^3),x, algorithm="fricas
```

```
[Out] 1/2*(3*a*c^5*d*x + 4*(2*b*c^3*d^3 + 3*a*c*d^5)*x^5 - (6*b*c^5*d +
 13*a*c^3*d^3)*x^3 - (a*c^5 + 4*(2*b*c^3*d^2 + 3*a*c*d^4)*x^4 - (
2*b*c^5 + 7*a*c^3*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c) - 2*(4*(2
*b*c^2*d^4 + 3*a*d^6)*x^6 - 5*(2*b*c^4*d^2 + 3*a*c^2*d^4)*x^4 + (
2*b*c^6 + 3*a*c^4*d^2)*x^2 - (4*(2*b*c^2*d^3 + 3*a*d^5)*x^5 - 3*(
2*b*c^4*d + 3*a*c^2*d^3)*x^3)*sqrt(d*x + c)*sqrt(d*x - c))*arctan
(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c))/(4*c^5*d^4*x^6 - 5*c^7*
d^2*x^4 + c^9*x^2 - (4*c^5*d^3*x^5 - 3*c^7*d*x^3)*sqrt(d*x + c)*s
qrt(d*x - c))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/x**3/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [A]** time = 0.295925, size = 285, normalized size = 2.44

$$\frac{\left(2\,bc^{2} + 3\,ad^{2}\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{2}}{2\,c}\right)}{c^{5}} - \frac{\left(bc^{2} + ad^{2}\right)\sqrt{dx+c}}{2\sqrt{dx-c}\,c^{5}} + \frac{2\left(bc^{2} + ad^{2}\right)}{\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^{2} + 2\,c\right)c^{4}} + \frac{2\left(ad^{2}\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^{6} - 4\,ac^{2}d^{2}\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^{2}\right)}{\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^{4} + 4\,c^{2}\right)^{2}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)*x^3),x, algorithm="giac")
```

```
[Out] (2*b*c^2 + 3*a*d^2)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/
c)/c^5 - 1/2*(b*c^2 + a*d^2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^5) +
2*(b*c^2 + a*d^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^4)
 + 2*(a*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^6 - 4*a*c^2*d^2*(sqrt
(d*x + c) - sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4
 + 4*c^2)^2*c^4)
```

## 3.277 $\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=119

$$-\frac{2d^2x\left(4ad^2+3bc^2\right)}{3c^6\sqrt{dx-c}\sqrt{c+dx}}+\frac{4ad^2+3bc^2}{3c^4x\sqrt{dx-c}\sqrt{c+dx}}+\frac{a}{3c^2x^3\sqrt{dx-c}\sqrt{c+dx}}$$

```
[Out] a/(3*c^2*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (3*b*c^2 + 4*a*d^2)/
(3*c^4*x*Sqrt[-c + d*x]*Sqrt[c + d*x]) - (2*d^2*(3*b*c^2 + 4*a*d^
2)*x)/(3*c^6*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

---

**Rubi [A]** time = 0.323861, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129

$$-\frac{2d^2x\left(4ad^2+3bc^2\right)}{3c^6\sqrt{dx-c}\sqrt{c+dx}}+\frac{4ad^2+3bc^2}{3c^4x\sqrt{dx-c}\sqrt{c+dx}}+\frac{a}{3c^2x^3\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] a/(3*c^2*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (3*b*c^2 + 4*a*d^2)/
(3*c^4*x*Sqrt[-c + d*x]*Sqrt[c + d*x]) - (2*d^2*(3*b*c^2 + 4*a*d^
2)*x)/(3*c^6*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

---

**Rubi in Sympy [A]** time = 17.9077, size = 105, normalized size = 0.88

$$\frac{a}{3c^2x^3\sqrt{-c+dx}\sqrt{c+dx}}+\frac{4ad^2+3bc^2}{3c^4x\sqrt{-c+dx}\sqrt{c+dx}}-\frac{2d^2x\left(4ad^2+3bc^2\right)}{3c^6\sqrt{-c+dx}\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/x**4/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a/(3*c**2*x**3*sqrt(-c + d*x)*sqrt(c + d*x)) + (4*a*d**2 + 3*b*c*
*2)/(3*c**4*x*sqrt(-c + d*x)*sqrt(c + d*x)) - 2*d**2*x*(4*a*d**2
+ 3*b*c**2)/(3*c**6*sqrt(-c + d*x)*sqrt(c + d*x))
```

---

**Mathematica [A]** time = 0.107597, size = 77, normalized size = 0.65

$$\frac{a\left(c^4+4c^2d^2x^2-8d^4x^4\right)+3bc^2x^2\left(c^2-2d^2x^2\right)}{3c^6x^3\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (3*b*c^2*x^2*(c^2 - 2*d^2*x^2) + a*(c^4 + 4*c^2*d^2*x^2 - 8*d^4*x
^4))/(3*c^6*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

---

**Maple [A]** time = 0.008, size = 73, normalized size = 0.6

$$\frac{-8\,ad^4x^4 - 6\,bc^2d^2x^4 + 4\,ac^2d^2x^2 + 3\,bc^4x^2 + ac^4}{3\,x^3c^6}\frac{1}{\sqrt{dx+c}}\frac{1}{\sqrt{dx-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/3*(-8*a*d^4*x^4-6*b*c^2*d^2*x^4+4*a*c^2*d^2*x^2+3*b*c^4*x^2+a*c
^4)/(d*x+c)^(1/2)/x^3/c^6/(d*x-c)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)*x^4),x, algorithm="maxima
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.235783, size = 193, normalized size = 1.62

$$\frac{6\,bd^2x^4 - ac^2 - \left(3\,bc^2 - 4\,ad^2\right)x^2 - 2\left(3\,bdx^3 + 2\,adx\right)\sqrt{dx+c}\sqrt{dx-c}}{3\left(8\,d^5x^8 - 12\,c^2d^3x^6 + 4\,c^4dx^4 - (8\,d^4x^7 - 8\,c^2d^2x^5 + c^4x^3)\sqrt{dx+c}\sqrt{dx-c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)*x^4),x, algorithm="fricas
```

```
[Out] 1/3*(6*b*d^2*x^4 - a*c^2 - (3*b*c^2 - 4*a*d^2)*x^2 - 2*(3*b*d*x^3
 + 2*a*d*x)*sqrt(d*x + c)*sqrt(d*x - c))/(8*d^5*x^8 - 12*c^2*d^3*
```

```
x^6 + 4*c^4*d*x^4 - (8*d^4*x^7 - 8*c^2*d^2*x^5 + c^4*x^3)*sqrt(d*
x + c)*sqrt(d*x - c))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/x**4/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [A]** time = 0.313585, size = 327, normalized size = 2.75

$$-\frac{\left(bc^2d + ad^3\right)\sqrt{dx+c}}{2\sqrt{dx-c}c^6} - \frac{2\left(bc^2d + ad^3\right)}{\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^2 + 2c\right)c^5}$$
$$-\frac{8\left(3\,bc^2d\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^8 + 3\,ad^3\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^8 + 24\,bc^4d\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^4 + 48\,ac^2d^3\left(\sqrt{dx+c} - \sqrt{d\right.\right.}{3\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^4 + 4\,c^2\right)^3 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)*x^4),x, algorithm="giac")
```

```
[Out] -1/2*(b*c^2*d + a*d^3)*sqrt(d*x + c)/(sqrt(d*x - c)*c^6) - 2*(b*c
^2*d + a*d^3)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^5) - 8
/3*(3*b*c^2*d*(sqrt(d*x + c) - sqrt(d*x - c))^8 + 3*a*d^3*(sqrt(d
*x + c) - sqrt(d*x - c))^8 + 24*b*c^4*d*(sqrt(d*x + c) - sqrt(d*x
 - c))^4 + 48*a*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 48*b*
c^6*d + 80*a*c^4*d^3)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2
)^3*c^4)
```

## 3.278 $\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=166

$$-\frac{3d^2\left(5ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^7}-\frac{3d^2\left(5ad^2+4bc^2\right)}{8c^6\sqrt{dx-c}\sqrt{c+dx}}+\frac{5ad^2+4bc^2}{8c^4x^2\sqrt{dx-c}\sqrt{c+dx}}+\frac{a}{4c^2x^4\sqrt{dx-c}\sqrt{c+dx}}$$

```
[Out] (-3*d^2*(4*b*c^2 + 5*a*d^2))/(8*c^6*Sqrt[-c + d*x]*Sqrt[c + d*x])
 + a/(4*c^2*x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (4*b*c^2 + 5*a*d^
2)/(8*c^4*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) - (3*d^2*(4*b*c^2 + 5
*a*d^2)*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c])/(8*c^7)
```

---

**Rubi [A]** time = 0.459333, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226

$$-\frac{3d^2\left(5ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^7}-\frac{3d^2\left(5ad^2+4bc^2\right)}{8c^6\sqrt{dx-c}\sqrt{c+dx}}+\frac{5ad^2+4bc^2}{8c^4x^2\sqrt{dx-c}\sqrt{c+dx}}+\frac{a}{4c^2x^4\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(x^5*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-3*d^2*(4*b*c^2 + 5*a*d^2))/(8*c^6*Sqrt[-c + d*x]*Sqrt[c + d*x])
 + a/(4*c^2*x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (4*b*c^2 + 5*a*d^
2)/(8*c^4*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) - (3*d^2*(4*b*c^2 + 5
*a*d^2)*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c])/(8*c^7)
```

---

**Rubi in Sympy [A]** time = 30.9584, size = 150, normalized size = 0.9

$$\frac{a}{4c^2x^4\sqrt{-c+dx}\sqrt{c+dx}}+\frac{5ad^2+4bc^2}{8c^4x^2\sqrt{-c+dx}\sqrt{c+dx}}-\frac{3d^2\left(5ad^2+4bc^2\right)}{8c^6\sqrt{-c+dx}\sqrt{c+dx}}-\frac{3d^2\left(5ad^2+4bc^2\right)\operatorname{atan}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{8c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/x**5/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a/(4*c**2*x**4*sqrt(-c + d*x)*sqrt(c + d*x)) + (5*a*d**2 + 4*b*c*
*2)/(8*c**4*x**2*sqrt(-c + d*x)*sqrt(c + d*x)) - 3*d**2*(5*a*d**2
 + 4*b*c**2)/(8*c**6*sqrt(-c + d*x)*sqrt(c + d*x)) - 3*d**2*(5*a*
d**2 + 4*b*c**2)*atan(sqrt(-c + d*x)*sqrt(c + d*x)/c)/(8*c**7)
```

---

**Mathematica [C]** time = 0.38189, size = 157, normalized size = 0.95

$$\frac{\frac{a\left(2c^5+5c^3d^2x^2-15cd^4x^4\right)+4bc^3x^2\left(c^2-3d^2x^2\right)}{x^4\sqrt{dx-c}\sqrt{c+dx}}+3i\left(5ad^4+4bc^2d^2\right)\log\left(\frac{-16c^6\sqrt{dx-c}\sqrt{c+dx}+16ic^7}{15ad^4x+12bc^2d^2x}\right)}{8c^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(x^5*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] ((4*b*c^3*x^2*(c^2 - 3*d^2*x^2) + a*(2*c^5 + 5*c^3*d^2*x^2 - 15*c
*d^4*x^4))/(x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (3*I)*(4*b*c^2*d^
2 + 5*a*d^4)*Log[((16*I)*c^7 - 16*c^6*Sqrt[-c + d*x]*Sqrt[c + d*x
])/(12*b*c^2*d^2*x + 15*a*d^4*x)])/(8*c^7)
```

---

**Maple [B]** time = 0.038, size = 387, normalized size = 2.3

$$\frac{1}{8\,c^6x^4}\left(15\,\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^6ad^6+12\,\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^6bc^2d^4-15\,\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2}}{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/8/c^6*(15*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^6*a
*d^6+12*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^6*b*c^2
*d^4-15*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^4*a*c^2
*d^4-12*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^4*b*c^4
*d^2-15*x^4*a*d^4*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)-12*x^4*b*c^2*d
^2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)+5*x^2*a*c^2*d^2*(-c^2)^(1/2)*
(d^2*x^2-c^2)^(1/2)+4*x^2*b*c^4*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)+
2*a*c^4*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/(-c^2)^(1/2)/x^4/(d^2*x
^2-c^2)^(1/2)/(d*x+c)^(1/2)/(d*x-c)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)*x^5),x, algorithm="maxima
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.304714, size = 707, normalized size = 4.26

$$-\frac{10\,ac^9dx - 48\left(4\,bc^3d^7 + 5\,acd^9\right)x^9 + 76\left(4\,bc^5d^5 + 5\,ac^3d^7\right)x^7 - \left(140\,bc^7d^3 + 143\,ac^5d^5\right)x^5 + 5\left(4\,bc^9d - 3\,ac^7d^3\right)x^3 - \left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)*x^5),x, algorithm="fricas
```

```
[Out] -1/8*(10*a*c^9*d*x - 48*(4*b*c^3*d^7 + 5*a*c*d^9)*x^9 + 76*(4*b*c
^5*d^5 + 5*a*c^3*d^7)*x^7 - (140*b*c^7*d^3 + 143*a*c^5*d^5)*x^5 +
 5*(4*b*c^9*d - 3*a*c^7*d^3)*x^3 - (2*a*c^9 - 48*(4*b*c^3*d^6 + 5
*a*c*d^8)*x^8 + 52*(4*b*c^5*d^4 + 5*a*c^3*d^6)*x^6 - (60*b*c^7*d^
2 + 43*a*c^5*d^4)*x^4 + (4*b*c^9 - 19*a*c^7*d^2)*x^2)*sqrt(d*x +
c)*sqrt(d*x - c) + 6*(16*(4*b*c^2*d^8 + 5*a*d^10)*x^10 - 28*(4*b*
c^4*d^6 + 5*a*c^2*d^8)*x^8 + 13*(4*b*c^6*d^4 + 5*a*c^4*d^6)*x^6 -
 (4*b*c^8*d^2 + 5*a*c^6*d^4)*x^4 - (16*(4*b*c^2*d^7 + 5*a*d^9)*x^
9 - 20*(4*b*c^4*d^5 + 5*a*c^2*d^7)*x^7 + 5*(4*b*c^6*d^3 + 5*a*c^4
*d^5)*x^5)*sqrt(d*x + c)*sqrt(d*x - c))*arctan(-(d*x - sqrt(d*x +
 c)*sqrt(d*x - c))/c))/(16*c^7*d^6*x^10 - 28*c^9*d^4*x^8 + 13*c^1
1*d^2*x^6 - c^13*x^4 - (16*c^7*d^5*x^9 - 20*c^9*d^3*x^7 + 5*c^11*
d*x^5)*sqrt(d*x + c)*sqrt(d*x - c))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**5/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [A]** time = 0.345015, size = 543, normalized size = 3.27

$$\frac{3\left(4\,bc^2d^2+5\,ad^4\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{4\,c^7}-\frac{\left(bc^2d^2+ad^4\right)\sqrt{dx+c}}{2\,\sqrt{dx-c}c^7}+\frac{2\left(bc^2d^2+ad^4\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)c^6}$$

$$+\frac{4\,bc^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+7\,ad^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+16\,bc^4d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}+60\,ac^2d^4\left(\sqrt{dx+c}-\sqrt{\right.}\right.}{2\left(\left(\sqrt{\right.}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x + c)^(3/2)*(d*x - c)^(3/2)*x^5),x, algorithm="giac")
```

```
[Out] 3/4*(4*b*c^2*d^2 + 5*a*d^4)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x
- c))^2/c)/c^7 - 1/2*(b*c^2*d^2 + a*d^4)*sqrt(d*x + c)/(sqrt(d*x
- c)*c^7) + 2*(b*c^2*d^2 + a*d^4)/(((sqrt(d*x + c) - sqrt(d*x - c
))^2 + 2*c)*c^6) + 1/2*(4*b*c^2*d^2*(sqrt(d*x + c) - sqrt(d*x - c
))^14 + 7*a*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^14 + 16*b*c^4*d^2
*(sqrt(d*x + c) - sqrt(d*x - c))^10 + 60*a*c^2*d^4*(sqrt(d*x + c)
 - sqrt(d*x - c))^10 - 64*b*c^6*d^2*(sqrt(d*x + c) - sqrt(d*x - c
))^6 - 240*a*c^4*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^6 - 256*b*c^
8*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 - 448*a*c^6*d^4*(sqrt(d*x
 + c) - sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4
*c^2)^4*c^6)
```

## 3.279 $\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

**Optimal**. Leaf size=40

$$\sqrt{cx-1}\sqrt{cx+1}+\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)$$

```
[Out] Sqrt[-1 + c*x]*Sqrt[1 + c*x] + ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x
]]
```

---

**Rubi [A]** time = 0.198525, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$\sqrt{cx-1}\sqrt{cx+1}+\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c^2*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] Sqrt[-1 + c*x]*Sqrt[1 + c*x] + ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x
]]
```

---

**Rubi in Sympy [A]** time = 9.2897, size = 34, normalized size = 0.85

$$\sqrt{cx-1}\sqrt{cx+1}+\operatorname{atan}\left(\sqrt{cx-1}\sqrt{cx+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c**2*x**2+1)/x/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] sqrt(c*x - 1)*sqrt(c*x + 1) + atan(sqrt(c*x - 1)*sqrt(c*x + 1))
```

---

**Mathematica [A]** time = 0.0540141, size = 42, normalized size = 1.05

$$\sqrt{cx-1}\sqrt{cx+1}-\tan^{-1}\left(\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c^2*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] Sqrt[-1 + c*x]*Sqrt[1 + c*x] - ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 +
c*x])]
```

---

**Maple [A]** time = 0.024, size = 53, normalized size = 1.3

$$1\sqrt{cx-1}\sqrt{cx+1}\left(\sqrt{c^2x^2-1}-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\right)\frac{1}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] (c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*((c^2*x^2-1)^(1/2)-
arctan(1/(c^2*x^2-1)^(1/2)))
```

---

**Maxima [A]** time = 1.52933, size = 34, normalized size = 0.85

$$\sqrt{c^2x^2-1}-\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2 + 1)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x),x, algorithm="maxima")
```

```
[Out] sqrt(c^2*x^2 - 1) - arcsin(1/(sqrt(c^2)*abs(x)))
```

---

**Fricas [A]** time = 0.241092, size = 127, normalized size = 3.18

$$-\frac{c^2x^2-\sqrt{cx+1}\sqrt{cx-1}cx-2\left(cx-\sqrt{cx+1}\sqrt{cx-1}\right)\arctan\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)-1}{cx-\sqrt{cx+1}\sqrt{cx-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2 + 1)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x),x, algorithm="fricas")
```

```
[Out] -(c^2*x^2 - sqrt(c*x + 1)*sqrt(c*x - 1)*c*x - 2*(c*x - sqrt(c*x +
 1)*sqrt(c*x - 1))*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) - 1
)/(c*x - sqrt(c*x + 1)*sqrt(c*x - 1))
```

---

**Sympy [A]** time = 31.06, size = 148, normalized size = 3.7

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{G_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c**2*x**2+1)/x/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2,
 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)) - meijerg(((3/4, 5/4, 1),
(1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1/(c**2*x**2))/(4*
pi**(3/2)) + I*meijerg(((-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4
, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*
pi**(3/2)) + I*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4, 3/4
), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)
)
```

---

**GIAC/XCAS [A]** time = 0.219661, size = 54, normalized size = 1.35

$$\sqrt{cx+1}\sqrt{cx-1} - 2\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c^2*x^2 + 1)/(sqrt(c*x + 1)*sqrt(c*x - 1)*x),x, algorithm="giac")
```

```
[Out] sqrt(c*x + 1)*sqrt(c*x - 1) - 2*arctan(1/2*(sqrt(c*x + 1) - sqrt(
c*x - 1))^2)
```

## 3.280 $\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}(c+dx^2)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx$

**Optimal**. Leaf size=53

$$\sqrt{bx-a}\sqrt{a+bx}\left(\frac{c}{a^2}+\frac{d}{b^2}\right)x^{-\frac{b^2c}{a^2d+b^2c}}$$

```
[Out] ((c/a^2 + d/b^2)*Sqrt[-a + b*x]*Sqrt[a + b*x])/x^((b^2*c)/(b^2*c
+ a^2*d))
```

---

**Rubi [A]** time = 0.297772, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018

$$\sqrt{bx-a}\sqrt{a+bx}\left(\frac{c}{a^2}+\frac{d}{b^2}\right)x^{-\frac{b^2c}{a^2d+b^2c}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/(x^((2*b^2*c + a^2*d)/(b^2*c + a^2*d))*Sqrt[-
a + b*x]*Sqrt[a + b*x]),x]
```

```
[Out] ((c/a^2 + d/b^2)*Sqrt[-a + b*x]*Sqrt[a + b*x])/x^((b^2*c)/(b^2*c
+ a^2*d))
```

---

**Rubi in Sympy [A]** time = 10.0874, size = 42, normalized size = 0.79

$$x^{-\frac{b^2c}{a^2d+b^2c}}\sqrt{-a+bx}\sqrt{a+bx}\left(\frac{d}{b^2}+\frac{c}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d*x**2+c)/(x**((a**2*d+2*b**2*c)/(a**2*d+b**2*c)))/(b*x-
a)**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] x**(-b**2*c/(a**2*d + b**2*c))*sqrt(-a + b*x)*sqrt(a + b*x)*(d/b*
*2 + c/a**2)
```

---

**Mathematica [C]** time = 4.26321, size = 1424, normalized size = 26.87

$$-\frac{d\left(da^2+b^2c\right)x^{-\frac{b^2c}{da^2+b^2c}}\left(\frac{d(a-bx)^2\sqrt{\frac{bx}{a}+1}F_1\left(-\frac{b^2c}{da^2+b^2c};-\frac{1}{2},\frac{1}{2};\frac{a^2d}{da^2+b^2c};\frac{bx}{a},-\frac{bx}{a}\right)a^3}{c\sqrt{1-\frac{bx}{a}}\left(2a^3dF_1\left(-\frac{b^2c}{da^2+b^2c};-\frac{1}{2},\frac{1}{2};\frac{a^2d}{da^2+b^2c};\frac{bx}{a},-\frac{bx}{a}\right)-b(da^2+b^2c)x\left(F_1\left(\frac{a^2d}{da^2+b^2c};-\frac{1}{2},\frac{3}{2};\frac{2da^2+b^2c}{da^2+b^2c};\frac{bx}{a},-\frac{bx}{a}\right)+{}_2F_1\left(\frac{1}{2},\frac{a^2}{2(da^2 \cdots}\right.\right.\right.}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x^2)/(x^((2*b^2*c + a^2*d)/(b^2*c + a^2*d))*Sqrt[-
a + b*x]*Sqrt[a + b*x]),x]
```

```
[Out] -((d*(b^2*c + a^2*d)*(-(((a - b*x)*(a + b*x)*Hypergeometric2F1[-1
/2, -(b^2*c)/(2*(b^2*c + a^2*d)), 1 - (b^2*c)/(2*(b^2*c + a^2*d))
, (b^2*x^2)/a^2])/c) + (a*b^2*(a - b*x)^2*Sqrt[1 + (b*x)/a]*Appel
lF1[-((b^2*c)/(b^2*c + a^2*d)), -1/2, 1/2, (a^2*d)/(b^2*c + a^2*d
), (b*x)/a, -((b*x)/a)])/(Sqrt[1 - (b*x)/a]*(2*a^3*d*AppellF1[-((
b^2*c)/(b^2*c + a^2*d)), -1/2, 1/2, (a^2*d)/(b^2*c + a^2*d), (b*x
)/a, -((b*x)/a)] - b*(b^2*c + a^2*d)*x*(AppellF1[(a^2*d)/(b^2*c +
 a^2*d), -1/2, 3/2, (b^2*c + 2*a^2*d)/(b^2*c + a^2*d), (b*x)/a, -
((b*x)/a)] + HypergeometricPFQ[{1/2, (a^2*d)/(2*(b^2*c + a^2*d))}
, {(b^2*c)/(b^2*c + a^2*d) + (3*a^2*d)/(2*(b^2*c + a^2*d))}, (b^2
*x^2)/a^2]))) + (a^3*d*(a - b*x)^2*Sqrt[1 + (b*x)/a]*AppellF1[-((
b^2*c)/(b^2*c + a^2*d)), -1/2, 1/2, (a^2*d)/(b^2*c + a^2*d), (b*x
)/a, -((b*x)/a)])/(c*Sqrt[1 - (b*x)/a]*(2*a^3*d*AppellF1[-((b^2*c
)/(b^2*c + a^2*d)), -1/2, 1/2, (a^2*d)/(b^2*c + a^2*d), (b*x)/a,
-((b*x)/a)] - b*(b^2*c + a^2*d)*x*(AppellF1[(a^2*d)/(b^2*c + a^2*
d), -1/2, 3/2, (b^2*c + 2*a^2*d)/(b^2*c + a^2*d), (b*x)/a, -((b*x
)/a)] + HypergeometricPFQ[{1/2, (a^2*d)/(2*(b^2*c + a^2*d))}, {(b
^2*c)/(b^2*c + a^2*d) + (3*a^2*d)/(2*(b^2*c + a^2*d))}, (b^2*x^2)
/a^2]))) + (a*b^2*(a + b*x)^2*Sqrt[1 - (b*x)/a]*AppellF1[-((b^2*c
)/(b^2*c + a^2*d)), 1/2, -1/2, (a^2*d)/(b^2*c + a^2*d), (b*x)/a,
-((b*x)/a)])/(Sqrt[1 + (b*x)/a]*(2*a^3*d*AppellF1[-((b^2*c)/(b^2*
c + a^2*d)), 1/2, -1/2, (a^2*d)/(b^2*c + a^2*d), (b*x)/a, -((b*x)
/a)] + b*(b^2*c + a^2*d)*x*(AppellF1[(a^2*d)/(b^2*c + a^2*d), 3/2
, -1/2, (b^2*c + 2*a^2*d)/(b^2*c + a^2*d), (b*x)/a, -((b*x)/a)] +
 HypergeometricPFQ[{1/2, (a^2*d)/(2*(b^2*c + a^2*d))}, {(b^2*c)/(
b^2*c + a^2*d) + (3*a^2*d)/(2*(b^2*c + a^2*d))}, (b^2*x^2)/a^2]))
) + (a^3*d*(a + b*x)^2*Sqrt[1 - (b*x)/a]*AppellF1[-((b^2*c)/(b^2*
c + a^2*d)), 1/2, -1/2, (a^2*d)/(b^2*c + a^2*d), (b*x)/a, -((b*x)
/a)])/(c*Sqrt[1 + (b*x)/a]*(2*a^3*d*AppellF1[-((b^2*c)/(b^2*c + a
^2*d)), 1/2, -1/2, (a^2*d)/(b^2*c + a^2*d), (b*x)/a, -((b*x)/a)]
+ b*(b^2*c + a^2*d)*x*(AppellF1[(a^2*d)/(b^2*c + a^2*d), 3/2, -1/
2, (b^2*c + 2*a^2*d)/(b^2*c + a^2*d), (b*x)/a, -((b*x)/a)] + Hype
rgeometricPFQ[{1/2, (a^2*d)/(2*(b^2*c + a^2*d))}, {(b^2*c)/(b^2*c
 + a^2*d) + (3*a^2*d)/(2*(b^2*c + a^2*d))}, (b^2*x^2)/a^2])))))/(
b^4*x^((b^2*c)/(b^2*c + a^2*d))*Sqrt[-a + b*x]*Sqrt[a + b*x]*Sqrt
[1 - (b^2*x^2)/a^2]))
```

**Maple [A]** time = 0.014, size = 66, normalized size = 1.3

$$\frac{x\left(a^{2}d+b^{2}c\right)}{a^{2}b^{2}}\sqrt{bx+a}\sqrt{bx-a}\left(x^{\frac{a^{2}d+2b^{2}c}{a^{2}d+b^{2}c}}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+a)^(1/2),
```

```
[Out] x*(a^2*d+b^2*c)*(b*x+a)^(1/2)/a^2/b^2/(x^((a^2*d+2*b^2*c)/(a^2*d+
b^2*c)))*(b*x-a)^(1/2)
```

---

**Maxima [A]** time = 1.85853, size = 107, normalized size = 2.02

$$\frac{\left(b^{2}c+a^{2}d\right)\sqrt{bx+a}\sqrt{bx-a}\,xe^{\left(-\frac{2\,b^{2}c\log(x)}{b^{2}c+a^{2}d}-\frac{a^{2}d\log(x)}{b^{2}c+a^{2}d}\right)}}{a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/(sqrt(b*x + a)*sqrt(b*x - a)*x^((2*b^2*c + a^2*d)/(b^2*c + a
```

```
[Out] (b^2*c + a^2*d)*sqrt(b*x + a)*sqrt(b*x - a)*x*e^(-2*b^2*c*log(x)/
(b^2*c + a^2*d) - a^2*d*log(x)/(b^2*c + a^2*d))/(a^2*b^2)
```

---

**Fricas [A]** time = 0.262433, size = 88, normalized size = 1.66

$$\frac{\left(b^{2}c+a^{2}d\right)\sqrt{bx+a}\sqrt{bx-a}\,x}{a^{2}b^{2}x^{\frac{2\,b^{2}c+a^{2}d}{b^{2}c+a^{2}d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/(sqrt(b*x + a)*sqrt(b*x - a)*x^((2*b^2*c + a^2*d)/(b^2*c + a
```

```
[Out] (b^2*c + a^2*d)*sqrt(b*x + a)*sqrt(b*x - a)*x/(a^2*b^2*x^((2*b^2*
c + a^2*d)/(b^2*c + a^2*d)))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(x**((a**2*d+2*b**2*c)/(a**2*d+b**2*c)))/(b*x-
a)**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{\sqrt{bx + a}\sqrt{bx - a}x^{\frac{2b^2c + a^2d}{b^2c + a^2d}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)/(sqrt(b*x + a)*sqrt(b*x - a)*x^((2*b^2*c + a^2*d)/(b^2*c + a
```

```
[Out] integrate((d*x^2 + c)/(sqrt(b*x + a)*sqrt(b*x - a)*x^((2*b^2*c +
a^2*d)/(b^2*c + a^2*d))), x)
```

## 3.281 $\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}} \, dx$

**Optimal**. Leaf size=36

$$\frac{\sqrt{1-x}\sin^{-1}(x)}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}}$$

```
[Out] (Sqrt[1 - x]*ArcSin[x])/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]])
```

---

**Rubi [A]** time = 0.0708708, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094

$$\frac{\sqrt{1-x}\sin^{-1}(x)}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + x]),x]

[Out] (Sqrt[1 - x]*ArcSin[x])/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]])
```

---

**Rubi in Sympy [A]** time = 8.37875, size = 31, normalized size = 0.86

$$\frac{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}\operatorname{asin}(x)}{\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+x)**(1/2)/(-1-x**(1/2))**(1/2)/(-1+x**(1/2))**(1/2),x)

[Out] sqrt(-sqrt(x) - 1)*sqrt(sqrt(x) - 1)*asin(x)/sqrt(-x + 1)
```

---

**Mathematica [A]** time = 0.947749, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}} \, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[1/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + x]),x]
```

```
[Out] Integrate[1/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + x]),
x]
```

---

**Maple [F]** time = 0.062, size = 0, normalized size = 0.

$$\int 1\frac{1}{\sqrt{1+x}}\frac{1}{\sqrt{-1-\sqrt{x}}}\frac{1}{\sqrt{-1+\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x)
```

```
[Out] int(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1)),x, algorithm="max
```

```
[Out] integrate(1/(sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1)), x
)
```

---

**Fricas [A]** time = 0.236123, size = 93, normalized size = 2.58

$$-i\,\log\left(\frac{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}+i\,x-1}{x}\right)+i\,\log\left(\frac{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}-i\,x-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1)),x, algorithm="fri
```

```
[Out] -I*log((sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1) + I*x -
1)/x) + I*log((sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1) -
 I*x - 1)/x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(1/2)/(-1-x**(1/2))**(1/2)/(-1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-sqrt(x) - 1)*sqrt(sqrt(x) - 1)*sqrt(x + 1)), x)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1)),x, algorithm="giac
```

```
[Out] Exception raised: TypeError
```

## 3.282 $\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx$

**Optimal**. Leaf size=75

$$-\frac{2\sqrt{a^2-b^2x}\tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$

```
[Out] (-2*Sqrt[a^2 - b^2*x]*ArcTan[Sqrt[a^2 - b^2*x]/Sqrt[a^2 + b^2*x]]
)/(b^2*Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]])
```

---

**Rubi [A]** time = 0.146345, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$

$$-\frac{2\sqrt{a^2-b^2x}\tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]]*Sqrt[a^2 + b^2*x]),x]
```

```
[Out] (-2*Sqrt[a^2 - b^2*x]*ArcTan[Sqrt[a^2 - b^2*x]/Sqrt[a^2 + b^2*x]]
)/(b^2*Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]])
```

---

**Rubi in Sympy [A]** time = 18.2102, size = 65, normalized size = 0.87

$$\frac{2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\,\operatorname{atan}\left(\frac{\sqrt{a^2+b^2x}}{\sqrt{a^2-b^2x}}\right)}{b^2\sqrt{a^2-b^2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*x+a**2)**(1/2)/(a-b*x**(1/2))**(1/2)/(a+b*x**(1/2))**(1
```

```
[Out] 2*sqrt(a - b*sqrt(x))*sqrt(a + b*sqrt(x))*atan(sqrt(a**2 + b**2*x
)/sqrt(a**2 - b**2*x))/(b**2*sqrt(a**2 - b**2*x))
```

---

**Mathematica [A]** time = 1.96338, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[1/(Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]]*Sqrt[a^2 + b^2*x]),x]
```

```
[Out] Integrate[1/(Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]]*Sqrt[a^2 + b
^2*x]), x]
```

---

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int 1\frac{1}{\sqrt{b^2x+a^2}}\frac{1}{\sqrt{a-b\sqrt{x}}}\frac{1}{\sqrt{a+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x)
```

```
[Out] int(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x
)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^2x+a^2}\sqrt{b\sqrt{x}+a}\sqrt{-b\sqrt{x}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)),x, algo
```

```
[Out] integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x
) + a)), x)
```

---

**Fricas [A]** time = 0.242236, size = 68, normalized size = 0.91

$$-\frac{2\,\arctan\left(-\frac{a^2-\sqrt{b^2x+a^2}\sqrt{b\sqrt{x}+a}\sqrt{-b\sqrt{x}+a}}{b^2x}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)),x, algo
```

```
[Out] -2*arctan(-(a^2 - sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*s
qrt(x) + a))/(b^2*x))/b^2
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x+a**2)**(1/2)/(a-b*x**(1/2))**(1/2)/(a+b*x**(1/2))**(1/2),x
```

```
[Out] Integral(1/(sqrt(a - b*sqrt(x))*sqrt(a + b*sqrt(x))*sqrt(a**2 + b
**2*x)), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^2x+a^2}\sqrt{b\sqrt{x}+a}\sqrt{-b\sqrt{x}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)),x, algo
```

```
[Out] integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x
) + a)), x)
```

# 3.283 $\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q \, dx$

**Optimal**. Leaf size=113

$$x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \left(c+dx^{2n}\right)^q \left(\frac{dx^{2n}}{c}+1\right)^{-q} F_1\left(\frac{1}{2n};-p,-q;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)$$

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q*AppellF1[1/(2*n)
, -p, -q, (2 + n^(-1))/2, (b^2*x^(2*n))/a^2, -((d*x^(2*n))/c)])/(
(1 - (b^2*x^(2*n))/a^2)^p*(1 + (d*x^(2*n))/c)^q)
```

---

**Rubi [A]** time = 0.23465, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \left(c+dx^{2n}\right)^q \left(\frac{dx^{2n}}{c}+1\right)^{-q} F_1\left(\frac{1}{2n};-p,-q;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q,x]
```

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q*AppellF1[1/(2*n)
, -p, -q, (2 + n^(-1))/2, (b^2*x^(2*n))/a^2, -((d*x^(2*n))/c)])/(
(1 - (b^2*x^(2*n))/a^2)^p*(1 + (d*x^(2*n))/c)^q)
```

---

**Rubi in Sympy [A]** time = 37.8593, size = 88, normalized size = 0.78

$$x\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\left(1+\frac{dx^{2n}}{c}\right)^{-q}(a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q \operatorname{appellf}_1\left(\frac{1}{2n},-p,-q,\frac{n+\frac{1}{2}}{n},\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a-b*x**n)**p*(a+b*x**n)**p*(c+d*x**(2*n))**q,x)
```

```
[Out] x*(1 - b**2*x**(2*n)/a**2)**(-p)*(1 + d*x**(2*n)/c)**(-q)*(a - b*
x**n)**p*(a + b*x**n)**p*(c + d*x**(2*n))**q*appellf1(1/(2*n), -p
, -q, (n + 1/2)/n, b**2*x**(2*n)/a**2, -d*x**(2*n)/c)
```

---

**Mathematica [A]** time = 0.370362, size = 0, normalized size = 0.

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q \, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q,x]
```

```
[Out] Integrate[(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q, x]
```

---

**Maple [F]** time = 0.813, size = 0, normalized size = 0.

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x)
```

```
[Out] int((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(dx^{2n}+c\right)^q (bx^n+a)^p (-bx^n+a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(dx^{2n}+c\right)^q (bx^n+a)^p (-bx^n+a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p,x, algorithm="fricas")
```

```
[Out] integral((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a-b*x**n)**p*(a+b*x**n)**p*(c+d*x**(2*n))**q,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(dx^{2n} + c\right)^q (bx^n + a)^p (-bx^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p,x, algorithm="giac")
```

```
[Out] integrate((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

## 3.284 $\int (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \, dx$

**Optimal**. Leaf size=87

$$x(a-bx^n)^p(a+bx^n)^p\left(a^2+b^2x^{2n}\right)^p\left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p}{}_2F_1\left(\frac{1}{4n},-p;\frac{1}{4}\left(4+\frac{1}{n}\right);\frac{b^4x^{4n}}{a^4}\right)$$

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p*Hypergeometr
ic2F1[1/(4*n), -p, (4 + n^(-1))/4, (b^4*x^(4*n))/a^4])/(1 - (b^4*
x^(4*n))/a^4)^p
```

---

**Rubi [A]** time = 0.150388, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114

$$x(a-bx^n)^p(a+bx^n)^p\left(a^2+b^2x^{2n}\right)^p\left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p}{}_2F_1\left(\frac{1}{4n},-p;\frac{1}{4}\left(4+\frac{1}{n}\right);\frac{b^4x^{4n}}{a^4}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p,x]
```

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p*Hypergeometr
ic2F1[1/(4*n), -p, (4 + n^(-1))/4, (b^4*x^(4*n))/a^4])/(1 - (b^4*
x^(4*n))/a^4)^p
```

---

**Rubi in Sympy [A]** time = 32.5251, size = 70, normalized size = 0.8

$$x\left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p}(a-bx^n)^p(a+bx^n)^p\left(a^2+b^2x^{2n}\right)^p{}_2F_1\left(\begin{matrix}-p,\frac{1}{4n}\\\frac{n+\frac{1}{4}}{n}\end{matrix}\middle|\frac{b^4x^{4n}}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a-b*x**n)**p*(a+b*x**n)**p*(a**2+b**2*x**(2*n))**p,x)
```

```
[Out] x*(1 - b**4*x**(4*n)/a**4)**(-p)*(a - b*x**n)**p*(a + b*x**n)**p*
(a**2 + b**2*x**(2*n))**p*hyper((-p, 1/(4*n)), ((n + 1/4)/n,), b*
*4*x**(4*n)/a**4)
```

---

**Mathematica [A]** time = 0.278254, size = 0, normalized size = 0.

$$\int (a-bx^n)^p(a+bx^n)^p\left(a^2+b^2x^{2n}\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p,x]
```

```
[Out] Integrate[(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p, x]
```

---

**Maple [F]** time = 0.806, size = 0, normalized size = 0.

$$\int (a - bx^n)^p (a + bx^n)^p \left(a^2 + b^2 x^{2n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x)
```

```
[Out] int((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(b^2 x^{2n} + a^2\right)^p (bx^n + a)^p (-bx^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p,x, algorithm="maxima
```

```
[Out] integrate((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 x^{2n} + a^2\right)^p (bx^n + a)^p (-bx^n + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p,x, algorithm="fricas
```

```
[Out] integral((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a-b*x**n)**p*(a+b*x**n)**p*(a**2+b**2*x**(2*n))**p,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(b^2 x^{2n} + a^2\right)^p (bx^n + a)^p (-bx^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p,x, algorithm="giac"
```

```
[Out] integrate((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

## 3.285 $\int \frac{(c+dx^{2n})^p}{(a-bx^n)(a+bx^n)}\,dx$

**Optimal**. Leaf size=76

$$\frac{x\left(c+dx^{2n}\right)^p\left(\frac{dx^{2n}}{c}+1\right)^{-p}F_1\left(\frac{1}{2n};1,-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)}{a^2}$$

```
[Out] (x*(c + d*x^(2*n))^p*AppellF1[1/(2*n), 1, -p, (2 + n^(-1))/2, (b^
2*x^(2*n))/a^2, -((d*x^(2*n))/c)])/(a^2*(1 + (d*x^(2*n))/c)^p)
```

---

**Rubi [A]** time = 0.168164, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097

$$\frac{x\left(c+dx^{2n}\right)^p\left(\frac{dx^{2n}}{c}+1\right)^{-p}F_1\left(\frac{1}{2n};1,-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^(2*n))^p/((a - b*x^n)*(a + b*x^n)),x]
```

```
[Out] (x*(c + d*x^(2*n))^p*AppellF1[1/(2*n), 1, -p, (2 + n^(-1))/2, (b^
2*x^(2*n))/a^2, -((d*x^(2*n))/c)])/(a^2*(1 + (d*x^(2*n))/c)^p)
```

---

**Rubi in Sympy [A]** time = 35.3518, size = 58, normalized size = 0.76

$$\frac{x\left(1+\frac{dx^{2n}}{c}\right)^{-p}\left(c+dx^{2n}\right)^p\operatorname{appellf}_1\left(\frac{1}{2n},1,-p,\frac{n+\frac{1}{2}}{n},\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c+d*x**(2*n))**p/(a-b*x**n)/(a+b*x**n),x)
```

```
[Out] x*(1 + d*x**(2*n)/c)**(-p)*(c + d*x**(2*n))**p*appellf1(1/(2*n),
1, -p, (n + 1/2)/n, b**2*x**(2*n)/a**2, -d*x**(2*n)/c)/a**2
```

---

**Mathematica [B]** time = 0.492794, size = 258, normalized size = 3.39

$$\frac{a^2c(2n+1)x\left(c+dx^{2n}\right)^pF_1\left(\frac{1}{2n};-p,1;1+\frac{1}{2n};-\frac{dx^{2n}}{c},\frac{b^2x^{2n}}{a^2}\right)}{\left(a^2-b^2x^{2n}\right)\left(2a^2dnpx^{2n}F_1\left(1+\frac{1}{2n};1-p,1;2+\frac{1}{2n};-\frac{dx^{2n}}{c},\frac{b^2x^{2n}}{a^2}\right)+2b^2cnx^{2n}F_1\left(1+\frac{1}{2n};-p,2;2+\frac{1}{2n};-\frac{dx^{2n}}{c},\frac{b^2x^{2n}}{a^2}\right)+a^2c(\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(c + d*x^(2*n))^p/((a - b*x^n)*(a + b*x^n)),x]
```

```
[Out] (a^2*c*(1 + 2*n)*x*(c + d*x^(2*n))^p*AppellF1[1/(2*n), -p, 1, 1 +
 1/(2*n), -((d*x^(2*n))/c), (b^2*x^(2*n))/a^2])/((a^2 - b^2*x^(2*
n))*(2*a^2*d*n*p*x^(2*n)*AppellF1[1 + 1/(2*n), 1 - p, 1, 2 + 1/(2
*n), -((d*x^(2*n))/c), (b^2*x^(2*n))/a^2] + 2*b^2*c*n*x^(2*n)*App
ellF1[1 + 1/(2*n), -p, 2, 2 + 1/(2*n), -((d*x^(2*n))/c), (b^2*x^(
2*n))/a^2] + a^2*c*(1 + 2*n)*AppellF1[1/(2*n), -p, 1, 1 + 1/(2*n)
, -((d*x^(2*n))/c), (b^2*x^(2*n))/a^2]))
```

---

**Maple [F]** time = 0.139, size = 0, normalized size = 0.

$$\int \frac{\left(c+d x^{2 n}\right)^{p}}{\left(a-b x^{n}\right)\left(a+b x^{n}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x)
```

```
[Out] int((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(d x^{2 n}+c\right)^{p}}{(b x^{n}+a)(b x^{n}-a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(d*x^(2*n) + c)^p/((b*x^n + a)*(b*x^n - a)),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^(2*n) + c)^p/((b*x^n + a)*(b*x^n - a)), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(d x^{2 n}+c\right)^{p}}{b^{2} x^{2 n}-a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(d*x^(2*n) + c)^p/((b*x^n + a)*(b*x^n - a)),x, algorithm="fricas")
```

```
[Out] integral(-(d*x^(2*n) + c)^p/(b^2*x^(2*n) - a^2), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(2*n))**p/(a-b*x**n)/(a+b*x**n),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(dx^{2\,n}+c\right)^{p}}{(bx^{n}+a)(bx^{n}-a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(d*x^(2*n) + c)^p/((b*x^n + a)*(b*x^n - a)),x, algorithm="giac")
```

```
[Out] integrate(-(d*x^(2*n) + c)^p/((b*x^n + a)*(b*x^n - a)), x)
```

## 3.286 $\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}} dx$

**Optimal**. Leaf size=96

$$-\frac{b^2x(np+n+1)\left(a-bx^{n/2}\right)^{p+1}\left(a+bx^{n/2}\right)^{p+1}\left(dx^n-\frac{a^2dn(p+1)}{b^2(np+n+1)}\right)^{-\frac{np+n+1}{n}}}{a^4dn(p+1)}$$

```
[Out] -((b^2*(1 + n + n*p)*x*(a - b*x^(n/2))^(1 + p)*(a + b*x^(n/2))^(1
 + p))/(a^4*d*n*(1 + p)*(-((a^2*d*n*(1 + p))/(b^2*(1 + n + n*p)))
 + d*x^n)^((1 + n + n*p)/n)))
```

---

**Rubi [A]** time = 0.361072, antiderivative size = 104, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, integrand size = 76, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$

$$-\frac{b^2x(np+n+1)\left(a^2-b^2x^n\right)\left(a-bx^{n/2}\right)^p\left(a+bx^{n/2}\right)^p\left(dx^n-\frac{a^2dn(p+1)}{b^2(np+n+1)}\right)^{-\frac{np+n+1}{n}}}{a^4dn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*((a^2*d*(1 + p))/(b^2*(1 + (-
1 - 2*n - n*p)/n)) + d*x^n)^((-1 - 2*n - n*p)/n),x]
```

```
[Out] -((b^2*(1 + n + n*p)*x*(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*(a^2 -
 b^2*x^n))/(a^4*d*n*(1 + p)*(-((a^2*d*n*(1 + p))/(b^2*(1 + n + n*
p))) + d*x^n)^((1 + n + n*p)/n)))
```

---

**Rubi in Sympy [A]** time = 51.4377, size = 102, normalized size = 1.06

$$-\frac{b^2x\left(a-bx^{\frac{n}{2}}\right)^p\left(a+bx^{\frac{n}{2}}\right)^p\left(a^2-b^2x^n\right)^{-p}\left(a^2-b^2x^n\right)^{p+1}\left(-\frac{a^2dn(p+1)}{b^2(np+n+1)}+dx^n\right)^{-p-1-\frac{1}{n}}(np+n+1)}{a^4dn\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((a-b*x**(1/2*n))**p*(a+b*x**(1/2*n))**p*(a**2*d*(1+p)/b**2/(1+(
n*p-2*n-1)/n)+d*x**n)**((-n*p-2*n-1)/n),x)
```

```
[Out] -b**2*x*(a - b*x**(n/2))**p*(a + b*x**(n/2))**p*(a**2 - b**2*x**n
)**(-p)*(a**2 - b**2*x**n)**(p + 1)*(-a**2*d*n*(p + 1)/(b**2*(n*p
```

```
+ n + 1)) + d*x**n)**(-p - 1 - 1/n)*(n*p + n + 1)/(a**4*d*n*(p +
1))
```

---

**Mathematica [A]** time = 0.92852, size = 0, normalized size = 0.

$$\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*((a^2*d*(1 + p))/(b^2*(1 + (-
1 - 2*n - n*p)/n)) + d*x^n)^((-1 - 2*n - n*p)/n),x]
```

```
[Out] Integrate[(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*((a^2*d*(1 + p))/(b
^2*(1 + (-1 - 2*n - n*p)/n)) + d*x^n)^((-1 - 2*n - n*p)/n), x]
```

---

**Maple [F]** time = 1.044, size = 0, normalized size = 0.

$$\int \left(a-bx^{\frac{n}{2}}\right)^p \left(a+bx^{\frac{n}{2}}\right)^p \left(\frac{a^2d\,(1+p)}{b^2}\left(1+\frac{-np-2\,n-1}{n}\right)^{-1}+dx^n\right)^{\frac{-np-2\,n-1}{n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-
2*n-1)/n)+d*x^n)^((-n*p-2*n-1)/n),x)
```

```
[Out] int((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p
-2*n-1)/n)+d*x^n)^((-n*p-2*n-1)/n),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^{\frac{1}{2}\,n}+a\right)^p \left(-bx^{\frac{1}{2}\,n}+a\right)^p \left(dx^n-\frac{a^2d(p+1)}{b^2\left(\frac{np+2\,n+1}{n}-1\right)}\right)^{-\frac{np+2\,n+1}{n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p*(d*x^n - a^2*d*(p + 1)/(b^2*((
(n*p + 2*n + 1)/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p*(d*x^n - a^2*d
*(p + 1)/(b^2*((n*p + 2*n + 1)/n - 1)))^(-(n*p + 2*n + 1)/n), x)
```

---

**Fricas [A]** time = 0.28891, size = 243, normalized size = 2.53

$$\frac{\left(\left(b^4np + b^4n + b^4\right)xx^{2n} - \left(2a^2b^2np + 2a^2b^2n + a^2b^2\right)xx^n + \left(a^4np + a^4n\right)x\right)\left(bx^{\frac{1}{2}n} + a\right)^p\left(-bx^{\frac{1}{2}n} + a\right)^p}{\left(a^4np + a^4n\right)\left(-\frac{a^2dnp + a^2dn - (b^2dnp + b^2dn + b^2d)x^n}{b^2np + b^2n + b^2}\right)^{\frac{np+2n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/(d*x^n - a^2*d*(p + 1)/(b^2*((
```

```
[Out] ((b^4*n*p + b^4*n + b^4)*x*x^(2*n) - (2*a^2*b^2*n*p + 2*a^2*b^2*n
 + a^2*b^2)*x*x^n + (a^4*n*p + a^4*n)*x)*(b*x^(1/2*n) + a)^p*(-b*
x^(1/2*n) + a)^p/((a^4*n*p + a^4*n)*(-(a^2*d*n*p + a^2*d*n - (b^2
*d*n*p + b^2*d*n + b^2*d)*x^n)/(b^2*n*p + b^2*n + b^2))^((n*p + 2
*n + 1)/n))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x**(1/2*n))**p*(a+b*x**(1/2*n))**p*(a**2*d*(1+p)/b**2/(1+(-
n*p-2*n-1)/n)+d*x**n)**((-n*p-2*n-1)/n),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{\frac{1}{2}n} + a\right)^p\left(-bx^{\frac{1}{2}n} + a\right)^p}{\left(dx^n - \frac{a^2d(p+1)}{b^2\left(\frac{np+2n+1}{n} - 1\right)}\right)^{\frac{np+2n+1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/(d*x^n - a^2*d*(p + 1)/(b^2*((
```

```
[Out] integrate((b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/(d*x^n - a^2*d
*(p + 1)/(b^2*((n*p + 2*n + 1)/n - 1)))^((n*p + 2*n + 1)/n), x)
```

# 4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```